Problem 7.2

May 12, 2024

1 Problem

Find the final derivatives for

$$
\begin{aligned}
f_0 &= \beta_0 + \omega_0 \cdot x_i \\
h_1 &= \sin[f_0] \\
f_1 &= \beta_1 + \omega_1 \cdot h_1 \\
h_2 &= \exp[f_1] \\
f_2 &= \beta_2 + \omega_2 \cdot h_2 \\
h_3 &= \cos[f_2] \\
f_3 &= \beta_3 + \omega_3 \cdot h_3 \\
l_i &= (f_3 - y_i)^2
\end{aligned}
$$

2 Answer

$$
\begin{aligned}
\frac{\partial l_i}{\partial f_3} &= 2(f_3 - y_i) \\
\frac{\partial l_i}{\partial h_3} &= \frac{\partial l_i}{\partial f_3}\frac{\partial f_3}{\partial h_3} = 2(f_3 - y_i) \cdot \omega_3 \\
\frac{\partial l_i}{\partial f_2} &= \frac{\partial l_i}{\partial h_3}\frac{\partial h_3}{\partial f_2} = 2(f_3 - y_i) \cdot \omega_3 \cdot -\sin[f_2] = -2\omega_3(f_3 - y_i)\sin[f_2] \\
\frac{\partial l_i}{\partial h_2} &= \frac{\partial l_i}{\partial f_2}\frac{\partial f_2}{\partial h_2} = -2\omega_3(f_3 - y_i)\sin[f_2] \cdot \omega_2 = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2] \\
\frac{\partial l_i}{\partial f_1} &= \frac{\partial l_i}{\partial h_2}\frac{\partial h_2}{\partial f_1} = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \\
\frac{\partial l_i}{\partial h_1} &= \frac{\partial l_i}{\partial f_1}\frac{\partial f_1}{\partial h_1} = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \omega_1 = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \\
\frac{\partial l_i}{\partial f_0} &= \frac{\partial l_i}{\partial h_1}\frac{\partial h_1}{\partial f_0} = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \cos[f_0]
\end{aligned}
$$