Problem 7.2

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May 12, 2024

1 Problem

Find the final derivatives for

$$f_0 = \beta_0 + \omega_0 \cdot x_i$$
$$h_1 = \sin[f_0]$$
$$f_1 = \beta_1 + \omega_1 \cdot h_1$$
$$h_2 = \exp[f_1]$$
$$f_2 = \beta_2 + \omega_2 \cdot h_2$$
$$h_3 = \cos[f_2]$$
$$f_3 = \beta_3 + \omega_3 \cdot h_3$$
$$l_i = (f_3 - y_i)^2$$

2 Answer

$$\frac{\partial l_i}{\partial f_3} = 2(f_3 - y_i)$$

$$\frac{\partial l_i}{\partial h_3} = \frac{\partial l_i}{\partial f_3}\frac{\partial f_3}{\partial h_3} = 2(f_3 - y_i) \cdot \omega_3$$

$$\frac{\partial l_i}{\partial f_2} = \frac{\partial l_i}{\partial h_3}\frac{\partial h_3}{\partial f_2} = 2(f_3 - y_i) \cdot \omega_3 \cdot -\sin[f_2] = -2\omega_3(f_3 - y_i)\sin[f_2]$$

$$\frac{\partial l_i}{\partial h_2} = \frac{\partial l_i}{\partial f_2}\frac{\partial f_2}{\partial h_2} = -2\omega_3(f_3 - y_i)\sin[f_2] \cdot \omega_2 = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2]$$

$$\frac{\partial l_i}{\partial f_1} = \frac{\partial l_i}{\partial h_2}\frac{\partial h_2}{\partial f_1} = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2] \cdot \exp[f_1]$$

$$\frac{\partial l_i}{\partial h_1} = \frac{\partial l_i}{\partial f_1}\frac{\partial f_1}{\partial h_1} = -2\omega_3\omega_2(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \omega_1 = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1]$$

$$\frac{\partial l_i}{\partial f_0} = \frac{\partial l_i}{\partial h_1}\frac{\partial h_1}{\partial f_0} = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \cos[f_0]$$

$$\frac{\partial l_i}{\partial \beta_0} = \frac{\partial l_i}{\partial f_0}\frac{\partial f_0}{\partial \beta_0} = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \cos[f_0]$$

$$\frac{\partial l_i}{\partial \omega_0} = \frac{\partial l_i}{\partial f_0}\frac{\partial f_0}{\partial \omega 0} = -2\omega_3\omega_2\omega_1(f_3 - y_i)\sin[f_2] \cdot \exp[f_1] \cdot \cos[f_0] \cdot x_i$$

The final derivatives are just multiplying by 1 and h_k respectively from looking up the omega or beta from the formula definitions.

```
dldbeta3 = 2 * (f3 - y) * 1
dldomega3 = 2 * (f3 - y) * h3
dldbeta2 = dldf2 * 1
dldomega2 = dldf2 * h2
dldbeta1 = dldf1 * 1
dldomega1 = dldf1 * h1
dldbeta0 = dldf0 * 1
dldomega0 = dldf0 * x
```