E-14a: Lab 3

Numpy | Pandas | Sklearn

Numpy

NumPy is the fundamental package for scientific computing with Python. It contains among other things:

- a powerful N-dimensional array object
- · sophisticated (broadcasting) functions
- tools for integrating C/C++ and Fortran code
- · useful linear algebra, Fourier transform, and random number capabilities

Besides its obvious scientific uses, NumPy can also be used as an efficient multi-dimensional container of generic data. Arbitrary data-types can be defined. This allows NumPy to seamlessly and speedily integrate with a wide variety of databases.

For more information, check this page: http://www.numpy.org/ (http://www.numpy.org/)

```
In [2]: # Load Library
import numpy as np
```

Let us explore numpy through examples.

Arrays

```
In [3]: a = np.array([1, 2, 3])
    print(a.shape)
    print(a[0], a[1], a[2])

a[0] = 5
    print(a)

(3,)
    1 2 3
    [5 2 3]

In []: b = np.array([[1, 2, 3], [4, 5, 6]])
    print(b.shape)
    print(b[0, 0], b[0, 1], b[1, 0])
```

```
In [4]: | a = np.zeros((2, 2))
        print(a)
        [[0. 0.]
         [0. 0.]]
In [5]: b = np.ones((1, 2))
        print(b)
        [[1. 1.]]
In [6]: e = np.random.random((2, 2))
        print(e)
        [[0.04502328 0.85775263]
         [0.99025505 0.58551765]]
        Array indexing
In [7]: a = np.array([[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12]])
        print(a)
        print(a[0, 1])
        [[ 1 2 3 4]
         [5 6 7 8]
         [ 9 10 11 12]]
        2
In [8]: b = a[:2, 1:3]
        print(b)
        [[2 3]
         [6 7]]
In [9]: a = np.array([[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12]])
        row_r1 = a[1, :]
        row_r2 = a[1:2, :]
        print(row_r1, row_r1.shape)
        print(row_r2, row_r2.shape)
        [5 6 7 8] (4,)
        [[5 6 7 8]] (1, 4)
```

Array math

```
In [10]: x = np.array([[1, 2], [3, 4]], dtype=np.float64)
         y = np.array([[5, 6], [7, 8]], dtype=np.float64)
          print(x + y)
         print(np.add(x, y))
         [[ 6. 8.]
          [10. 12.]]
         [[ 6. 8.]
          [10. 12.]]
In [11]: | print(x - y)
         print(np.subtract(x, y))
         [[-4. -4.]
          [-4. -4.]]
         [[-4. -4.]
          [-4. -4.]]
In [12]: | print(x * y)
         print(np.multiply(x, y))
         [[ 5. 12.]
          [21. 32.]]
         [[ 5. 12.]
          [21. 32.]]
In [13]: | print(x / y)
         print(np.divide(x, y))
                       0.33333333]
         [[0.2
          [0.42857143 0.5
                                 ]]
         [[0.2
                       0.33333333]
           [0.42857143 0.5
                                 ]]
In [14]: | print(np.sqrt(x))
         [[1.
                       1.41421356]
          [1.73205081 2.
                                 ]]
In [15]: x = np.array([[1, 2], [3, 4]])
          print(np.sum(x))
          print(np.sum(x, axis=0))
         print(np.sum(x, axis=1))
         10
         [4 6]
         [3 7]
```

Broadcasting

```
In [18]: x = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9], [10, 11, 12]])
v = np.array([1, 0, 1])
y = np.empty_like(x)

for i in range(4):
    y[i, :] = x[i, :] + v

print(y)

[[ 2  2  4]
  [ 5  5  7]
  [ 8  8  10]
  [11  11  13]]
```

Plotting

Matplotlib is a Python 2D plotting library which produces publication quality figures in a variety of hardcopy formats and interactive environments across platforms. Matplotlib can be used in Python scripts, the Python and IPython shells, the Jupyter notebook, web application servers, and four graphical user interface toolkits.

Matplotlib tries to make easy things easy and hard things possible. You can generate plots, histograms, power spectra, bar charts, errorcharts, scatterplots, etc., with just a few lines of code. For examples, see the sample plots and thumbnail gallery: https://matplotlib.org/index.html (https://matplotlib.org/index.html)

For simple plotting the pyplot module provides a MATLAB-like interface, particularly when combined with IPython. For the power user, you have full control of line styles, font properties, axes properties, etc, via an object oriented interface or via a set of functions familiar to MATLAB users.

```
In [19]: import matplotlib.pyplot as plt

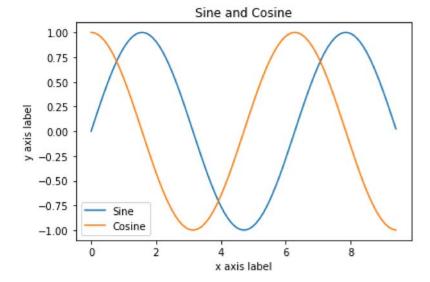
x = np.arange(0, 3 * np.pi, 0.1)
y = np.sin(x)

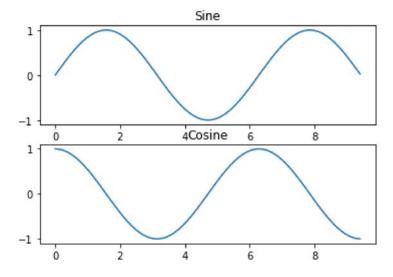
plt.plot(x, y)
plt.show()
```

<Figure size 640x480 with 1 Axes>

```
In [20]: x = np.arange(0, 3 * np.pi, 0.1)
y_sin = np.sin(x)
y_cos = np.cos(x)

plt.plot(x, y_sin)
plt.plot(x, y_cos)
plt.xlabel('x axis label')
plt.ylabel('y axis label')
plt.title('Sine and Cosine')
plt.legend(['Sine', 'Cosine'])
plt.show()
```





Pandas

Pandas is an open source, BSD-licensed library providing high-performance, easy-to-use data structures and data analysis tools for the Python programming language.

pandas is a NumFOCUS sponsored project. This will help ensure the success of development of pandas as a world-class open-source project, and makes it possible to donate to the project.

For more information, check this page: https://pandas.pydata.org/)

```
In [22]: # Load Library
import pandas as pd

In [23]: # Import dataset
dataset = pd.read_csv("Boston_March2018.csv")
```

```
dataset.head()
In [24]:
Out[24]:
             MLSNUM STATUS LISTPRICE SOLDPRICE
                                                     LISTDATE SOLDDATE EXPIREDDATE DOM DTO
          0 72049670
                          SLD
                                239900.0
                                            247000.0
                                                       8/6/2016
                                                                  3/1/2018
                                                                                  NaN
                                                                                        295
                                                                                              295
           1 72056522
                          SLD
                                338000.0
                                            338000.0
                                                      8/22/2016
                                                                  3/1/2018
                                                                                  NaN
                                                                                        545
                                                                                              515
          2 72080286
                          SLD
                                2999999.0
                                           2950000.0
                                                     10/12/2016
                                                                  3/1/2018
                                                                                  NaN
                                                                                        504
                                                                                              493
           3 72118879
                          SLD
                                2600000.0
                                           2600000.0
                                                      2/14/2017
                                                                  3/1/2018
                                                                                  NaN
                                                                                         28
                                                                                               14
                                            525000.0
           4 72124101
                          SLD
                                525000.0
                                                      2/28/2017
                                                                  3/1/2018
                                                                                  NaN
                                                                                        867
                                                                                              287
          5 rows × 38 columns
In [25]: dataset.columns
Out[25]: Index(['MLSNUM', 'STATUS', 'LISTPRICE', 'SOLDPRICE', 'LISTDATE', 'SOLDDATE',
                 'EXPIREDDATE', 'DOM', 'DTO', 'ADDRESS', 'CITY', 'STATE', 'ZIP', 'AREA',
                 'BEDS', 'BATHS', 'SQFT', 'AGE', 'LOTSIZE', 'AGENTNAME', 'OFFICENAME',
                 'OFFICEPHONE', 'SHOWINGINSTRUCTIONS', 'REMARKS', 'STYLE', 'LEVEL',
                 'GARAGE', 'HEATING', 'COOLING', 'ELEMENTARYSCHOOL', 'JUNIORHIGHSCHOOL',
                 'HIGHSCHOOL', 'OTHERFEATURES', 'PROPTYPE', 'STREETNAME', 'HOUSENUM1',
                 'HOUSENUM2', 'PHOTOURL'],
                dtype='object')
In [26]:
          dataset.LISTPRICE.head()
Out[26]: 0
                239900.0
          1
                338000.0
          2
               2999999.0
          3
               2600000.0
          4
                525000.0
          Name: LISTPRICE, dtype: float64
In [27]:
         dataset['LISTPRICE'].head()
Out[27]:
         0
                239900.0
                338000.0
          1
          2
               2999999.0
          3
               2600000.0
                525000.0
          Name: LISTPRICE, dtype: float64
```

```
columns_i_want=['LISTPRICE', 'SOLDPRICE']
In [28]:
          dataset[columns_i_want].head()
Out[28]:
              LISTPRICE SOLDPRICE
           0
               239900.0
                           247000.0
           1
               338000.0
                           338000.0
               2999999.0
                          2950000.0
           2
              2600000.0
                          2600000.0
           3
               525000.0
                           525000.0
In [29]:
          dataset.PROPTYPE.unique()
Out[29]: array(['MF', 'CC', 'SF'], dtype=object)
          dataset.iloc[:2]
In [30]:
Out[30]:
              MLSNUM STATUS LISTPRICE SOLDPRICE LISTDATE SOLDDATE EXPIREDDATE DOM DTO
           0 72049670
                          SLD
                                  239900.0
                                              247000.0
                                                        8/6/2016
                                                                   3/1/2018
                                                                                     NaN
                                                                                           295
                                                                                                 295
           1 72056522
                          SLD
                                 338000.0
                                             338000.0
                                                       8/22/2016
                                                                   3/1/2018
                                                                                           545
                                                                                                 515
                                                                                     NaN
          2 rows × 38 columns
In [31]:
          dataset.iloc[:2, :2]
Out[31]:
              MLSNUM STATUS
           0 72049670
                          SLD
           1 72056522
                          SLD
          dataset.iloc[:, 4:6].head()
In [32]:
Out[32]:
              LISTDATE SOLDDATE
           0
                8/6/2016
                           3/1/2018
           1
               8/22/2016
                           3/1/2018
           2 10/12/2016
                           3/1/2018
           3
               2/14/2017
                           3/1/2018
               2/28/2017
                           3/1/2018
```

```
In [33]: dataset.SOLDPRICE < 1000000</pre>
Out[33]: 0
                    True
          1
                    True
          2
                   False
          3
                   False
          4
                    True
          5
                    True
          6
                    True
          7
                    True
          8
                   False
          9
                    True
          10
                    True
          11
                    True
          12
                   False
          13
                   False
          14
                   False
          15
                    True
                   False
          16
          17
                    True
          18
                    True
In [34]:
          dataset[dataset.SOLDPRICE < 1000000].head()</pre>
Out[34]:
              MLSNUM STATUS LISTPRICE SOLDPRICE LISTDATE SOLDDATE EXPIREDDATE DOM
                                                                                                  DTO
           0 72049670
                           SLD
                                  239900.0
                                               247000.0
                                                          8/6/2016
                                                                     3/1/2018
                                                                                       NaN
                                                                                              295
                                                                                                   295
           1 72056522
                           SLD
                                  338000.0
                                               338000.0
                                                         8/22/2016
                                                                     3/1/2018
                                                                                       NaN
                                                                                              545
                                                                                                   515
           4 72124101
                           SLD
                                  525000.0
                                               525000.0
                                                                     3/1/2018
                                                                                             867
                                                                                                   287
                                                        2/28/2017
                                                                                       NaN
           5 72133120
                           SLD
                                  468000.0
                                               479000.0
                                                                                              273
                                                                                                   273
                                                         3/20/2017
                                                                     3/1/2018
                                                                                       NaN
           6 72148511
                           SLD
                                  209900.0
                                               176505.0
                                                        4/19/2017
                                                                     3/1/2018
                                                                                       NaN
                                                                                              311
                                                                                                   287
          5 rows × 38 columns
In [35]: | np.sum(dataset.SOLDPRICE), np.mean(dataset.SOLDPRICE)
```

Out[35]: (2464492263.65, 476598.7746374008)

In [36]: dataset[(dataset.SOLDPRICE < 1000000) & (dataset.PROPTYPE == "SF")]</pre> Out[36]: MLSNUM STATUS LISTPRICE SOLDPRICE LISTDATE SOLDDATE EXPIREDDATE DOM **4** 72124101 SLD 525000.0 525000.0 NaN 867 2/28/2017 3/1/2018 **5** 72133120 SLD 468000.0 479000.0 3/20/2017 3/1/2018 NaN 273 **6** 72148511 SLD 209900.0 176505.0 4/19/2017 3/1/2018 NaN 311 **7** 72153413 SLD 549900.0 530000.0 3/1/2018 261 4/26/2017 NaN 72166889 SLD 130000.0 130000.0 3/1/2018 185 5/16/2017 NaN **10** 72166942 SLD 1150000.0 945000.0 5/18/2017 3/1/2018 NaN 194

In [37]: dataset.describe()

Out[37]:

		MLSNUM	LISTPRICE	SOLDPRICE	EXPIREDDATE	DOM	DTO	
С	ount	5.171000e+03	5.171000e+03	5.171000e+03	0.0	5171.000000	5171.000000	5171.
n	nean	7.225305e+07	4.807745e+05	4.765988e+05	NaN	69.591762	48.977374	2062.
	std	4.473714e+04	4.945971e+05	5.016361e+05	NaN	92.763384	68.484702	1119.
	min	7.115216e+07	1.990000e+04	2.300000e+04	NaN	0.000000	0.000000	101.
	25%	7.224754e+07	2.499000e+05	2.450000e+05	NaN	18.000000	6.000000	1747.
	50%	7.226724e+07	3.748000e+05	3.700000e+05	NaN	35.000000	20.000000	2048.
	75%	7.227533e+07	5.499000e+05	5.515000e+05	NaN	94.000000	73.000000	2360.
	max	7.230167e+07	1.050000e+07	1.511000e+07	NaN	1562.000000	938.000000	34452.
4								

In [41]: grouped_by_type = dataset.groupby("PROPTYPE")
grouped_by_type

Out[41]: <pandas.core.groupby.groupby.DataFrameGroupBy object at 0x00000154BEBE6630>

```
dataset.groupby("PROPTYPE").describe()
In [43]:
Out[43]:
                       AGE
                                                                                    BATHS
                                          std
                                                     min 25%
                                                               50%
                                                                      75%
                                                                            max
                       count
                              mean
                                                                                    count
                                                                                           mean
           PROPTYPE
                   CC
                       1444.0
                                49.587950
                                          53.831568
                                                     0.0
                                                          11.0
                                                                32.0
                                                                       93.0
                                                                            1019.0
                                                                                   1444.0
                                                                                           1.745845
                   MF
                        484.0
                               108.140496
                                          29.322242
                                                     5.0
                                                          98.0
                                                                113.0
                                                                      118.0
                                                                             283.0
                                                                                     484.0
                                                                                           2.927686
                                60.889608
                                          53.487955
                                                     0.0
                                                          30.0
                                                                57.0
                                                                            1863.0
                       3243.0
                                                                       82.0
                                                                                   3243.0
                                                                                           2.024206
          3 rows × 112 columns
In [40]:
           dataset.groupby("PROPTYPE").mean()
Out[40]:
                       MLSNUM
                                     LISTPRICE
                                                   SOLDPRICE
                                                                  EXPIREDDATE DOM
                                                                                           DTO
                                                                                                      ZIP
           PROPTYPE
```

TODO: http://pandas.pydata.org/pandas-docs/stable/merging.html)

7.225306e+07 464208.570802 456815.676056

7.225364e+07 512072.209661

7.225119e+07 498397.169421

Sklearn

CC

MF

Sklearn provides simple and efficient tools for data mining and data analysis. It is accessible to everybody, and reusable in various contexts. Built on NumPy, SciPy, and matplotlib.

515437.729363

493278.811983

NaN

NaN

54.799169

55.506198

NaN 78.280604 54.210607

39.804017

41.280992

210

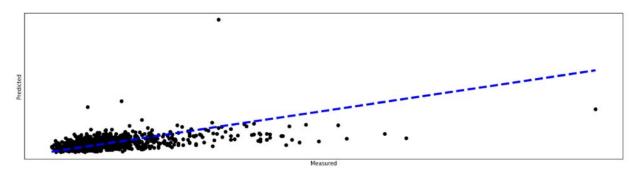
199

205

Open source, commercially usable - BSD license. Take a look at: http://scikit-learn.org/stable/ (http://scikit-learn.org/stable/)

```
In [49]: # Load tools from sklearn library
    from sklearn import datasets, linear_model
    from sklearn.metrics import mean_squared_error, r2_score
In [63]: dataset=dataset.fillna(-9999)
```

```
In [64]: # Run Linear Regresison
         # Use features: 'BEDS', 'BATHS', 'SQFT', 'AGE', 'LOTSIZE'
         prices = dataset['SOLDPRICE']
         features = dataset[['BEDS', 'BATHS', 'SQFT', 'AGE', 'LOTSIZE']]
         print ("Orginal shape")
         print (features.shape, prices.shape)
         print ()
         features = features.fillna(-9999)
         # Split data using the above example
         features train = features[:4000]
         features test = features[4000:]
         # Split the targets into training/testing sets
         prices train = prices[:4000]
         prices_test = prices[4000:]
         print ("Train shape")
         print (features_train.shape, prices_train.shape)
         print ()
         print ("Test shape")
         print (features_test.shape, prices_test.shape)
         regr.fit(features_train, prices_train)
         # Make predictions using the testing set
         prices_pred = regr.predict(features_test)
         # Explained variance score: 1 is perfect prediction
         print('Variance score: %.2f' % r2_score(prices_test, prices_pred))
         # Plot outputs
         plt.figure(figsize=(20, 5))
         plt.scatter(prices_test, prices_pred, color='black')
         plt.plot([prices_test.min(), prices_test.max()], [prices_test.min(), prices_test.
         plt.ylabel('Predicted')
         plt.xlabel('Measured')
         plt.xticks(())
         plt.yticks(())
         plt.show()
         Orginal shape
         (5171, 5) (5171,)
         Train shape
         (4000, 5) (4000,)
         Test shape
         (1171, 5) (1171,)
         Variance score: 0.02
```



```
In [53]: #load tools from sklearn library

from sklearn import datasets, linear_model
    from sklearn.metrics import mean_squared_error, r2_score

# Run Linear Regresssion
# Use feature: 'BEDS', 'BATHS', 'SQFT', 'AGE'
    prices=dataset['SOLDPRICE']
    features=dataset[['BEDS','BATHS','SQFT','AGE']]
    print("Original shape")
    print(features.shape, prices.shape)
    print()
```

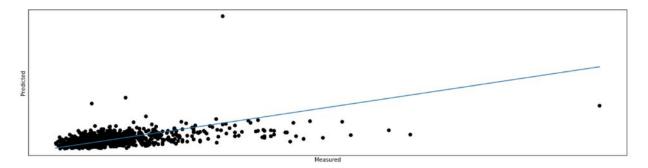
Original shape (5171, 4) (5171,)

```
In [61]: # Split data using the above example
         features train=features[:4000]
         features_test=features[4000:]
         #Split the targets into training/testing sets
         prices_train=prices[:4000]
         prices test=prices[4000:]
         regr=linear model.LinearRegression()
         print ("Train shape")
         print(features_train.shape, prices_train.shape)
         print ()
         print ("Test shape")
         print (features_test.shape, prices_test.shape)
         regr.fit(features train, prices train)
         #Make predictions using the testing set
         regr.fit(features train, prices train)
         prices_pred=regr.predict(features_test)
         #Explain variance score: 1 is perfect prediction
         print('Variance Score: %.2f' % r2_score(prices_test, prices_pred))
         #Plot outputs
         plt.figure(figsize=(20,5))
         plt.scatter(prices_test, prices_pred, color='black')
         plt.plot([prices_test.min(), prices_test.max()], [prices_test.min(),prices_test.m
         plt.ylabel('Predicted')
         plt.xlabel('Measured')
         plt.xticks(())
         plt.yticks(())
         plt.show()
         Train shape
```

```
Train shape
(4000, 4) (4000,)

Test shape
(1171, 4) (1171,)

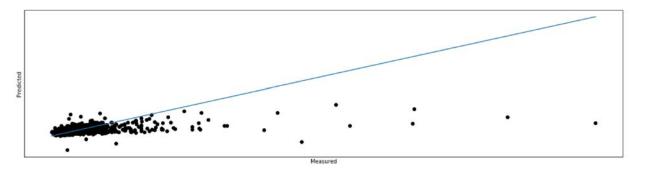
Variance Score: 0.02
```



```
In [71]: target_df=dataset['SOLDPRICE']
  features_df=dataset[['BEDS','BATHS','SQFT','AGE','LOTSIZE']]
```

```
In [84]:
         # Use sklearn train/test/split
         from sklearn.model_selection import train_test_split
         x_train, x_test, y_train,y_test = train_test_split(dataset[['BEDS','BATHS','SQFT'
         regr.fit(features_train, prices_train)
         #Make predictions using the testing set
         regr.fit(x_train,y_train)
         y pred=regr.predict(x test)
         #Explain variance score: 1 is perfect prediction
         print('Variance Score: %.2f' % r2_score(y_test, y_pred))
         #Plot outputs
         plt.figure(figsize=(20,5))
         plt.scatter(y test, y pred, color='black')
         plt.plot([y_test.min(), y_test.max()], [y_test.min(),y_test.max()])
         plt.ylabel('Predicted')
         plt.xlabel('Measured')
         plt.xticks(())
         plt.yticks(())
         plt.show()
```

Variance Score: 0.25



In [75]: features_train

Out[75]:

	BEDS	BATHS	SQFT	AGE	LOTSIZE
3945	3	3.5	1467	119	1690.0
1964	5	2.0	1842	118	4356.0
605	4	2.0	1846	65	10000.0
3046	3	1.0	1226	118	3244.0
4655	7	4.0	3468	98	3684.0
184	3	1.5	1300	33	4438.0
750	4	1.5	2158	58	10018.0
3228	4	3.0	1892	59	10635.0
696	4	2.0	1880	118	9583.0
3237	2	1.5	1116	36	-9999.0
3421	4	3.0	4265	38	43560.0
4838	2	1.0	760	54	-9999.0
2930	2	1.5	2951	67	12593.0
3262	2	1.0	1129	220	-9999.0
2902	3	1.0	1104	118	10000.0
3507	1	1.0	348	45	0.0
2008	4	2.5	2171	46	95396.0
3030	5	2.0	1832	29	8255.0
3388	4	2.0	2354	87	27007.0
2274	3	2.0	1631	119	-9999.0
4798	4	2.0	2499	65	12720.0
1341	3	3.0	2262	24	-9999.0
3485	3	1.0	1056	64	15002.0
465	3	1.0	1333	78	6476.0
4248	3	1.0	1716	58	26943.0
3298	3	2.0	1440	32	5000.0
1589	3	2.5	1632	0	33247.0
84	3	1.0	960	61	10018.0
4058	2	2.5	1901	1	-9999.0
2320	3	2.0	1414	63	18731.0
4504	4	2.5	2624	25	30056.0
1681	2	2.0	1300	12	-9999.0
3755	2	2.0	1192	12	-9999.0

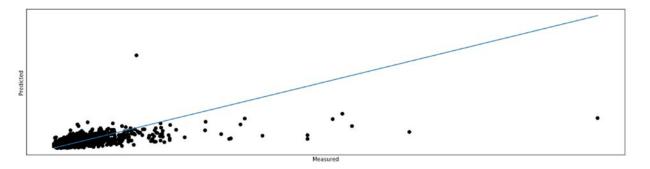
	BEDS	BATHS	SQFT	AGE	LOTSIZE
3210	3	2.0	1533	96	5181.0
243	4	3.5	3225	2	75386.0
2983	3	2.5	1836	38	22178.0
1494	1	1.0	843	1	-9999.0
633	4	2.0	1382	52	30000.0
3489	4	2.0	2500	33	7333.0
4211	4	2.5	2452	100	611147.0
2585	3	2.0	1645	61	10001.0
3647	3	2.0	2402	48	30502.0
3331	4	1.0	1338	92	19642.0
1097	3	3.5	1790	118	-9999.0
2106	5	3.0	2207	98	1380.0
772	4	2.5	3256	23	11646.0
2107	2	2.5	2003	133	217965.0
2349	2	1.0	480	58	4792.0
3950	4	1.5	1536	98	9638.0
329	4	2.5	3068	19	8962.0
1650	4	2.0	1440	25	22893.0
2812	2	2.0	1926	65	88427.0
688	3	2.5	1952	98	15718.0
1883	2	2.0	948	34	-9999.0
3310	2	2.0	1301	1	-9999.0
4746	2	1.0	594	118	-9999.0
3978	4	2.0	1608	58	47916.0
314	4	2.5	2800	59	46500.0
3395	1	1.0	378	48	-9999.0
1793	2	2.0	1600	48	-9999.0

3464 rows × 5 columns

In [90]: from sklearn.model_selection import train_test_split

```
In [92]:
         # YOUR TURN
         # Use categorical variables
         # new_dataset_name = pandas_library.get_dummies(dataset, columns=[list_columns_na
         df_dummies = pd.get_dummies(dataset, columns=['PROPTYPE'])
         features=['BEDS','BATHS','SQFT','LOTSIZE','PROPTYPE_CC', 'PROPTYPE_MF', 'PROPTYPE
         target='SOLDPRICE'
         x_train, x_test, y_train,y_test = train_test_split(df_dummies[features],df_dummie
         regr.fit(features_train, prices_train)
         #Make predictions using the testing set
         regr.fit(x train,y train)
         y_pred=regr.predict(x_test)
         #Explain variance score: 1 is perfect prediction
         print('Variance Score: %.2f' % r2_score(y_test, y_pred))
         #Plot outputs
         plt.figure(figsize=(20,5))
         plt.scatter(y test, y pred, color='black')
         plt.plot([y_test.min(), y_test.max()], [y_test.min(),y_test.max()])
         plt.ylabel('Predicted')
         plt.xlabel('Measured')
         plt.xticks(())
         plt.yticks(())
         plt.show()
```

Variance Score: 0.27



```
In []: plt.figure(figsize=(10, 7))
    resid = y_test - regr.predict(X_test)

plt.axhline(y=0, linestyle='-', linewidth=2, color="r")
    plt.scatter(x=y_pred, y=resid, alpha=0.5, s=3)

plt.title("Residual plot")
    plt.ylabel(r"Y-$\hat Y$")
    plt.xlabel(r"$\hat Y$")
    plt.tight_layout()
    plt.show()
```