Problem 7.6

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1 Problem

Show that for

$$\boldsymbol{z} = \boldsymbol{\beta} + \boldsymbol{\Omega}\boldsymbol{h}$$

$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{h}} = \boldsymbol{\Omega}^T$$

2 Answer

Using numerator-layout notation. $\boldsymbol{z}$ is $m \times 1$ and $\boldsymbol{h}$ is $n \times 1$. $\boldsymbol{\Omega}$ is $m \times n$.

$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{h}} = \begin{bmatrix} \frac{\partial z_1}{h_0} & \frac{\partial z_1}{h_1} & \frac{\partial z_1}{h_2} & \cdots & \frac{\partial z_0}{h_n} \\ \frac{\partial z_2}{h_0} & \frac{\partial z_2}{h_1} & \frac{\partial z_2}{h_2} & \cdots & \frac{\partial z_1}{h_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_m}{h_0} & \frac{\partial z_m}{h_1} & \frac{\partial z_m}{h_2} & \cdots & \frac{\partial z_m}{h_n} \end{bmatrix}$$

$$z_i = \beta_i + \sum_j \omega_{ij} h_j$$

$$\text{Therefore } \frac{\partial z_i}{\partial h_j} = \omega_{ij}$$

Looking at the top row of the matrix, it is comprised of each ω_{0j} of the first row of Ω. However, the text book uses the denominator layout. This transposes our above matrix.

$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{h}} = \begin{bmatrix} \frac{\partial z_1}{h_1} & \frac{\partial z_2}{h_1} & \frac{\partial z_3}{h_1} & \cdots & \frac{\partial z_m}{h_1} \\ \frac{\partial z_1}{h_2} & \frac{\partial z_2}{h_2} & \frac{\partial z_3}{h_2} & \cdots & \frac{\partial z_m}{h_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_1}{h_n} & \frac{\partial z_2}{h_n} & \frac{\partial z_3}{h_n} & \cdots & \frac{\partial z_m}{h_n} \end{bmatrix}$$

Looking at the top row of the matrix, it is comprised of each ω_{j0} of the first column of Ω. And therefore the answer is a matrix comprising ω_{ji} which is $\boldsymbol{\Omega}^T$.