



Cointegration Models

Rob Reider
Adjunct Professor, NYU-Courant
Consultant, Quantopian

What is Cointegration?

- ullet Two series, P_t and Q_t can be random walks
- ullet But the linear combination $P_t-c\ Q_t$ may not be a random walk!
- If that's true
 - $P_t c \ Q_t$ is forecastable
 - lacksquare P_t and Q_t are said to be cointegrated

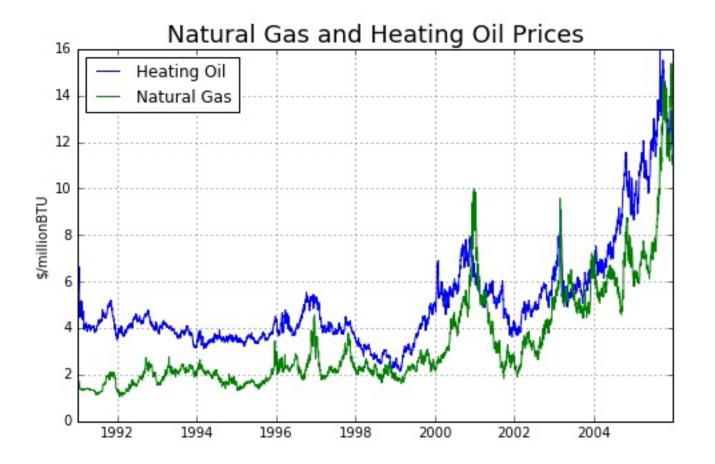
Analogy: Dog on a Leash

- $P_t = \text{Owner}$
- $ullet Q_t = \mathsf{Dog}$
- Both series look like a random walk
- Difference, or distance betweem them, looks mean reverting
 - If dog falls too far behind, it gets pulled forward
 - If dog gets too far ahead, it gets pulled back



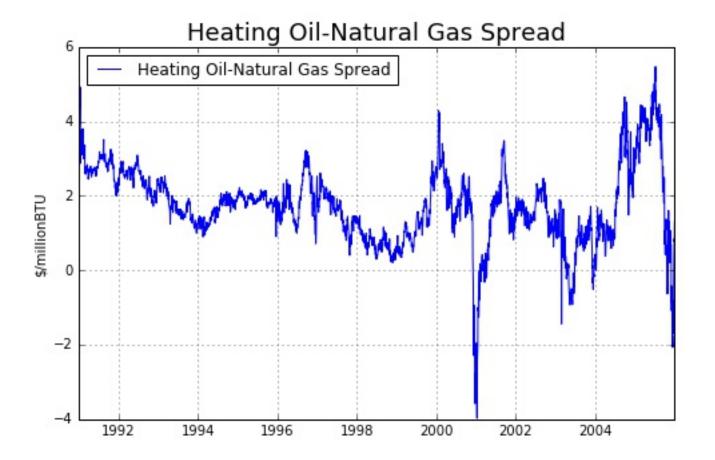
Example: Heating Oil and Natural Gas

Heating Oil and Natural Gas both look like random walks...



Example: Heating Oil and Natural Gas

• But the spread (difference) is mean reverting





What Types of Series are Cointegrated?

- Economic substitutes
 - Heating Oil and Natural Gas
 - Platinum and Paladium
 - Corn and Wheat
 - Corn and Sugar
 - ...
 - Bitcoin and Ethereum?
- How about competitors?
 - Coke and Pepsi?
 - Apple and Blackberry? No! Leash broke and dog ran away

Two Steps to Test for Cointegration

- ullet Regress P_t on Q_t and get slope c
- ullet Run Augmented Dickey-Fuller test on $P_t-c\ Q_t$ to test for random walk
- Alternatively, can use coint function in statsmodels that combines both steps

from statsmodels.tsa.stattools import coint
coint(P,Q)



Let's practice!





Case Study: Climate Change

Rob Reider
Adjunct Professor, NYU-Courant
Consultant, Quantopian



Analyzing Temperature Data

- Temperature data:
 - New York City from 1870-1916
 - Downloaded from National Oceanic and Atmospheric Administration (NOAA)
- Convert index to datetime object
- Plot data



Analyzing Temperature Data

- Test for Random Walk
- Take first differences
- Compute ACF and PACF
- Fit a few AR, MA, and ARMA models
- Use Information Criterion to choose best model
- Forecast temperature over next 30 years



Let's practice!





Congratulations

Rob Reider
Adjunct Professor, NYU-Courant
Consultant, Quantopian



Advanced Topics

- GARCH Models
- Nonlinear Models
- Multivariate Time Series Models
- Regime Switching Models
- State Space Models and Kalman Filtering
- •



Keep practicing!