Appendix N: An Advection-Diffusion Model Extension of the Lorenz-96 Model

A low-order model with sensitive dependence on initial conditions, low computational cost, and bounded state variables is useful for testing DA algorithms. The traditional Lorenz-96 models (refs to original) has been used in many ensemble DA studies including (Sakov, Anderson, standard set).

It can be extended to include an additional 40 variables, qm, that represent concentrations of a tracer and are collocated with the standard variables, xm, m=1,…,M. The xm are treated as scaled anomalous velocities so that where is a specified constant mean velocity, is a multiplying constant that sets the average magnitude of wind perturbations, and is an anomalous velocity at gridpoint m. Velocities are expressed with units of nondimensional distance per nondimensional time. A nondimensional location of m is assigned to each gridpoint in the model so that the distance between neighboring gridpoints is 1 (note that this is different than many previous papers where the distance between gridpoints is defined as 1/M).

The qm are treated as dimensionless concentration of a tracer that is advected by the total velocity vm using an upstream semi-lagrangian method.

The concentration is also subject to a time invariant smoothing diffusion, a specified source that is a function of time and grid point, and an exponential sink with a constant coefficient.

Smoothing: This is necessary to avoid the formation of shocks with the semi-lagrangian dynamics and is implemented as a (1, -2, 1) smoother. The form of this is

. I think we can neglect this in an initial implementation.

Specified Source: A source that is a function of gridpoint and assimilation time is specified as S(m, t) with units of amount per time. For initial tests, this has been set to S(m, t) = 100 if m=1 and 0 for all other m’s. This is a constant source at a single grid point.

Exponential Sink: This is simulating some sort of removal of the tracer that is proportional to the amount of the tracer. The e-folding rate in terms of non-dimensional time is so that the amount of tracer removed each time is .

Numerical implementation details: This can be added into a standard L96 model implementation. Before updating the standard variables at each time step, do the following for the tracer variables.

1. Advection: Compute the velocity at each gridpoint.
2. Do an upstream semi-Lagrangian advection. Treating the grid interval as 1, locate the upstream location for Find the bounding grid points and linearly interpolate between them to get the updated value for the tracer at each point m.
3. Numerical diffusion is applied. Skip this for now.
4. The source term is added in at each grid point.
5. The exponential sink is applied at each grid point.

Some initial values for constants:

Values that seem to produce useful results:

Mean velocity

Velocity perturbation scaling

E-folding coefficient for damping:

Source rate at single grid point 100