Topik-topik Tugas Besar (*Final Project*)
Pemodelan Sistem
(untuk IF-37-02 dan IF-37-05)

Daftar Isi

1 Pendahuluan

Pada tugas besar (*final project*) dalam kuliah Pemodelan Sistem (CSG3J3), mahasiswa diharapkan dapat memodelkan suatu sistem dengan kemampuan bernalar, matematika, pemrograman, dan penyelesaian masalah (*problem solving*) yang telah dipelajari sebelumnya pada perkuliahan-perkuliahan dasar, seperti: Kalkulus, Dasar Algoritma dan Pemrograman, Algoritma dan Struktur Data, Matematika Diskret, Probabilitas dan Statistika, dan Desain dan Analisis Algoritma. Mahasiswa tidak diharuskan melakukan pemodelan sistem dengan pendekatan tertentu seperti yang diajarkan dalam perkuliahan, karena hal tersebut justru dapat membatasi kreativitas mahasiswa dalam pendekatan permasalahan. Masalah sistem yang dimodelkan hendaknya menarik dan non trivial, meskipun dapat dilakukan secara non-deterministik.

Untuk membantu mahasiswa dalam menemukan masalah-masalah pemodelan yang menarik dan non-trivial, pengajar mengambil beberapa soal pemodelan yang pernah dimuat pada kompetisi-kompetisi pemodelan maupun pemrograman kompetitif. Soal-soal pemodelan yang diberikan pada dokumen ini diharapkan menjadi rujukan mahasiswa dalam menentukan sistem yang akan dimodelkan dalam tugas besar. Soal-soal tersebut diambil dari dua kompetisi, yaitu *Mathematical Constest in Modeling* (MCM [MCM]) dan *International Collegiate Programming Contest* (ACM-ICPC [ICPC]), keduanya merupakan kompetisisi internasional tingkat mahasiswa. Beberapa soal dimodifikasi agar sesuai dengan perkuliahan ini.

Kriteria pengerjaan tugas besar adalah:

1. Setiap kelompok paling banyak memuat empat mahasiswa.
2. Setiap kelompok di satu kelas tidak mengerjakan tugas besar dengan topik yang sama.
3. Registrasi pemilihan topik dilakukan pada forum diskusi yang telah ditentukan di `idea`.
4. Untuk mempermudah, setiap kelompok memilih satu soal pada dokumen ini untuk dikerjakan, jika tidak, maka kelompok tersebut boleh melakukan modifikasi terhadap masalah yang dijelaskan pada soal. Mo-

difikasi dilakukan agar permasalahan yang diangkat menjadi lebih menarik.

5. Perlu diketahui bahwa topik-topik yang diberikan merupakan suatu acuan (*guideline*), jadi pengerjaan pemodelan yang dilakukan oleh mahasiswa diharapkan tidak harus sama persis dengan pengerjaan soal kompetisi.

6. Kelompok yang ingin mengajukan topik sendiri harus menyampaikannya pada forum diskusi di `idea`.

7. Beberapa topik hanya dapat dikerjakan oleh kelompok yang beranggotakan tiga orang atau kurang (topik 6, 7, 8). Topik-topik tersebut adalah topik yang tidak terlalu sulit. Namun mahasiswa diharapkan memilih topik-topik lain terlebih dulu. Kelompok yang beranggotakan tiga orang atau kurang dapat memilih topik manapun.

8. Soal bertanda * merupakan *challenging problem*. Mahasiswa diharapkan memiliki pengetahuan yang cukup terkait Matematika Diskrit, Probabilitas dan Statistika, serta Desain dan Analisis Algoritma.

9. Setiap mahasiswa diharapkan sudah memiliki kelompok dan sudah memilih topik untuk dikerjakan pada akhir pekan ke-4 perkuliahan (kecuali jika ada revisi pengumuman lain dari pengajar).

10. Hal-hal lain, termasuk revisi topik (bila ada), akan disampaikan melalui `idea`.

2 The Emergency-Facilities Location Problem

(*1986 MCM Problem*)

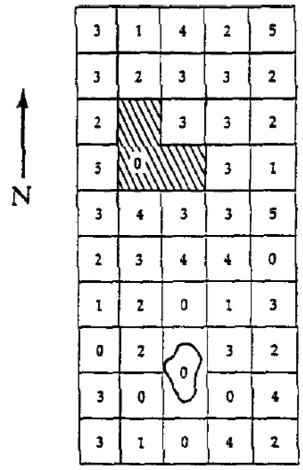

Figure 1: A map of Rio Rancho, with number of emergencies in 1985 indicated for each block.

The township of Rio Rancho has hitherto not had its own emergency facilities. It has secured funds to erect two emergency facilities in 1986, each of which will combine ambulance, fire, and police services. Figure 1. indicates the demand, or number of emergencies per square block, for 1985. The “L” region in the north is an obstacle, while the rectangle in the south is a part with a shallow pond. It takes an emergency vehicle an average of 15 seconds to go one block in the N–S direction and 20 seconds in the E–W direction. Your task it to locate the two facilities so as to minimize the total response time.

- Assume that the demand is concentrated at the center of the block and that the facilities will be located on corners.
- Assume that the demand is uniformly distributed on the streets bordering each block and that the facilities may be located anywhere on the streets.

3 The Aircraft Queueing Problem

(*1989 MCM Problem*)

A common procedure at airports is to assign aircraft to runways on a first-come-first-served basis. That is, as soon as an aircraft is ready to leave the gate ("push back"), the pilot calls ground control and is added to the queue. Suppose that a control tower has access to a fast online database with the following information for each aircraft :

- the time it is scheduled for push back;
- the time it actually pushes back;
- the number of passengers on board;
- the number of passengers who are scheduled to make a connection at the next stop, as well as the time to make that connection; and
- the schedule time of arrival at its next stop.

Assume that there are seven types of aircraft with passenger capacities varying from 100 to 400 in steps of 50. Develop and analyze a mathematical model that takes into account both the travelers' and airlines' satisfaction.

4 The Emergency Power-Restoration Problem

(*1992 MCM Problem*)

Power companies serving coastal regions must have emergency-response systems for power outages due to storms. Such systems require the input of data that allow the time and cost required for restoration to be estimated and the "value" of the outage judged by objective criteria. In the past, Hypothetical Electric Company (HECO) has been criticized in the media for its lack of a prioritization scheme.

You are a consultant to HECO power company. HECO possesses a computerized database with real-time access to service calls that currently require the information:

- time of report,
- type of requestor,
- estimated number of people affected, and
- location (x, y).

Crew sites are located at coordinates $(0, 0)$ and $(40, 40)$, where x and y are in miles. The region serviced by HECO is within $-65 < x < 65$ and $-50 < y < 50$. The region is largely metropolitan with an excellent road network. Crews must return to their dispatch site only at the beginning and end of shift. Company policy requires that no work be initiated until the storm leaves the area, unless the facility is a commuter railroad or hospital, which may be processed immediately if crews are available.

HECO has hired you to develop the objective criteria and schedule the work for the storm restoration requirements listed in Figure 2 and 3 using the work force described in Figure 4. Note that the first call was received at 4:20 A.M. and that the storm left the area at 6:00 A.M. Also note that many outages were not reported until much later in the day.

HECO has asked for a technical report for their purposes and an "executive summary" in laymen's terms that can be presented to the media. Further,

Time (A.M.)	Location	Type	# Affected	Estimated Repair Time (hrs for crew)
4:20	(−10, 30)	Business (cable TV)	?	6
5:30	(3, 3)	Residential	20	7
5:35	(20, 5)	Business (hospital)	240	8
5:55	(−10, 5)	Business (railroad sys.)	25 workers; 75,000 commuters	5
6:00	All-clear given; storm leaves area; crews can be dispatched			
6:05	(13, 30)	Residential	45	2
6:06	(5, 20)	Area*	2000	7
6:08	(60, 45)	Residential	?	9
6:09	(1, 10)	Government (city hall)	?	7
6:15	(5, 20)	Business (shopping mall)	200 workers	5
6:20	(5, −25)	Government (fire dept.)	15 workers	3
6:20	(12, 18)	Residential	350	6
6:22	(7, 10)	Area*	400	12
6:25	(−1, 19)	Industry (newspaper co.)	190	10
6:40	(−20, −19)	Industry (factory)	395	7
6:55	(−1, 30)	Area*	?	6
7:00	(−20, 30)	Government (high school)	1200 students	3
7:00	(40, 20)	Government (elementary school)	1700	?
7:00	(7, −20)	Business (restaurant)	25	12
7:00	(8, −23)	Government (police station & jail)	125	7

Figure 2: Storm restoration requirements (4:20 A.M to 7:00 A.M). *Area signifies a combination of two or more of the other classification types.

7:05	(25, 15)	Government (elementary school)	1900	5
7:10	(−10, −10)	Residential	?	9
7:10	(−1, 2)	Government (college)	3000	8
7:10	(8, −25)	Industry (computer manuf.)	450 workers	5
7:10	(18, 55)	Residential	350	10
7:20	(7, 35)	Area*	400	9
7:45	(20, 0)	Residential	800	5
7:50	(−6, 30)	Business (hospital)	300	5
8:15	(0, 40)	Business (several stores)	50	6
8:20	(15, −25)	Government (traffic lights)	?	3
8:35	(−20, −35)	Business (bank)	20	5
8:50	(47, 30)	Residential	40	?
9:50	(55, 50)	Residential	?	12
10:30	(−18, −35)	Residential	10	10
10:30	(−1, 50)	Business (civic center)	150	5
10:35	(−7, −8)	Business (airport)	350 workers	4
10:50	(5, −25)	Government (fire dept.)	15	5
11:30	(8, 20)	Area*	300	12

Figure 3: Storm restoration requirements (7:05 A.M to 11:30 A.M). *Area signifies a combination of two or more of the other classification types.

- Dispatch locations at $(0, 0)$ and $(40, 40)$.
- Crews consist of three trained workers.
- Crews report to the dispatch location only at the beginning and end of their shifts.
- One crew is scheduled for duty at all times on jobs assigned to each dispatch location. These crews would normally be performing routine assignments. Until the "storm leaves the area," they can be dispatched for "emergencies" only.
- Crews work 8-hr shifts.
- There are six crew teams available at each location.
- Crews can work only one overtime shift in a work day and receive time-and-a-half for overtime.

Figure 4: Crew descriptions.

they would like recommendations for the future. To determine your prioritized scheduling system, you will have to make additional assumptions. Detail those assumptions. In the future, you may desire additional data. If so, detail the information desired.

5 The Communications Network Problem

(*1994 MCM Problem*)

In your company, information is shared among departments on a daily basis. This information includes the previous day's sales statistics and current production guidance. It is important to get this information out as quickly as possible.

Suppose that a communications network is to be used to transfer blocks of data (files) from one computer to another. As an example, consider the graph model in Figure 5.

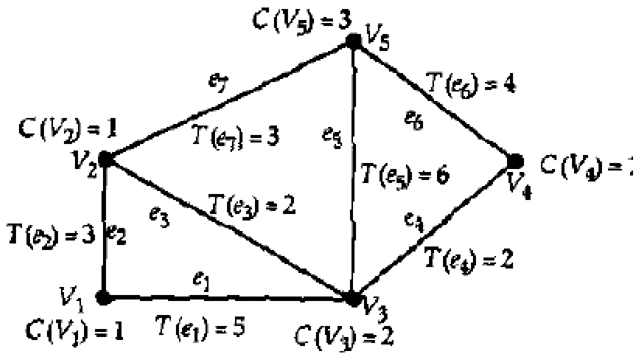

Figure 5: Example of a file transfer network.

Vertices $V_1, V_2, \ldots, V_m$ represent computers, and edges $e_1, e_2, \ldots, e_n$ represent files to be transferred (between computers represented by edge endpoints). $T(e_x)$ is the time that it takes to transfer file e_x, and $C(V_y)$ is the capacity of the computer represented by V_y to transfer files simultaneously. A file transfer involves the engagement of both computers for the entire time that it takes to transfer the file. For example, $C(V_y) = 1$ means that computer V_y can be involved in only one transfer at a time.

We are interested in scheduling the transfers in an optimal way, to minimize the total time that it takes to complete them all. This minimum total time is called the $makespan$. Consider the three following situations for your company.

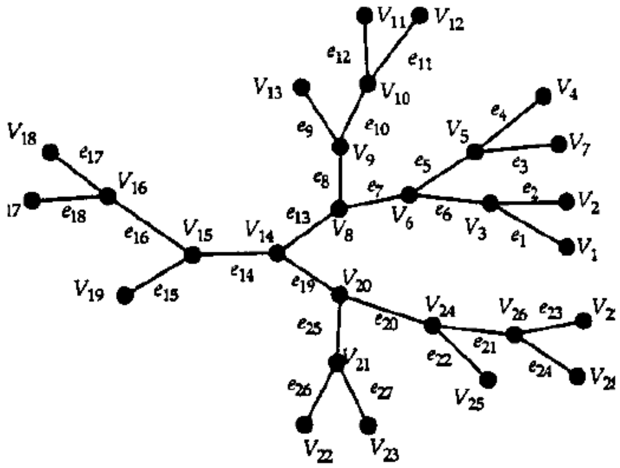

Figure 6: Network for situations A and B.

5.1 Situation A

Your corporation has 28 departments. Each department has a computer, each of which is represented by a vertex in Figure 6. Each day, 27 files must be transferred, represented by the edges in Figure 6. For this network, $T(e_x) = 1$ and $C(V_y) = 1$ for all x and y. Find an optimal schedule and the $makespan$ for the given network. Can you prove to your supervisor that your $makespan$ is the smallest possible (optimal) for the given network?

Describe your approach to solving the problem. Does your approach work for the general case, that is, where $T(e_x)$, $C(V_y)$, and the graph structure are arbitrary?

5.2 Situation B

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$T(e_x)$	3.0	4.1	4.0	7.0	1.0	8.0	3.2	2.4	5.0	8.0	1.0	4.4	9.0	3.2
x	15	16	17	18	19	20	21	22	23	24	25	26	27	
$T(e_x)$	2.1	8.0	3.6	4.5	7.0	7.0	9.0	4.2	4.4	5.0	7.0	9.0	1.2	

Figure 7: File transfer time data for situation B.

Suppose that your company changes the requirements for data transfer.

You must now consider the same basic network structure (again, see Figure 6) with different types and sizes of files. These files take the amount of time to transfer indicated in Figure 7 by the $T(e_x)$ terms for each edge. We still have $C(V_y) = 1$ for all y. Find an optimal schedule and the $makespan$ for the new network. Can you prove that your $makespan$ is the smallest possible for the new network? Describe your approach to solving this problem. Does your approach work for the general case? Comment on any peculiar or unexpected results.

5.3 Situation C

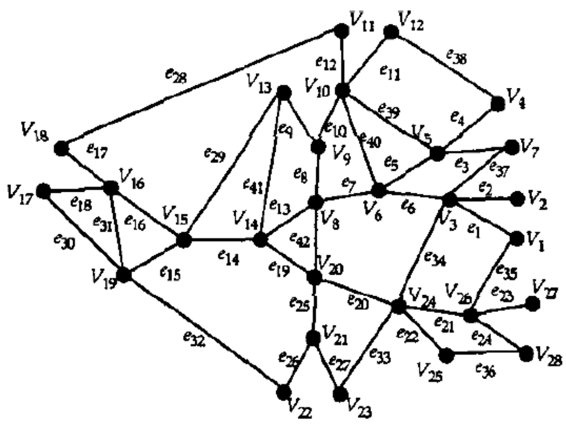

Figure 8: Network for situation C.

x	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
$T(e_x)$	6.0	1.1	5.2	4.1	4.0	7.0	2.4	9.0	3.7	6.3	6.6	5.1	7.1	3.0	6.1

Figure 9: File transfer time data for situation C, for the added transfers.

Your corporation is considering expansion. If that happens, there are several new files (edges) that will need to be transferred daily. This expansion will also include an upgrade of the computer system. Some of the 28 departments will get new computers that can handle more than one transfer at a time. All of these changes are indicated in Figure 8, 9, and 10. What is the best schedule and $makespan$ that you can find? Can you prove that your

y	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$C(V_y)$	2	2	1	1	1	1	1	1	2	3	1	1	1	2
y	15	16	17	18	19	20	21	22	23	24	25	26	27	28
$C(V_y)$	1	2	1	1	1	1	1	2	1	1	1	2	1	1

Figure 10: Computer capacity data for situation C.

$makespan$ is the smallest possible for this network? Describe your approach to solving the problem. Comment on any peculiar or unexpected results.

6 The Contest Judging Problem

(*1996 MCM Problem,* only available for group with < 4 persons)

6.1 Background Information

When determining the winner of a competition like the Mathematical Contest in Modeling, there is generally a large number of papers to judge. Let's say that there are $P = 100$ papers. A group of J judges is collected to accomplish the judging. Funding for the contest constrains both the number of judges that can be obtained and the amount of time that they can judge. For example, if $P = 100$, then $J = 8$ is typical.

Ideally, each judge would read all papers and rank-order them, but there are too many papers for this. Instead, there are a number of screening rounds in which each judge reads some number of papers and gives them scores. Then some selection scheme is used to reduce the number of papers under consideration: if the papers are rank-ordered, then the bottom 30% that each judge rank-orders could be rejected. Alternatively, if the judges do not rank-order the papers, but instead give them numerical scores (say, from 1 to 100), then all papers falling below some cutoff level could be rejected.

The new pool of papers is then passed back to the judges, and the process is repeated. A concern is that the total number of papers that each judge reads must be substantially less than P. The process is topped when there are only W papers left. These are the winners. Typically, for $P = 100$, we have $W = 3$.

6.2 Problem

Your task is to determine a selection scheme, using a combination of rank-ordering, numerical scoring, and other methods, by which the final W papers will include only papers from among the "best" $2W$ papers. (By "best" we assume that there is an absolute rank-ordering to which all judges would agree.) For example, the top three papers found by your method will consist entirely of papers from among the "best" six papers. Among all such methods, the one that requires each judge to read the least number of papers is desired.

Note the possibility of systematic bias in a numerical scoring scheme. For example, for a specific collection of papers, one judge could average 70 points, while another could average 80 points. How would you scale your scheme to accommodate for changes in the contest parameters (P, J, and W)?

7 Grade Inflation

(*1998 MCM Problem,* only available for group with < 4 persons)

7.1 Background Information

Some college administrators are concerned about the grading at A Better Class (ABC) college. On average, the faculty at ABC have been giving out high grades (the average grade now given out is an A-), and it is impossible to distinguish between the good and mediocre students. The terms of a very generous scholarship only allow the top 10% of the students to be funded, so a class ranking is required.

The dean had the thought of comparing each student to the other students in each class, and using this information to build up a ranking. For example, if a student obtains an A in a class in which all students obtain an A, then this student is only "average" in this class. On the other hand, if a student obtains the only A in a class, then that student is clearly "above average". Combining information from several classes might allow students to be placed in deciles (top 10%, next 10%, etc.) across the college.

7.2 Problem

- Assuming that the grades given out are (A+, A, A-, B+, . . .) can the dean's idea be made to work?
- Assuming that the grades given out are only (A, B, C, . . .) can the dean's idea be made to work?
- Can any other schemes produce a desired ranking?
- A concern is that the grade in a single class could change many student's deciles. Is this possible?

8 Unlawful Assembly

(*1999 MCM Problem,* only available for group with < 4 persons)

Many public facilities have signs in rooms used for public gatherings which state that it is “unlawful” for the rooms to be occupied by more than a specified number of people. Presumably, this number is based on the speed with which people in the room could be evacuated from the room’s exits in case of an emergency. Similarly, elevators and other facilities often have “maximum capacities” posted.

Develop a mathematical model for deciding what number to post on such a sign as being the “lawful capacity”. As part of your solution discuss criteria, other than public safety in the case of a fire or other emergency, that might govern the number of people considered “unlawful” to occupy the room (or space). Also, for the model that you construct, consider the differences between a room with movable furniture such as a cafeteria (with tables and chairs), a gymnasium, a public swimming pool, and a lecture hall with a pattern of rows and aisles. You may wish to compare and contrast what might be done for a variety of different environments: elevator, lecture hall, swimming pool, cafeteria, or gymnasium. Gatherings such as rock concerts and soccer tournaments may present special conditions. Apply your model to one or more public facilities at your institution (or neighboring town). Compare your results with the stated capacity, if one is posted. If used, your model is likely to be challenged by parties with interests in increasing the capacity.

9 Radio Channel Assignments*

(*2000 MCM Problem*)

We seek to model the assignment of radio channels to a symmetric network of transmitter locations over a large planar area, so as to avoid interference. One basic approach is to partition the region into regular hexagons in a grid (honeycomb-style), as shown in Figure 11, where a transmitter is located at the center of each hexagon.

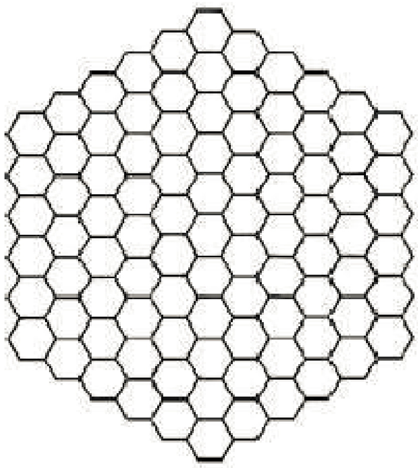

Figure 11: The pattern of regular hexagons in a grid (honeycomb-style).

An interval of the frequency spectrum is to be allotted for transmitter frequencies. The interval will be divided into regularly spaced channels, which we represent by integers $1, 2, 3, \ldots$. Each transmitter will be assigned one positive integer channel. The same channel can be used at many locations, provided that interference from nearby transmitters is avoided. Our goal is to minimize the width of the interval in the frequency spectrum that is needed to assign channels subject to some constraints. This is achieved with the concept of a $span$. The $span$ is the minimum, over all assignments satisfying the constraints, of the largest channel used at any location. It is not required that every channel smaller than the $span$ be used in an assignment that attains the $span$. Let s be the length of a side of one of the hexagons. We concentrate on the case that there are two levels of interference.

9.1 Requirement A

There are several constraints on frequency assignments. First, no two transmitters within distance $4s$ of each other can be given the same channel. Second, due to spectral spreading, transmitters within distance $2s$ of each other must not be given the same or adjacent channels: their channels must differ by at least 2. Under these constraints, what can we say about the $span$?

9.2 Requirement B

Repeat Requirement A, assuming the grid in the example spreads arbitrarily far in all directions.

9.3 Requirement C

Repeat Requirements A and B, except assume now more generally that channels for transmitters within distance $2s$ differ by at least some given integer k, while those at distance at most $4s$ must still differ by at least one. What can we say about the $span$ and about efficient strategies for designing assignments, as a function of k?

9.4 Requirement D

Consider generalizations of the problem, such as several levels of interference or irregular transmitter placements. What other factors may be important to consider?

10 Airline Overbooking

(*2002 MCM Problem*)

> You are all packed and ready to go on a trip to visit your best friend in New York City. After you check in at the ticket counter, the airline clerk announces that your flight has been overbooked. Passengers need to check in immediately to determine if they still have a seat.

Historically, airlines know that only a certain percentage of passengers who have made reservations on a particular flight will actually take that flight. Consequently, most airlines overbooked, that is, they take more reservations than the capacity of the aircraft. Occasionally, more passengers will want to take a flight than the capacity of the plane leading to one or more passengers being bumped and thus unable to take the flight for which they had reservations.

Airlines deal with bumped passengers in various ways. Some are given nothing, some are booked on later flights on other airlines, and some are given some kind of cash or airline ticket incentive.

Consider the overbooking issue in light of the current situation:

1. Less flights by airlines from point A to point B.
2. Heightened security at and around airports.
3. Passengers' fear.
4. Loss of billions of dollars in revenue by airlines to date.

Build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbooks a particular flight so that the company's revenue is maximized. Insure that your model reflects the issues above, and consider alternatives for handling "bumped" passengers. Additionally, write a short memorandum to the airline's CEO summarizing your findings and analysis.

11 A Faster QuickPass System

(*2004 MCM Problem*)

"QuickPass" systems are increasingly appearing to reduce people's time waiting in line, whether it is at tollbooths, amusement parks, or elsewhere. Consider the design of a QuickPass system for an amusement park. The amusement park has experimented by offering QuickPasses for several popular rides as a test. The idea is that for certain popular rides you can go to a kiosk near that ride and insert your daily park entrance ticket, and out will come a slip that states that you can return to that ride at a specific time later. For example, you insert your daily park entrance ticket at 1:15 P.M., and the QuickPass states that you can come back between 3:30 and 4:30 P.M. when you can use your slip to enter a second, and presumably much shorter, line that will get you to the ride faster. To prevent people from obtaining QuickPasses for several rides at once, the QuickPass machines allow you to have only one active QuickPass at a time.

You have been hired as one of several competing consultants to improve the operation of QuickPass. Customers have been complaining about some anomalies in the test system. For example, customers observed that in one instance QuickPasses were being offered for a return time as long as 4 hours later. A short time later on the same ride, the QuickPasses were given for times only an hour or so later. In some instances, the lines for people with QuickPasses are nearly as long and slow as the regular lines.

The problem then is to propose and test schemes for issuing QuickPasses in order to increase people's enjoyment of the amusement park. Part of the problem is to determine what criteria to use in evaluating alternative schemes. Include in your report a non-technical summary for amusement park executives who must choose between alternatives from competing consultants.

12 Wheel Chair Access at Airports

(*2006 MCM Problem*)

One of the frustrations with air travel is the need to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to easily walk to a different flight's waiting area. One of the ways that an airline can make the transition easier is to provide a wheel chair and an escort to those people who ask for help. It is generally known well in advance which passengers require help, but it is not uncommon to receive notice when a passenger first registers at the airport. In rare instances an airline may not receive notice from a passenger until just prior to landing.

Airlines are under constant pressure to keep their costs down. Wheel chairs wear out and are expensive and require maintenance. There is also a cost for making the escorts available. Moreover, wheel chairs and their escorts must be constantly moved around the airport so that they are available to people when their flight lands. In some large airports the time required to move across the airport is nontrivial. The wheel chairs must be stored somewhere, but space is expensive and severely limited in an airport terminal. Also, wheel chairs left in high traffic areas represent a liability risk as people try to move around them. Finally, one of the biggest costs is the cost of holding a plane if someone must wait for an escort and becomes late for their flight. The latter cost is especially troubling because it can affect the airline's average flight delay which can lead to fewer ticket sales as potential customers may choose to avoid an airline.

Epsilon Airlines has decided to ask a third party to help them obtain a detailed analysis of the issues and costs of keeping and maintaining wheel chairs and escorts available for passengers. The airline needs to find a way to schedule the movement of wheel chairs throughout each day in a cost effective way. They also need to find and define the costs for budget planning in both the short and long term.

Epsilon Airlines has asked your consultant group to put together a bid to help them solve their problem. Your bid should include an overview and analysis of the situation to help them decide if you fully understand their problem. They require a detailed description of an algorithm that you would like to implement which can determine where the escorts and wheel chairs should be and how they should move throughout each day. The goal is to keep the total costs as low as possible. Your bid is one of many that the airline will consider. You must make a strong case as to why your solution is the best and show that it will be able to handle a wide range of airports under a variety of circumstances.

Your bid should also include examples of how the algorithm would work for a large (at least four concourses), a medium (at least two concourses), and a small airport (one concourse) under high and low traffic loads. You should determine all potential costs and balance their respective weights. Finally, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid, your report should include projections of potential costs and needs in the future with recommendations to meet future needs.

13 The Airplane Seating Problem

(*2007 MCM Problem*)

Airlines are free to seat passengers waiting to board an aircraft in any order whatsoever. It has become customary to seat passengers with special needs first, followed by first-class passengers (who sit at the front of the plane). Then coach and business-class passengers are seated by groups of rows, beginning with the row at the back of the plane and proceeding forward.

Apart from consideration of the passengers' wait time, from the airline's point of view, time is money, and boarding time is best minimized. The plane makes money for the airline only when it is in motion, and long boarding times limit the number of trips that a plane can make in a day. The development of larger planes, such as the Airbus A380 (800 passengers), accentuate the problem of minimizing boarding (and deboarding) time.

Devise and compare procedures for boarding and deboarding planes with varying numbers of passengers: small ($85 - 210$), midsize ($210 - 330$), and large ($450 - 800$). Prepare an executive summary, not to exceed two single-spaced pages, in which you set out your conclusions to an audience of airline executives, gate agents, and flight crews.

14 Designing a Traffic Circle

(*2009 MCM Problem*)

Many cities and communities have traffic circles – from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a technical summary of not more than two double-spaced pages that explains to a traffic engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

15 Repeater Coordination*

(*2011 MCM Problem*)

The VHF radio spectrum involves line-of-sight transmission and reception. This limitation can be overcome by “repeaters,” which pick up weak signals, amplify them, and retransmit them on a different frequency. Thus, using a repeater, low-power users (such as mobile stations) can communicate with one another in situations where direct user-to-user contact would not be possible.

However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies. In addition to geographical separation, the “continuous tone-coded squelch system” (CTCSS), sometimes nicknamed “private line” (PL), technology can be used to mitigate interference problems. This system associates to each repeater a separate sub-audible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to received signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.

For a circular flat area of radius 40 miles radius, determine the minimum number of repeaters necessary to accommodate $1,000$ simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency, and there are 54 different PL tones available.

How does your solution change if there are $10,000$ users? Discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

16 Camping along The Big Long River

(*2012 MCM Problem*)

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar-powered rubber rafts, which travel on average 4 m.p.h. or motorized boats, which travel on average 8 m.p.h. The trips range from 6 to 18 nights of camping on the river, start to finish. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, X trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are Y camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season? The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time. In addition to your one page summary sheet, prepare a one page memo to the managers of the river describing your key findings.

17 Unloading Commuter Trains

(*2014 HiMCM Problem*)

Trains arrive often at a central station, the nexus for many commuter trains from suburbs of larger cities on a “commuter” line. Most trains are long (perhaps 10 or more cars long). The distance a passenger has to walk to exit the train area is quite long. Each train car has only two exits, one near each end so that the cars can carry as many people as possible. Each train car has a center aisle and there are two seats on one side and three seats on the other for each row of seats.

To exit a typical station of interest, passengers must exit the car, and then make their way to a stairway to get to the next level to exit the station. Usually these trains are crowded so there is a “fan” of passengers from the train trying to get up the stairway. The stairway could accommodate two columns of people exiting to the top of the stairs.

Most commuter train platforms have two tracks adjacent to the platform. In the worst case, if two fully occupied trains arrived at the same time, it might take a long time for all the passengers to get up to the main level of the station.

Build a mathematical model to estimate the amount of time for a passenger to reach the street level of the station to exit the complex. Assume there are n cars to a train, each car has length d. The length of the platform is p, and the number of stairs in each staircase is q.

Use your model to specifically optimize (minimize) the time traveled to reach street level to exit a station for the following:

1. **Requirement 1**. One fully occupied train’s passengers to exit the train, and ascend the stairs to reach the street access level of the station.

2. **Requirement 2**. Two fully occupied trains’ passengers (all passengers exit onto a common platform) to exit the trains, and ascend the stairs to reach the street access level of the station.

3. **Requirement 3**. If you could redesign the location of the stairways along the platform, where should these stairways be placed to minimize the time for one or two trains’ passengers to exit the station?

4. **Requirement 4**. How does the time to street level vary with the number s of stairways that one builds?

5. **Requirement 5**. How does the time vary if the stairways can accommodate k people, k an integer greater than one?

18 Fare and Balanced

(*2009 World Finals ACM-ICPC*)

Handling traffic congestion is a difficult challenge for young urban planners. Millions of drivers, each with different goals and each making independent choices, combine to form a complex system with sometimes predictable, sometimes chaotic behavior. As a devoted civil servant, you have been tasked with optimizing rush-hour traffic over collections of roads.

All the roads lie between a residential area and a downtown business district. In the morning, each person living in the residential area drives a route to the business district. The morning commuter traffic on any particular road travels in only one direction, and no route has cycles (morning drivers do not backtrack).

Each road takes a certain time to drive, so some routes are faster than others. Drivers are much more likely to choose the faster routes, leading to congestion on those roads. In order to balance the traffic as much as possible, you are to add tolls to some roads so that the perceived "cost" of every route ends up the same. However, to avoid annoying drivers too much, you must not levy a toll on any driver twice, no matter which route he or she takes.

Figure 12 shows a collection of five roads that form routes from the residential area (at intersection 1) to the downtown business district (at intersection 4). The driving cost of each road is written in large blue font. The dotted arrows show the three possible routes from 1 to 4. Initially the costs of the routes are 10, 8 and 12. After adding a toll of cost 2 to the road connecting 1 and 4 and a toll of cost 4 to the road connecting 3 and 4, the cost of each route becomes 12.

You must determine which roads should have tolls and how much each toll should be so that every route from start to finish has the same cost (driving time cost + possible toll) and no route contains more than one toll road. Additionally, the tolls should be chosen so as to minimize the final cost. In some settings, it might be impossible to impose tolls that satisfy the above conditions.

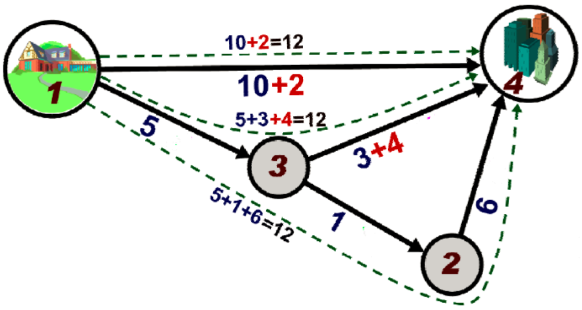

Figure 12: Roads connecting residential area at intersection 1 to business district at intersection 4.

19 Asteroid Rangers*

(*2012 World Finals ACM-ICPC*)

The year is 2112 and humankind has conquered the solar system. The Space Ranger Corps have set up bases on any hunk of rock that is even remotely inhabitable. Your job as a member of the Asteroid Communications Ministry is to make sure that all of the Space Ranger asteroid bases can communicate with one another as cheaply as possible. You could set up direct communication links from each base to every other base, but that would be prohibitively expensive. Instead, you want to set up the minimum number of links so that everyone can send messages to everyone else, potentially relayed by one or more bases. The cost of any link is directly proportional to the distance between the two bases it connects, so this doesn't seem that hard of a problem.

There is one small difficulty, however. Asteroids have a tendency to move about, so two bases that are currently very close may not be so in the future. Therefore as time goes on, you must be willing to switch your communication links so that you always have the cheapest relay system in place. Switching these links takes time and money, so you are interested in knowing how many times you will have to perform such a switch.

A few assumptions make your task easier. Each asteroid is considered a single point. Asteroids always move linearly with a fixed velocity. No asteroids ever collide with other asteroids. Also, any relay system that becomes optimal at a time t_0 will be uniquely optimal for any time s satisfying $t < s < t + 10^{-6}$. The initial optimal relay system will be unique.

20 Shortest Flight Path*

(*2012 World Finals ACM-ICPC*)

Commercial flights are statistically quite safe (in terms of number of deaths per passenger-kilometer, only going to the moon is safer). But there are still reasons for precautions and safety regulations. An early such rule was the so-called "60-minute rule," which required that a two-engine plane must always be within 60 minutes of the nearest adequate airport along its entire flight path. A variety of similar rules have existed, but at their core, they remain the same: the flight path can not take the airplane more than a certain maximum allowed distance from the nearest airport. With these restrictions, planes cannot always use a direct route for flying from one airport to another.

In this problem we will compute the shortest flight path between two airports while adhering to a maximum allowed distance rule. In Figure 13, which illustrates the first sample test case, any flight route has to stay within the three circles. Thus a plane going from airport 2 to airport 3 has to detour from the direct route via the region around airport 1. Note that the plane would not necessarily have to go to airport 1 itself.

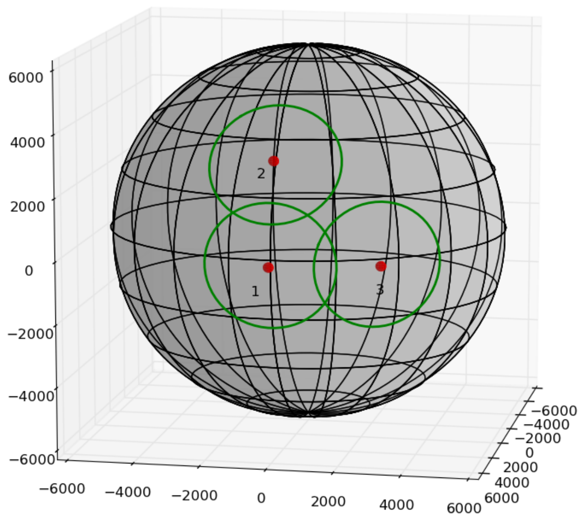

Figure 13: An example of a flight path.

Things are further complicated by the fact that planes have limited fuel supply, and to go longer distances they may need to make a stopover at intermediate airports. Thus, depending on the fuel capacity, a plane going from airport 2 to airport 3 in Figure 13 might have to stop over at airport 1 (or the fuel capacity might be too low even to go to airport 1, in which case the trip would be impossible to make).

You may use the following simplifying assumptions:

1. The surface of the earth is a sphere of radius 6370 km.

2. Both time and fuel consumption are directly proportional to distance traveled. In other words we are interested only in total distance traveled.

3. The difference in distance caused by planes flying at different altitudes is negligible. Thus, effectively,we assume them to be flying along the earth's surface.

4. A plane may stop for refueling at as many intermediate airports as needed, each time getting a full tank.

20.1 Input

The first line of each test case contains two integers N and R, where $2 \leq N \leq 25$ is the number of airports and $1 \leq R \leq 10000$ is the maximum allowed flight distance (in km) from the nearest airport. Each of the next N lines contains two integers ϕ and θ, satisfying $0 \leq \phi < 360$ and $-90 \leq \theta \leq 90$, the longitude and latitude (respectively) of an airport, in degrees. The airports are numbered according to their order in the input starting from one. No two airports are at the same position.

Following this is a line containing an integer Q, satisfying $1 \leq Q \leq 100$. Each of the next Q lines contains three integers s, t, c satisfying $1 \leq s, t \leq N$, $s \neq t$, and $1 \leq c \leq 50000$, indicating a plane going from airport s to airport t with a fuel capacity yielding a range of c km.

20.2 Output

For each test case, display the case number followed by one line for each query containing the length in km of the shortest flight path between airport s and t, subject to the fuel constraint c. Display the length accurate to three decimal places. If there is no permissible path between the two airports, then display the word `impossible` instead.

You may assume the answer is numerically stable for perturbations of up to $0.1\ \mathrm{km}$ of R or c. The ACM-ICPC input and output sample is depicted in Figure 14.

Sample Input	**Output for Sample Input**
3 2000 0 0 0 30 30 0 3 2 3 5000 2 3 4000 2 3 3000 2 10000 45 45 225 -45 2 1 2 50000 2 1 50000	Case 1: 4724.686 6670.648 impossible Case 2: impossible impossible

Figure 14: Input-output sample for Shortest Flight Path problem.

21 Surely You Congest

(*2015 World Finals ACM-ICPC*)

You are in charge of designing an advanced centralized traffic management system for smart cars. The goal is to use global information to instruct morning commuters, who must drive downtown from the suburbs, how best to get to the city center while avoiding traffic jam.

Unfortunately, since commuters know the city and are selfish, you cannot simply tell them to travel routes that take longer than normal (otherwise they will just ignore your directions). You can only convince them to change to different routes that are equally fast.

The city's network of roads consists of intersections that are connected by bidirectional roads of various travel times. Each commuter starts at some intersection, which may vary from commuter to commuter. All commuters end their journeys at the same place, which is downtown at intersection 1. If two commuters attempt to start travelling along the same road in the same direction at the same time, there will be congestion; you must avoid this. However, it is fine if two commuters pass through the same intersection simultaneously or if they take the same road starting at different times.

Determine the maximum number of commuters who can drive downtown without congestion, subject to all commuters starting their journeys at exactly the same time and without any of them taking a suboptimal route.

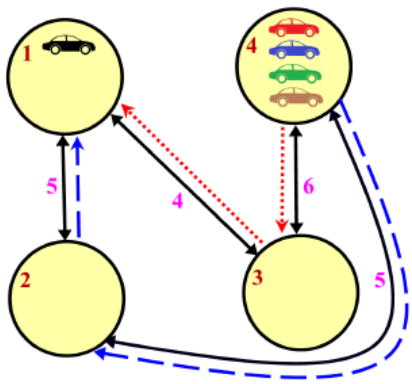

Figure 15: Ilustation of maximum number of commuters who can reach downtown without congestion.

In Figure 15, cars are shown in their original locations. One car is already downtown. Of the cars at intersection 4, one can go along the dotted route through intersection 3, and another along the dashed route through intersection 2. But the remaining two cars cannot reach downtown while avoiding congestion. So a maximum of 3 cars can reach downtown with no congestion.

21.1 Input

The input consists of a single test case. The first line contains three integers n, m, and c, where n ($1 \leq n \leq 25000$) is the number of intersections, m ($0 \leq m \leq 50000$) is the number of roads, and c ($0 \leq c \leq 1000$) is the number of commuters. Each of the next m lines contains three integers x_i, y_i, and t_i describing one road, where x_i and y_i ($1 \leq x_i, y_i, n$) are the distinct intersections the road connects, and t_i ($1 \leq t_i \leq 10000$) is the time it takes to travel along that road in either direction. You may assume that downtown is reachable from every intersection. The last line contains c integers listing the starting intersections of the commuters.

21.2 Output

Display the maximum number of commuters who can reach downtown without congestion. The ACM-ICPC input and output sample is depicted in Figure 16.

Sample Input 1	Sample Output 1
3 3 2 1 2 42 2 3 1 2 3 1 2 3	2

Sample Input 2	Sample Output 2
4 4 5 1 2 5 1 3 4 4 2 5 4 3 6 4 4 4 4 1	3

Figure 16: Input-output sample for Surely You Congest problem.

22 Surveillance*

(*2015 World Finals ACM-ICPC*)

The International Corporation for Protection and Control (ICPC) develops efficient technology for, well, protection and control. Naturally, they are keen to have their own headquarters protected and controlled. Viewed from above, the headquarters building has the shape of a convex polygon. There are several suitable places around it where cameras can be installed to monitor the building. Each camera covers a certain range of the polygon sides (building walls), depending on its position. ICPC wants to minimize the number of cameras needed to cover the whole building.

22.1 Input

The input consists of a single test case. Its first line contains two integers n and k ($3 \leq n \leq 10^6$ and $1 \leq k \leq 10^6$), where n is the number of walls and k is the number of possible places for installing cameras. Each of the remaining k lines contains two integers a_i and b_i ($1 \leq a_i, b_i \leq n$). These integers specify which walls a camera at the i-th place would cover. If $a_i \leq b_i$ then the camera covers each wall j such that $a_i \leq j \leq b_i$. If $a_i > b_i$ then the camera covers each wall j such that $a_i \leq j \leq n$ or $1 \leq j \leq bi$.

22.2 Output

Display the minimal number of cameras that suffice to cover each wall of the building. The ranges covered by two cameras may overlap. If the building cannot be covered, display `impossible` instead. The ACM-ICPC input and output sample is depicted in Figure

Sample Input 1

```
100 7
1 50
50 70
70 90
90 40
20 60
60 80
80 20
```

Sample Output 1

```
3
```

Sample Input 2

```
8 2
8 3
5 7
```

Sample Output 2

```
impossible
```

Sample Input 3

```
8 2
8 4
5 7
```

Sample Output 3

```
2
```

Figure 17: Input-output sample for Surveillance problem.

23 Weather Reports*

(*2015 World Finals ACM-ICPC*)

You have been hired by the Association for Climatological Measurement, a scientific organization interested in tracking global weather trends over a long period of time. Of course, this is no easy task. They have deployed many small devices around the world, designed to take periodic measurements of the local weather conditions. These are cheap devices with somewhat restricted capabilities. Every day they observe which of the four standard kinds of weather occurred: sunny, cloudy, rainy, or foggy. After every n of these observations have been made, the results are reported to the main server for analysis. However, the massive number of devices has caused the available communication bandwidth to be overloaded. The Association needs your help to come up with a method of compressing these reports into fewer bits.

For a particular device's location, you may assume that the weather each day is an independent random event, and you are given the predicted probabilities of the four possible weather types. Each of the 4^n possible weather reports for a device must be encoded as a unique sequence of bits, such that no sequence is a prefix of any other sequence (an important property, or else the server would not know when each sequence ends). The goal is to use an encoding that minimizes the expected number of transmitted bits.

23.1 Input

The first line of input contains an integer $1 \leq n \leq 20$, the number of observations that go into each report. The second line contains four positive floating-point numbers, p_{sunny}, p_{cloudy}, p_{rainy}, and p_{foggy}, representing the respective weather probabilities. These probabilities have at most 6 digits after the decimal point and sum to 1.

23.2 Output

Display the minimum expected number of bits in the encoding of a report, with an absolute or relative error of at most 10^{-4}. The ACM-ICPC input and output sample is depicted in Figure 18.

Sample Input 1	Sample Output 1
2 0.9 0.049999 0.05 0.000001	1.457510

Sample Input 2	Sample Output 2
20 0.25 0.25 0.25 0.25	40.000000

Figure 18: Input-output sample for Weather Reports problem.

Referensi

[MCM] https://www.comap.com/undergraduate/contests/mcm/previous-contests.php

[ICPC] https://icpc.baylor.edu/worldfinals/problems