



INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Describe Model

Rob Reider

Adjunct Professor, NYU-Courant
Consultant, Quantopian



Mathematical Description of MA(1) Model

$$R_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

- Since only one lagged error on right hand side, this is called:
 - MA model of order 1, or
 - MA(1) model
- MA parameter is θ
- Stationary for all values of θ



Interpretation of MA(1) Parameter

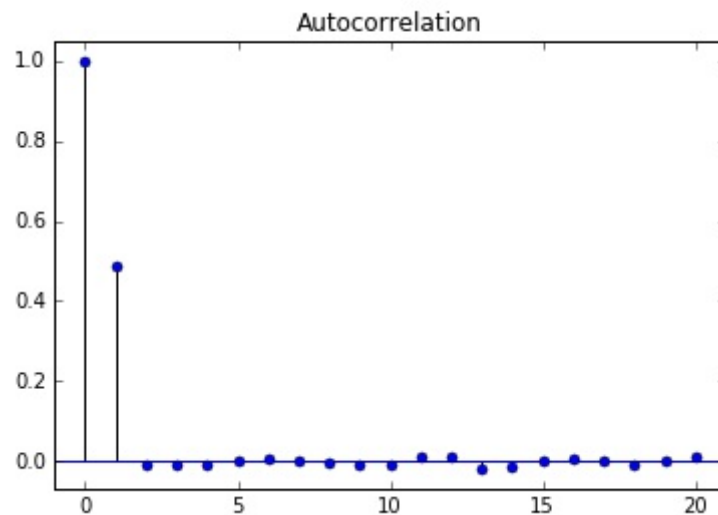
$$R_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

- Negative θ : One-Period Mean Reversion
- Positive θ : One-Period Momentum
- Note: One-period autocorrelation is $\theta/(1 + \theta^2)$, not θ

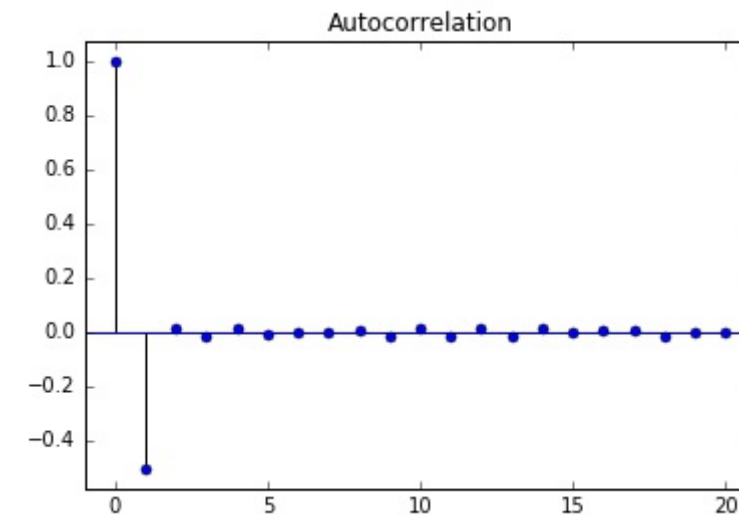


Comparison of MA(1) Autocorrelation Functions

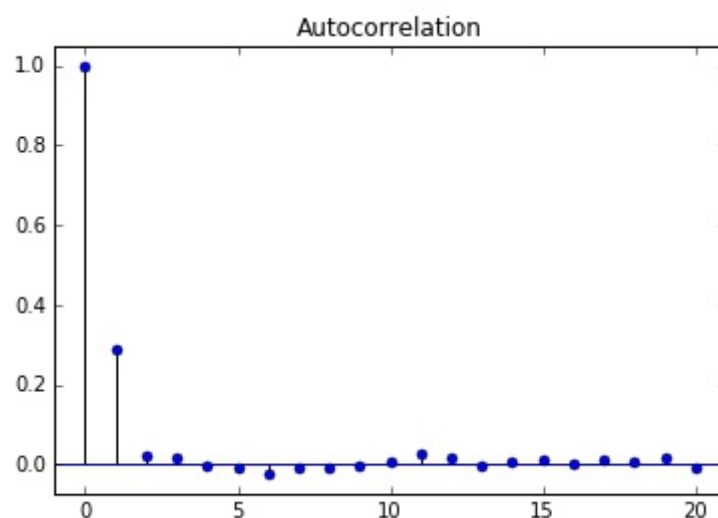
- $\theta = 0.9$



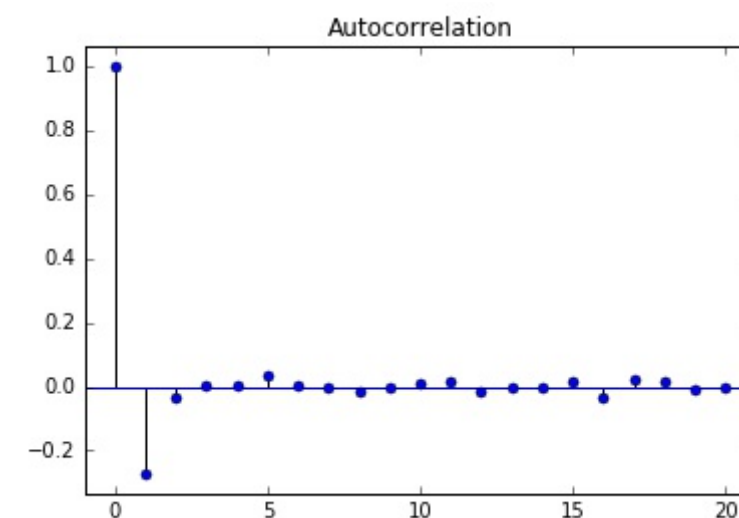
- $\phi = -0.9$



- $\phi = 0.5$



- $\phi = -0.5$



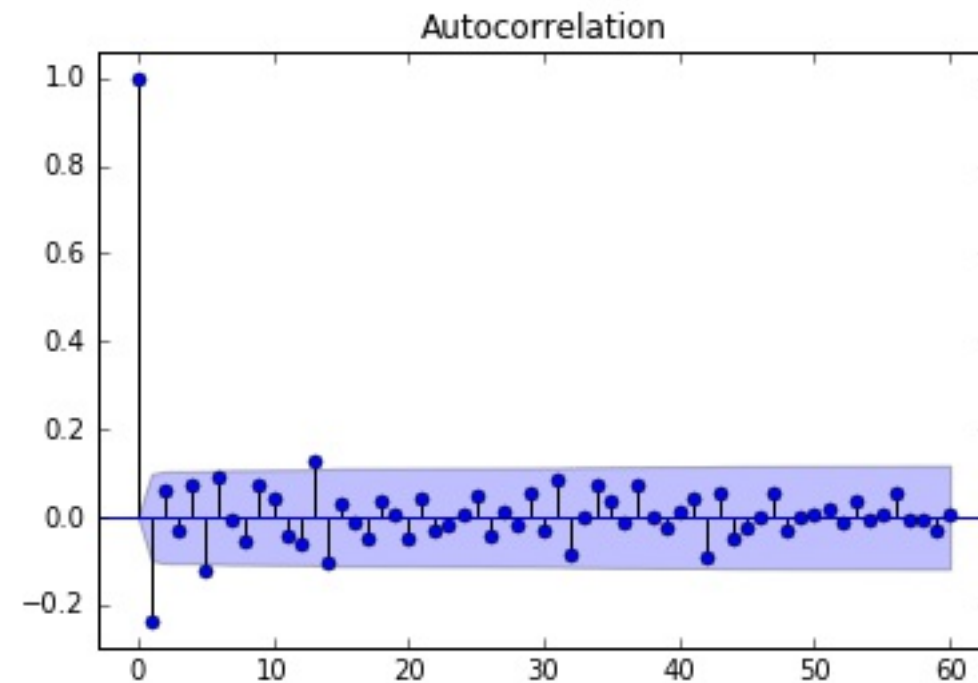


Example of MA(1) Process: Intraday Stock Returns





Autocorrelation Function of Intraday Stock Returns





Higher Order MA Models

- MA(1)

$$R_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1}$$

- MA(2)

$$R_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2}$$

- MA(3)

$$R_t = \mu + \epsilon_t - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \theta_3 \epsilon_{t-3}$$

- ...



Simulating an MA Process

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1])
ma = np.array([1, 0.5])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```




INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Let's practice!



INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Estimation and Forecasting an MA Model

Rob Reider

Adjunct Professor, NYU-Courant
Consultant, Quantopian



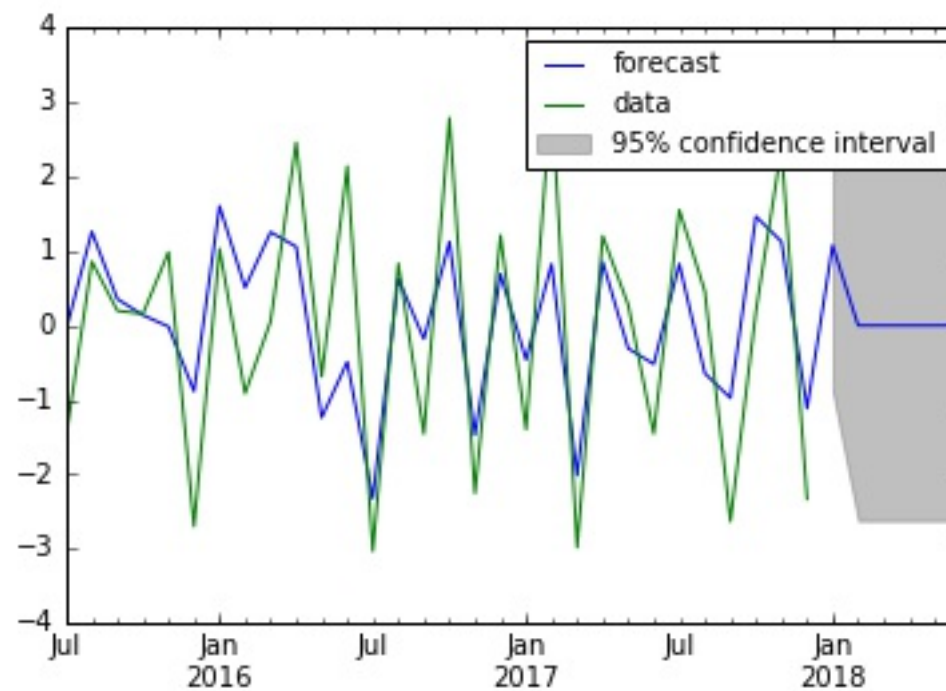
Estimating an MA Model

- Same as estimating an AR model (except `order=(0,1)`)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(0,1))
result = mod.fit()
```

Forecasting an MA Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```





INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Let's practice!



INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

ARMA models

Rob Reider

Adjunct Professor, NYU-Courant
Consultant, Quantopian



ARMA Model

- ARMA(1,1) model:

$$R_t = \mu + \phi R_{t-1} + \epsilon_t + \theta \epsilon_{t-1}$$



Converting Between ARMA, AR, and MA Models

- Converting AR(1) into an MA(infinity)

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

$$R_t = \mu + \phi(\mu + R_{t-2} + \epsilon_{t-1}) + \epsilon_t$$

$$R_t = \mu + \phi(\mu + \phi(\mu + R_{t-3} + \epsilon_{t-2}) + \epsilon_{t-1}) + \epsilon_t$$

$$\vdots$$

$$R_t = \mu + \epsilon_t + \phi \epsilon_{t-1} + \phi^2 \epsilon_{t-2} + \phi^3 \epsilon_{t-3} + \phi^4 \epsilon_{t-4} + \dots$$



INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Let's practice!