1 Problem

Compute the derivatives of the least squares loss $L[\phi]$ with respect to ϕ_0 and ϕ_1 for the Gabor model equation 6.8.

2 Answer

$$f[x,\phi] = sin(\phi_0 + 0.06 \cdot \phi_1 x) \cdot exp\left(\frac{-(\phi_0 + 0.06 \cdot \phi_1 x)^2}{32.0}\right) \tag{1}$$

$$L[\phi] = \sum_{i=1}^{I} (f[x_i,\phi] - y_i)^2 \tag{2}$$

$$\tag{3}$$

$$\text{Let } u_i = f[x_i,\phi] - y_i$$

$$\text{Then } L[\phi] = \sum_{i=1}^{I} (u_i)^2$$

$$\frac{\partial L[\phi]}{\partial \phi_0} = \sum_{i=1}^{I} \frac{\partial L[\phi]}{\partial u_i} \cdot \frac{\partial u_i}{\partial \phi_0}$$

$$\text{Let us break it down } \frac{\partial L[\phi]}{\partial u_i} = 2 \cdot u_i = 2 \cdot \left(sin(\phi_0 + 0.06 \cdot \phi_1 x) \cdot exp\left(\frac{-(\phi_0 + 0.06 \cdot \phi_1 x_i)^2}{32.0}\right) - y_i \right)$$

$$\frac{\partial u_i}{\partial \phi_0} = \frac{\partial sin(\phi_0 + 0.06 \cdot \phi_1 x) \cdot exp\left(\frac{-(\phi_0 + 0.06 \cdot \phi_1 x_i)^2}{32.0}\right)}{\partial \phi_0}$$

$$\text{Let } p = sin(\phi_0 + 0.06 \cdot \phi_1 x)$$

$$\text{Let } q = exp\left(\frac{-(\phi_0 + 0.06 \cdot \phi_1 x_i)^2}{32.0}\right)$$

$$\text{So that } \frac{\partial u_i}{\partial \phi_0} = p\frac{\partial q}{\partial \phi_0} + q\frac{\partial p}{\partial \phi_0}$$