Systems Simulation
Single-server queuing system

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Example (Grocery simulation)

- In a small grocery, customers arrive at the counter at random times.
- The interarrival time (in minutes) is an integer value and belongs to the **uniform** distribution (for simplicity) in the interval 1-8.
- Each interarrival time has the same probability to occurring
  - We already know how to calculate the equally likely probability
- Also, we need to compute the cumulative probability

Interarrival Time [minute]	Probability	Cumulative Probability
1		
2		
3		
4		
5		
6		
7		
8		

Cont.

- The cumulative probability calculation is important as it helps to determine the interarrival time, based on a generated random number 0-1
  - split the range into intervals

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.125	0.125	001–125
2	0.125	0.250	126–250
3	0.125	0.375	251–375
4	0.125	0.500	376–500
5	0.125	0.625	501–625
6	0.125	0.750	626–750
7	0.125	0.875	751–875
8	0.125	1.000	876–000

- We assign random digits (integer intervals) based on the cumulative probabilities.
- This is used for educational purposes. We can make this based on the cumulative probability directly

Cont.

- Now, for each customer in our simulation, we generate a random number and assign a random interarrival time based on the distribution in the previous table

Customer	Random Digits	Time between Arrivals (Minutes)	Customer	Random Digits	Time between Arrivals (Minutes)
1	–	–	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

Cont.

- Also, the service time is another variable that needs to be simulated.
  - it should have its distribution, which can be figured out from historical data
- For this example, let us assume we have the following distribution of the service time, where the service time varies between 1-6 minutes

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.10		
2	0.20		
3	0.30		
4	0.25		
5	0.10		
6	0.05		

Cont.

- Like interarrival time, we generate service time for each of the customers in our system
  - The first customer has a service time, but no interarrival time

Customer	Random Digits	Service Time (Minutes)	Customer	Random Digits	Service Time (Minutes)
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	79	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

Cont.

- Now we can start calculating (filling) the simulation table

These two columns come from the previous distributions

They are not necessarily to be filled in the table at once, we can fill them line by line

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0				
2	8		1					
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Performance measurements

- There are some simple mathematical formulas that can help you filling the table

Time Service Begins (TSB)=max(Arrival Time (AT), Previous Customer's Service End Time (PCSET))

Waiting Time in Queue (WTIQ)=Time Service Begins (TSB)−Arrival Time (AT)

Service End Time (SET) =Time Service Begins (TSB) + Service Time (ST)

Time in System (TIS)=Service End Time (SET) −Arrival Time (AT)

Idle Time (IT) =Time Service Begins (TSB) −Previous Customer's Service End Time (PCSET) (for each customer after the first where this calculation is positive).

Cont.

TSB=max(AT, PCSET)
SET = TSB + ST
WTIQ= TSB − AT
TIS = SET − AT
IT = TSB − PCSET

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1					
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

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Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6		4					
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
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1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1		3					
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Cont.

TSB=max(AT, PCSET)
SET = TSB + ST
WTIQ= TSB − AT
TIS = SET − AT
IT = TSB − PCSET

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8		2					
6	3		4					
7	8		5					
8	7		4					
9	2		5					
10	3		3					
11	1		3					
12	1		5					
13	5		4					
14	6		1					
15	3		5					
16	8		4					
17	1		3					
18	2		3					
19	4		2					
20	5		3					
Total	82		68					

Cont.

You can simulate based on the number of customers (e.g., run for 20 customers)

OR for based on a duration (e.g., run for

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	—	0	4	0	4	0	4	0
2	8	8	1	8	9	0	1	4
3	6	14	4	14	18	0	4	5
4	1	15	3	18	21	3	6	0
5	8	23	2	23	25	0	2	2
6	3	26	4	26	30	0	4	1
7	8	34	5	34	39	0	5	4
8	7	41	4	41	45	0		2
9	2	43	5	45	50			
10	3	46	3	50	53			
11	1	47	3	53	56			
12	1	48	5	56	61			
13	5	53	4	61	65			
14	6	59	1	65	66			
15	3	62	5	66	71	4	9	0
16	8	70	4	71	75	1	5	0
17	1	71	3	75	78	4	7	0
18	2	73	3	78	81	5	8	0
19	4	77	2	81	83	4	6	0
20	5	82	3	83	86	1	4	0
Total	82		68			56	124	18

Total Run Time of Simulation

Useful measurements

- One can use the previous table to extract useful metrics to evaluate the system:

- Average waiting time:

$$AWT = \frac{total\ time\ customers\ waited\ in\ the\ queue}{total\ number\ of\ customers} = \frac{56}{20} = 2.8\ minutes$$

- Probability to wait

$$PTW = \frac{nmber\ of\ custmers\ who\ waited}{total\ number\ of\ customers} = \frac{13}{20} = 0.65$$

Cont.

- Probability of idle server

$$PIS = \frac{total\ idle\ time}{total\ run - time\ of\ the\ simulation} = \frac{18}{86} = 0.21$$

- Probability of busy time

$$PBS = 1 - PIS$$

Cont.

- Average service time:

$$AST = \frac{total\ service\ time}{totla\ number\ of\ customers} = \frac{68}{20} = 3.4\ minutes$$

This value should be similar to the expected value of the service time distribution $E(S)$

$$E(S) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.25) + 5(0.1) + 6(0.05) = 3.2\ minutes$$

- Average time between arrivals

$$ATBA = \frac{sum\ of\ the\ interarrival\ time}{number\ of\ customers\ (arrivals) - \mathbf{1}\ (\boldsymbol{first\ arrival\ has\ not\ value})} = \frac{82}{19} = 4.3\ minutes$$

Cont.

- Average waiting time for the actual waited customers

$$Actual - AWT = \frac{total\ time\ customers\ waited\ in\ the\ queue}{total\ number\ of\ customers\ who\ waited\ (non\ zero\ wait\ time)} = \frac{56}{20} = \quad 4.3\ minutes$$

- **A**verage **t**ime spent **i**n the **s**ystem

$$ATIS = \frac{total\ time\ sent\ in\ system\ by\ all\ custmers}{total\ number\ of\ customers} = \frac{124}{20} = 6.4\ minutes$$

Example

- Create a simulation table to simulate a coffee shop given the following distributions of the **interarrival time** and the **service time**

Interarrival Time (minutes)	Probability
1	0.10
2	0.15
3	0.20
4	0.25
5	0.20
6	0.10

Service Time (minutes)	Probability
3	0.20
4	0.25
5	0.30
6	0.15
7	0.10

Example

Create a simulation table to simulate a coffee shop given the following distributions of the **interarrival time** and the **service time**

Interarrival Time (minutes)	Probability
1	0.15
2	0.15
3	0.20
4	0.30
5	0.20

Service Time (minutes)	Probability
3	0.30
4	0.25
5	0.30
6	0.15

Customer	*Random Digits*	*Service Time (Minutes)*
1	84	
2	10	
3	74	
4	53	
5	17	
6	79	

Customer	*Random Digits*	*Service Time (Minutes)*
1	84	
2	10	
3	74	
4	53	
5	17	
6	79	

Example

Create a simulation table to simulate a coffee shop given the following distributions of the **interarrival time** and the **service time**

Interarrival Time (minutes)	Probability	cumulative probability	Random Digit Assignment
1	0.15		
2	0.15		
3	0.20		
4	0.30		
5	0.20		

Example

Create a simulation table to simulate a coffee shop given the following distributions of the **interarrival time** and the **service time**

Service Time (minutes)	Probability	cumulative probability	Random Digit Assignment
3	0.30		
4	0.25		
5	0.30		
6	0.15		

Example

Create a simulation table to simulate a coffee shop given the following distributions of the **interarrival time** and the **service time**

Customer	Random Digits	Time between Arrivals (Minutes)
1		
2	10	
3	74	
4	53	
5	17	
6	79	

Customer	Random Digits	Service Time (Minutes)
1	84	
2	10	
3	74	
4	53	
5	17	
6	79	

Simulation System

Performance Measure

Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1								
2								
3								
4								
5								
6								