

数字信号处理

作业4.

$$1. \begin{cases} x[n] = 0.5^n u[n] + u[-n-1] \\ y[n] = 0.75^n u[n] \end{cases}$$

$$(1) \textcircled{1} X(z) = X_1(z) + X_2(z)$$

$$(2) \begin{cases} X_1(z) = \frac{1}{1-0.5z^{-1}} & |z| > 0.5 \\ X_2(z) = \frac{-1}{1-z^{-1}} & |z| < 1 \end{cases}$$

$$\therefore X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-z^{-1}} \quad |z| \in (\frac{1}{2}, 1)$$

$$\textcircled{2} Y(z) = \frac{1}{1-0.75z^{-1}} \quad |z| > 0.75$$

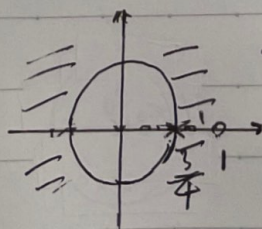
$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{-2z + 3 - z^{-1}}{1 - 0.75z^{-1}} \quad |z| \in (\frac{3}{4}, +\infty)$$

$$= -2z + \frac{1.5 - z^{-1}}{1 - 0.75z^{-1}}$$

$$= -2z + \frac{4}{3} + \frac{\frac{1}{6}}{1 - 0.75z^{-1}}$$

$$\therefore h[x] = -2\delta[n-1] + \frac{4}{3}\delta[n] + \frac{1}{6} \cdot (0.75)^n \cdot u[n]$$

$H(z)$ 的零极点图



极点: $\frac{3}{4}$

零点: 1

ROC: $(\frac{3}{4}, +\infty)$

(3). 差分方程

$$H(z) = \frac{-2z + 3 - z^{-1}}{1 - \frac{3}{4}z^{-1}} = \frac{Y(z)}{X(z)}$$

∴ 差分方程 $y[n] - \frac{3}{4}y[n-1] = -2x[n+1] + 3x[n] - x[n-1]$

(4). 稳定性 : 稳定.

因果性 : 非因果.

2. $\because \arg [H(e^{j\omega})]_{\text{ap}} = \frac{1-r^2}{|1-re^{j\theta}e^{-j\omega}|^2} > 0 \quad (4.3.5)$

将 $z=1$ 代入 4.3.6 得
($\omega=0$)

$$H_{\text{ap}}(e^{j0}) = H_{\text{ap}}(1) = A > 0 \quad \text{常数}$$

$$\therefore \arg [H_{\text{ap}}(e^{j\omega})] = 0$$

由 $\arg [H_{\text{ap}}(e^{j\omega})] > 0$ 知. $\arg [H_{\text{ap}}(e^{j\omega})]$ 是递减函数

$$\therefore \arg [H_{\text{ap}}(z)] \leq 0$$