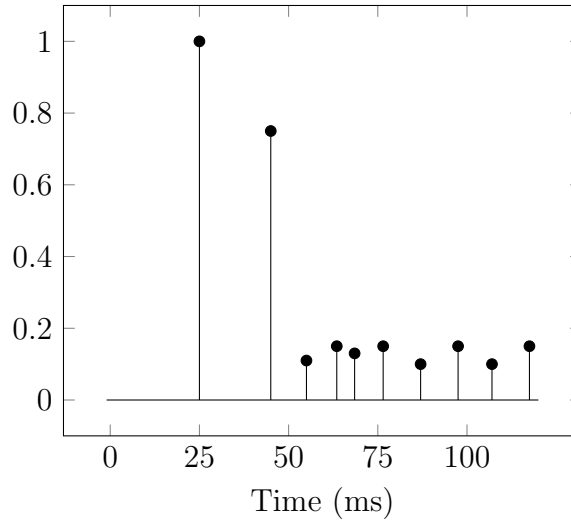


Homework 9

Due April 29 at 11 pm

1. (Impulse response of a room) We are interested in analyzing the acoustic characteristics of an auditorium during a concert. To this end, we want to study the sound traveling from the singer's location to a seat in the last row.
 - (a) Under what assumptions would the sound propagation be characterized by the convolution of the emitted sound and an impulse response?
 - (b) Assume the assumptions hold, and we measure the following impulse response:



Why do you think the response is zero before 25 ms?

- (c) What do the two first spikes in the impulse response correspond to?
 - (d) Suggest a way to measure the impulse response by bursting a balloon. What are the possible limitations of this approach?
 - (e) Suggest a way to measure the impulse response using a device that produces pure tones (i.e. sinusoidal sounds).
2. (Discrete filter) Let us index the DFT coefficients of the N -dimensional vectors from $-(N-1)/2$ to $(N-1)/2$ (assuming N is odd). We define the bandlimited signals in this space as those for which the nonzero Fourier coefficients are zero beyond a certain value k_c , i.e. $x \in \mathbb{C}^N$ is bandlimited if $\hat{x}[k] = 0$ for all $|k| > k_c$. Let y be the vector with the smallest ℓ_2 norm such that $x * y = x$ for all bandlimited vectors with cut-off frequency k_c (where k_c is a fixed integer smaller than $(N-1)/2$). Derive an explicit expression for the entries of y , showing that they are real valued.
3. (PCA of stationary vector) Let \tilde{x} be a wide-sense stationary vector with real-valued autocovariance vector $a_{\tilde{x}}$, with covariance matrix $\Sigma_{\tilde{x}}$. In the notes we showed that the eigenvectors and eigenvalues of $\Sigma_{\tilde{x}}$ are complex exponentials and the DFT coefficients of

$a_{\tilde{x}}$ respectively. Here we will show that we can derive an equivalent real-valued eigendecomposition because the autocovariance vector is real. We will assume that N is an odd number.

- (a) Show that the DFT coefficients of $a_{\tilde{x}}$ are real, and satisfy $\hat{a}_{\tilde{x}}[k] = \hat{a}_{\tilde{x}}[N - k]$ for $k = 1, \dots, \frac{N-1}{2}$.
- (b) Show that

$$\Sigma_{\tilde{x}}[j_2, j_1] = \frac{1}{N} \sum_{k=0}^{N-1} \hat{a}_{\tilde{x}}[k] \exp\left(\frac{i2\pi k(j_2 - j_1)}{N}\right). \quad (1)$$

- (c) Show that $\Sigma_{\tilde{x}}$ has the following decomposition

$$\Sigma_{\tilde{x}}[j_2, j_1] = \sum_{k=0}^{N-1} \lambda_k u_k[j_1] u_k[j_2], \quad (2)$$

where the eigenvectors correspond to the orthonormal vectors

$$\frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}, \quad \sqrt{\frac{2}{N}} \begin{bmatrix} 1 \\ \cos\left(\frac{2\pi k}{N}\right) \\ \dots \\ \cos\left(\frac{2\pi k(N-1)}{N}\right) \end{bmatrix}, \quad \sqrt{\frac{2}{N}} \begin{bmatrix} 0 \\ \sin\left(\frac{2\pi k}{N}\right) \\ \dots \\ \sin\left(\frac{2\pi k(N-1)}{N}\right) \end{bmatrix}, \quad 1 \leq k \leq \frac{N-1}{2}.$$

and report the value of $\lambda_0, \dots, \lambda_k$.

4. (Household electricity usage) Given the rise of smart electricity meters and the wide adoption of electricity generation technology like solar panels, there is a wealth of electricity usage data available. Here we will explore a household electricity usage dataset which represents a time series of power-related variable.
 - (a) Load the data file `household_power_consumption_days.csv` in Python and plot it. Compute and plot the autocorrelation of these data using Definition 4.1 in the note for the first 3 weeks and first 3 years. Provide explanations for each of the plots.
 - (b) Apply DFT to the autocorrelation and give a plot of the magnitudes of the computed DFT coefficients. What do you notice from the plot? (Hint: You might want to zoom in to see spikes around zero frequency clearly.)
 - (c) Based on what you have observed, what are the advantages and disadvantages of modeling this time series data as stationary?