NPTEL MOOC

PROGRAMMING, DATA STRUCTURES AND ALGORITHMS IN PYTHON

Week 4, Lecture 1

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$O(n^2)$ sorting algorithms

- Selection sort and insertion sort are both $O(n^2)$
- $O(n^2)$ sorting is infeasible for n over 5000

A different strategy?

- Divide array in two equal parts
- Separately sort left and right half
- Combine the two sorted halves to get the full array sorted

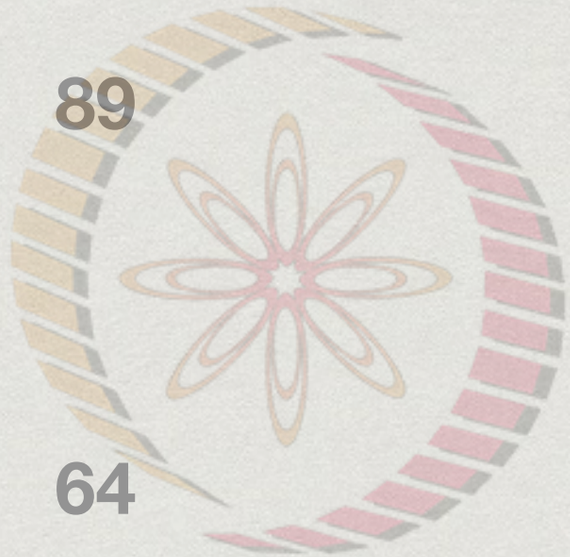
Combining sorted lists

- Given two sorted lists `A` and `B`, combine into a sorted list `C`
  - Compare first element of `A` and `B`
  - Move it into `C`
  - Repeat until all elements in `A` and `B` are over
- Merging `A` and `B`

Merging two sorted lists

32 74 89

21 55 64



Merging two sorted lists

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Merging two sorted lists

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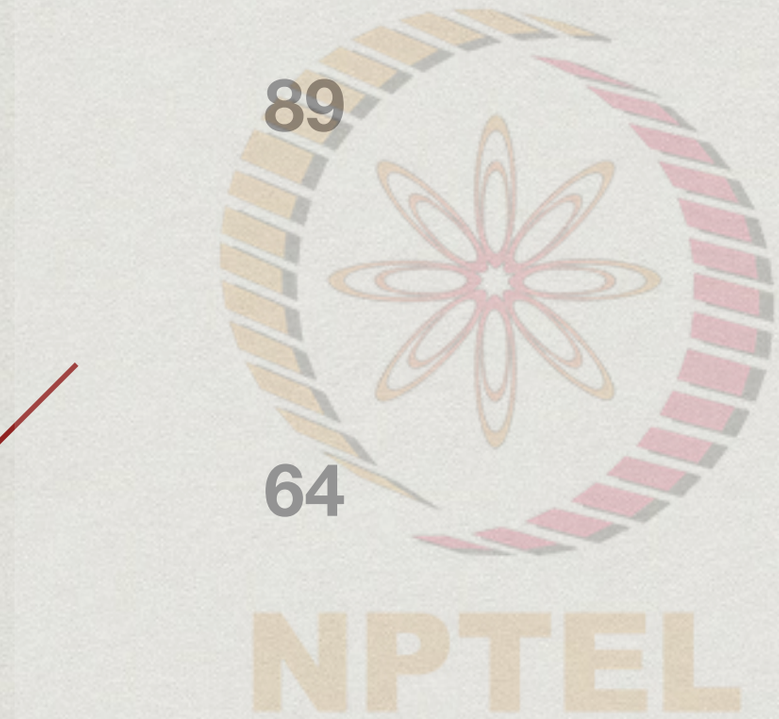
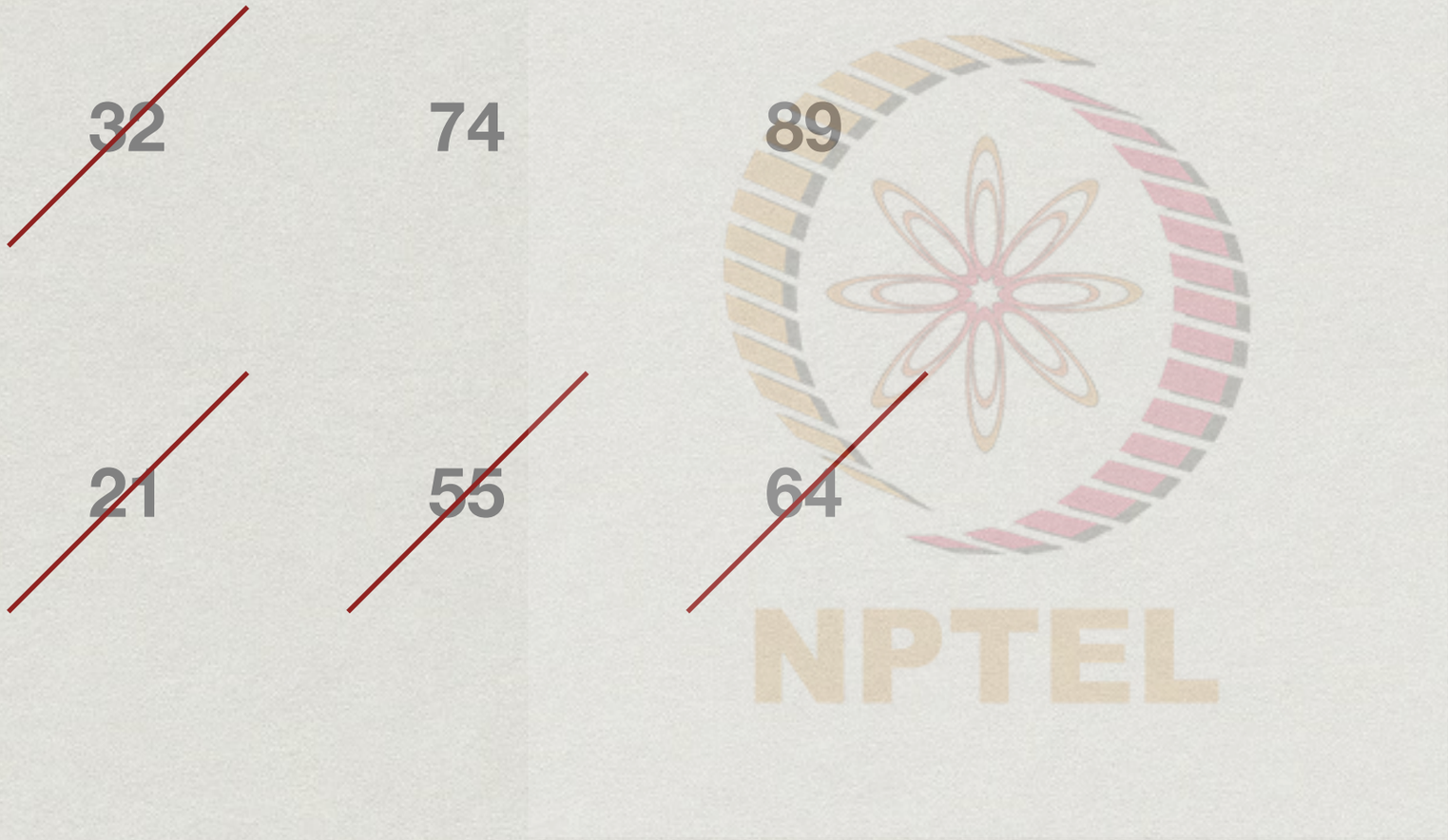
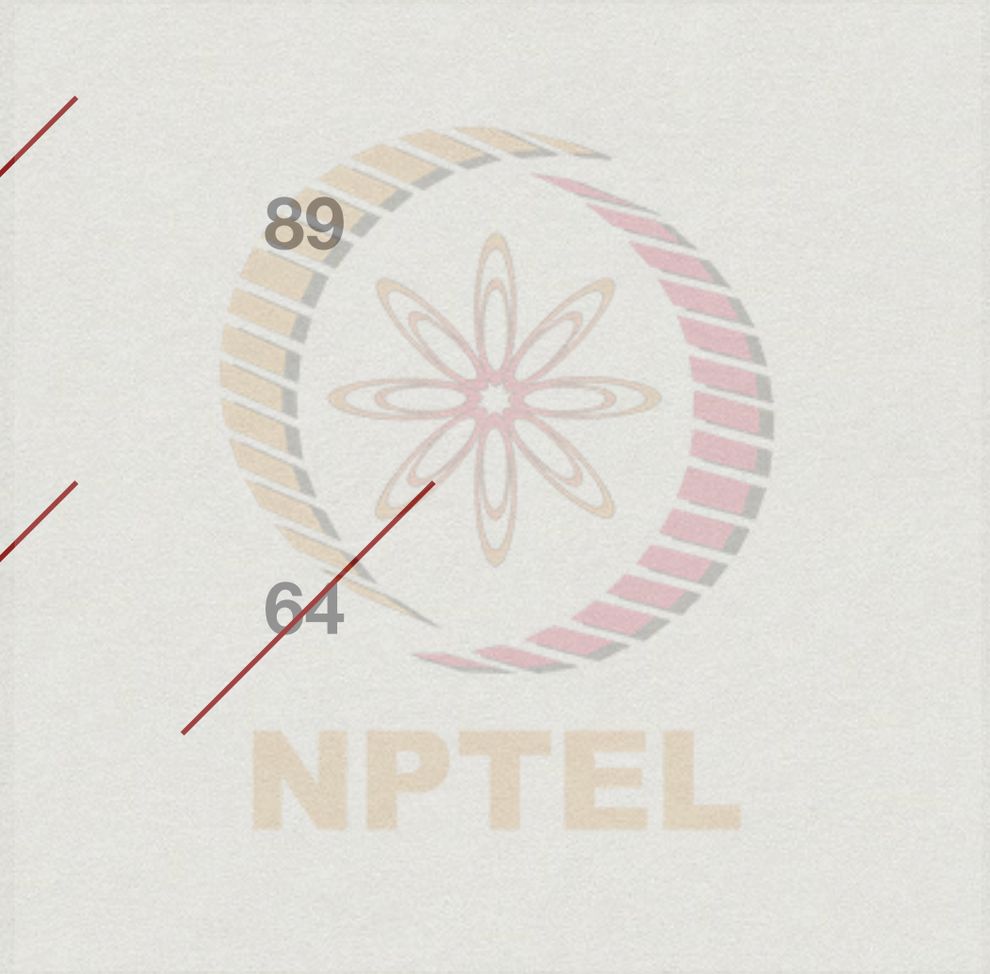


Merging two sorted lists



Merging two sorted lists



21 32 55 64 74

Merging two sorted lists

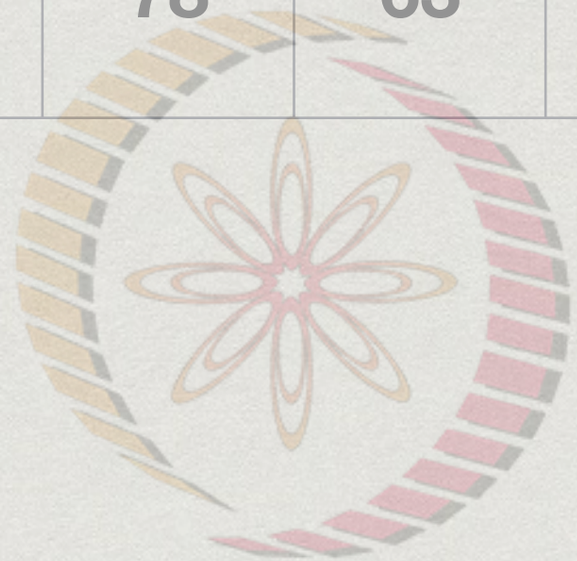
21 32 55 64 74 89

Merge Sort

- Sort `A[0:n//2]`
- Sort `A[n//2:n]`
- Merge sorted halves into `B[0:n]`
- How do we sort the halves?
  - Recursively, using the same strategy!

Merge Sort

43	32	22	78	63	57	91	13



Merge Sort

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43	32	22	78

Merge Sort

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Merge Sort

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Merge Sort

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Merge Sort

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Merge Sort

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Merge Sort


43 32 22 78 63 57 91 13
22 32 43 78 13 57 63 91
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Merge Sort


13 22 32 43 57 63 78 91
22 32 43 78
13 57 63 91
32 43
22 78
57 63
13 91
43 32 22 78 63 57 91 13

Divide and conquer

- Break up problem into disjoint parts
- Solve each part separately
- Combine the solutions efficiently

Merging sorted lists

Combine two sorted lists `A` and `B` into `C`

- If `A` is empty, copy `B` into `C`
- If `B` is empty, copy `A` into `C`
- Otherwise, compare first element of `A` and `B` and move the smaller of the two into `C`
- Repeat until all elements in `A` and `B` have been moved

Merging

```
def merge(A,B): # Merge A[0:m],B[0:n]
  (C,m,n) = ([],len(A),len(B))
  (i,j) = (0,0) # Current positions in A,B
  while i+j < m+n: # i+j is number of elements merged so far
    if i == m:  # Case 1: A is empty
      C.append(B[j])
      j = j+1
    elif j == n: # Case 2: B is empty
      C.append(A[i])
      i = i+1
    elif A[i] <= B[j]: # Case 3: Head of A is smaller
      C.append(A[i])
      i = i+1
    elif A[i] > B[j]: # Case 4: Head of B is smaller
      C.append(B[j])
      j = j+1
  return(C)
```

Merging, wrong

```
def mergewrong(A,B): # Merge A[0:m],B[0:n]
  (C,m,n) = ([],len(A),len(B))
  (i,j) = (0,0) # Current positions in A,B
  while i+j < m+n:
  # i+j is number of elements merged so far
    # Combine Case 1, Case 4
    if i == m or A[i] > B[j]:
      C.append(B[j])
      j = j+1
    # Combine Case 2, Case 3:
    elif j == n or A[i] <= B[j}:
      C.append(A[i])
      i = i+1
  return(C)
```

Merge Sort

To sort `A[0:n]` into `B[0:n]`

- If n is 1, nothing to be done
- Otherwise
  - Sort `A[0:n//2]` into `L` (left)
  - Sort `A[n//2:n]` into `R` (right)
  - Merge `L` and `R` into `B`

Merge Sort

```
def mergesort(A,left,right):
    # Sort the slice A[left:right]

    if right - left <= 1: # Base case
        return(A[left:right])

    if right - left > 1:   # Recursive call

        mid = (left+right)//2

        L = mergesort(A,left,mid)
        R = mergesort(A,mid,right)

        return(merge(L,R))
```