Problem 7.4

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1 Problem

Calculate the derivative $\frac{\partial l_i}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]}$ for the least squares loss function $l_i = (y_i - f[\boldsymbol{x}_i, \boldsymbol{\phi}])^2$.

2 Answer

$$\begin{aligned} \operatorname{Let} \ u &= (y_i - f[\boldsymbol{x}_i, \boldsymbol{\phi}]) \\ l_i &= u^2 \end{aligned}$$
 Using the Chain Rule
$$\frac{\partial l_i}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]} = \frac{\partial l_i}{\partial u} \cdot \frac{\partial u}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]}$$
 We have
$$\frac{\partial u}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]} = -1$$

$$\frac{\partial l_i}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]} = 2 \cdot u \cdot -1$$

$$\frac{\partial l_i}{\partial f[\boldsymbol{x}_i, \boldsymbol{\phi}]} = -2 \left(y_i - f[\boldsymbol{x}_i, \boldsymbol{\phi}] \right)$$