CS 419 Homework 2

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2.1

The logistic growth model is defined as the following:

$$X' = rX(1 - \frac{X}{K}) \tag{1}$$

which is equivalent to

$$\frac{dX}{dt} = rX(1 - \frac{X}{K}) \tag{2}$$

We can rewrite this equation by seperating the r and X.

$$rdt = \frac{dX}{X(1 - X/K)} \tag{3}$$

Before we can integrate, we need to fix the right side of the equation. We take the dX out, and use A and B as constants.

$$\frac{A}{P} + \frac{B}{(1 - X/K)} \tag{4}$$

With a common denominator, this turns into

$$A(1 - X/K) + BX = A + X(B - A/K) = 1 \tag{5}$$

This means that

$$B = \frac{A}{K}, andA = 1 \tag{6}$$

Now we can subsitute A and B back in:

$$\frac{1}{X} + \frac{1/K}{(1 - X/K)} \tag{7}$$

This simplifies back to

$$rdt = \frac{dX}{X} + \frac{dX/K}{(1 - X/K)} \tag{8}$$

Now we interegrate both sides:

$$\int r dt = \int \frac{dX}{X} + \int \frac{dX/K}{(1 - X/K)} \tag{9}$$

$$rt + c = ln(X) + \int \frac{dX/K}{(1 - X/K)} \tag{10}$$

If we use u = 1 - X/K and du = -1/X dX for integration with substitution, it turns out to be

$$\int \frac{-du}{u} = -ln(u) = -ln(1 - X/K) \tag{11}$$

Which can be rewritten as

$$rt + c = ln(X) - ln(1 - X/K) = ln(\frac{X}{(1 - X/K)}) \tag{12}$$

Now to eliminate the ln, we can take both sides to the e. We use C to represent the constant.

$$Ce^{rt} = \frac{X}{1 - X/K} \tag{13}$$

Now we can solve for X

$$(1 - \frac{X}{K})Ce^{rt} = X \tag{14}$$

$$Ce^{rt} = X(1 + \frac{Ce^{rt}}{K}) \tag{15}$$

$$X = \frac{Ce^{rt}}{1 + \frac{Ce^{rt}}{K}} \tag{16}$$

2.3

```
function xchange = logistic(t, tau)

    r = 1;
    k = 1000;
    x = calcx(r, t, k);
    xtau = calcx(r, (t-tau), k);
    h = 1;

    xchange = r*x*(1-x/k)-(h*xtau);

function x=calcx(r, t, x)

    x = (exp(r*t)/(1+(exp(r*t)/k)));
```

If we run this script with the following code we can see our results

```
tspan = [0 20]
x0 = 1
[t,x] = ode45('logistic', tspan, tau)
plot(t, x, xlabel('Time'), ylabel('Population Density'))
```

After running this model with various values of tau, we see as the value of tau gets larger, the carrying capacity of the population becomes lower and lower.