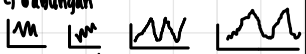
Pendahuluan

1. Data deret waktu → data yg diamati berdasarkan urutan waktu dg rentang yg sama
2. Data deret waktu digunakan jika diduga ada keragaman besar akibat faktor waktu (faktor lain juga dipengaruhi oleh waktu)
3. Data deret waktu secara teoritis:
   $X_t = b_1 Z_1(t) + b_2 Z_2(t) + b_k Z_k(t) + \epsilon_t$
   b_k = parameter ke-k
   $Z_k(t)$ = fungsi ke-k pada t
   ϵ_k = komponen acak ke-k
4. Karakteristik Data Deret Waktu
   a) Stasioner → rataan dan ragam konstan
   co: model klasik dan ada SK
   b) Tidak stasioner → rataan dan ragam tidak konstan
   co: model ML dan tidak ada SK
5. Pola Data Deret Waktu
   a) Horizontal → ragam/fluktuasi di sekitar rata-rata yang konstan
   b) Musiman → dipengaruhi oleh waktu musiman (bulanan, mingguan, tahunan)
   co: sirup hijriyah
   c) Siklis → seperti musiman tetapi fluktuasi jangka panjang. co: sirup maehi
   d) Trend → kenaikan/penurunan jangka panjang
   e) Gabungan



konstan | trend | musiman | siklis

6. Ruang Lingkup Analisis Data DW
   a) Pemulusan
   b) Pemodelan
   c) Peramalan
7. Proses Peramalan
   a) Problem definition
   b) Data collection
   c) Data Analysis
   d) Forecasting Model development
   e) Model validation
   f) Model selection & fitting
   g) Monitoring Forecast Model performance
8. Metode Analisis DW
   a) ARIMA → fungsi deret waktu, pendekatan model identifikasi, penaksiran awal param
   b) Regresi dg dummy variabel
   c) Bayesian → state space based on model dinamis linear (diagnostik pengaruh)
   d) Metode smoothing → mengurangi ketidakteraturan data (musiman)
9. Metode Peramalan Kuantitatif
   a) Pemulusan (smoothing)
   1) SMA → stasioner 3) SES
   2) DMA → trend 4) DES
   5) Winter (aditif + multiplikatif) ↳ musiman
   b) Pemodelan Box Jenkins (ARIMA)
10. Metode Peramalan Kualitatif
    ↳ didasarkan penilaian dan intuisi bukan pengolahan data historis
11. Ukuran seberapa baik metode
    a) MAD (Mean Absolute Deviation)
    $MAD = \frac{1}{n}\sum|X_i - \hat{X}_i|$
    b) MSD (Mean Squared Deviation)
    $MSD = \frac{1}{n}\sum(X_i - \hat{X}_i)^2$
    c) MAPE (Mean Absolute Percentage Error)
    $MAPE = \frac{1}{n}\sum\left|\frac{X_i - \hat{X}_i}{X_i}\right| \cdot 100\%$
    d) AIC
    e) BIC | ukuran memilih model terbaik

SMA & DMA

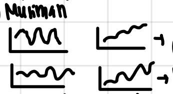
1. Sekilas tentang Pemulusan
   a) Prinsip dasar: mengenal pola dg menghaluskan variasi lokal
   b) Prinsip penghalusan berupa rata²
   c) fitted value / data aktual ↳ memprediksi data masa depan
   $e_t = y_t - \hat{y}_t$
   d) forecast value → memprediksi data yang hilang based on data sebelum
   $e_t(1) = y_t - \hat{y}_t(t-1)$
2. SMA
   a) Ide: data suatu periode dipengaruhi data periode sebelumnya
   b) data → stasioner/konstan
   c) Prinsip:
   1) data smoothing ke-t = rata-rata m buah data dari t hingga t-m+1
   $S = \frac{1}{m}\sum_{i=t-m+1}^{t} X_i$
   2) data smoothing t → nilai forecasting ke-t+1
   $F_t = S_{t-1}$ dan $F_{n,h} = S_n$
   3) m besar → pola lebih halus
   d) $Var(S_t) < Var(X_t)$
3. DMA
   a) mirip SMA
   b) data trend
   c) Prinsip penghalusan
   1) Penghalusan tahap 1
   $S_{1,t} = \frac{1}{m}\sum_{i=t-m+1}^{t} X_i$
   2) Penghalusan tahap II
   $S_{2,t} = \frac{1}{m}\sum_{i=t-m+1}^{t} S_{1,t}$
   3) Forecasting
   $A_t = 2S_{1,t} - S_{2,t}$, $B_t = \frac{2}{m-1}(S_{1,t} - S_{2,t})$
   $F_{2,t,t+h} = A_t + B_t(h)$ h↑ t↓ semakin ↓ semakin ↑

SES & DES

1. Exponential Smoothing
   a) Data yang baru-baru ini lebih berpengaruh dari data yg lama
   b) Pemulusan dg pembobotan menurun secara exponential
   c) MA → pembobotan sama tapi dalam kenyataan tidak begitu
2. SES
   a) Smoothing ke-t
   $\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$
   λ → parameter pemulus : $0 < \lambda < 1$
   λ=0 → bobot baru ↑, λ=1 → bobot lama ↑
   b) Smoothing periode ke-t → nilai forecast periode ke-(T+τ)
   $\hat{y}_{T+\tau}(T) = \tilde{y}_T$
   c) Residual
   $e_T(1) = y_{T+1} - \hat{y}_{T+1}(T)$
   d) Hitung MAPE, MAD, MSE
   e) SSE paling kecil dg λ berapa
3. DES
   a) Smoothing ke-T dg 2 tahap
   1) Tahap level
   $\tilde{y}_T^{(1)} = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$
   2) Tahap trend
   $\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)}$
   b) Smoothing ke-T → forecast ke-(T+τ)
   $\hat{y}_{T+\tau}(T) = (2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}) + \tau\frac{\lambda}{1-\lambda}(\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)})$
   $= (2 + \frac{\lambda}{1-\lambda}\tau)\tilde{y}_T^{(1)} - (1 + \frac{\lambda}{1-\lambda}\tau)\tilde{y}_T^{(2)}$
   c) Residual
   $e_T(1) = y_{T+1} - \hat{y}_{T+1}(T)$
   d) Hitung MAPE, MAD, MSE
   e) SSE paling kecil dg λ berapa

Winter

1. Winter
   a) Triple exponential
   b) Musiman



→ Aditif (ragam sama)
→ Multiplikatif (ragam beda)
no trend | trend

   c) Beda DES dg Holt Winter
   DES : $\hat{y}_{t+h}(t) = L_t + T_t h$
   HW : $\hat{y}_{t+h}(t) = \underbrace{L_t}_{\text{level}} + \underbrace{B_t h}_{\text{trend}} + \underbrace{S_{t+h-m}}_{\text{seasonal}}$
2. Winter Aditif
   a) Smoothing t+h
   $L_t = \alpha(Y_t - S_{t-m}) + (1-\alpha)(L_{t-1} + b_{t-1})$
   $b_t = \gamma(L_t - L_{t-1}) + (1-\gamma)b_{t-1}$
   $S_t = \delta(Y_t - L_t) + (1-\delta)S_{t-m}$
   b) Forecasting t+h
   $\hat{Y}_{t+h}(t) = \underbrace{L_t}_{\text{intercep}} + \underbrace{b_t}_{b_0}h + S_{t+h-m}$
   c) Step:
   1) Ketahui nilai L_t = intercept + $b_t = b_0$
   2) $\hat{Y}_t = L_t + b_t(t) \to S_t = Y_t - \hat{Y}_t \to \bar{S}_{(i)}$ → $\Sigma S_t = 0$
   3) $\hat{Y}_{T+p}(T) = \hat{a}_T + p\hat{b}_T + S_{n_{T+p-L}}$, $T=0, p=1$
   4) Dugaan e_T, b_T, S_{nT}
3. Winter Multiplikatif
   a) Smoothing ke T+h
   $L_t = \alpha\left(\frac{Y_t}{S_{t-m}}\right) + (1-\alpha)(L_{t-1} + B_{t-1})$
   $B_t = \gamma(L_t - L_{t-1}) + (1-\gamma)B_{t-1}$
   $S_t = \delta\left(\frac{Y_t}{L_t}\right) + (1-\delta)S_{t-m}$
   b) Forecast ke t+h
   $\hat{Y}_{t+h}(t) = (L_t + B_t h)S_{t+h-m}$
   c) Step:
   1) Ketahui nilai L_t = intercept + $b_t = b_0$
   2) $\hat{Y}_t = L_t + b_t(t) \to S_{0,t} = Y_t/\hat{Y}_t \to \bar{S}_{(i)}$ → $CF = m/\Sigma \bar{S}_{(i)}$
   3) $\hat{Y}_{T+p}(T) = \hat{a}_T + p\hat{b}_T + S_{n_{T+p-L}}$, $T=0, p=1$
   4) Dugaan e_T, b_T, S_{nT}

Model Regresi DW

1. Review Model Regresi
   a) Ordo X = 1 → linier dalam parameter → X dan Y hubungannya linier
   b) Perubahan Y → perubahan X
   c) $Y = X\beta + \epsilon$
   d) Asumsi
   1) Hubungan linier
   2) ε peubah acak bebas thdp X
   3) $\epsilon \sim N(0, \sigma^2) \to E(\epsilon) = 0 + Var(\epsilon) = \sigma^2$
   4) ε tidak berkorelasi satu sama lain $E(\epsilon_i\epsilon_j) = 0$ / $cov(\epsilon_i, \epsilon_j) = 0$
   5) Tidak ada multikolinieritas antar X
   e) Jika εi berkorelasi 1 sama lain:
   1) Hasil OLS tetap tidak berbias tapi bukan lagi ragam paling minimum
   2) ε berkorelasi diri · underestimate (lebih kecil dari nilai sebenarnya) dan S_{b_1} kecil
   3) SK berbasis uji t dan F sudah tidak tepat ($t_{hit} = b_1/S_{b_1}$ → cenderung tolak H0)
   f) Model Regresi DW
   $Y_t = \beta_0 + \beta_1 X_1(t) + \dots + \beta_k X_k(t) + \epsilon_t$
   g) Residual plot
   1) Cek ε~N → residual vs percent/hist
   2) Cek homoscedasticity → residual vs fitted value (tidak berpola)
   3) Cek autokorelasi → residual vs observation (lebar pita sama)
   - ACF
   - PACF } ada autokorelasi → korelasi $e_t - e_{t-1}$ di luar SK

Penanganan Autokorelasi

1. Review
   a) Sebab autokorelasi
   1) Ada peubah yang tidak disertakan
   2) Misspesifikasi model
   3) Measurement error
   b) Jika asumsi tidak terpenuhi
   1) Penduga masih tak bias dan konsisten
   2) Ukuran contoh besar → asumsikan normal
   3) Penduga tidak efisien (bukan penduga tak bias terbaik (BLUE))
   Ragam tidak masuk kategori BLUE
   4) Penduga galat baku tidak reliable sehingga uji T dan F tidak valid
2. Deteksi Autokorelasi
   a) Pendekatan Grafik
   1) Plot sisaan vs order → tidak membentuk pola tertentu
   2) ACF dan PACF → tidak ada yg signifikan
   b) Uji Durbin Watson
   1) $H_0: \phi = 0$, $H_1: \phi >< \neq 0$
   2) Statistik uji
   $d = \sum_{t=2}^{t=n}(e_t - e_{t-1})^2 / \sum_{t=1}^{t=n} e_t^2$
   3) Asumsi Durbin Watson
   a. Ada intersep pada model regresi
   b. Peubah X tetap (fixed)
   c. Galat → AR ordo-1
   $u_t = \rho u_{t-1} + v_t$, ρ = koef autokor (0-1)
   d. ε~N
   e. Lag Y tidak disertakan sebagai penjelas ↳ model regresi tidak boleh AR
   4) 2 titik kritis → dL dan dU
   $0 < d < 4$
   d→0 : autokorelasi +
   d→4 : autokorelasi −
   d→2 : belum cukup bukti

+	incon	tidak	incon	−

0 dL dU 2 4-dU 4-dL 4

3. Penanganan Autokorelasi
   a) Cochrane Orcutt
   b) Hildreth Lu
   c) Regresi peubah lag
4. Cochrane Orcutt
   a) Regresikan Y terhadap X → diperoleh galat e_t
   b) Menduga koefisien korelasi ordo ke-1 ($\hat\rho$) dengan meregresikan e_t dg e_{t-1}
   $e_t = \rho e_{t-1} + u_t$
   c) Transformasi X dan Y
   $Y_t^* = Y_t - \hat\rho Y_{t-1}$, $X_{ti}^* = X_{ti} - \hat\rho X_{t-1,i}$
   d) Meregresikan Y^* dan X^* diperoleh $\beta_0^*, \beta_1^*, \dots$
   e) Hitung $\hat\beta_0 = \frac{\hat\beta_0^*}{1-\hat\rho}$, subst $\hat\beta_0, \hat\beta_1^*, \dots$ ke pers (a) diperoleh e_t baru
   f) Ulangi tahap (b) hingga (e) → hingga $\hat\rho$ konvergen
5. Hildreth Lu
   a) Menentukan spesifikasi nilai ρ
   b) Menentukan SSE dari tiap ρ
   c) Pilih ρ dg SSE minimum
   d) Menentukan spesifikasi nilai ρ di sekitar ρ yg dipilih
   e) Ulangi tahap (a) hingga diperoleh ρ dg SSE paling minimum

Regresi dg Peubah Lag

1. Model Regresi
   a) Static Regresi
   $Y_t = \beta_0 + \beta_1 X_{t,1} + \dots + \beta_k X_{t,k} + u_t$
   b) Regresi w/ distributed lag
   ↳ ada faktor lain dimana periode sebelum berpengaruh thdp Y
   $Y_t = \beta_0 + \beta_1 X_{t,1} + \beta_2 X_{t-1,1} + \beta_3 X_{t-2,1} + u_t$
   c) Dynamic/AR model → regresi dg dirinya sendiri (hanya dipengaruhi Y_t periode before)
   $Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t$
2. Model AR
   ↳ Y dipengaruhi X waktu sekarang serta dipengaruhi Y satu waktu sebelum
   $Y_t = \alpha + \beta X_t + \gamma Y_{t-1} + u_t + \beta_2 X_{t-1}$
3. Lag
   ↳ waktu yg dibutuhkan X dalam memengaruhi Y
4. Pendekatan Koyck untuk DL model
   a) Asumsi → semakin jauh jarak lag X dari periode sekarang maka semakin kecil pengaruh lag thdp Y
   b) Metode → menduga model dinamis DL dg asumsi semua koef β punya tanda sama.
   c) Model Koyck paling umum dari model DL → geometric lag
5. Model Koyck
   ↳ koef menurun secara geometris
   $\beta_k = \beta_0 C^k, k = 0, 1, \dots$
   C = rata-rata penurunan (0-1)
   1-c = kecepatan penyesuaian
   Model: $Y_t = \alpha(1-C) + \beta_0 X_t + C Y_{t-1} + V_t$
   step:
   a) Regresi Y_t dg Y_{t-1} dan X_t
   $\alpha(1-C)$ = inter, β_0 = koef X_t, C = koef Y_{t-1}
   b) Cari $\alpha, \beta_0, \beta_1, \beta_2, \dots$
   c) $Y = \alpha + \beta_0 X_{t-1} + \beta_1 X_{t-1} + \beta_2 X_{t-2} \dots$ ↳ model lag
   d) Y_t interpret
6. Koyck → X_t, X_{t-1}, X_{t-2} → ada lag beda
   ARDL → X_t, Y_{t-1} → lag sama
7. ARDL
   ARDL → dipengaruhi Y_{t-1}
   ↘ bisa ditambah dengan melihat pengaruh peubah lain
   syarat : plot time series sama polanya dengan Y_t meskipun beda arah (// atau ⊥)
8. Ukuran kebaikan model
   ↳ deviance

Fundamental Concept

1. Time series and stokastik proses
   a) A stokastik proses → pengumpulan nilai dari variabel acak berdasarkan waktu dg jarak yang sama
   b) Simple stokastik proses : random walk
2. Means : $\mu_t = E(Y_t)$
3. Variance : $Var(X) \geq 0$
   $Var(a+bX) = b^2 Var(X)$
   $Var(X+Y) = Var(X) + Var(Y) + \underbrace{2Cov(X,Y)}_{=0}$
   $Var(X) = E(X^2) - (E(X))^2$
4. Covariance
   $cov(a+bX, c+dY) = bd\,cov(X,Y)$
   $cov(X+Y, Z) = cov(X,Z) + cov(Y,Z)$
   $cov(X,X) = Var(X)$, $cov(X,Y) = cov(Y,X)$
   X, Y independent → $cov(X,Y) = 0$
5. Correlation
   $\rho = corr(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, $-1 \leq \rho \leq 1$
   $corr(a+bX, c+dY) = sign(bd)\,corr(X,Y)$
   $sign(bd) = \begin{cases} 1, & bd > 0 \\ 0, & bd = 0 \\ -1, & bd < 0 \end{cases}$
6. Autocovariance
   $\gamma_{t,s} = cov(Y_t, Y_s) = E[(Y_t - \mu_t)(Y_t - \mu_s)]$
   $= E(Y_t Y_s)$
7. Autocorrelation
   $\rho_{t,s} = corr(Y_t, Y_s) = \frac{\gamma_{t,s}}{\sqrt{\gamma_{t,t} \cdot \gamma_{s,s}}}$
8. AutoCov, AutoCorr
   $\gamma_{t,t} = Var(Y_t)$ $\rho_{t,t} = 1$
   $\gamma_{t,s} = \gamma_{s,t}$ $\rho_{t,s} = \rho_{s,t}$
   $|\gamma_{t,s}| \leq \sqrt{\gamma_{t,t} \cdot \gamma_{s,s}}$ $|\rho_{t,s}| \leq 1$
9. Random Walk = langkah acak
   ↳ $e \sim iid(0, \sigma^2)$, e = noise/galat
   $Y_t = Y_{t-1} + e_t \Rightarrow Y_t = e_1 + e_2 + \dots + e_t$
   a) Mean
   $\mu_t = E(e_1 + e_2 + \dots + e_t) = 0$
   b) Variance
   $Var(Y_t) = Var(e_1 + e_2 + \dots + e_t) = t\sigma_e^2$
   c) Covariance
   $\gamma_{t,s} = cov(Y_t, Y_s) = cov(e_1 + \dots + e_t, e_{t+1} + e_s)$
   $= t\sigma^2_e$, $1 \leq t \leq s$
   d) Correlation
   $\rho_{t,s} = t\sigma^2_e / \sqrt{t\sigma^2_e s\sigma^2_e} = \sqrt{t/s}$
   e) Random walk → trend : μ, Var ≠ lama
   f) A MA
   $Y_t = \frac{e_t + e_{t-1}}{2}$, $\mu_t = 0$
   $cov(Y_t, Y_s) = \begin{cases} 0.5\sigma^2_e, & |t-s| = 0 \\ 0.25\sigma^2_e, & |t-s| = 1 \\ 0, & |t-s| > 1 \end{cases}$
   $\rho_{t,s} = \begin{cases} 1, & |t-s| = 0 \\ 0.5, & |t-s| = 1 \\ 0, & |t-s| > 1 \end{cases}$
   g) Stasionarity
   ↳ sebelum forecasting perlu distasionerkan agar μ, Var konstan
   a) $E(Y_t) = E(Y_{t-k})$
   b) $Var(Y_t) = Var(Y_{t-k})$
   c) $cov(Y_{t-k}, Y_{s-k}) = cov(Y_{t-s}, Y_0)$
   $= \gamma_0|t-s|$
   d) $\rho_k = corr(Y_t, Y_{t-k})$, $cov(Y_t, Y_{t-k}) = \gamma_k$
   $\rho_k = \gamma_k/\gamma_0$
   e) $\gamma_0 = Var(Y_t)$ $\rho_0 = 1$
   $\gamma_k = \gamma_{-k}$ $\rho_k = \rho_{-k}$
   $|\gamma_k| \leq \gamma_0$ $|\rho_k| \leq 1$
   e_t konstan maka
   a) $\mu_t = E(e_t) = 0$
   b) $\gamma_k = \begin{cases} Var(e_t) = \sigma^2_e, & k = 0 \\ 0, & k \neq 0 \end{cases}$
   c) $\rho_k = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$

Stationary time series : MA(q)

1. General Linear Proses
   a) $Y_t = e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \dots$
   $\psi_j = \phi^j$
   $Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \dots$
   b) $E(Y_t) = E(e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \dots)$
   $= 0$
   c) $Var(Y_t) = Var(e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \dots)$
   $\sigma^2_e + \phi^2\sigma^2_e + \phi^4\sigma^2_e + \dots = \frac{\sigma^2_e}{1-\phi^2}$
   d) $cov(Y_t, Y_{t-k}) = \phi^k\sigma^2_e / 1 - \phi^2$
   e) $corr(Y_t, Y_{t-k}) = \phi^k$
2. MA(q)
   a) $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q}$
   b) MA(1)
   1) $Y_t = e_t - \theta_1 e_{t-1}$
   2) $\mu_t = E(Y_t) = 0$
   3) $Var(Y_t) = \gamma_0 = \sigma^2_e + \theta_1^2\sigma^2_e$
   $= \sigma^2_e(1 + \theta_1^2)$
   4) $cov(Y_t, Y_{t-k}) = \gamma_k = \begin{cases} -\theta_1\sigma^2_e, & k = 1 \\ 0, & k \geq 2 \end{cases}$
   5) $corr(Y_t, Y_{t-k}) = \rho_k = \begin{cases} -\theta_1/(1+\theta_1^2), & k = 1 \\ 0, & k \geq 2 \end{cases}$
   6) θ = − → naik, θ = + → turun
   c) MA(2)
   1) $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$
   2) $\mu_t = E(Y_t) = 0$
   3) $Var(Y_t) = \gamma_0 = \sigma^2_e(1 + \theta_1^2 + \theta_2^2)$
   4) $cov(Y_t, Y_{t-k}) = \begin{cases} (-\theta_1 + \theta_1\theta_2)\sigma^2_e, & k = 1 \\ -\theta_2\sigma^2_e, & k = 2 \\ 0, & k \geq 3 \end{cases}$
   5) $\rho_k = \begin{cases} 1, & k = 0 \\ \frac{-\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2}, & k = 1 \\ \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2}, & k = 2 \\ 0, & k \geq 3 \end{cases}$
   d) General MA(q)
   1) $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q}$
   2) $E(Y_t) = 0$
   3) $Var(Y_t) = \gamma_0 = (1 + \theta_1^2 + \dots + \theta_q^2)\sigma^2_e$
   4) $\rho_k = \begin{cases} -\theta_k + \theta_1\theta_{k+1} + \theta_2\theta_{k+2} + \dots + \theta_{q-k}\theta_q \\ 0, & k > q \end{cases}$

Stationary time series : AR(p)

1. AR(p)
   a) $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t$
2. AR(1)
   a) $Y_t = \phi_1 Y_{t-1} + e_t$
   b) $E(Y_t) = 0$
   c) $E(e_t, Y_{t-1}) = 0$, $E(e_t, Y_{t-2}) = 0$
   $E(e_t Y_{t-k}) = E(e_t)E(Y_{t-k}) = 0$
   $E(Y_{t-k}Y_t) = \phi E(Y_{t-k}Y_{t-1}) + E(e_t Y_{t-k})$
   d) $Var(Y_t) = \phi_1^2\gamma_0 + \sigma^2_e$
   $\gamma_0 = \frac{\sigma^2_e}{1-\phi^2}$, $\phi^2 < 1$ / $|\phi| < 1$
   e) $cov(Y_t, Y_{t-k}) = \gamma_k = \phi\gamma_{k-1}$
   $= \phi^k(\sigma^2_e/1-\phi^2)$
   f) $\rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k$
   g) General linear proses AR(1)
   $Y_t = e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \dots + \phi^k Y_{t-k}$
3. AR(2)
   a) $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$
   b) $E(e_t Y_{t-1}) = 0$, $E(e_t Y_{t-2}) = 0$
   c) $\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2}$
   d) $\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}$
   $\rho_1 = \phi_1/1-\phi_2$
   $\rho_2 = \frac{\phi_2(1-\phi_2) + \phi_1^2}{1-\phi_2}$
4. ARMA(p,q)
   a) $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} + \dots - \theta_q e_{t-q}$
5. ARMA(1,1)
   a) $Y_t = \phi_1 Y_{t-1} + e_t \quad \theta_1 e_{t-1}$
   b) $E(e_t Y_t) = \sigma^2_e$
   $E(e_{t-1}Y_t) = (\phi - \theta)\sigma^2_e$

c) $\gamma_0 = \phi\gamma_1 + (1-\theta(\phi-\theta))\sigma^2_e$
$\gamma_1 = \phi\gamma_0 - \theta\sigma^2_e$
$\gamma_k = \phi\gamma_{k-1},\ k \geq 2$
$\gamma_0 = \frac{1-2\phi\theta+\theta^2}{1-\phi^2}\sigma^2_e$

d) $\rho_k = \begin{cases} 0, & k=0 \\ \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1}, & k \geq 1 \end{cases}$

6. Stationarity AR(1)
$|\phi| < 1$

7. Stationarity AR(2)
$\phi_1+\phi_2<1 \quad \phi_2-\phi_1<1 \quad |\phi_2|<1$

8. Invertibility
a) MA(1): $Y_t = e_t - \theta_1 e_{t-1}$
b) $e_t = Y_t + \theta_1 e_{t-1}$
$= Y_t + \theta_1(Y_{t-1} + \theta e_{t-2})$
$= Y_t + \theta Y_{t-1} + \theta^2 e_{t-2}$
$= Y_t + \theta Y_{t-1} + \theta^2 Y_{t-2} + \theta^3 e_{t-3}$
$Y_t = -(\theta Y_{t-1} + \theta^2 Y_{t-2} + \dots) + e_t$
c) MA(1) invert bila $|\theta| < 1$
d) MA(2): $-1 < \theta_2 < 1$, $\theta_1+\theta_2 > -1$, $\theta_1 - \theta_2 < 1$

Model Non Stationarity : ARIMA (

1. Differencing
a) AR(1) : $Y_t = \phi Y_{t-1} + e_t$
$|\phi| > 1 \to$ AR(1) non stationary
$\phi = 1$: $Y_t = Y_{t-1} + e_t$
$Y_t - Y_{t-1} = e_t$
$\nabla^1 Y_t = e_t$

$E(\nabla^1 Y_t) = 0$
$Var(\nabla^1 Y_t) = \sigma^2_e$ } $\to$ stationary

b) Backshift (B)
$B(Y_t) = Y_{t-1}$ $\quad B^3(Y_t) = Y_{t-3}$
$B^2(Y_t) = Y_{t-2}$ $\quad B^k(Y_t) = Y_{t-k}$

c) Backward (∇)
$\nabla = 1 - B$
$\nabla^2 = (1-B)^2 = 1 - 2B + B^2$

$\nabla Y_t = (1-B)Y_t = Y_t - Y_{t-1}$
$\nabla^2 Y_t = (1-B)^2 Y_t = (1-2B+B^2)Y_t$
$= Y_t - 2Y_{t-1} + Y_{t-2}$

2. ARIMA
$W_t = \phi_1 W_{t-1} + \phi_2 W_{t-2} + \dots + \phi_p W_{t-p}$
$+ e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q}$
$Y_t - Y_{t-1} = \phi_1(Y_{t-1} - Y_{t-2}) + \phi_2(Y_{t-2} - Y_{t-3}) + \dots + \phi_p(Y_{t-p} - Y_{t-p-1})$
$+ e_t - \theta_1 e_{t-1} - \dots - \theta_q e_{t-q}$
$Y_t = (1+\phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} + (\phi_3 - \phi_2)Y_{t-3} + \dots + (\phi_p - \phi_{p-1})Y_{t-p} - \phi_p Y_{t-p-1} + e_t - \theta_1 e_{t-1} - \dots - \theta_q e_{t-q}$

3. IMA (1,1)
$d=1, q=1$
$Y_t = Y_{t-1} + e_t - \theta e_{t-1} \to$ non stat
$Y_t - Y_{t-1} = e_t - \theta e_{t-1}$
$W_t = e_t - \theta e_{t-1} \to$ stat

4. IMA(2,2)
$d=2,\ q=2$
$Y_t = 2Y_{t-1} - Y_{t-2} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \to$ non stat
$Y_t - 2Y_{t-1} + Y_{t-2} = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$
$\nabla^2 Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$
$W_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \to$ stat

5. ARI (1,1)
$p=1 \quad d=1$
$\nabla Y_t = \phi \nabla Y_{t-1} + e_t \to$ stat
$Y_t - Y_{t-1} = \phi(Y_{t-1} - Y_{t-2}) + e_t$
$Y_t = (1+\phi)Y_{t-1} - \phi Y_{t-2} + e_t \to$ non stat

6. ARIMA
a) ARIMA (0,0,1) : $Y_t = e_t - \theta_1 e_{t-1}$
b) ARIMA (0,1,1) : $\nabla Y_t = e_t - \theta e_{t-1}$
c) ARIMA (1,0,0) : $Y_t = \phi Y_{t-1} + e_t$
d) ARIMA (1,1,0) : $\nabla Y_t = \phi \nabla Y_{t-1} + e_t$
e) ARIMA (0,2,2) = $\nabla^2 Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$
f) ARIMA (1,1,1) = $\nabla Y_t = \phi \nabla Y_{t-1} + e_t - \theta_1 e_{t-1}$

7. Transformation

Common Box-Cox Transformations	
Lambda	Suitable Transformation
-2	$Y^{-2} = 1/Y^2$
-1	$Y^{-1} = 1/Y^1$
-0.5	$Y^{-0.5} = 1/(Sqrt(Y))$
0	log(Y)
0.5	$Y^{0.5} = Sqrt(Y)$
1	$Y^1 = Y$
2	Y^2

Model Spesification

1. ACF $\to \rho$ seperti sebelum
MA(q) $\to$ cut off lag q. after q dianggap 0
AR(p) $\to$ tails off (sinus) after lag p
ARMA(p,q) $\to$ tails off

2. PACF
$\rho_j = \underbrace{\phi_{k1}}_{partial}\rho_{j-1} + \phi_{k2}\underbrace{\rho_{j-2}}_{autocorr} + \dots$
MA(q) $\to$ tails off setelah lag q
AR(p) $\to$ cut off lag p, after p = 0
ARMA(p,q) $\to$ tails off
SK PACF : $\pm 2/\sqrt{n}$ ($\pm z_{\alpha/2}$ SE)

3. EACF
4. drift slowly $\to$ nonstat

Parameter Estimation

1. Momen
a) AR (1)
$\hat{\phi} = r_1 = \rho_1$
b) AR(2)
$\rho_1 = \phi_1 + \rho_1\phi_2 \to r_1 = \phi_1 + r_1\phi_2$
$\rho_2 = \rho_1\phi_1 + \phi_2 \to r_2 = r_1\phi_1 + \phi_2$
$\hat{\phi}_1 = \frac{r_1(1-r_2)}{1-r_1^2}$, $\hat{\phi}_2 = \frac{r_2 - r_1^2}{1-r_1^2}$
c) AR(p)
$\phi_1 + r_1\phi_2 + r_2\phi_3 + \dots + r_{p-1}\phi_p = r_1$
$r_1\phi_1 + \phi_2 + r_1\phi_3 + \dots + r_{p-2}\phi_p = r_2$
$r_{p-1}\phi_1 + r_{p-2}\phi_2 + r_{p-3}\phi_3 + \phi_p = r_p$
d) MA(1)
$\rho_1 = -\theta/(1+\theta^2)$
$\hat{\theta} = -1 + \sqrt{1-4r_1^2}/2r_1$
e) ARMA (1,1)
$\rho_k = \frac{(1-\theta\phi)(\phi-\theta)}{1-2\theta\phi+\theta^2}\phi^{k-1}, k \geq 1$
$\rho_2/\rho_1 = \phi$
$\hat{\phi} = r_2/r_1$
$r_1 = (1-\theta\hat{\phi})(\hat{\phi}-\theta)/1-2\theta\hat{\phi}+\theta^2$
f) Noise Variance (AR(p))
$\hat{\sigma}^2_e = (1 - \hat{\phi}_1 r_1 - \hat{\phi}_2 r_2 - \hat{\phi}_p r_p)s^2$
$s^2 = \frac{1}{n-1}\sum_1^n (Y_t - \bar{Y})^2$
AR(1): $\hat{\sigma}^2_e = (1-r_1^2)s^2$, $\hat{\phi} = r_1$
g) Noise Variance (MA(q))
$\hat{\sigma}^2_e = s^2 / 1 + \hat{\theta}_1^2 + \hat{\theta}_2^2 + \dots + \hat{\theta}_q^2$
$s^2 = \frac{1}{n-1}\sum_1^n (Y_t - \bar{Y})^2$
ARMA(1,1): $\hat{\sigma}^2_e = \frac{1-\hat{\phi}^2}{1-2\hat{\phi}\hat{\theta}+\hat{\theta}^2}s^2$

2. LSE
a) AR(1)
$S_c(\phi,\mu) = \sum[(Y_t - \mu) - \phi(Y_{t-1} - \mu)]^2$
$\mu = \frac{1}{(n-1)(1-\phi)}(\sum Y_t - \phi\sum Y_{t-1})$
n >>
$\mu = \frac{1}{1-\phi}(\bar{Y} - \phi\bar{Y}) = \bar{Y}$
$\hat{\phi} = \frac{\sum(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum(Y_{t-1} - \bar{Y})^2}$

3. ML
a) AR(1)
$Var(\hat{\phi}) = \frac{1-\phi^2}{n}$
b) AR(2)
$Var(\hat{\phi}_1) = var(\hat{\phi}_2) = \frac{1-\phi_2^2}{n}$
$Corr(\hat{\phi}_1, \hat{\phi}_2) = -\phi_1/1-\phi_2 = -\rho_1$
c) MA(1) = $Var(\hat{\theta}) = 1-\theta^2/n$
d) MA(2) = $Var(\hat{\theta}_1) = var(\hat{\theta}_2) = 1-\theta_2^2/n$
$corr(\hat{\theta}_1, \hat{\theta}_2) = \theta_1/1-\theta_2$
e) ARMA (1,1)
$Var(\hat{\phi}) = (1-\phi^2/n)(1-\phi\theta/\phi-\theta)^2$
$Var(\hat{\theta}) = (1-\theta^2/n)(1-\phi\theta/\phi-\theta)^2$
$Corr(\hat{\phi}, \hat{\theta}) = \sqrt{(1-\phi)^2(1-\theta^2)}/1-\phi\theta$

Model Diagnostik

1. Ljung box test
H_0 : e uncorrelated
H_1 : e correlated
$Q_* = n(n+2)\left(\frac{\hat{r}_1^2}{n-1} + \frac{\hat{r}_2^2}{n-2} + \dots + \frac{\hat{r}_k^2}{n-k}\right)$

2. Box pierce
$Q = n(\hat{r}_1^2 + \hat{r}_2^2 + \dots + \hat{r}_k^2)$
$Q/Q_* \sim$ Chi square $_{k-p-q}$

3. Over fitting
AIC kecil, log likelihood tinggi, σ^2 estimated kecil

4. Forecasting
a) AR(1)
1) $\hat{Y}_t(1) = \underbrace{\mu}_{intercept} + \underbrace{\phi}_{koef}(Y_t - \mu)$
$Y_{t+1} - \mu = \phi(Y_{t+1-1} - \mu) + e_{t+1}$
$Y_{t+2} - \mu = \phi(Y_{t+1} - \mu) + e_{t+2}$
$Y_t(2) = \mu + \phi(Y_t(1) - \mu) + e_{t+2}$
2) $Var(e_t(l)) = \sigma^2_e\left(\frac{1-\phi^{2l}}{1-\phi^2}\right)$
$Var(e_t(1)) = \frac{\sigma^2_e}{1-\phi^2}$,
$\Rightarrow Var(e_t(1)) \approx Var(Y_t) = \gamma_0$

b) MA(1)
$Y_t = \mu + e_t - \theta e_{t-1}$
$Y_{t+1} = \mu + e_{t+1} - \theta e_t$
$\hat{Y}_t(1) = \mu - \theta e_t$
$\hat{Y}_t(2) = \mu$
$\hat{Y}_t(l) = \mu$, $l > 1$

$S_n = \hat{Y} \pm 1.96 \cdot SE$
SE