Exit Strategies from Quantitative Easing programs
Preliminary and Incomplete*

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Abstract

I study the macroeconomic effects of unwinding the central bank balance sheet, considering the trade-offs from fiscal-monetary policy interactions. I construct a Regime-Switching NK-DSGE calibrated to the US economy before the COVID-19 crisis. Reducing the central bank balance sheet reduces inflation at the cost of increasing debt/GDP. Effects on output, inflation, and debt depend on how aggressive the strategy (Quantitative Tightening (QT) or Sell-off of bonds) is and the configuration of conventional fiscal and monetary policies.

Keywords: Monetary Policy; Fiscal and monetary policy mix; Quantitative Easing; Quantitative Tightening

JEL Classification: E31, E52, E58, E62, E63

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1 Introduction

After the great recession and COVID-19 crises, short-term interest rates in developed economies approached the Zero Lower Bound (ZLB), leaving central banks without their conventional monetary policy instrument. As a result, they started to apply Unconventional Monetary Policies, mainly Quantitative Easing (QE), to stimulate the economy through interest rates of longer horizons[1]. This expansion in the central bank's balance sheet increased different liquidity measures, like bank deposits and central bank reserves. This happened with massive fiscal expansions, which increased fiscal deficits and debt/GDP ratios to levels not seen since the second world war. As the economies recover and inflation rates reach values not seen for forty years in the developed world, it is unclear how to unwind the economic stimulus introduced during the crisis.

This paper studies the macroeconomic effects of unwinding the central bank balance sheet, considering the trade-offs from fiscal-monetary policy interactions. It studies exit strategies characterized by different speeds (for instance, Quantitative Tightening (QT) or Sell-off of bonds) under different configurations of the conventional fiscal and monetary policy mix. To answer the question, I construct a Regime-Switching NK-DSGE and calibrated it to the US economy before the COVID-19 crisis. I simulate the economy to generate a crisis that resembles the one caused by the COVID-19 pandemic in the US and the policy response. In this context, I study different strategies to reduce the Central Bank Balance Sheet and their macroeconomic effects.

The model has five agents: households, financial intermediaries, firms, central bank, and fiscal authority. Quantitative Easing is central bank purchases of long-term government bonds to Households, paid by issuing reserves to financial intermediaries. The model exhibits market segmentation in the public debt market and a leverage constraint in financial intermediaries, as in Elenev et al. (2021). Due to these frictions, Quantitative Easing policies generate portfolio rebalancing effects in Households and Financial Intermediaries, causing real effects. When the Central Bank does QE, it floods financial intermediaries with reserves,

[1] As the empirical literature on quantitative easing macroeconomic effects has shown, these policies have a substantial impact on long-term interest rates, output, and inflation. See, for instance: Vissing-Jorgensen (2021) for effects on yields, Bhattarai and Neely (Forthcoming), for a review of macroeconomic effects of these policies.

which relaxes its leverage constraint. This allows them to issue more deposits to households. On the other hand, the central bank's intervention in the long-term debt market as a new buyer exerts pressure on its price. This gives incentives to households to sell their position of long-term bonds and replace them for deposits, which they value by their liquidity effects, even if they pay a lower return. The downward pressures on nominal returns decrease incentives for saving, working as a demand shock for households, generating an increase in labor, output, and inflation.

Conventional monetary and fiscal policies consist of a Taylor rule for the short-term interest rate and a fiscal rule for taxes. Parameters in these rules switch between three policy regimes, as in Bianchi and Melosi (2017) and Bianchi and Melosi (2022). Fiscal-monetary policy mix alternates between a monetary-led regime, a fiscal-led regime, and a Zero Lower Bound regime. In the first regime, the central bank reacts strongly to inflation deviations from the target, and the fiscal authority accommodates the primary fiscal surplus to stabilize the debt. This behaviour was characterized as *active* monetary policy and *passive* fiscal policy in Leeper (1991) and the literature initiated thereafter. In the second regime, the fiscal authority disregards the objective of stabilizing debt to stimulate the economy. As a result, the monetary authority does not react strongly enough to inflation deviations, letting it stabilize debt in real terms (*active* fiscal policy and *passive* monetary authority). Finally, the ZLB regime represents a crisis regime, where the monetary authority is left with no room to stimulate the economy by decreasing the short-term nominal interest rate since it reached the effective lower bound. The fiscal authority stimulates the economy by not reacting to debt deviations. This last regime constitutes an extreme version of the fiscally led regime. The economy presents recurrent regime switches following a transition matrix. Differently from Bianchi and Melosi (2017), the transition to and from the ZLB regime is endogenous.

The probability of entering the ZLB regime decreases with the nominal interest rate, being equal to one when the rate is below or very close to one. The likelihood of exiting the ZLB is increasing in a Shadow Interest rate. The Shadow Interest rate is equal to the Taylor Rule that the Central Bank follows in the Monetary led regime, without the lower bound. The endogeneity surrounding this regime allows the agents in the economy to form rational expectations toward the occurrence of this regime based on their information on macroeconomic variables like inflation, output, and nominal interest rates. The endogenization of this transition probability constitutes one of the contributions of this paper.

The main contribution of this paper is showing that the impact of Central Bank Balance Sheets policies (either QE or QT and Sell-Off of bonds) depends on the fiscal-monetary policy mix. Even though asset-purchasing programs are typically studied as a purely monetary problem, this paper shows that the fiscal and monetary authorities are critical during these programs. First, QE provides additional fiscal space in the short run since it reduces new debt issuance. The Central Bank intervention increases the long-term debt price, depressing the interest rate payments the government has to pay and increasing the central bank's profits, which it transfers to the treasury. On top of this, it fosters a faster recovery, reducing the primary fiscal deficit. All these factors allow the treasury to decrease debt/GDP. Second, as a counterpart of the central bank's balance sheet expansions, liquidity increases substantially, both in the hand of commercial banks (reserves) or non-bank private agents (deposits or currency). The extent to which the increase in liquidity affect output and inflation rate depends on the fiscal-monetary policy mix. QE is more expansive at the Zero Lower Bound and under the fiscally led regime when the inflationary impact is also more significant.

How the central bank balance sheet's size is reduced matters for inflation, output, and debt dynamics. When the monetary authority starts shrinking its holdings of long-term government bonds, either without repurchasing the ones that mature (QT) or actively selling its position in the secondary market (Sell-off of bonds), it decreases inflation at the cost of an increase in the ratio of debt/GDP. These effects increase when the exit strategy is more aggressive. A sell-off of bonds that reduces the balance sheet from 20% of GDP to 5% in 7 years stabilizes inflation around the target of 2% while preventing the central bank from increasing nominal short-term interest rates in 2p.p. (5.3% to 3.3%). However, the economy ends up with 6p.p. more debt/GDP than a less aggressive policy. The disinflationary benefits of a sell-off of bonds are less apparent when the economy exits the crisis towards a fiscally led regime, where the government does not raise taxes to counteract the increase in debt.

This paper contributes to several strands of the literature. In particular, the fast-growing literature that studies the macroeconomic effects of unconventional monetary policies in general, and Quantitative Easing programs in particular, through different channels. As summarized by Bhattarai and Neely (Forthcoming), Kuttner (2018), the literature has emphasized different channels through which these policies can affect variables like output and inflation. In particular, portfolio rebalancing effects, signaling channels of future interest rates, and stock markets, among others. A crucial missing piece in these papers is the role of fiscal policy. To isolate the effect of monetary policies, the literature typically simplifies fiscal authority behavior. It assumes it can perfectly accommodate taxes to satisfy the government budget constraint at all periods, leaving the role of inflation stabilization to the central bank. However, under these assumptions, there is no role for policy uncertainty or interchange in policy objectives among the different authorities, and inflationary surprises to stabilize debt are entirely ruled out. This paper fills that gap by allowing the fiscal-monetary policy mix to play a role in shaping the macroeconomic effects of unconventional policies.

Elenev et al. (2021) ask whether monetary policy can create fiscal capacity. In their setting, fiscal capacity depends on the probability of shifting fiscal policy from active to passive. In this sense, I share with them the objective of studying the effects of unconventional monetary policies while allowing fiscal policy to shift between regimes. The main difference with is paper is that they assume and calibrate a fiscal limit beyond which the fiscal policy starts increasing taxes to ensure debt sustainability. This assumption prevents inflationary dynamics from arising in the model since all the agents in the economy are rational and know the government will never inflate away part of the debt. My main contribution here is to study exit strategies from Quantitative Easing programs when we relax this assumption.

Many authors studied the transmission mechanisms of Quantitative Easing programs in general equilibrium models, either with financial frictions, like in Gertler and Karadi (2011), Sims and Wu (2021), Sims et al. (2020), Del Negro et al. (2017), among others; with market segmentation and/ or portfolio adjustment costs as the main mechanisms to break the non-arbitrage condition between short-term and long-term bonds, as in Chen et al. (2012), Harrison (2017). Furthermore, a recent paper by Cui and Sterk (2021) studies the liquidity effects of asset purchasing programs in a model with heterogeneous agents (HANK model). The main contribution to this strand of the literature provides richer modeling of the fiscal sector in the context of a Dynamic Stochastic General Equilibrium (DSGE) model.

By giving a central role to the interaction or coordination of fiscal and monetary policies, this article relates to papers that study fiscal-monetary policy mix, typically in normal times and as simple rules, as in Leeper (1991), Schmitt-Grohé and Uribe (2007), Leeper and Leith (2016), and to the literature of the Fiscal Theory of the Price Level as in Cochrane (2001), Cochrane (2021). While allowing the policy mix to change among regimes, this paper relates to the literature on regime switches in policy rules, as in Bianchi (2013), and to the literature that studies the role of these regimes at the zero lower bound (ZLB), as in Bianchi and Melosi (2017), Bianchi and Melosi (2022), Bianchi and Melosi (2019), among others. The contribution to this branch of the literature is the study of unconventional monetary policies together with the conventional Taylor rule on short-term interest rates.

This paper focuses on the macroeconomic effects of reducing the central bank balance sheet. In a broad sense, I share the question with Wen et al. (2014), Harrison (2017), Sims et al. (2020), Bonciani and Oh (2021). However, this paper differs from these in many aspects. Wen et al. (2014) studies the exit strategy from QE programs emphasizing the importance of the timing and the pace of the exit, and the private sector expectations of the exit, as I do, but the focus relies on the impact on firms, there is no role for the fiscal authority. Harrison (2017) studies the optimal QE policy in a DSGE-NK model with portfolio adjustment cost, and Sims et al. (2020) studies optimal simple and implementable QE rules through minimizing a quadratic loss-function in a DSGE-NK model with financial frictions. Bonciani and Oh (2021) extends the work of Sims et al. (2020) by showing that the central bank's loss function depends on the central bank's asset purchases volatility. Crucial differences are that these articles focus on central bank purchases of corporate/ private sector bonds, assume a limited role in fiscal policy, and do not allow for changes in the conduct of the fiscal policy rule. Foerster (2015) shows that private agents' expectations about the exit strategy from a QE program impact the initial effectiveness of the policy in a Markov-switching dynamic stochastic general equilibrium (MS-DSGE) model with a financial sector.

The remainder of the paper goes as follows. In the following section, I provide motivating evidence of the importance of the fiscal authority's behavior during Quantitative Easing programs, looking at data for the US during the COVID-19 crisis. Section 3 presents the model, section 4 discuss the calibration, functional forms, and solution method. In section 5, I present the quantitative results of the model and explain the main transmission mechanism of QE. 6 constitutes the paper's main section, where I simulate the crisis and present the different exit strategies from QE. Finally, 7 concludes.

2 COVID-19 crisis and policy response in the US

In March 2020, the COVID-19 pandemic hit economies worldwide, generating an unprecedented macroeconomic crisis. As a response, central banks in leading developed economies, particularly the US, started to stimulate the economy through cuts in short-term interest rates until they touched the zero lower bound. In parallel, central banks started to purchase assets, mainly treasuries, expanding their balance sheet at a breakneck speed. As can be seen in figure 1, the assets in the Federal Reserve increased from 4 trillion dollars in the third quarter of 2019 to around 8 trillion dollars one year later.

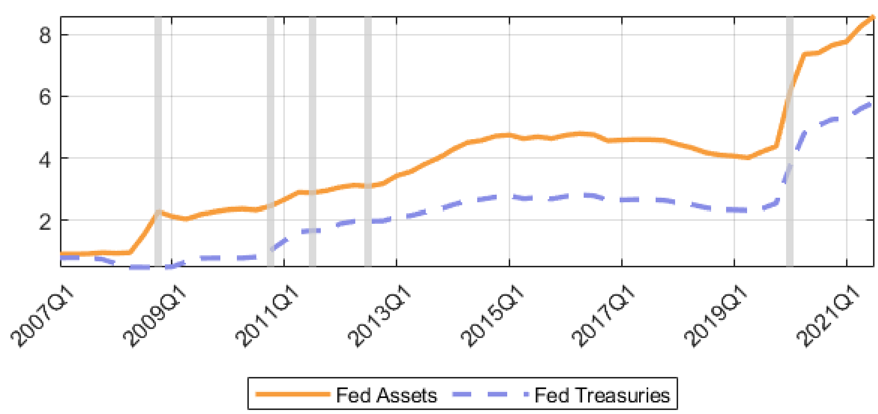

Figure 1: Federal Reserve Balance Sheet

Assets in the Federal Reserve balance sheet (orange line) and treasuries in the balance sheet (dotted light-blue line). Variables in trillions of dollars. Source: FRED and US Financial Accounts. Grey vertical lines show the start of a Quantitative Easing program.

The central bank intervention in the treasuries' market impacted spreads of different maturities, from 1 to 30 years. Vissing-Jorgensen (2021) provides empirical evidence that the FED purchases were causal to the spike in treasury prices.

One natural question that arises in this context is: who was selling the treasuries to the FED during this period of financial stress in the treasuries markets?. In figure 2 we have the net purchases or treasuries from the start of the COVID-19 crisis. They are net flows of treasuries, i.e., net of revaluation effects that take into account the change in treasuries' prices[2]. Different colors represent different agents in the economy. When the bar is above zero, the agent was purchasing treasuries during the period, while if it is below zero, it represents net sales. The blue line represents the net debt issuance from the US Treasury, and it is the sum of all the bars in a corresponding period.

Two observations arise from that figure. First, the total debt issuance increased during the whole period and was especially important during the first two quarters of 2020. Second, the monetary authority's purchases of government bonds played a vital role during this period, sustaining the demand for treasuries when main economic agents like the household sector, mutual funds, and the rest of the world were selling their bond holdings. Furthermore, together with the evidence in Vissing-Jorgensen (2021), we can argue that the FED purchases provided fiscal space in the short run, allowing the fiscal authority to accommodate a fiscal shock while maintaining the price of debt at high levels. The fact that the primary seller of bonds to the FED was the fiscal authority is relevant. It shows that the Quantitative Easing program that started in March 2020 consisted not only of an expansion of the central bank's balance sheet and maturity shortens in bonds in the hands of private agents. On top of that, this policy allowed the treasury to accommodate a fiscal expansion during the worse moment of the crisis. In the empirical application, I study the macroeconomic implications of this fact through the lens of the model.

[2] The conclusions do not change when we consider changes in treasuries. See Appendix, section 8.1.

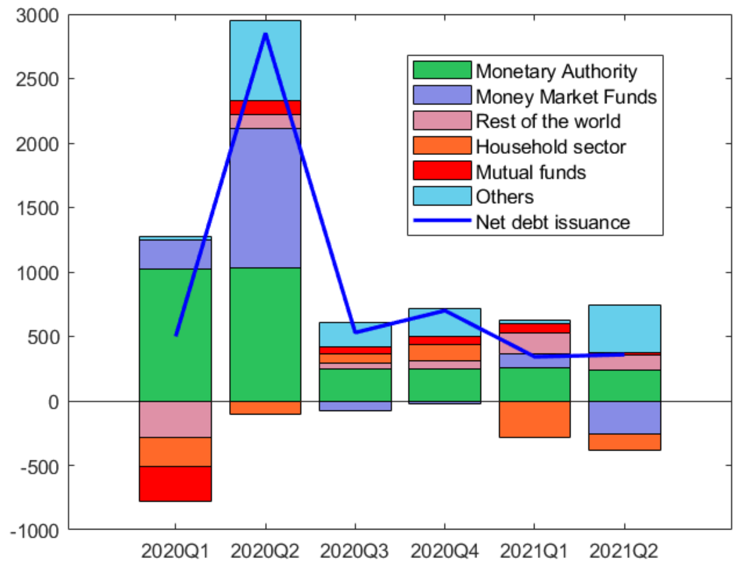

Figure 2: Net purchases of treasuries

Source: US Financial Accounts. Data in billions of dollars. Flows, net of revaluation effects.

When the FED performs quantitative easing policies, it purchases assets. The counterpart of this operation is the issuance of bank reserves (or money in commercial banks' hands) that increase the central bank's liabilities, expanding the central bank's balance sheet. If commercial banks were the original owners of the bonds (case 1), this operation would leave its balance sheet unchanged since they increase one asset (reserves) while decreasing another (treasuries). However, suppose the original owner of the bond was the non-bank private sector (case 2), like households or mutual funds. In that case, the bank acts as a mere intermediary of this policy. The result would be an increase in deposits or currency (money in private hands), together with an increase in reserves and the central bank's balance sheet. As a result, different measures of money increase after quantitative easing policies under different cases; either bank money in the form of reserves (case 1) or non-bank private money (case 2). This liquidity expansion, also emphasized by Cui and Sterk (2021), and the owner of this liquidity, shapes the transmission mechanism of QE policies.

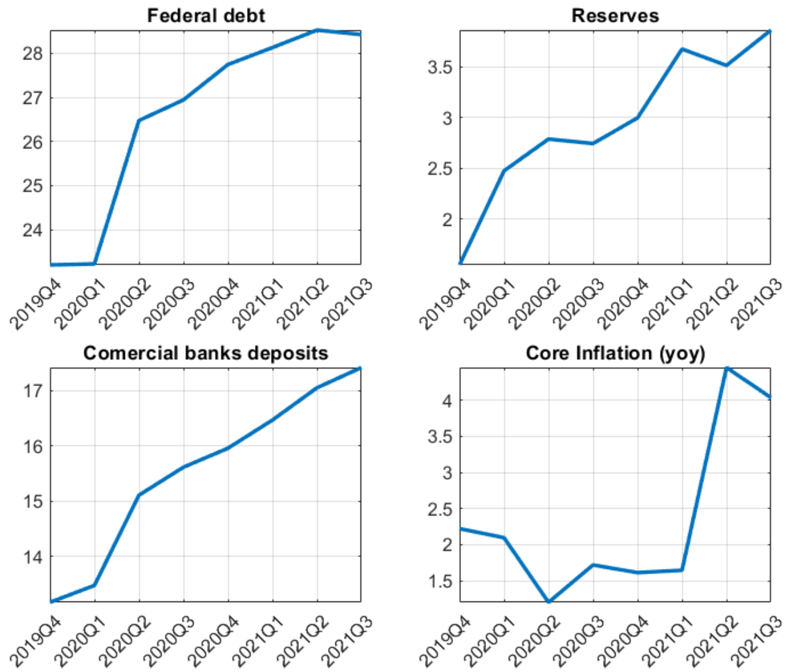

Figure 3: Macroeconomic variables: public debt, liquidity, inflation

US Federal Debt, Reserves and Deposits, in trillions of dollars. Core inflation in %. Source debt, deposits and inflation data: FRED. Source reserves: US financial accounts, release December 2021.

In figure 3, we can see the central bank's balance sheet expansion took place together with a massive expansion in federal government debt and liquidity, both in the form of bank reserves and deposits[3]. Other measures of monetary aggregates present significant spikes during this period too. Some of these variables, like public debt, are at values over GDP not seen since the Second World War.[4] On top of this, inflation rates, either measured as yearly changes in core CPI or general CPI measures, are well above the target, presenting an extra challenge for policymakers. How public debt will be repaid and how (and if) the extra liquidity should be retired from the economy in an inflationary environment are policy questions I assess in this paper.

[3]In the appendix, section 8.1, I show this last feature differs from what happened after the Quantitative Easing programs after the Great Recession.

[4]See section 8.1 for a historical plot of this variable.

3 Model

The model is a Markov-Switching NK-DSGE model with five agents: Firms, Households, Monetary Authority, Fiscal Authority, and Financial Intermediaries (FI). There are three assets in the economy: Short-term public bonds B_t^S, with one-period maturity and price Q_t^S; deposits D_t, with price Q_t^D; and long-term public bonds: B_t^L, with geometrically decaying maturity δ, as in Hatchondo and Martinez (2009). They pay a coupon κ every period, and its price is Q_t^L.

As standard in the literature that assesses the transmission mechanisms of quantitative easing policies, there is segmentation in bonds' markets.[5] I follow Elenev et al. (2021) in assuming that the Household cannot invest in short-term public bonds, that are hold exclusively by financial intermediaries and the central bank; while financial intermediaries do not invest in long-term public bonds. Short-term bonds in the model stand for treasury bills and central bank reserves indistinctly since both assets share liquidity and return properties. The assumption that financial intermediaries exclusively hold them in the model makes this asset close to 'money in banks' hands.

Market segmentation interacts in the model with two nominal frictions. First, prices are sticky as in Rotemberg (1982) and the New-Keynesian literature. Second, deposits provide liquidity services to households that increase their utility, as in the tradition of money in the Utility function models. I assume this friction is in terms of deposits since they are the most liquid asset the Household can invest in this economy. The inclusion of variables that stand for reserves and monetary aggregates in the model, together with the frictions above, allows the model to generate the main macroeconomic effects of quantitative easing policies that we observe in the data.

Two authorities constitute the government: the treasury and the central bank. Each authority has its budget constraint and policy instrument(s). The presence of the two authorities allows the model to generate realistic dynamics of quantitative easing policies, where

[5]See, for instance, Chen et al. (2012), and Bhattarai and Neely (Forthcoming) for a comprehensive analysis on different mechanisms to break the non-arbitrage condition between different assets in the economy, and thus, the neutrality result from Wallace (1981).

the central bank finances the purchases of government bonds through the issuance of bank reserves.

Next, I describe each agent and its optimization problem.

3.1 Households

There is a continuum of measure one of homogeneous households that live infinite periods in the economy. A representative agent chooses consumption c_t, labor n_t, deposits D_t^H, and long-term public bonds $B_t^{L,H}$ to maximize its lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \nu_t \beta^t U\left(c_t, \frac{D_t^H}{P_t}, n_t\right)$$

ν_t is a preference shock, that follows an AR(1) process. Deposits are the most liquid asset that a Household can purchase. I assume agents derive utility from the liquidity services this asset provides. The utility function is monotone increasing in consumption and deposits, monotone decreasing in labor, and satisfies Inada conditions on all variables.

Long term bonds pay geometrically decaying coupons, as in Hatchondo and Martinez (2009). A bond B_t^L issued at time t, pays the sequence of coupons: κ, $\kappa(1-\delta)$, $\kappa(1-\delta)^2, \ldots,$ where $\kappa > 0$ and $\delta \in (0,1)$. This last parameter controls the debt maturity, where $\delta = 1$ corresponds to a short term bond, and $\delta = 0$ represents a consol. This maturity specification allows to reduce the number of state variables in the model, since a bond issued at time $j-k$ is equivalent to $(1-\delta)^k$ bonds issued at period t, and hence the state variable B_{t-1}^L represents total long-term debt in equivalent newly issued long-term bonds.

Every period, the representative Household pays taxes (τ_t), receives nominal dividend payments from financial intermediaries Div_t, firm's profits since households are the owners Π_t^f, and rebates $\tilde{\Pi}_t$. When she invests in long-term bonds, she pays a portfolio adjustment cost of $\Phi(.)$ on top of the asset's price [6]

The optimization problem of a representative household is the following.

[6] This assumption helps the model to generate a positive term premium in long-term public bonds.

$$
\max_{c_t, n_t, D_t^H, B_t^{L,H}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \nu_t U\left(c_t, \frac{D_t^H}{P_t}, n_t\right)
$$

$$
\begin{aligned}
P_t c_t + Q_t^D D_t^H + B_t^{L,H} Q_t^L + \Phi_L\left(\frac{B_t^{L,H}}{P_t}\right) P_t &= W_t n_t + D_{t-1}^H + \cdots \\
+B_{t-1}^{L,H}\left[\kappa + (1-\delta) Q_t^L\right] + \Pi_t^f + Div_t + \tilde{\Pi}_t - \tau_t P_t & \qquad (1) \\
D_t^H &\geq 0 \\
B_t^{L,H} &\geq 0
\end{aligned}
$$

Π_t are transfers from different agents in the economy, P_t price level, W_t is the nominal wage. Under this specification, the expected return on a long-term bonds purchased at period t is: $R_{t,t+1}^L = \mathbb{E}_t \frac{\kappa + (1-\delta) Q_{t+1}^L}{Q_t^L}$.

Notice that the non-negativity condition on deposits does not bind at the optimum even if they pay a lower return, given the preferences' assumptions. Finally, the last constraint is the non-negativity conditions for public bonds since the Household cannot go short on them.

Associate the multiplier $\beta^t \lambda_t$ to the budget constraint. The system of equilibrium conditions that characterizes the households' optimization problem solution is the following:

$$
\frac{-U_n\left(c_t, \frac{D_t^H}{P_t}, n_t\right)}{U_c\left(c_t, \frac{D_t^H}{P_t}, n_t\right)} = \frac{W_t}{P_t}
$$

$$
Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{U_d\left(c_t, \frac{D_t^H}{P_t}, n_t\right) P_t}{U_c\left(c_t, \frac{D_t^H}{P_t}, n_t\right)}
$$

$$
Q_t^L + \Phi_L'\left(\frac{B_t^{L,H}}{P_t}\right) = \mathbb{E}_t \mathcal{M}_{t,t+1}\left[\kappa + (1-\delta) Q_{t+1}^L\right]
$$

Where $\mathcal{M}_{t,t+1}$ is the stochastic discount factor between period t and $t+1$.

$$
\mathcal{M}_{t,t+1} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} = \beta \mathbb{E}_t \frac{U_c\left(c_{t+1}, \frac{D_{t+1}^H}{P_{t+1}}, n_{t+1}\right)}{U_c\left(c_t, \frac{D_t^H}{P_t}, n_t\right)} \frac{1}{\pi_{t+1}}
$$

Together with the budget constraint 1, they characterize the solution to the Household's optimization problem[7]. Define $\pi_t = \frac{P_t}{P_{t-1}}$ as the inflation rate of period t. Notice that even without the presence of portfolio adjustment cost, there will exist a spread between the prices of deposits and long-term bonds. Households are willing to invest in deposits, even though they provide a lower return because they derive utility from them. The presence of portfolio adjustment costs in bonds generates a term spread and prevents the Household from fully exploiting the arbitrage opportunities in the assets' markets. The literature has shown this feature to be vital for the transmission mechanism of quantitative easing policies.

3.2 Firms

The productive sector is divided into two levels: final good producers and intermediate goods producers.

3.2.1 Final good producer

A representative firm produces the domestic final good y_t from varieties y_i, for i $\in [0, 1]$.

$$y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

ε is the elasticity of substitution between varieties. The optimization problem of the representative firm is the following:

$$\max_{y_t, \{y_{i,t}\}_{i\in[0,1]}} P_t y_t - \int_0^1 P_{i,t} y_{i,t} di$$
$$\text{s.t } y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

And the optimal demand function for variety i is given by the following expression:

$$y_{i,t} = y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \tag{2}$$

[7] The non-negativity condition in public bond purchases ensures the transversality condition is satisfied.

3.2.2 Intermediate goods firms

Intermediate goods firms are monopolistically competitive in the goods market. Each firm produces a variety i according to a linear production function:

$$y_{i,t} = Z_t n_{i,t} \tag{3}$$

Where Z_t is a mean reverting TFP shock, common to all varieties, with law of motion:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \sigma_z \varepsilon_t^z,\ \varepsilon_t^z \sim N(0,1)$$

When changing prices, firm i is subject to a quadratic adjustment cost in prices, as in Rotemberg (1982):

$$\frac{\phi^P}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - \pi^*\right)^2 y_t$$

where ϕ_P measures the degree of nominal price rigidity, and y_t is aggregate output, given by:

$$y_t = \int_0^1 y_{i,t} di$$

The nominal dividends of firm i at period t that are transferred by the Household are given by:

$$\Pi_{i,t}^f = P_{i,t} y_{i,t} - W_t n_{i,t} - \frac{\phi^P}{2}\left(\frac{P_{i,t}}{P_{i,t-1}} - \pi^*\right)^2 y_t P_t \tag{4}$$

Firms maximize the discounted sum of profits using the households' discount factor, and subject to technology 3, and demand 2.

Their optimization problem is the following:

$$\max_{P_{i,t},n_{i,t}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k} \Pi^f_{i,t+k}$$

$$\text{s.t. } y_{i,t} = z_t n_{i,t}$$

$$y_{i,t} = y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon}$$

$$\Pi^f_{i,t} = P_{i,t} y_{i,t} - W_t n_{i,t} - \frac{\phi^P}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - \pi^* \right)^2 P_t y_t$$

In equilibrium, all firms behave symmetrically, and we have $P_{i,t} = P_t$. Thus, aggregate profits are given by:

$$\Pi^f_t = P_t y_t - W_t n_t - \frac{\phi^P}{2} (\pi_t - \pi^*)^2 y_t P_t$$

Their optimization problem is characterized by the following conditions, where MC_t is the multiplier of equation 2.

$$1 - \varepsilon + \varepsilon \frac{MC_t}{P_t} = \phi_P (\pi_t - \pi^*) \pi_t - \phi_P \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \frac{y_{t+1}}{y_t} \pi^2_{t+1} (\pi_{t+1} - \pi^*) \right] \tag{5}$$

$$W_t = MC_t Z_t \tag{6}$$

The first condition is the New-Keynesian Phillips curve, where MC_t is the nominal marginal cost.

3.3 Financial Intermediaries

In this section, I follow a simplified version of the financial intermediaries' description in Elenev et al. (2021).

A representative agent in this sector starts the period t with a net worth W^I_t. Every period t, it pays dividends to households, given by a fraction τ^I of its net wealth minus the equity raised that period: A_t.

$$Div_t = \tau^I W^I_t - A_t \tag{7}$$

This agent can invest in public short-term bonds $B_t^{S,I}$ at price Q_t^S. $B_t^{S,I}$ is the sum of treasury bills (treasuries with a maturity of up to a year) and central bank reserves. The reason for adding these assets into one variable is that they have similar risk and return properties and can be substitutes from the financial intermediaries' point of view. Its liabilities are given by deposits from households D_t^I. The following expression then gives the balance sheet:

$$(1-\tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \tag{8}$$

Where $\Phi_A(.)$ is a convex cost of issuing new equity. These costs are rebated in a lump-sum fashion to the household.

The net wealth at period t is:

$$W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \tag{9}$$

In line with Basel regulation, financial intermediaries are subject to a leverage constraint that states that their debt (in this case, deposits) can be, at most, a fraction ν of its assets.

$$D_t^I \leq \nu B_t^{S,I} \tag{10}$$

The optimization problem of a financial intermediary consists on maximizing the discounted sum of dividends subject to restrictions 8, 9, and 10. I assume the financial intermediary discounts its future flows using the Household's stochastic discount factor.

$$\begin{aligned}
&\max_{A_t, D_t^I, B_t^{S,I}} \mathbb{E}_0 \sum_{k=0}^{\infty} \mathcal{M}_{t,t+k}\left(\tau^I W_t^I - A_t\right) \\
\text{s.t. } &(1-\tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \\
&W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \\
&D_t^I \leq \nu B_t^{S,I}
\end{aligned}$$

Define η_t as the Balance sheet multiplier and μ_t as the Leverage constraint multiplier. Using the first order condition with respect to equity A_t to substitute out the multiplier η_t, we obtain the system of equations that characterize the financial intermediary's optimization problem, together with 7, 8, 9, 10 [8]:

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t \left(1 - \Phi_A'(A_t)\right) \tag{11}$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \nu \mu_t \left(1 - \Phi_A'(A_t)\right) \tag{12}$$

Where $\tilde{\mathcal{M}}_{t,t+1}$ is the stochastic discount factor for financial intermediaries, defined as:

$$\tilde{\mathcal{M}}_{t,t+1} \equiv \mathcal{M}_{t,t+1} \left(1 - \Phi_A'(A_t)\right) \left(\tau^I + \frac{1 - \tau^I}{1 - \Phi_A'(A_{t+1})}\right)$$

Notice that the spread between deposits and short-term bonds is a function of the leverage constraint multiplier μ_t. Since $\nu < 1$, the return on short-term bonds is higher than the short-term bonds' return when the leverage constraint binds.

3.4 Monetary Authority

The Central Bank performs conventional and unconventional monetary policies. The conventional monetary policy sets the short-term nominal interest rate R_t, subject to a Zero Lower Bound restriction. This rate is defined as the inverse of the short-term public debt price:

$$R_t \equiv \frac{1}{Q_t^S} \tag{13}$$

The unconventional monetary policy consists of central bank balance sheet policies. In particular, the central bank purchases long-term government debt $B_t^{L,CB}$ in exchange for reserves ($B_t^{S,CB}$), as in the data since the great recession.[9] I define this policy as Quantitative Easing in the model.

The following expression gives the Central Bank's balance sheet:

$$\frac{B_{t-1}^{S,CB}}{P_t} + \frac{B_{t-1}^{L,CB}}{P_t} \left[\kappa + (1-\delta) Q_t^L\right] = Q_t^S \frac{B_t^{S,CB}}{P_t} + Q_t^L \frac{B_t^{L,CB}}{P_t} + \Lambda_t^{CB}$$

[8] See appendix, section 8.2 for further details.

[9] In particular, the data counterpart of $B_t^{S,CB}$ in the model is given by the net central bank position between treasury bills and reserves. When this variable is negative, the central bank issuance of reserves is more significant than its treasury bill purchases.

where Λ_t^{CB} are net dividends or profits in real terms. I assume that profits Λ_t^{CB} are transferred to the fiscal authority. [10]

Finally, the central bank is subject to a revenue neutrality constraint, which is in line with the data. It states that when the central bank increases its assets by purchasing long-term bonds, it has to outset the operation through a decrease in its net position of short-term assets.

$$Q_t^L B_t^{L,CB} + Q_t^S B_t^{S,CB} = 0 \tag{14}$$

When it performs QE, we would expect $B_t^{S,CB} < 0$, representing the increase in reserves issuance.

3.5 Fiscal Authority

The treasury consumes g_t, and obtains resources from three different sources. It collects tax revenues from households in a lump-sun fashion, τ_t, receives dividends from the central bank Λ_t^{CB}, and issues debt, whose total real value is $\frac{B_t}{P_t}$. This debt is composed by short-term bonds B_t^S, and long-term bonds B_t^L. The total debt issuance is: [11]

$$B_t = Q_t^S B_t^S + Q_t^L B_t^L \tag{15}$$

The period budget constraint of the fiscal authority is:

$$\underbrace{\tau_t - g_t}_{\equiv s_t} + \frac{B_t}{P_t} + \Lambda_t^{CB} = \frac{B_{t-1}^S}{P_t} + \frac{B_{t-1}^L}{P_t}\left[\kappa + (1-\delta)Q_t^L\right]$$

Where s_t is the primary fiscal surplus of period t. Replacing Λ_t^{CB} from the central bank balance sheet and using 14, I obtain a consolidated budget constraint:

$$s_t + \frac{B_t}{P_t} = \underbrace{\frac{B_{t-1}^S - B_{t-1}^{S,CB}}{P_t}}_{A} + \underbrace{\frac{B_{t-1}^L - B_{t-1}^{L,CB}}{P_t}}_{B}\left[\kappa + (1-\delta)Q_t^L\right] \tag{16}$$

[10] For simplicity, in this paper I abstract from asymmetries in the transfer of central bank's profits to the treasury.

[11] Notice that the real amount of short-term bonds is $Q_t^S \frac{B_t^S}{P_t}$ and the one of long-term bonds is $Q_t^L \frac{B_t^L}{P_t}$.

The terms 'A' and 'B' are the outstanding short and long-term public debt in the private's hands. Notice that through the purchases of long-term public bonds, the central bank reduces the pressure on fiscal accounts for the consolidated government. However, this is not necessarily the case when we consider the net position of short-term assets. For instance, if $B_t^{S,CB} < 0$, implying reserves issuance, then the consolidated government debt of short maturity is increasing with this policy. In this sense, quantitative easing policies can be interpreted as a maturity swap.

Government consumption is given by the following expression:

$$g_t = \theta(y^* - y_t) + (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \varepsilon_t^g,\ \varepsilon_t^g \sim N(0,1)$$

with $0 < \theta < 1$, representing the fiscal multiplier. Notice that this term generates a counter-cyclical behavior of government consumption, introducing fiscal stimulus when the economy is at a recession. Government consumption goods are thrown into the ocean. Finally, the maturity composition of newly issued government debt is constant in book value terms, with a fraction $\bar{\mu}$ of debt being long-term.

$$\frac{B_t^S}{B_t^L} = \frac{1-\bar{\mu}}{\bar{\mu}} \tag{17}$$

3.6 Policy rules

To close the model, I assume that fiscal and monetary authorities follow rules to set the policy instruments: τ_t, R_t and b_t^{CB}. First, since the objective of this paper is to study the effects of quantitative easing policies under different interactions of conventional fiscal and monetary policies, I assume that central bank purchases of long-term bonds follow an AR(1) process:

$$b_t^{L,CB} = (1-\rho^{QE})b_*^{L,CB} + \rho^{QE} b_{t-1}^{L,CB} + \sigma^{QE}\epsilon_t^{QE} \tag{18}$$

Where $b_{\underset{*}{L,CB}}$ is the average size of these asset position at the steady state. Increases of the central bank balance sheet are random and unrelated to the economic conditions. This assumption implies that QE constitutes an alternative policy instrument of the central bank, and not a substitute. Furthermore, it prevents the instrument from altering the determinacy properties of the model.

I follow the literature on fiscal-monetary policy interactions, and assume that fiscal and monetary authorities follow rules to determine the conventional policy instruments. In particular, the nominal short-term interest rate follows the Taylor rule 20, reacting to output and inflation deviations from its steady state values, together with an autorregresive parameter ρ_R and a monetary shock ϵ_t^m with standard deviation σ_t^m. The monetary shock stands for interest rate deviations from the reaction function. The intensity of interest rate reaction to output and inflation deviations are characterized by policy parameters α_y and α_π, respectively. Finally, $\bar{R}$ is the mean of the nominal interest rate.

For fiscal policy, I assume that taxes τ_t react to total real debt deviations from its steady state value b, and has an autorregresive coefficient ρ_τ, as in 19. The elasticity of tax deviations to debt deviations is characterized by parameter γ.

$$\tau_t - \tau^* = \rho_\tau(\xi_t)(\tau_{t-1} - \tau^*) + (1 - \rho_\tau(\xi_t))\,\gamma(\xi_t)(b_{t-1} - b^*) \tag{19}$$

$$\frac{R_t}{\bar{R}(\xi_t)} = \left(\frac{R_{t-1}}{\bar{R}(\xi_t)}\right)^{\alpha_R(\xi_t)} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\alpha_\pi(\xi_t)} \left(\frac{y_t}{y^*}\right)^{\alpha_y(\xi_t)}\right]^{1-\alpha_R(\xi_t)} e^{\sigma_M(\xi_t)\epsilon_t^M} \tag{20}$$

The aforementioned parameters depend on a discrete shock ξ_t, that follows a Markov process. I follow Bianchi and Melosi (2017) and assume that this shock can take three values, representing the three regimes through which the economy fluctuates.

The first regime, the *monetary led (M)* regime, is characterized by a strong interest rate reaction to inflation deviations (high α_π), satisfying the Taylor principle, and a strong tax reaction to debt (high enough γ)[12]. This regime is associated to an *active monetary policy* and *passive fiscal policy*, in Leeper (1991) terminology. Under this regime, the monetary authority accommodates the nominal interest rate more than proportionally to changes in

[12]In Appendix, I present, for a given calibration, the combinations of values α_π and γ that give rise to determinacy and present precise values for these parameters at each regime.

the inflation rate, to stabilize the inflation rate, while the treasury passively adjusts taxes to stabilize real debt.

The second regime, is the *fiscally led regime (F)* where the fiscal authority's main objective is to stabilize the economy, and not the real debt (low γ), and then the central bank allows the inflation rate to deviate from target in order to stabilize debt in real terms. In this case, the Taylor principle is not satisfied, and this regime can be characterized by *passive monetary policy* and *active fiscal policy* in the sense of Leeper (1991).

The third regime is the *Zero Lower Bound regime* (ZLB). It is characterized by an unreactive nominal short-term interest rate, that remains fixed at its effective lower bound, and by a fiscal policy that is unreactive to the level of real debt. This can be considered as an extreme form of the fiscally led regime, since we have $\alpha_\pi = \gamma = 0$. As in Bianchi and Melosi (2017), this regime represents a *crisis regime*, where the economy enters due to the realization of bad shocks that drive the economy to a recession. However, differently from Bianchi and Melosi (2017), the economy enters the zero lower bound regime endogenously, when the central bank cannot decrease further the short-term interest rate, due the existence of a lower bound.

3.7 Transition probabilities

Assume the Markov-switching shock ξ_t depends on the realization of two random variables ξ_t^P and ξ_t^C. When $\xi_t^C = 1$, the economy suffers a crisis and moves to the zero lower bound regime. When $\xi_t^C = 0$, the economy is in *normal times* and the government can set fiscal and monetary policies without restriction. In this case, the variable ξ_t^P determines the regime in place. $\xi_t^P = M$ stands for the monetary led regime, $\xi_t^P = F$ for the fiscally led regime, and it evolves according to the transition matrix:

$$PP = \begin{bmatrix} p_{mm} & 1 - p_{mm} \\ 1 - p_{ff} & p_{ff} \end{bmatrix}$$

where $p_{ij} = P\left(\xi_{t+1}^P = j | \xi_t^P = i\right)$

As can be seen from the transition matrix PP, during normal times, the probability of being in one regime or the other is constant and fully exogenous. The realization of this shock represents the outcome of a policy game between the fiscal and the monetary authority, where the winner is the ‘active’ authority.

Define q as the probability of entering to the ZLB regime, $q = P(\xi_{t+1}^C = 1|\xi_t^C = 0)$ and r as the probability of moving out of the crisis regime $q = P(\xi_{t+1}^C = 0|\xi_t^C = 1)$.

The transition matrix for ξ_t is then:

$$P = \begin{bmatrix} (1-q)PP & q[1;1] \\ r[p_{mm};(1-p_{mm})] & (1-r) \end{bmatrix}$$

q and r are endogenous processes. The probability of entering to the ZLB regime, is a decreasing function of the nominal interest rate:

$$q = P(\xi_{t+1}^C = 1|\xi_t^C = 0) = f(R_t)$$

Intuitively, it is a function that generates zero probability of switching when the gross nominal interest rate is higher than one, and it increases then R_t approaches the value 1.

The probability of leaving the crisis regime, r, is a function of a shadow interest rate R_t^S:

$$r = P(\xi_{t+1}^C = 0|\xi_t^C = 1) = g(R_t^S)$$

The shadow interest rate represents the interest rate that would hold in the economy, if this were always at monetary dominance regime, without a lower bound restriction:

$$\frac{R_t}{\bar{R}\left(\xi_t^P = M\right)} = \left(\frac{R_{t-1}}{\bar{R}\left(\xi_t^P = M\right)}\right)^{\alpha_R\left(\xi_t^P=M\right)} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\alpha_\pi\left(\xi_t^P=M\right)} \left(\frac{y_t}{y^*}\right)^{\alpha_y\left(\xi_t^P=M\right)}\right]^{1-\alpha_R\left(\xi_t^P=M\right)} e^{\sigma_M\left(\xi_t^P=M\right)\epsilon_t^M} \tag{21}$$

This assumption reflects the fact that the probability of leaving the zero lower bound regime is not independent of the economy conditions. For instance, when output and/or inflation recovers, the Shadow interest rate increases, increasing the likelihood that the central bank would start increasing interest rates.

The endogenous transition probabilities to and out of the zero lower bound matter for agents' expectations. Contrary to a model with fully exogenous Markov, in this model agents know the probability of tightening monetary policy increases with output and inflation even at the zero lower bound.

4 Functional forms, calibration and solution

4.1 Functional forms

In this section, I present the functional forms assumed in the numerical exercise. I assume the following CRRA utility function for households that depends positively on consumption and real deposits, and negatively on labor.

$$U(c_t, d_t^H, n_t) = \frac{\left[c_t^{1-\varphi}\left(d_t^H\right)^{\varphi}\right]^{1-\sigma}}{1-\sigma} - \psi\frac{n_t^{\eta}}{\eta}$$

Where $d_t^H = \frac{D_t^H}{P_t}$ are real deposits.

The portfolio adjustment cost for long-term bonds is quadratic:

$$\Phi_L\left(b_t^{L,H}\right) = \frac{\phi_L}{2}\left(\frac{b_t^{L,H}}{b^{L,H}}\right)^2$$

where $b_t^{L,H} = \frac{B_t^{L,H}}{P_t}$, and $b^{L,H}$ is the steady state value of public long-term bonds held by households.

The convex cost of issuing equity are the following:

$$\Phi_A(A_t) = \frac{\chi}{2}\frac{A_t^2}{P_t}$$

Finally, the endogenous transition probabilities to and out the zero lower bound regime are assumed to follow a logistic distribution as in Benigno et al. (2020) and Bocola (2016). As in their models, the economy's transition between regimes is a logistic function of a subset of the endogenous variables of the model. They are given by the following expressions:

$$q = P(\xi_{t+1}^C = 1|\xi_t^C = 0) = \frac{\exp\left\{-\gamma^q\left(R_t - 1\right)\right\}}{1 + \exp\left\{-\gamma^q\left(R_t - 1\right)\right\}}$$

where $\gamma^q > 0$ is a constant. And,

$$r = P(\xi_{t+1}^C = 0|\xi_t^C = 1) = \frac{\exp\left\{-\gamma^r\left(R_t^S - 1\right)\right\}}{1 + \exp\left\{-\gamma^r\left(R_t^S - 1\right)\right\}}$$

with $\gamma^r < 0$.

4.2 Calibration

I work with quarterly data for the US. In this section, I present the calibration. In table 1, I present the calibration of all the model parameters, with their source or target. Some parameters have direct counterparts in the data. For instance, $\bar{R}$, the average short-term nominal interest rate is set equal to 1.011, as in quarterly data for the period 1980-2021[13]. The average of long-term public bonds purchased by the central bank, $b_*^{L,CB}$ are set to 0.014 to match the average 7% annual ratio of Federal Reserve total treasuries to output ratio, in market value terms, for the period 1980-2021. Finally, the parameter that characterizes the collateral ratio in the financial intermediaries' leverage constraint, ν, is set to 0.97 as in the Basel regulation[14].

Some parameters are taken from the literature. For instance, the risk aversion parameter σ is set to 2, as is standard in the literature. The inverse of Frisch elasticity η, is set to 3 as in Leeper et al. (2021), $\bar{\mu}$, that is the proportion of long-term bonds in book values issued by the treasury, is 0.67 as in Elenev et al. (2021) and θ, that is 0.27 as in Bianchi and Melosi (2017). The convex cost of issue equity for financial intermediaries, χ is 22 as in Elenev et al. (2021). The coupon payment of long-term bonds, κ, that includes the interest and the matured part fraction of bonds, is normalized to 1.

[13]Notice that, in the data, this value corresponds to the average nominal interest rate, including periods where the interest rate was at the effective lower bound.

[14]In the data, the balance sheet of financial intermediaries includes a broader set of assets that can be used as collateral, than the ones included in the model (T-bills and central bank reserves). This assumption, although restrictive since it introduces a tight relationship between deposits and reserves, is maintained for simplicity.

Other parameters are calibrated to match first-order moments in the data. β takes the value 0.996 to match the average return of a real risk-free asset from Jordà et al. (2017). δ is set to 0.0357, to roughly match the average maturity of public bonds with maturity longer than 1 year, 7 years. The parameter that characterizes the quadratic portfolio adjustment cost of long-term bonds, ϕ_L is calibrated to 0.005, to match the average spread between long and short term bonds of 3.2% in the period 1980Q1-2021Q4 at the steady state[15].

The preference parameter φ is set to 0.0025 to generate a simulated mean of annualized debt to GDP ratio close to the one in the data before the COVID-19 crisis, around 90% in 2020Q1. The fraction of financial intermediaries' wealth paid to households as dividends is calibrated to 0.82 to match the average spread between short-term interest rate and deposits, of 0.31% from Drechsler et al. (2017)[16]. The elasticity of substitution between varieties ϵ is set to 7, generating an average markup of 17%, and the parameter ϕ^P that characterizes Rotemberg adjustment costs is set to 150, to roughly match the average inflation standard deviation during the period 1980-2021. Finally, the preference parameter ψ is calibrated to normalize labor to one at the steady state.

[15]This spread in the model is the difference between the return on the long-term bond $R^L = \frac{\kappa+(1-\delta)Q^L}{Q^L}$ and the short-term interest rate R.

[16]The model does not prevent the net return on deposits to be negative at the zero lower bound regime. This could be motivated in the data, with the fact that during this period, although banks did not charge fees on deposits, some of them increased their account maintenance's charges to customers.

	Description	Value	Source or target
β	Discount factor	0.996	Jordà et al. (2017)
$\bar{R}$	Average interest rate	1.011	Av. Data 1980-2021
$\bar{\mu}$	Proportion of long-debt	0.67	Elenev et al. (2021)
δ	Maturity parameter	0.0357	Maturity long bonds (7 years)
κ	Coupon Payment	1	Normalization
ϕ_L	Portfolio adjustment cost	0.005	10-year yield (1980-2021)
σ	Risk aversion	2	Standard
η	Inverse Frisch elasticity	3	Leeper et al. (2021)
ψ	Preference parameter	1.339	Normalization labor
φ	Preference parameter	0.0025	Debt/GDP (1980-2021)
τ^I	Dividends distribution	0.81	Spread T-bill to deposits
χ	Equity cost	22	Elenev et al (2021)
ν	Leverage constraint FI	0.97	Basel regulation
ϕ^P	Prices adjustment cost	150	Inflation volatility (1980-2021)
ϵ	Elasticity of subst. varieties	7	Markup 17%
$b_*^{L,CB}$	Average CB Balance sheet	0.0140	$\frac{Q^L b_*^{LC,B}}{4y} = 7\%$ 1980-2021
θ	Government spending	0.27	Bianchi and Melosi (2017)

Table 1: Calibration: model parameters

Table 2 presents the calibration for persistence and standard deviation of the exogenous processes in the model. They were jointly calibrated to match some second order moments in the data, for the period 1980-2021. The comparison between moments in the data with the simulated moments are presented in the following section.

Parameter	Description	Value
ρ_{QE}	Persistence QE	0.9
ρ_ν	Persistence preference	0.9
ρ_z	Persistence TFP	0.9
ρ_G	Persistence gov. spending	0.96
σ_{QE}	Dispersion QE	0.002
σ_ν	Dispersion preference	0.008
σ_z	Dispersion TFP	0.002
σ_G	Dispersion gov. spending	0.0026

Table 2: Calibration: exogenous processes

Table 3 presents the regime-switching parameters correspondent to the fiscal and monetary policy rules from section 3.6. The first set of parameters corresponds to the Taylor rule 20, and the correspondent parameters at each regime. The second set correspond to the parameter values for the Shadow interest rate 21. These parameters are equal to the Taylor rule's parameters at the monetary regime, and they are not regime-dependent. They were included in the table for completeness. The third set of parameters in this table correspond to the fiscal rule 19. The parameters in this table, with the exception of mean interest rates in all regimes, come from Bianchi and Melosi (2017). Since a parameter estimation goes beyond the scope of this paper, I take the parameters correspondent to conventional fiscal and monetary policy rules from this article that presents the same regimes and performs a Bayesian Estimation of the correspondent parameters using data for the US until the Great Recession[17]. The average interest rates out of the zero lower bound is $\bar{R}$, explained in table 1. At the zero lower bound, I set the average interest rate to 1.0005, as the average quarterly Effective federal funds rate observe in period: 2008Q4-2017Q1 and 2020Q1-2022Q1.

	Description	MD	FD	ZLB
α_R	Taylor rule	0.86	0.67	0.2
α_π	Taylor rule	1.6	0.64	0
α_y	Taylor rule	0.51	0.27	0
σ^M	Taylor rule	0.0025	0.0025	0.00025
R	Taylor rule	R^*	R^*	1.0005
$\alpha_{R,s}$	Shadow R	-	-	0.86
$\alpha_{\pi,s}$	Shadow R	-	-	1.6
α_y	Shadow R	-	-	0.9
$\sigma^{M,s}$	Shadow R	-	-	0.0025
R^S	Shadow R	R^*	R^*	R^*
γ	Fiscal rule	0.0712	0	0
α_τ	Fiscal rule	0.96	0.69	0.69

Table 3: Calibration: regime-dependent policy parameters .

Table 4 presents parameters relative to transition probabilities between regimes. Exogenous parameters from matrix P, p_{mm} and p_{ff} come from Bianchi and Melosi (2017). Parameters γ^r and γ^q give the steepness of the logistic function. They were calibrated to obtain similar ergodic probability of the ZLB regime as in Bianchi and Melosi (2017), and to minimize the cases at which the economy is at this regime with a gross interest rate below one.

[17]Since this estimation does not include the COVID-19 crisis and abstracts from the existence of unconventional policies and a financial intermediary sector, I perform parameter sensitivity analysis for these parameters in appendix (to be completed)

Parameter	Value	Source or target
γ^q	500	Average prob. of ZLB regime
γ^r	-200	Average prob. of ZLB regime
p_{mm}	0.9923	Bianchi and Melosi (2017)
p_{ff}	0.9923	Bianchi and Melosi (2017)

Table 4: Calibration: transition probabilities

Figure 4 shows the corresponding probabilities for values of the nominal interest rate (left) or shadow interest rate (right) between 0.95 and 1.02. A positive value for γ^q generates a higher probability of entering to the zero lower bound when the interest rate approaches is below one, and lower when it is above one. Notice that for this calibration, the probability of entering this regime when R is 0.987 or lower is one.

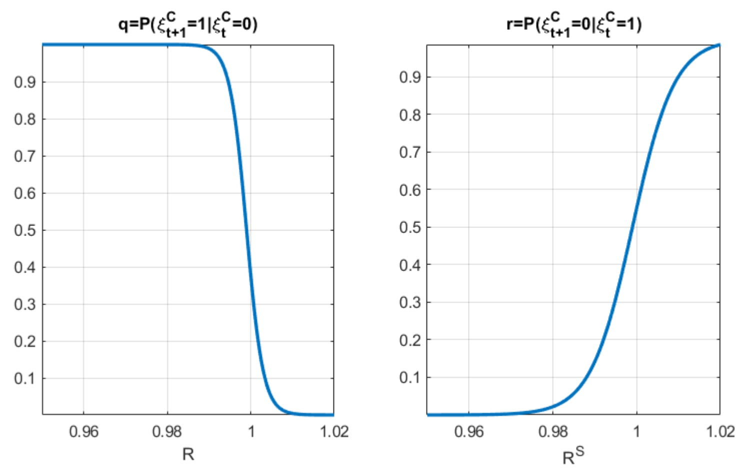

Figure 4: Endogenous transition probabilities to and out the zero lower bound.

For the probability of exiting the zero lower bound, the parameter γ^r is negative, generating a monotone increasing probability of leaving the crisis regimen when the Shadow interest rate is higher.

4.3 Solution method

I solve the model in real terms through second order perturbation methods for endogenous Markov Switching DSGE models, following Benigno et al. (2020). In the solution, I assume the leverage constraint for financial intermediaries is binding at all regimes. The approximation point for the solution method consists in a weighted average of steady states in different regimes, as authors in Benigno et al. (2020) explain. The weights are the ergodic means of corresponding regimes. In appendix 8.4 I present the steady state equations and describe the approximation point.

5 Quantitative Results

5.1 Second order moments

In this section I present some second order moments for selected variables, to evaluate how the model performs in terms of generated volatility and cyclical properties. Data variables were demeaned to make them comparable with their model counterpart, where there is no growth. Empirical moments were calculated using quarterly data for the period 1980Q2-2021Q3. NIPA variables are real and per capital. Debt, inflation and interest rate are annualized. Data moments were calculated from a simulation of one million periods, and they correspond to averages of the three regimes. In this sample, the economy is in the monetary dominance regime 52% of the time, 37% in fiscal dominance and 11% in the zero lower bound.

The following table compares the standard deviation (in %) and correlation with output growth. Output, consumption and debt are in logarithmic differences. Inflation and term spread are annualized gross rates.

	d Ln y_t	**d Ln** c_t	**d Ln** $b_t^S + b_t^L$	**Inflation**	**Term spread**
Standard deviation (in %)					
Data	1.3	1.4	1.7	2.8	1.5
Model	0.6	0.8	1.6	2.5	1.4
Correlation with d Ln y_t					
Data	1.00	0.90	-0.33	0.44	-0.11
Model	1.00	0.81	-0.29	0.23	-0.05

Table 5: Second order moments in data and model

Note: Growth rates for output, consumption, debt in the data are quarterly logarithmic differences, and demeaned. They are real and per capita. Inflation is the quarterly logarithmic difference, annualized. Term spread is the difference between annual 10-year treasury yield and annual federal funds rate. Model moments obtained from a simulation with 100.000 periods.

As can be seen from the table, the model generates the correct ranking in volatilities, and generates standard deviation for debt, inflation and spread, similar to the ones in data. However, it generates around half of volatility in output and consumption. In terms of cyclical properties, the model generates the correct sign in correlation with output for all the variables in the table, and close magnitudes to the empirical ones.

5.1.1 Unconditional second order moments at different regimes

Table 6 show means and volatility measures for debt to GDP ratio, inflation, nominal short-term interest rate (R_t) and nominal return on long-term bonds (R_t), conditional on regimes. They reflect features typically highlighted by the literature that studies fiscal-monetary policy mix. Fiscal dominance regime is characterized by higher debt to GDP, interest rates, and more volatile inflation and interest rate. Debt to GDP, on the contrary, is more volatile in dominance regime. The ZLB is characterized by a very stable short-term interest rate close to one, and almost nil inflation, but quite volatile (standard deviation 2.6%).

	MD		**FD**		**ZLB**	
	Mean	**Std(%)**	**Mean**	**Std(%)**	**Mean**	**Std(%)**
Debt to GDP	0.95	8.5	1.04	3.6	0.98	5.6
Inflation	1.02	1.6	1.02	3.1	1.00	2.6
Interest rate (R)	1.03	1.5	1.04	2.3	1.00	0.2
Long-run return (R^L)	1.04	1.5	1.05	2.7	1.02	1.6

Table 6: Data moments conditional on regimes

Note: Data generated moments, from a sample of 100.000 periods. The model is simulated for a long sample where the regime at place is stochastic. Moments at each regime are obtained conditioning the economy being on the corresponding regime at a given period. Debt to GDP is $\frac{b}{4y}$, inflation and returns are annualized since the model is solved quarterly.

5.2 The transmission mechanism of Quantitative Easing

This section sheds light in the model's transmission mechanism of an increase in central bank bond purchases, $b_t^{L,CB}$, given by an exogenous shock ϵ_t^{QE}, following 18. I show the simulated path for endogenous variables when there is a one standard deviation shock in the central bank purchases of long-term bonds minus the counterfactual path for those variables, in an scenario without the shock. This shock implies increasing the real balance sheet to GDP ratio in 1.3p.p., i.e., increasing from its steady state of 7% to 8.3%, a conservative shock. I consider three different scenarios, where the economy is at a given regime, and it remains at the same regime for 16 quarters in both path (with and without the shock). Even though in the simulation exercise there is no regime change, every period agents in the economy expect the economy to evolve according to transition matrix 3.7.

5.2.1 Regime dependent Quantitative Easing shock

Figures 19 and 20 show evolution of endogenous variables at monetary dominance regime (continue blue line), fiscal dominance regime (dotted red line) and zero lower bound regime (dashed yellow line). They are presented as absolute deviations from a path without increase in central bank purchases, in percentage.

When the central bank increases its purchases of long-term bonds (b_t^{CB}), it pays through reserves issuance to financial intermediaries, due to the revenue neutrality constraint. This relaxes the leverage constraint 10, allowing them to increase deposits to households.

The pressure the central bank intervention exerts in the long-term bonds market drives its price up, and then the expected long-run return on these assets ($\mathbf{E}_t R^L_{t,t+1}$) falls. This increase in the price gives incentives households to rebalance their portfolio, selling their long-term bonds position and increasing deposits. The increase in deposits give the household an extra liquidity service, generating incentives to consume more, since they are complements in the utility function. On impact, without an increase in output supply, this increase in consumption demand is inflationary.

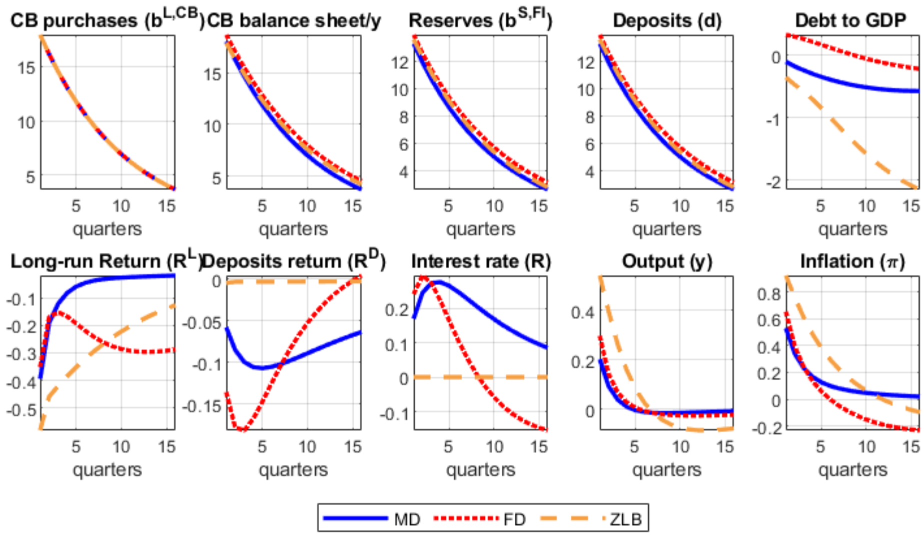

Figure 5: Impact of a Quantitative Easing shock conditional on a regime

Note: log deviations (in %) in a simulated path when there is a one standard deviation shock in the central bank purchases of long-term bonds ($\epsilon_t^{QE} = 1$) to the the counterfactual path for those variables, in an scenario without shock ($\epsilon_t^{QE} = 0$). Path conditional on monetary dominance regime (continue blue line), fiscal dominance regime (dotted red line) and zero lower bound regime (dashed yellow line) for 16 periods.

At the zero lower bound scenario, nominal short-term interest rate (R_t) is fixed, the nominal returns on long bonds and deposits (R_t^d, in appendix) fall. Since inflation increases, real returns fall, exacerbating the expansionary effects in the economy.

For constant total factor productivity z, firms react to the increase in demand by hiring more labor to increase production. Real wages increase, increasing marginal costs, and simulating households to increase labor supply.

At the end, the QE shock is expansionary and inflationary at all regimes. However, at fiscal or monetary dominance regime, the central bank raises the nominal interest rate following the corresponding Taylor rule to counteract the initial inflationary effects of the shock. In those scenarios, the fall in real rates is lower, ameliorating the expansionary effects on consumption, labor and inflation. Furthermore, since the fall in $\mathbf{E}_t R_{t,t+1}^L$ is lower, the portfolio rebalancing effect is lower. This implies that the household intertemporal decision between savings and consumption is moderate. Notice that in both cases, the household absorbes the total increment in deposits due to the raise in reserves in the market. Thus, it adjusts its saving through the position of long-term bonds.

Regarding the treasury behaviour, the expansionary effect of central bank purchases triggers a fall in government spending due to automatic stabilizers. Hence, primary fiscal surplus increases on impact, in all regimes. This effect is more significant in the zero lower bound, where output increase more. Interest payments decrease in long-term bonds decrease in all scenarios, due to the fall in $\mathbf{E}_t R^L_{t,t+1}$. Out of the zero lower bound, this effect is ameliorated by the increase in R_t. The third source of resources for the treasury is given by the central bank profits (Λ^{CB}_t), that increase on impact in all scenarios. The final result is an increase in fiscal space, interpreted as a fall in debt (b^S_t and b^L_t) and real debt (b_t) at the ZLB an monetary dominance regime. At the fiscal dominance regime however, real debt increases on impact. Under monetary dominance regime, taxes fall following the fiscal rule.

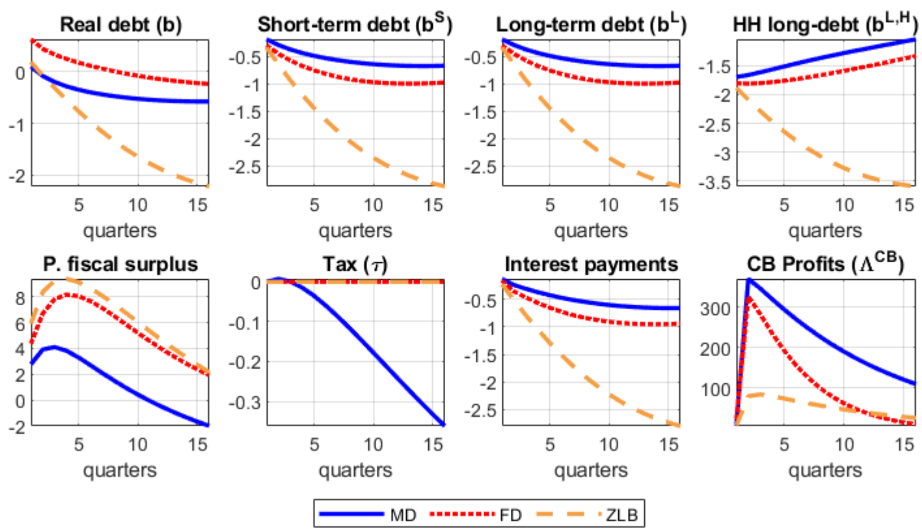

Figure 6: Impact of a Quantitative Easing shock conditional on a regime: fiscal variables

Note: log deviations (in %) in a simulated path when there is a one standard deviation shock in the central bank purchases of long-term bonds ($\epsilon^{QE}_t = 1$) to the the counterfactual path for those variables, in an scenario without shock ($\epsilon^{QE}_t = 0$). Path conditional on monetary dominance regime (continue blue line), fiscal dominance regime (dotted red line) and zero lower bound regime (dashed yellow line) for 16 periods.

This exercise shows that the expansionary effects of quantitative easing policies are of considerate size under a crisis regime, where output increases almost 0.6 p.p. and inflation

0.1p.p. with respect to the counterfactual scenario without intervention. However, its effects on output and inflation are milder on fiscal and monetary dominance regime, where output increases 0.3 and 0.2 p.p. and inflation 0.6 and 0.5p.p., being considerably less persistent in the first case. This simple exercise gives us some insights regarding the power of QE to expand the economy, and allow us to hypothesize how effective could be to contract the economy through the reversal operation.

6 Exit strategies from Quantitative easing programs

As has been stated by policy makers, financial analyst and academics, there is high uncertainty regarding the impacts of the implementation of Quantitative Easing programs and even more, the unwinding of the economic stimulus introduced through these measures. The main objective of this paper is to contribute to the understanding of the extent to which different strategies regarding the size of the central bank balance sheet generate a contractionary effect on the economy, and the mechanisms behind them. To do that, in this section I simulate the economy to generate a crisis that resembles the one that took place in US from the first quarter of 2020, when the COVID-19 pandemic started. First, I show the model can generate a crisis with similar characteristics than the data: a strong fall in output and short-term interest rate, until it reaches the zero lower bound, together with a rise in debt to GDP ratio and a fall in the inflation rate. In this context, I simulate a Quantitative Easing program that increases the ratio of central bank balance sheet over output in 10p.p., and compare the macroeconomic dynamics with and without this program. Then, I study three different strategies for managing the central bank balance sheet size: *tapering, quantitative tightening (QT)* and a *sell-off* of bonds.

Tapering is defined as a situation where the stock of long-term bonds at the central bank ($b_t^{L,CB}$) remains constant, implying $b_t^{L,CB} = b_{t-1}^{L,CB}$. The size of the central bank balance sheet over output ratio is endogenous, since it is affected by the price of long-term bonds (Q_t^L) and output y_t.

Quantitative tightening is defined as a strategy where the central bank does not repurchase the bonds that mature every period. The stock of bonds at the central bank will then decrease at rate δ, that is the average maturity of long-term bonds, the unique kind of bonds the monetary authority purchases. This implies the stock of bonds follows: $b_t^{L,CB} = (1-\delta)\, b_{t-1}^{L,CB}$.

Finally, the strategy called *sell-off* of bonds is any path of shocks that generates $b_t^{L,CB} < (1-\delta)\, b_{t-1}^{L,CB}$, implying that the stock of bonds at the central bank decreases a faster rate that the maturity rate.

6.1 The crisis development and the QE program

I simulate the model in 50.000 samples of 40 periods, under two scenarios, "Baseline" and "Quantitative Easing", with the following characteristics. In the baseline, the economy is at the approximation point at t=1, and in the monetary dominance regime. At periods 2 and 3, the economy is hit by strong negative preference and TFP shocks. Since both shocks are persistent, they remain under their steady state value for the whole sample. From t=2 onward, the regime at place is stochastic, following transition matrix 3.7. In this scenario, QE, monetary policy and fiscal policy shocks are random. After the initial hit during periods 2 and 3, preference and TFP shocks follow random paths.

The quantitative easing scenario shares the same characteristics as Baseline, except that the path for QE shocks is not random, and it is imposed to generate an increase in the annualized central bank balance sheet to output of around 10p.p. in the first 6 periods. After period 6, the balance sheet remains constant (tapering strategy).

Figure 7 shows the mean of simulated variables, for the first 20 periods for the economy without QE (continue blue line) and with QE (dotted orange line). The negative preference and technology shock generate a strong fall in output growth, that reaches its bottom of almost -11p.p. in period three but it is not fully recovered for nine periods. Interest rate starts falling immediately and reaches the lower bound at period 3.

The main striking features of the crisis with quantitative easing are the following. First, there is a milder fall in output, that falls 1p.p. less than without the program. Second, there is a faster and stronger recovery, even though output growth is negative for the same number of periods. In both cases, there is a period with around 4% of deflation at period 2, but under the QE scenario, inflation recovers much faster. It reaches a peak of 8.5% at period six, the same period at which the balance sheet reaches its maximum value of 20.6% of GDP. Without this program, inflation increases less at the recovery, and reaches a peak of 5.2% three periods later.

In both cases, real debt increases quickly when the crisis stars, reaching values of around 120% of GDP. However, it decreases fast following the output and prices behaviour. The fall in real debt is more significant under QE program, for three reasons: first, central bank purchases increase central bank profits that are transferred to the treasury; second, the central bank intervention decreases the long-run return on bonds, and thus interest payments on debt are lower; and third, the faster recovery triggers a fall in government spending drove by automatic stabilizers, so that the primary fiscal deficit is lower. These three factors allow the government to issue less debt during the crisis, than in a counterfactual without the QE program.

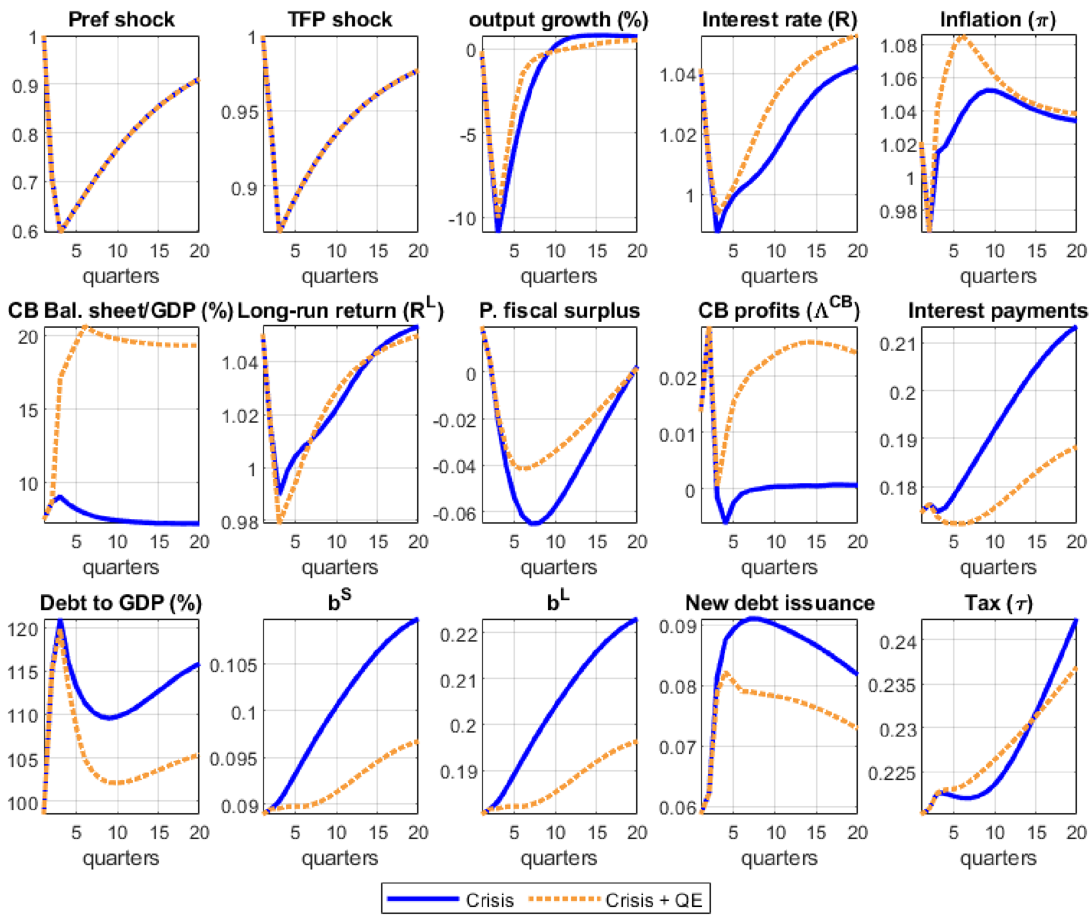

Figure 7: Simulated crisis

Note: Average from 50.000 samples. Blue continuous line is the scenario without QE program. Orange dotted line is the scenario with a QE program that increases the annualized central bank balance sheet to output of around 10p.p. Output growth, balance sheet and Debt to GDP are in percentages. Interest rate (R), long-run return (R^L), inflation, debt are annualized.

Finally, figure 8 shows, per period, the percentage of samples in which the economy was at the zero lower bound regime. Notice it is zero at periods one (initial condition) and two. The reason for this it that the endogenous probability of moving to the zero lower bound regime at $t+1$ depends on the interest rate at t. From period four onward the economy is at the ZLB with high probability. The frequency of this regime is considerably lower under the scenario with QE program, and it is almost nil from period eleven.

To summarize, the unconventional monetary policy intervention generates a faster recovery, a shorter duration of the crisis regime, lower stock of debt, and larger inflation dynamics.

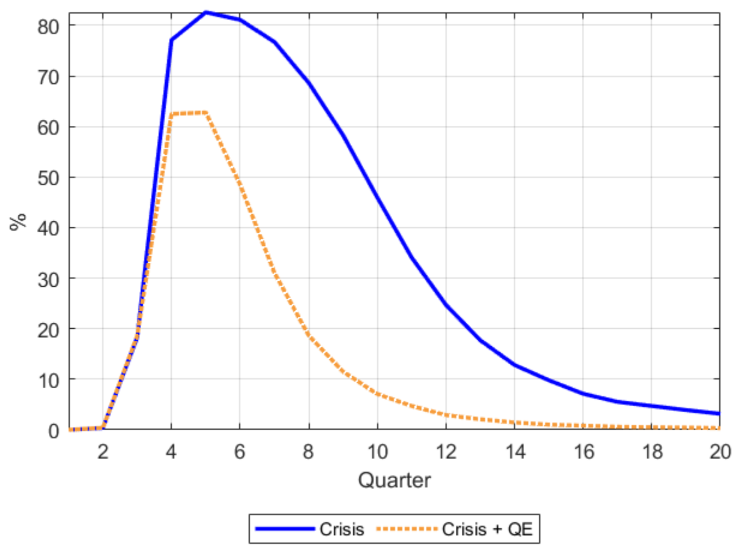

Figure 8: Percentage of simulated samples at the ZLB regime, per period.

6.2 Central bank balance sheet exit strategies

In this section I compare the path for endogenous variables under three scenarios. The orange dotted line is the tapering scenario, same to the one in figure 7. This baseline simulation is compared with two alternative strategies for the size of the central bank balance sheet. The green continuous line is the scenario with Quantitative Tightening (QT) from period 12 onward. Light-blue dashed line is the scenario with sell-off of bonds from period 12 onward[18]. The figure shows mean values for each variable across samples. The unwinding of economic stimulus is assumed to start two years after the crisis. However, notice that the central bank started the monetary contraction with conventional interest rate instruments some periods earlier (as showed in figure 4). This last explains why inflation is already close to 4% when the unwinding of the central bank balance sheet starts.

[18]For this scenario I assumed a QE show equal to -0.2 for every t from t=12.

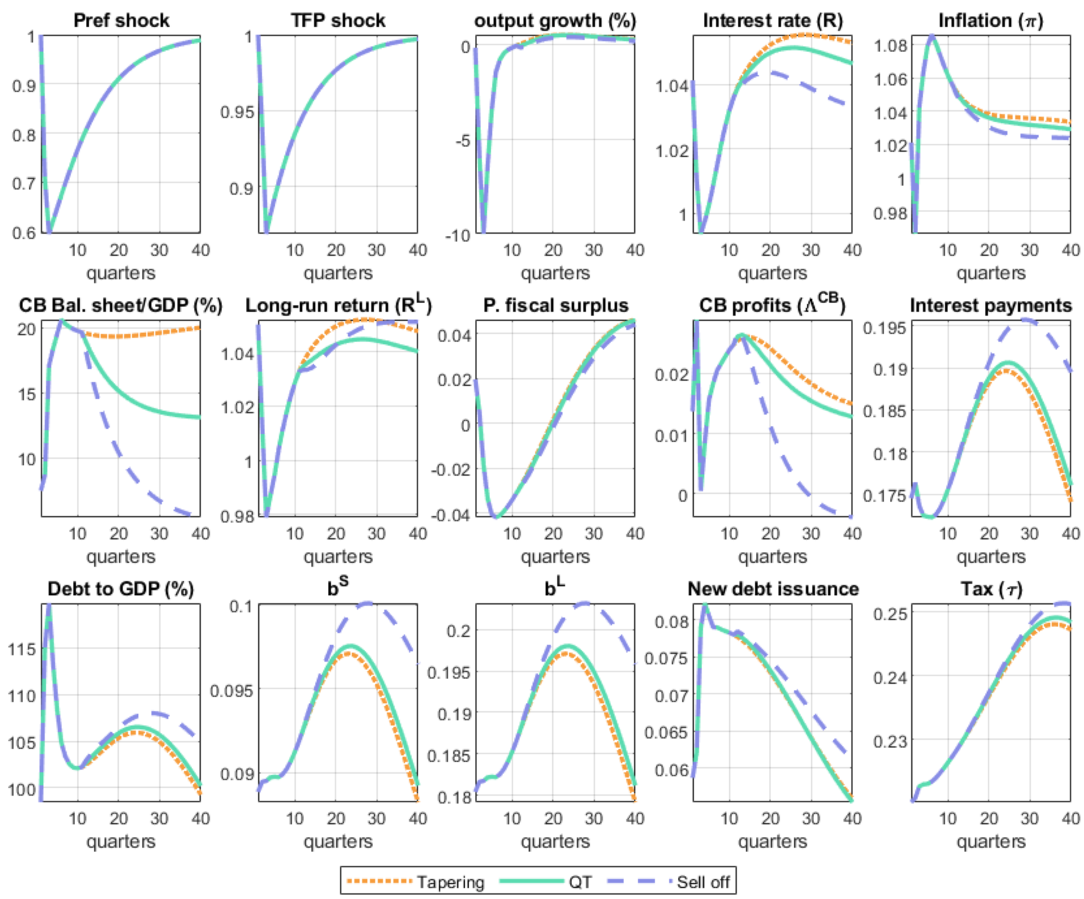

Figure 9: Crisis and exit strategies from QE.

Note: Average from 50.000 samples. Orange dotted line is the tapering scenario, same to the one in figure 7. Green continuous line is the scenario with Quantitative Tightening (QT) from period 12 onward. Light-blue dashed line is the scenario with sell-off of bonds from period 12 onward. Output growth, balance sheet and Debt to GDP are in percentages. Interest rate (R), long-run return (R^L), inflation, debt are annualized.

First, notice that given the average maturity of the debt, δ, the balance sheet over GDP with QT falls around 6 p.p. in 7 years. This strategy doesn't seem to generate significant differences in output and inflation with respect to the tapering alternative. As a result, the difference in short-term interest rate path is lower to half p.p. The behaviour of real debt/y, primary fiscal surplus, and the rest of fiscal variables do not defer much either. The effectiveness of this strategy depends on the average maturity of the central bank balance sheet that imposes a speed at which it decreases.

A more aggressive strategy regarding the CB balance sheet, with a fast sell-off of bonds, allows the CB to increase the short-term interest rate is 2p.p. less (5.3 vs 3.3%). It stabilizes inflation much faster, since it falls to 4% in 4 periods, and then stabilizes in 2.3%, close to the average inflation in the economy, while it is 3.4% in the tapering scenario, even with interest rate 2p.p. higher.

The sell-off of bonds generate a bust in long-term bonds' prices generating a sudden increase in the interest payment the government has to face, a fall in CB profits that are transferred to the treasury, and lower primary fiscal surplus. For this reason, the government issues more debt, and the final deleveraging of the economy is less substantial under this scenario. The ratio of real debt to GDP is 105.3% in the sell-off scenario, while it is 99% at the tapering one.

In conclusion, an aggressive sell-off of bonds is more effective in taking the inflation rate down to the target, requiring lower increases in the short-term interest rate. However, it has the downside of a lower deleveraging of the economy.

These conclusions are robust to alternative scenarios like imposing a longer duration of the ZLB regime, or starting the shrinking in central bank balance sheet at the same time the economy leaves the ZLB regime[19].

7 Concluding Remarks

I study different exit strategies for reducing the Central Bank Balance Sheet in a model that generates fiscal-monetary policy trade-offs.

The impact of Central Bank Balance Sheets policies (either QE or QT and Sell-Off of bonds) depends on the fiscal-monetary policy mix. First, QE provides additional fiscal space in the short run since it reduces new debt issuance. Second, as a counterpart of the central bank's balance sheet expansions, liquidity increases substantially, both in the hand of commercial banks (reserves) or non-bank private agents (deposits or currency). The extent

[19] In the appendix, I show this alternative scenarios. In the first case, I work with the same simulation but plot only selected paths where the economy remains at the ZLB for one year (i.e. from quarter 3 to 7). In the second case, I plot the paths where the economy starts Quantitative Tightening or Sell-off of bonds at period 7, imposing the exit from the ZLB regime at the same time.

to which the increase in liquidity affect output and inflation rate depends on the fiscal-monetary policy mix. QE is more expansive at the Zero Lower Bound and under the fiscally led regime when the inflationary impact is also more significant.

How the central bank balance sheet's size is reduced matters for inflation, output, and debt dynamics. When the monetary authority starts shrinking its holdings of long-term government bonds, either without repurchasing the ones that mature (QT) or actively selling its position in the secondary market (Sell-off of bonds), it decreases inflation at the cost of an increase in the ratio of debt/GDP. These effects increase when the exit strategy is more aggressive. A sell-off of bonds that reduces the balance sheet from 20% of GDP to 5% in 7 years stabilizes inflation around the target of 2% while preventing the central bank from increasing nominal short-term interest rates in 2p.p. (5.3% to 3.3%). However, the economy ends up with 6p.p. more debt/GDP than a less aggressive policy. The disinflationary benefits of a sell-off of bonds are less apparent when the economy exits the crisis towards a fiscally led regime, where the government does not raise taxes to counteract the increase in debt.

8 Appendix

8.1 Extra plots for US case

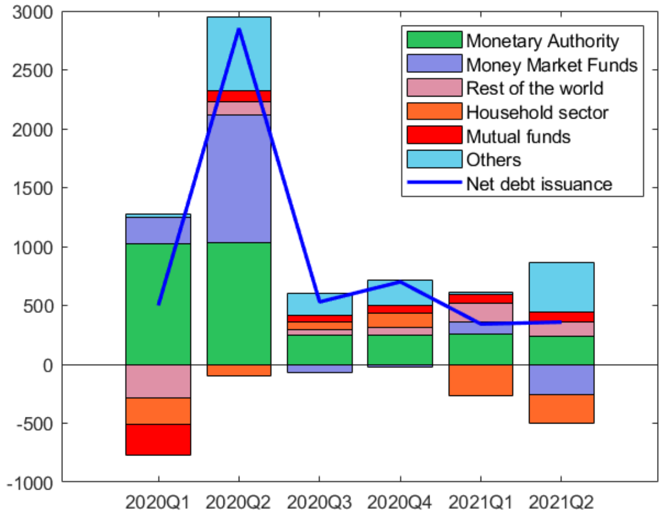

Figure 10: Change in Treasuries' holdings

Source: US Financial Accounts. Data in billions of dollars. Contain revaluation effects.

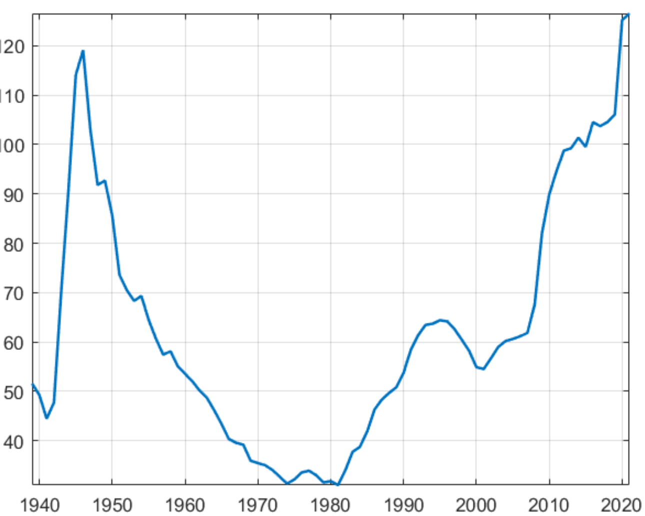

Figure 11: Public debt to GDP in US: historical perspective

Source: FRED. Annual Gross Federal Debt as a Percent of GDP.

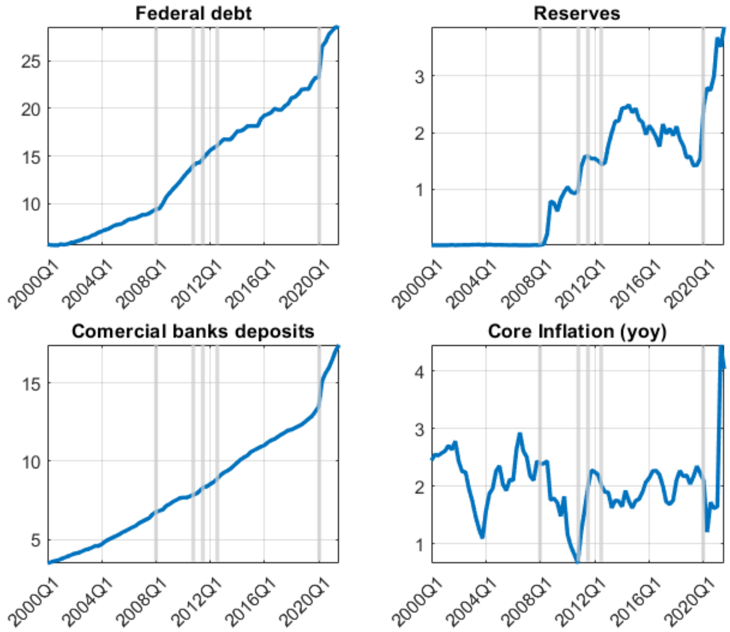

Figure 12: Macroeconomic variables: public debt, liquidity, inflation. Great recession and COVID-19 crisis.

US Federal Debt, Reserves and Deposits, in trillions of dollars. Core inflation in %. Source debt, deposits and inflation data: FRED. Source reserves: US financial accounts, release December 2021.

8.2 Model: Financial Intermediary optimization problem

The optimization problem of a representative Financial Intermediary can be written in sequential form as follows:

$$
\begin{aligned}
V^I\left(W_t^I\right) = \max_{A_t, D_t^I, F_t^I, B_t^{S,I}} \tau^I W_t^I - A_t + \mathbb{E}_t\left[\mathcal{M}_{t,t+1} V^I\left(W_{t+1}^I\right)\right] \\
\text{s.t. } (1-\tau^I)W_t^I + A_t - \Phi_A(A_t) + Q_t^D D_t^I = Q_t^S B_t^{S,I} \\
W_t^I = B_{t-1}^{S,I} - D_{t-1}^I \\
D_t^I \leq \nu B_t^{S,I}
\end{aligned}
$$

Define η_t as the Balance sheet multiplier and μ_t as the Leverage constraint multiplier. The first order conditions are given by the following system:

$$A_t : -1 + \eta_t - \eta_t \chi \frac{A_t}{P_t} = 0$$
$$D_t^I \colon \eta_t Q_t^D - \mu_t - \mathbb{E}_t \mathcal{M}_{t,t+1} V_w^I \left(W_{t+1}^I\right) = 0$$
$$B_t^{S,I} : -\eta_t Q_t^D + \mu_t \nu + \mathbb{E}_t \mathcal{M}_{t,t+1} V_w^I \left(W_{t+1}^I\right) = 0$$

And the envelope condition:

$$V_w^I \left(W_{t+1}^I\right) = \tau^I + \eta_t (1 - \tau^I)$$

Using the first order condition with respect to A_t to substitute out the multiplier η_t, we obtain the system of equations that characterize the financial intermediary's optimization problem, together with 7, 8, 9, **??**:

$$Q_t^D = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \mu_t \left(1 - \chi a_t\right) \tag{22}$$

$$Q_t^S = \mathbb{E}_t \tilde{\mathcal{M}}_{t,t+1} + \nu \mu_t \left(1 - \chi a_t\right) \tag{23}$$

Where $\tilde{\mathcal{M}}_{t,t+1}$ is the stochastic discount factor for financial intermediaries, defined as:

$$\tilde{\mathcal{M}}_{t,t+1} \equiv \mathcal{M}_{t,t+1} \left(1 - \chi \frac{A_t}{P_t}\right) \left(\tau^I + \frac{1 - \tau^I}{1 - \chi \frac{A_{t+1}}{P_{t+1}}}\right)$$

8.3 Equilibrium conditions

In this section I present the system of equilibrium conditions with functional forms and in real terms.

Definitions: $d_t = \frac{D_t}{P_t}$, $b_t^{j,i} = \frac{b_t^{j,i}}{P_t}$, $w_t = \frac{W_t}{P_t}$, $div_t = \frac{Div_t}{P_t}$, $\pi_t^f = \frac{\Pi_t^f}{P_t}$, $a_t = \frac{A_t}{P_t}$, $mc_t = \frac{MC_t}{P_t}$, $w_t^I = \frac{W_t^I}{P_t}$, $\pi_t = \frac{P_t}{P_{t-1}}$, for $j \in \{L, S\}$, $i \in \{H, CB, FI\}$.

8.3.1 Households

$$c_t + Q_t^D d_t^H + b_t^{L,H} Q_t^L = w_t n_t - \tau_t + \frac{d_{t-1}^H}{\pi_t} + \frac{b_{t-1}^{L,H}}{\pi_t}\left[\kappa + (1-\delta)Q_t^L\right] + div_t + \pi_t^f + \frac{\chi}{2}a_t^2 \tag{1}$$

$$\frac{\gamma n_t^\eta}{\left[c_t^{1-\varphi}\left(d_t^H\right)^\varphi\right]^{-\sigma}(1-\varphi)c_t^{-\varphi}(d_t^H)^\varphi} = w_t \tag{2}$$

$$Q_t^D = \mathbb{E}_t \mathcal{M}_{t,t+1} + \frac{\varphi c_t}{(1-\varphi)d_t^H} \tag{3}$$

$$Q_t^L + \phi^L \frac{b_t^{L,H}}{(b^{L,H})^2} = \mathbb{E}_t \mathcal{M}_{t,t+1}\left[\kappa + (1-\delta)Q_{t+1}^L\right] \tag{4}$$

$$\mathcal{M}_{t,t+1} = \frac{\beta\lambda_{t+1}}{\lambda_t} = \beta\mathbb{E}_t \frac{\left[c_{t+1}^{1-\varphi}\left(d_{t+1}^H\right)^\varphi\right]^{-\sigma} c_{t+1}^{-\varphi}(d_{t+1}^H)^\varphi}{\left[c_t^{1-\varphi}\left(d_t^H\right)^\varphi\right]^{-\sigma} c_t^{-\varphi}(d_t^H)^\varphi}\frac{1}{\pi_{t+1}} \tag{5}$$

8.3.2 Firms

$$y_t = z_t n_t \tag{6}$$

$$\pi_t^f = y_t - w_t n_t - \frac{\phi^P}{2}\left(\pi_t - 1\right)^2 y_t \tag{7}$$

$$1 - \varepsilon + \varepsilon mc_t = \phi_P\left(\pi_t - \pi^*\right)\pi_t - \phi_P \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\frac{y_{t+1}}{y_t}\pi_{t+1}^2\left(\pi_{t+1} - \pi^*\right)\right] \tag{8}$$

$$w_t = mc_t z_t \tag{9}$$

8.3.3 Financial Intermediaries

$$div_t = \tau^I w_t^I - a_t \tag{10}$$

$$(1-\tau^I)w_t^I + a_t - \frac{\chi}{2}a_t^2 + Q_t^D d_t^I = Q_t^S b_t^{S,I} \tag{11}$$

$$w_t^I = \frac{b_{t-1}^{S,I}}{\pi_t} - \frac{d_{t-1}^I}{\pi_t} \tag{12}$$

$$d_t^I \le \nu b_t^{S,I} \tag{13}$$

$$Q_t^D = \mathbb{E}_t\tilde{\mathcal{M}}_{t,t+1} + \mu_t\left(1 - \chi a_t\right) \tag{14}$$

$$Q_t^S = \mathbb{E}_t\tilde{\mathcal{M}}_{t,t+1} + \nu\mu_t\left(1 - \chi a_t\right) \tag{15}$$

$$\tilde{\mathcal{M}}_{t,t+1} = \mathcal{M}_{t,t+1}\left(1-\chi a_t\right)\left(\tau^I + \frac{1-\tau^I}{1-\chi a_{t+1}}\right) \tag{16}$$

8.3.4 Government

$$\frac{1}{R_t} = Q_t^S \tag{17}$$

$$Q_t^S b_t^{S,CB} + Q_t^L b_t^{L,CB} = 0 \tag{18}$$

$$b_t = Q_t^S b_t^S + Q_t^L b_t^L \tag{19}$$

$$b_t^S = (1-\bar{\mu})/\bar{\mu} b_t^L \tag{20}$$

$$g_t = \theta(y^* - y_t) + (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \varepsilon_t^g,\ \varepsilon_t^g \sim N(0,1) \tag{21}$$

8.3.5 Market clearing conditions

$$c_t + g_t + \frac{\phi_P}{2}\left(\pi_t - \pi^*\right)^2 y_t = y_t \tag{22}$$

$$d_t^H = d_t^I \tag{23}$$

$$b_t^S = b_t^{S,I} + b_t^{S,CB} \tag{24}$$

$$b_t^L = b_t^{L,H} + b_t^{L,CB} \tag{25}$$

8.3.6 Policy rules

$$\tau_t - \tau^* = \rho_\tau\left(\xi_t\right)\left(\tau_{t-1} - \tau^*\right) + \left(1-\rho_\tau\left(\xi_t\right)\right)\gamma\left(\xi_t\right)\left(b_{t-1} - b^*\right) \tag{16}$$

$$\frac{R_t}{R\left(\xi_t\right)} = \left(\frac{R_{t-1}}{R\left(\xi_t\right)}\right)^{\alpha_R(\xi_t)}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\alpha_\pi(\xi_t)}\left(\frac{y_t}{y^*}\right)^{\alpha_y(\xi_t)}\right]^{1-\alpha_R(\xi_t)} e^{\sigma_M(\xi_t)\epsilon_t^M} \tag{27}$$

$$b_t^{L,CB} = (1-\rho^{QE})b_*^{L,CB} + \rho^{QE} b_{t-1}^{L,CB} + \sigma^{QE}\epsilon_t^{QE} \tag{28}$$

Law of motion for exogenous processes: z_t, ν_t.

Transition matrix for Markov-switching shock ξ_t.

8.3.7 Definition of auxiliary variables used in figures and tables

Return on long-term bond:

$$R^L_{t,t+1} = \mathbb{E}_t \frac{\kappa + (1-\delta)Q^L_t}{Q^L_{t+1}}$$

Convenience yield:

$$cy_t = R_t - \frac{1}{\mathcal{M}_{t,t+1}}$$

Annual debt to GDP ratio:

$$\frac{b_t}{4y_t}$$

Primary fiscal surplus:

$$s_t = \tau_t - g_t$$

Total interest rate payments:

$$(R_{t-1} - 1)\,\frac{b^S_{t-1}}{\pi_t} + (\kappa - \delta)\,\frac{b^L_{t-1}}{\pi_t}$$

Private interest rate payments:

$$(R_{t-1} - 1)\,\frac{b^{S,FI}_{t-1}}{\pi_t} + (\kappa - \delta)\,\frac{b^{L,H}_{t-1}}{\pi_t}$$

Total debt service:

$$\frac{b^S_{t-1}}{\pi_t} + \kappa\frac{b^L_{t-1}}{\pi_t}$$

8.4 Steady state and approximation point

At the steady state, shocks are equal to zero: $\epsilon^\nu_t = 0$, $\epsilon^z_t = 0$, $\epsilon^g_t = 0$, $\epsilon^m_t = 0$, $\epsilon^{QE}_t = 0$.

Then: $z^{ss} = 1$, $\nu^{ss} = 1$, $g^{ss} = \bar{g}$, $b^{L,CB,ss} = b^{L,CB}_*$.

There is one endogenous variable which steady state value differs among regimes. For monetary ($\xi^C_t = 0$ and $\xi^P_t = M$) or fiscal ($\xi^C_t = 0$ and $\xi^P_t = F$) dominance regimes, we have: $R_t = R^*$ in absence of shocks. For the Zero lower bound regime ($\xi^C_t = 1$), $R_t = 1.0005$. Define p^M, p^F, p^C as the ergodic probability of monetary dominance, fiscal dominance or crisis regime, respectively. Where: $p^M + p^F + p^C = 0$. The short-term nominal interest rate at the approximation point is obtained as follows:

$$R^{ss} = \left(p^M + p^F\right) R^* + \left(1 - p^M - p^F\right) 1.0005$$

8.5 Additional results

8.5.1 "Impulse response functions" for other shocks

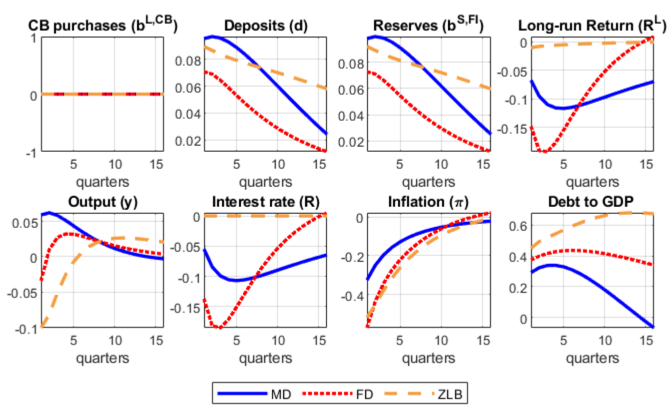

Figure 13: Impact of a TFP shock conditional on a regime

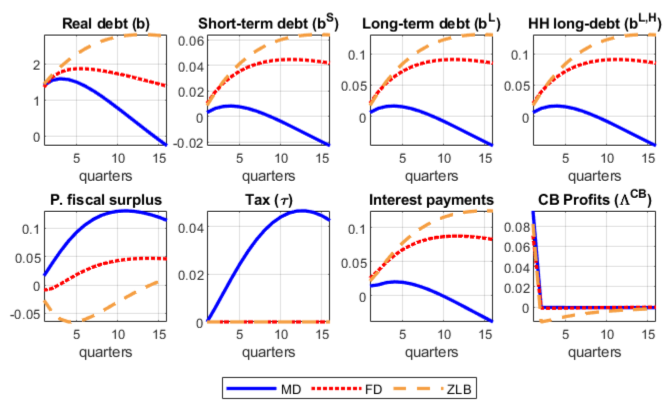

Figure 14: Impact of a TFP shock conditional on a regime: fiscal variables

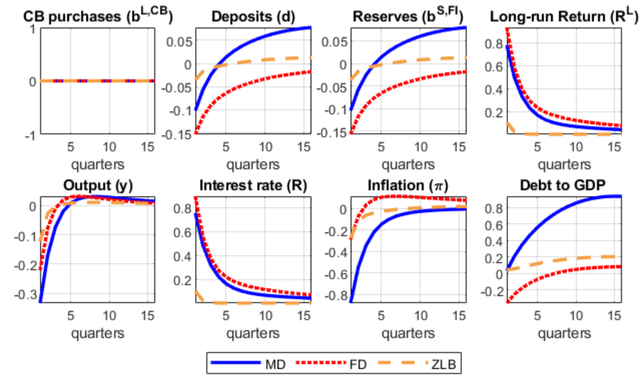

Figure 15: Impact of a monetary policy shock conditional on a regime

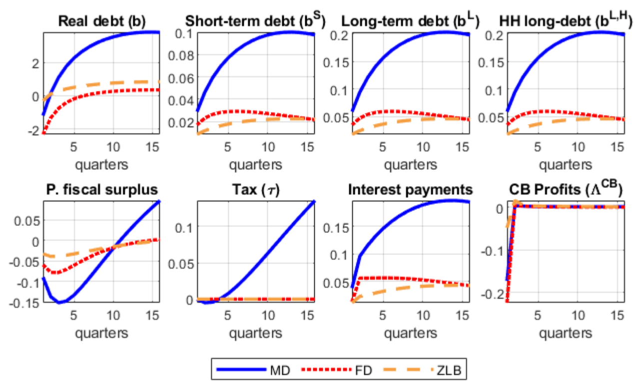

Figure 16: Impact of a monetary policy shock conditional on a regime: fiscal variables

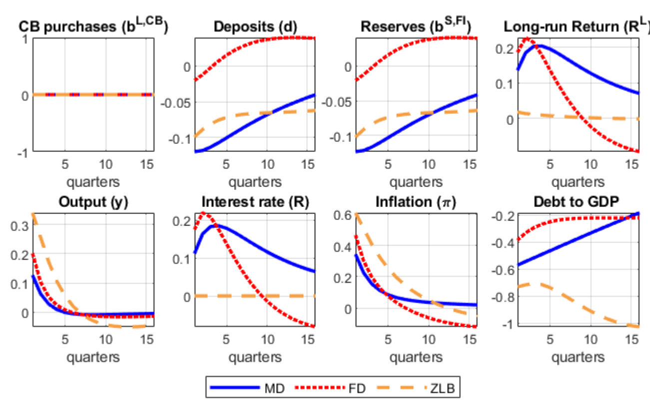

Figure 17: Impact of a preference shock conditional on a regime

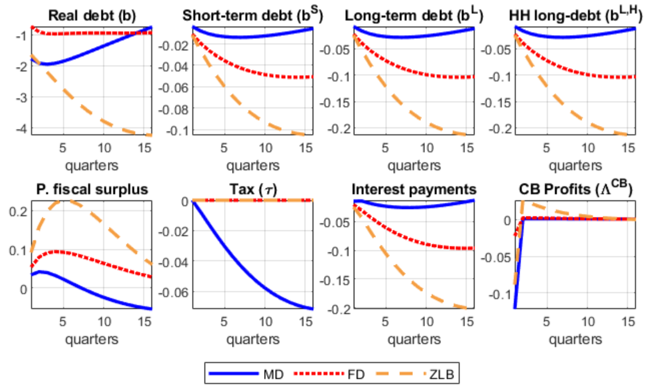

Figure 18: Impact of a preference shock conditional on a regime: fiscal variables

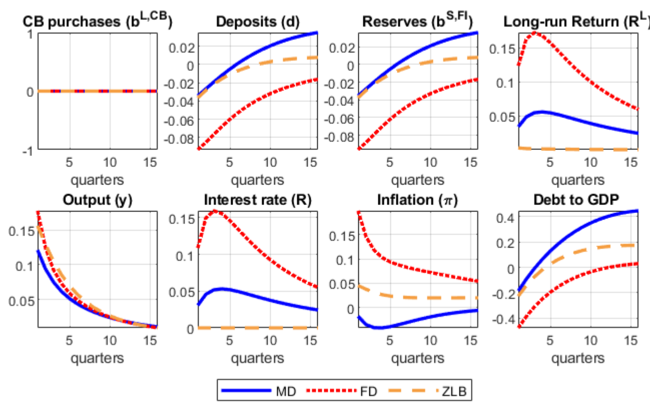

Figure 19: Impact of a government spending shock conditional on a regime

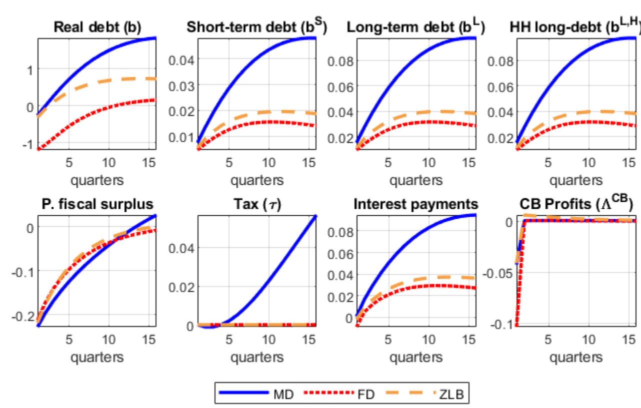

Figure 20: Impact of a government spending shock conditional on a regime: fiscal variables

8.5.2 Exit strategies: Alternative Scenarios

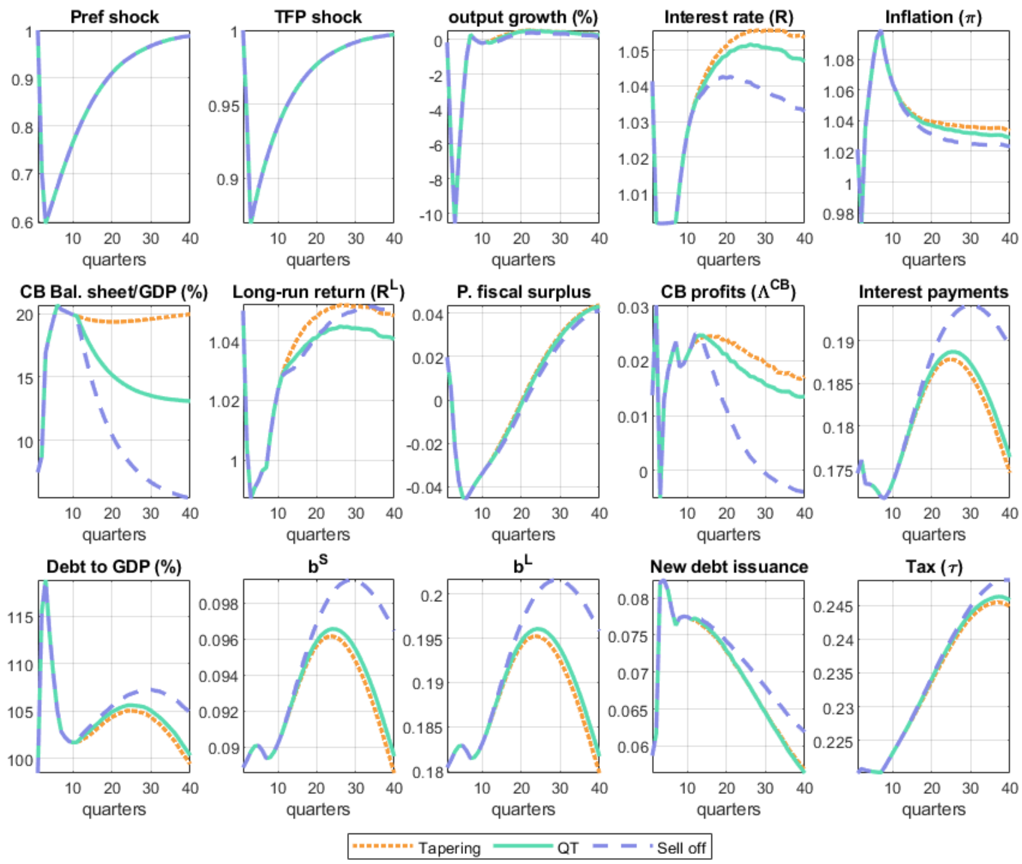

Figure 21: Crisis and exit strategies from QE.

Note: Simulation of 50.000 samples. The figure shows the average for the selected paths where the economy was at the ZLB in quarters 3-7 (endogenously, 1310 samples). Orange dotted line is the tapering scenario, same to the one in figure 7. Green continuous line is the scenario with Quantitative Tightening (QT) from period 12 onward. Light-blue dashed line is the scenario with sell-off of bonds from period 12 onward. Output growth, balance sheet and Debt to GDP are in percentages. Interest rate (R), long-run return (R^L), inflation, debt are annualized.

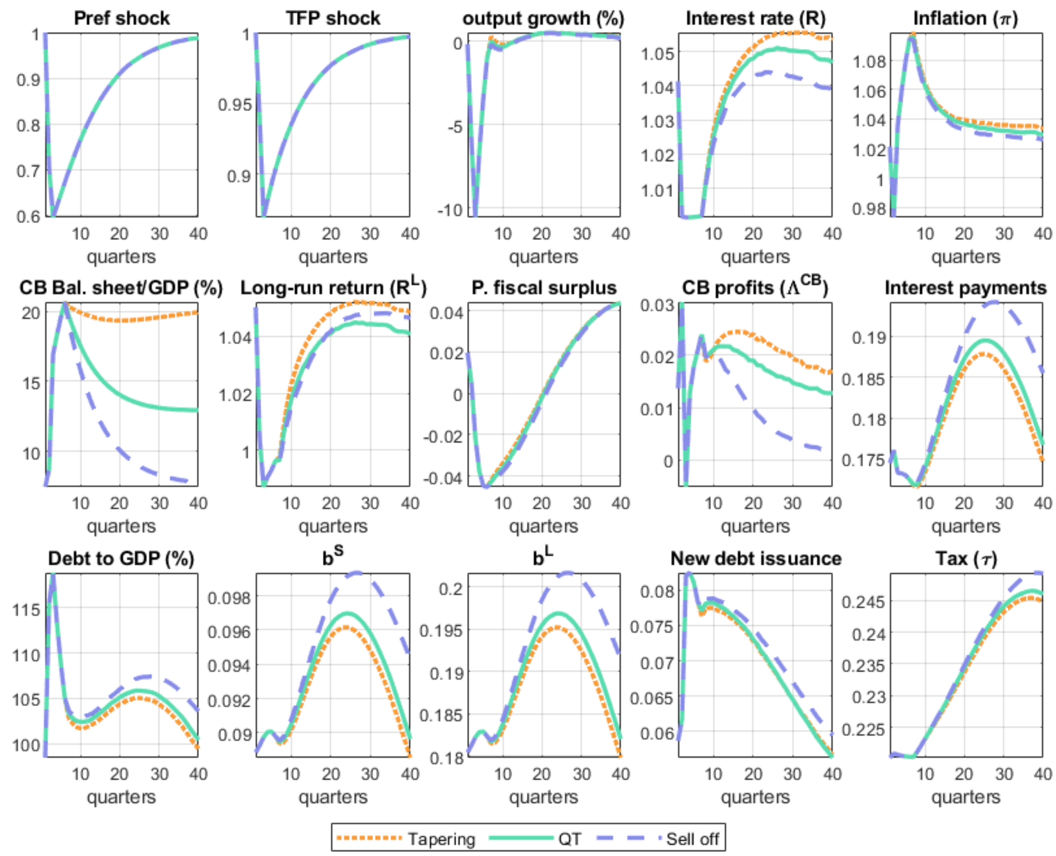

Figure 22: Crisis and exit strategies from QE.

Note: Simulation of 50.000 samples. The figure shows the average for the selected paths where the economy was at the ZLB in quarters 3-7 (endogenously). Orange dotted line is the tapering scenario, same to the one in figure 7. Green continuous line is the scenario with Quantitative Tightening (QT) from period 7 onward. Light-blue dashed line is the scenario with sell-off of bonds from period 7 onward. Output growth, balance sheet and Debt to GDP are in percentages. Interest rate (R), long-run return (R^L), inflation, debt are annualized.

8.6 Data sources

The following table presents data sources for plots in section 2 and calibration.

Variable	**Source**	**Table**
Deposits in depositary institutions	FRED	
Monetary aggregate M1	FRED	
CPI	FRED	
US GDP Implicit Price Deflator, Index 2015=100	FRED	
Federal reserve Assets	US Financial Accounts	L109
Federal Reserve Total treasuries	US Financial Accounts	L110
Federal Reserve Treasury bills	US Financial Accounts	L111
Federal Reserve Other treasuries	US Financial Accounts	L112
Federal Reserve Total liabilities	US Financial Accounts	L113
Federal Reserve Reserves	US Financial Accounts	L114
Checkable Accounts in Federal Reserve	US Financial Accounts	L115
Total public debt	FRED	
Effective federal funds rate	FRED	
10-year Yield	FRED	
1-year Yield	FRED	
US GDP	BEA	NIPA
Private consumption	BEA	NIPA
Government spending	BEA	NIPA
US Population	FRED	

Table 7: Data sources

9 Bibliography

BENIGNO, G., A. FOERSTER, C. OTROK, AND A. REBUCCI (2020): "Estimating macroeconomic models of financial crises: An endogenous regime-switching approach," Tech. rep., National Bureau of Economic Research.

BHATTARAI, S. AND C. J. NEELY (Forthcoming): "An Analysis of the Literature on International Unconventional Monetary Policy," Journal of Economic Literature.

BIANCHI, F. (2013): "Regime switches, agents' beliefs, and post-World War II US macroeconomic dynamics," Review of Economic studies, 80, 463–490.

BIANCHI, F. AND L. MELOSI (2017): "Escaping the great recession," American Economic Review, 107, 1030–58.

——— (2019): "The dire effects of the lack of monetary and fiscal coordination," Journal of Monetary Economics, 104, 1–22.

——— (2022): "Inflation as a Fiscal Limit," .

BOCOLA, L. (2016): "The pass-through of sovereign risk," Journal of Political Economy, 124, 879–926.

BONCIANI, D. AND J. OH (2021): "Optimal monetary policy mix at the zero lower bound," .

CHEN, H., V. CÚRDIA, AND A. FERRERO (2012): "The macroeconomic effects of large-scale asset purchase programmes," The economic journal, 122, F289–F315.

COCHRANE, J. H. (2001): "Long-term debt and optimal policy in the fiscal theory of the price level," Econometrica, 69, 69–116.

——— (2021): "The fiscal theory of the price level," Manuscript. URL.

CUI, W. AND V. STERK (2021): "Quantitative Easing with Heterogenous Agents," .

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017): "The great escape? A quantitative evaluation of the Fed's liquidity facilities," American Economic Review, 107, 824–57.

Drechsler, I., A. Savov, and P. Schnabl (2017): "The deposits channel of monetary policy," The Quarterly Journal of Economics, 132, 1819–1876.

Elenev, V., T. Landvoigt, P. J. Shultz, and S. Van Nieuwerburgh (2021): "Can Monetary Policy Create Fiscal Capacity?" Tech. rep., National Bureau of Economic Research.

Foerster, A. T. (2015): "Financial crises, unconventional monetary policy exit strategies, and agents expectations," Journal of Monetary Economics, 76, 191–207.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," Journal of monetary Economics, 58, 17–34.

Harrison, R. (2017): "Optimal quantitative easing," .

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," Journal of international Economics, 79, 117–125.

Jordà, Ò., M. Schularick, and A. M. Taylor (2017): "Macrofinancial history and the new business cycle facts," NBER macroeconomics annual, 31, 213–263.

Kuttner, K. N. (2018): "Outside the box: Unconventional monetary policy in the great recession and beyond," Journal of Economic Perspectives, 32, 121–46.

Leeper, E. M. (1991): "Equilibria under 'active'and 'passive'monetary and fiscal policies," Journal of monetary Economics, 27, 129–147.

Leeper, E. M. and C. Leith (2016): "Understanding inflation as a joint monetary–fiscal phenomenon," in Handbook of Macroeconomics, Elsevier, vol. 2, 2305–2415.

Leeper, E. M., C. Leith, and D. Liu (2021): "Optimal time-consistent monetary, fiscal and debt maturity policy," Journal of Monetary Economics, 117, 600–617.

Rotemberg, J. J. (1982): "Sticky prices in the United States," Journal of political economy, 90, 1187–1211.

Schmitt-Grohé, S. and M. Uribe (2007): "Optimal simple and implementable monetary and fiscal rules," Journal of monetary Economics, 54, 1702–1725.

Sims, E. and J. C. Wu (2021): "Evaluating central banks' tool kit: Past, present, and future," Journal of Monetary Economics, 118, 135–160.

Sims, E., J. C. Wu, and J. Zhang (2020): "The four equation new keynesian model," The Review of Economics and Statistics, 1–45.

Vissing-Jorgensen, A. (2021): "The treasury market in spring 2020 and the response of the federal reserve," Tech. rep., National Bureau of Economic Research.

Wallace, N. (1981): "A Modigliani-Miller theorem for open-market operations," The American Economic Review, 71, 267–274.

Wen, Y. et al. (2014): "When and how to exit quantitative easing," Federal Reserve Bank of St. Louis Review, 96, 243–265.