2248-CSE-5311-005

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Hands On 6

Average runtime complexity of non-random pivot Quicksort.

Recurrence relation

$T(n) =$ Partitioning + recursive calls.

Partitioning $= O(n)$

Recursive cost (Pivot at index k)

$$T(k) + T(n-k-1)$$

$$\therefore \; T(n) = \sum_{i=0}^{n-1} \left(T(i) + T(n-i-1) \right) + O(n)$$

For average case: The pivot divides the array in roughly 2 equal parts:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + O\left(\frac{n}{2}\right)\right) + O(n)$$

$$T(n) = 8T\left(\frac{n}{8}\right) + 3(O(n))$$

General form

After $\log(n)$ steps: $T(n) = 2^k T\left(\frac{n}{2^k}\right) + k(O(n))$ | $k = \log(n)$

$$T(n) = n\,T\left(\frac{n}{2^{\log(n)}}\right) + \log(n)\,O(n)$$

$$T(n) = n\,T(1) + O(n\log(n))$$

$$\therefore \; T(n) = O(n\log(n))$$