Chapter 4:

4.2 A sample space *S* consists of five simple events with these probabilities:
***P*(*E*1)= *P*(*E*2)=0.15, *P*(*E*3)=0.4, *P*(*E*4)= 2*P*(*E*5)**

a. Find the probabilities for simple events *E*4 and *E*5.

$$\sum P(x) = 1 \rightarrow 0.15 + 0.15 + 0.40 + 2x + x = 1 \rightarrow x = \frac{0.30}{3} = 0.10.$$

P(E4)=0.20, P(E5)=0.10.

b. Find the probabilities for these two events:
A: {*E*1, *E*3, *E*4}, P(A)=0.75,
B :{*E*2, *E*3} , P(B)=0.55,

c. List the simple events that are either in event *A* or event *B* or both.
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.75 + 0.55 - 0.40 = 0.90.$

d. List the simple events that are in both event *A* and event *B.*
$P(A \cap B) = 0.40$

4.3 A sample space contains 10 simple events: *E*1, *E*2, . . . , *E*10.
If *P*(*E*1) =3*P*(*E*2) = 0.45 and the remaining simple events are equiprobable, find the probabilities of these remaining simple events.

$$\sum P(x) = 1 \rightarrow 0.45 + 0.15 + 8x = 1 \rightarrow x = \frac{0.4}{8} = 0.05.$$

4.21 Choosing People In how many ways can you select five people from a group of eight if the order of selection is important?
nPr

8P5=6720

4.22 Choosing People, again: In how many ways can you select two people from a group of 20 if the order of selection is **not important**?
nCr

20C2=190 ways

4.23 Dice Three dice are tossed. How many simple events are in the sample space?

6*6*6=216

4.26 What to Wear? You own 4 pairs of jeans, 12 clean T-shirts, and 4 wearable pairs of sneakers. How many outfits (jeans, T-shirt, and sneakers) can you create?

4*12*4=192 outfits

4.42 An experiment can result in one of five equally likely simple events, *E*1, *E*2, . . . , *E*5. Events *A*, *B*, and *C* are defined as follows:
***A*: *E*1, *E*3 ,,,, *P*(*A*)=0.4, *B*: *E*1, *E*2, *E*4, *E*5,,, *P*(*B*)=0.8, *C*: *E*3, *E*4 ,,,,,*P*(*C*)=0.4,**
Find the probabilities associated with these compound events by listing the simple events in each.

- **a.** $P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6$
- **b.** $P(A \cap B) = P(E1) = 0.20.$
- **c.** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.2 = 1.$
- **d.** $P(A|B) = \frac{0.2}{0.8}.$
- **e.** $P(A \cap B)^c = 1 - P(A \cap B) = 1 - 0.2 = 0.20.$

4.49 Suppose that *P*(*A*)=0.4 and *P*(*B*)=0.2. If events *A* and *B* are independent, find these probabilities:

a. $P(A \cap B) = 0.4 * 0.2 = 0.08.$
b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.2 - 0.08 = 0.52.$

4.50 Suppose that *P*(*A*) =0.3 and *P*(*B*)=0.5. If events *A* and *B* are mutually exclusive, find these probabilities:

a. $P(A \cap B) = 0.$
b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.5 = 0.80.$

4.51 Suppose that $\boldsymbol{P(A) = 0.4 \;\; and \; P(A \cap B) = 0.12}$.

- **a.** Find *P*(*B*|*A*)=0.12 * 0.4
- **b.** Are events *A* and *B* mutually exclusive? No.

c. If *P*(*B*) =0.3, are events *A* and *B* independent? Yes.
$P(A \cap B) = P(A) * P(B) = 0.3 * 0.4 = 0.12$

<u>4.69. Bayes' Rule</u> :

A sample is selected from one of two populations, *S*1 and *S*2, with probabilities *P*(*S*1)=0 .7 and *P*(*S*2)= 0.3. If the sample has been selected from *S*1, the probability of observing an event *A* is *P*(*A*|*S*1)=0.2. Similarly, if the sample has been selected from *S*2, the probability of observing *A* is *P*(*A*|*S*2)=0.3.

- **a. If a sample is randomly selected from one of the two populations, what is the probability that event *A* occurs?**

$$P(A) = 0.7 * 0.2 + 0.3 * 0.3 = 0.23$$

- **b. If the sample is randomly selected and event *A* is observed, what is the probability that the sample was selected from population *S*1?**

$$P(S1|A) = 0.14/0.23$$

4.83 Probability Distribution II :A random variable *x* can assume five values: 0, 1, 2, 3, 4. A portion of the probability distribution is shown here:

x	**P(x)**	**X*P(x)**	**X^2 *P(x)**
0	**0.1**	**0**	**0**
1	**0.3**	**0.3**	**0.3**
2	**0.3**	**0.6**	**1.2**
3	**a**	**0.6**	**1.8**
4	**0.1**	**0.4**	**1.6**
S	**1**	**1.90**	**4.90**

a. Find *p*(3)=a=1-0.8=0.20.

b. **b.** Calculate the population mean, variance, and standard deviation.

$$\mu = mean = \sum x * P(x) = 1.9$$

$$EX^2 = \sum x^2 * P(x) = 4.9$$

$$\sigma^2 = EX^2 - \mu^2 = 4.9 - (1.9)^2 = 1.29$$

d. What is the probability that *x* is greater than 2? P(x>2)=0.3
e. What is the probability that *x* is 3 or less? P(x<=3)=0.9

Chapter 5:

5.7 : Let *x* be a binomial random variable with *n*=7, *p*=0.3. Find these values:

1) $P(x \geq 1) = 1 - P(0) = 1 - {}_0^7C\ (0.3^0)(0.7^7) = 0.92$

2) $P(x \leq 6) = 1 - P(7) = 1 - {}_7^7C\ (0.3^7)(0.7^0) = 0.9998$

3) $mean = \mu = np = 7 * 0.3 = 2.1$

4) $variance = \sigma^2 = np(1-p) = 7 * .3 * .7 = 1.47$

5.40 Let x be a Poisson random variable with mean 2.5/day. Calculate probabilities:

a. $P(x \leq 2) = P(0) + P(1) + P(2) = \frac{e^{-2.5}\ (2.5)^0}{0!} + \frac{e^{-2.5}\ (2.5)^1}{1!} + \frac{e^{-2.5}\ (2.5)^2}{2!}$

b. $P(x \geq 2) = 1 - P(0) - P(1)$

$$= 1 - \frac{e^{-2.5}\ (2.5)^0}{0!} - \frac{e^{-2.5}\ (2.5)^1}{1!}$$

c. $variance = \sigma^2 = 2.5.$

d. $P(x = 20)\ in\ week. \quad \mu = 7 * 2.5 = 17.5,, P(x = 20) = \frac{e^{-17.5}\ (17.5)^{20}}{20!}$

<u>5.41: Poisson vs. Binomial</u>:

Let x be a binomial random variable with n =20 and p= .1.

a. Calculate P(x<= 2) using binomial probability.

$$P(x \leq 2) = P(0) + P(1) + P(2)$$
$$= C_0^7\,(0.1)^0(0.9)^7 + C_1^7\,(0.1)^1(0.9)^6 + C_2^7\,(0.1)^2(0.9)^5 = 0.70$$

b) Use the Poisson approximation to calculate P(x<=2).

$$\mu = np = 20 * 0.1 = 2$$

5.51: Let x be a hyper-Geometric random variable with N=15, n= 3, and M=4. Calculate

a. $P(0) = \frac{C_0^4 C_3^{11}}{C_3^{15}} = 0.36$

b. $P(1) = \frac{C_1^4 C_2^{11}}{C_3^{15}} = 0.48$

c. $P(2) = \frac{C_2^4 C_1^{11}}{C_3^{15}} =$

d. $P(3) = \frac{C_3^4 C_0^{11}}{C_3^{15}} =$

e. $mean$

$$\mu = \frac{nM}{N} = \frac{3 * 4}{15}$$

f. $variance$

$$\sigma^2 = \frac{nM(N-M)(N-n)}{NN(N-1)} = \frac{3 * 4 * 11 * 12}{15 * 15 * 14}$$

Chapter 6:

6.5: Find the following probabilities for the standard normal random variable z, Z:Normal(0,1):

1. $P(-1.43 < Z < 0.68) = P(Z < 0.68) - P(Z < -1.43)$
$= 0.7517 - 0.0764 = 0.6753$

2. $P(0.58 < Z < 1.74) = P(Z < 1.74) - P(Z < 0.58)$
$= 0.9591 - 0.7190 = 0.2401$

3. $P(Z > 1.34) = 1 - P(Z < 1.34) = \mathbf{1 - 0.9099 = 0.0901}$

6.7+6.11: Find the following Z_o for the standard normal random variable z, Z:Normal(0,1):

1. $P(Z < z_o) = 0.025,\ z_o = -1.96$

2. $P(Z > z_o) = 0.025,\ P(Z < z_o) = 0.975,\ z_o = 1.96$

3. $P(-z_o < Z < z_o) = 0.8262$

$$P(Z < z_o) = 0.8262 + \frac{0.1738}{2} = 0.9131, \quad z_o = 1.36$$

4. $90th\ percentile = P(Z < z_o) = 0.90,\ z_o = 1.28$

5. $98th\ percentile = P(Z < z_o) = 0.98,\ z_o = 2.05$

6.12: Find the following probabilities for the normal random variable X, $X: Normal(\mu = 10,\ \sigma = 2)$.

1. $P(X < 10.6) = P\left(Z < \frac{10.6-10}{2}\right) = P(Z < 0.3) = 0.6179.$

2. $P(X > 13.5) = 1 - P\left(Z < \frac{13.5-10}{2}\right) = 1 - P(Z < 1.75) = 0.0401.$

3. $P(9.4 < X < 10.6) = P\left(\frac{9.4-10}{2} < Z < \frac{10.6-10}{2}\right) =$

$$= P(Z < 0.3) - P(Z < -0.3) = 0.6179 - 0.3821 = 0.2358.$$

4. $90th\ percentile = P(X < x_o) = 0.90,$

$$P\left(Z < \frac{xo-10}{2}\right) = 0.90, \quad \frac{xo-10}{2} = 1.28,, xo = 12.56$$

6.37 Let x be a binomial random variable with n=25 and p=0.3. X:Bin(25, 0.3)

1. $mean = nP = \mathbf{25 * 0.3 = 7.5}$

2. $\boldsymbol{variance = \sigma^2 = np(1-P) = 25 * 0.3 * 0.7 = 5.25,, \sigma = 2.29}.$

3. $Use\ the\ Normal\ Approximation\ to\ find\ \ P(X \leq 9) =$

$$= P\left(Z < \frac{9.5 - 7.5}{2.29}\right) = P(Z < 0.87) = 0.8078.$$

4. $Use\ the\ Normal\ Approximation\ to\ find\ \ P(X \geq 6) = 1 - P\left(Z < \frac{5.5 - 7.5}{2.29}\right)$

$$= 1 - P(Z < -0.87) = 1 - 0.1922 = 0.8078.$$

5. $Use\ the\ Normal\ Approximation\ to\ find\ \ P(6 \leq X \leq 9) =$

$$= P\left(Z < \frac{9.5 - 7.5}{2.29}\right) - P\left(Z < \frac{5.5 - 7.5}{2.29}\right) = P(Z < 0.87) - P(Z < -0.87)$$

$$= 0.8078 - 0.1922 = 0.6156.$$