

Outline

- Ensemble learning
- Ensemble learning approaches
- Bagging
 - Bag of decision trees
 - Random forest
- Boosting
 - Adaboost
- Stacking

What is Ensemble Learning

- Ensemble learning is a machine learning technique that combines the predictions of multiple models to achieve better accuracy and performance.
- These models, also known as weak learners or base learners, may individually be simple, but together they can form a strong predictive system
 - ▶ If we want to perform better on a task, we need more workers.
- Consider ensemble learning like asking multiple experts for an opinion. While each expert may have limitations, combining their insights leads to a more accurate decision.



Why Use Ensemble Learning

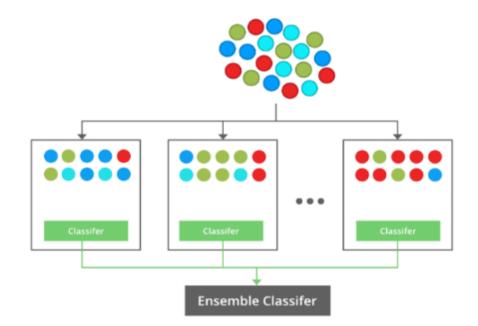
- Reducing Bias and Variance: Individual models often suffer from bias or variance.
 - bias means the model makes strong assumptions about the data
 - e.g., A linear regression model used for a highly non-linear relationship will have a high bias because it assumes a linear relationship where none exists (underfitting)
 - variance: high variance means the model makes significant changes in prediction even with small changes in the dataset (overfitting)
 - BOTH LEAD TO POOR GENERALIZATION
- ► Ensemble learning strikes a balance, creating models that generalize better on unseen data.

Why Use Ensemble Learning

- Error Reduction: By averaging the predictions of multiple models, ensembles help cancel out errors or inconsistencies of individual models, improving reliability.
- Robustness: The variability in weak learners means that if one model performs poorly in a specific scenario, other models can compensate for this weakness.

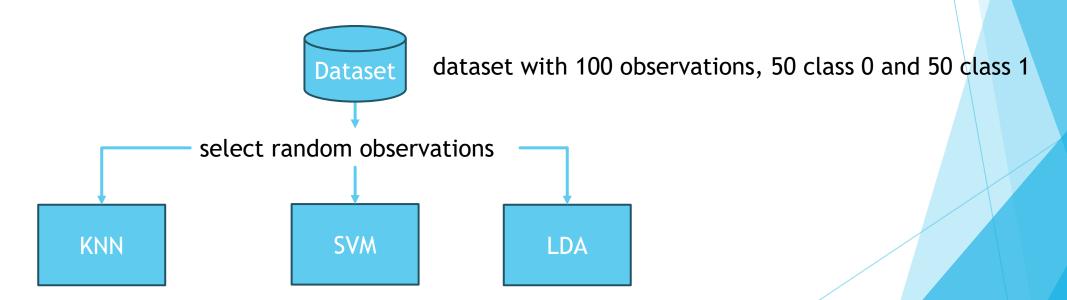
Bagging: Bootstrap aggregating

- Multiple models are trained on random subsets of the data, often with replacement (this is called bootstrapping).
 - Each one of these models is called a base or weak learner
 - Although, in bagging, they might not be weak
- Each model is trained independently, and their results are averaged (or voted) to make a final prediction



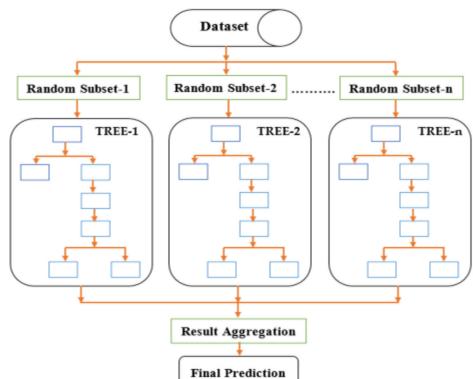
Bagging: Bootstrap aggregating

- Weak learner is a model that performs slightly better than a random guess
 - > just over 50%, 51%, for example
- The simplest ensemble learning approach is to aggregate the power of various classifiers like KNN, SVM and LDA (Strong learners)



Bagged trees

- A bag of trees is an ensemble learning method that trains a collection of trees of different subsets (samples) of the dataset
- The samples are selected randomly with replacement
- The trees inside this bag are trained using heuristics like *information gain* or *Gini-index*



Code example Classification

```
from sklearn.ensemble import BaggingClassifier
from sklearn.tree import DecisionTreeClassifier
from sklearn.datasets import load iris
from sklearn.model selection import train test split
from sklearn.metrics import accuracy score
iris = load iris()
X, y = iris.data, iris.target
X train, X test, y train, y test = train test split(X, y, test size=0.3,
random state=42)
base tree = DecisionTreeClassifier()
bagged trees = BaggingClassifier(base estimator=base tree, n estimators=100,
random state=42)
bagged trees.fit(X train, y train)
y pred = bagged trees.predict(X test)
print(f"Accuracy: {accuracy score(y test, y pred):.4f}")
```

Code example Regression

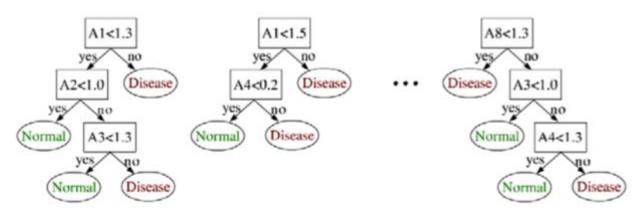
```
from sklearn.ensemble import BaggingRegressor
from sklearn.tree import DecisionTreeRegressor
from sklearn.datasets import fetch california housing
from sklearn.model selection import train test split
from sklearn.metrics import mean squared error
data = fetch california housing()
X, y = data.data, data.target
X train, X test, y train, y test = train test split(X, y, test size=0.3, random state=42)
base tree = DecisionTreeRegressor() bagged trees reg =
BaggingRegressor(base estimator=base tree, n estimators=100, random state=42)
bagged trees reg.fit(X train, y train)
y pred = bagged trees reg.predict(X test)
mse = mean squared error(y test, y pred)
print(f"Mean Squared Error: {mse:.4f}")
```

Bagged trees Cont.

- Bagged trees, reduces variance by training trees on different subsets of the data
- However, in the bagged trees, there is a high chance that different trees may make similar splits
 - as they have access to all features
- Also, the computational cost is high, especially when the dataset is huge with high dimensionality

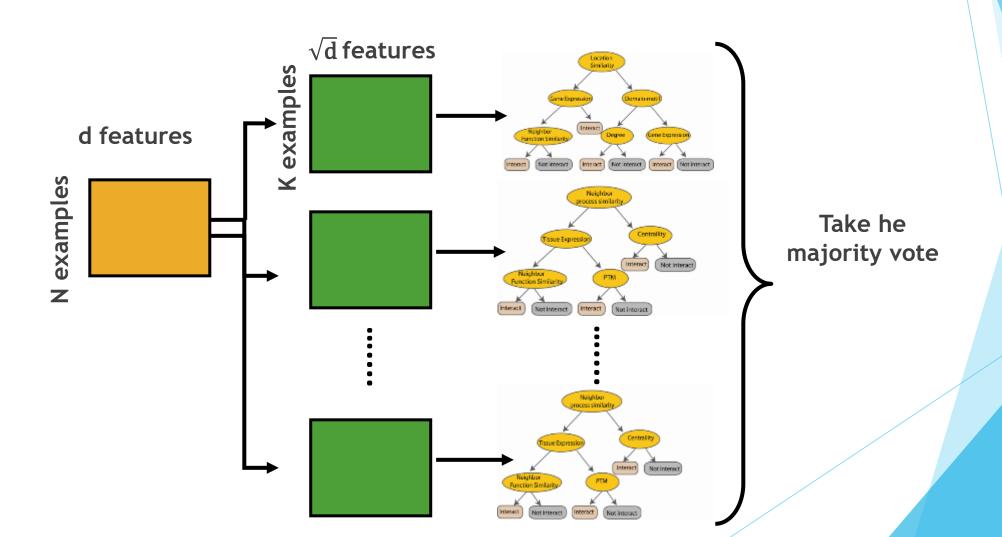
Random Forest

- Random Forest RF, is similar to the bagged trees. However, it adds another layer of randomness
- The other layer of randomness is at the feature level.
- It takes \sqrt{d} features for each subset, $\frac{d}{3}$ for regression problems
- This layer of randomness increases the diversity of the model and improves generalization to new unseen data



Each tree uses a random selection of $m \approx \sqrt{d}$ features $\{A_{i_j}\}_{j=1}^m$ chosen from *all* features A_1, A_2, \ldots, A_d

Random Forest Cont.



Code Example Random Forest

Classification

```
from sklearn.ensemble import RandomForestClassifier
from sklearn.datasets import load_iris
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score

iris = load_iris()
X, y = iris.data, iris.target

X_train, X_test, y_train, y_test = train_test_split(X, y,
test_size=0.3, random_state=42)

RF = RandomForestClassifier(n_estimators=100,
random_state=42)
class_weight='balanced'
RF.fit(X_train, y_train)

y_pred = RF.predict(X_test)
print(f"Accuracy: {accuracy_score(y_test, y_pred):.4f}")
```

Regression

from sklearn.ensemble import RandomForestRegressor

```
from sklearn.datasets import fetch california housing
from sklearn.model selection import train test split
from sklearn.metrics import mean squared error
data = fetch california housing()
X, y = data.data, data.target
X train, X test, y train, y test = train test split(X, y,
test size=0.3, random state=42)
rf regressor = RandomForestRegressor(n estimators=100,
random state=42)
rf regressor.fit(X train, y train)
y pred = rf regressor.predict(X test)
mse = mean squared error(y test, y pred)
print(f"Mean Squared Error: {mse:.4f}")
```

Important notes

- RF and bagged trees choose the samples (bootstrapped subsets) randomly with replacement
 - Some samples might appear multiple times and some might not appear in the subset
- It might happen by chance that the majority of examples are drawn from 1 class, which might affect the performance
 - maybe all samples come from 1 class
 - ▶ The high number of estimators (weak learners mitigates the bad effect
- to further ensure this will not affect the performance, use balance subsets
 - This applies to classification tasks

```
RF = RandomForestClassifier(class_weight='balanced' n_estimators=100, random_state=42)
```

In bagged trees the class_weight is added to the estimator it self

```
base_tree = DecisionTreeClassifier(class_weight='balanced', random_state=42)
bagging_classifier = BaggingClassifier(base_estimator=base_tree, n_estimators=100, random_state=42)
```

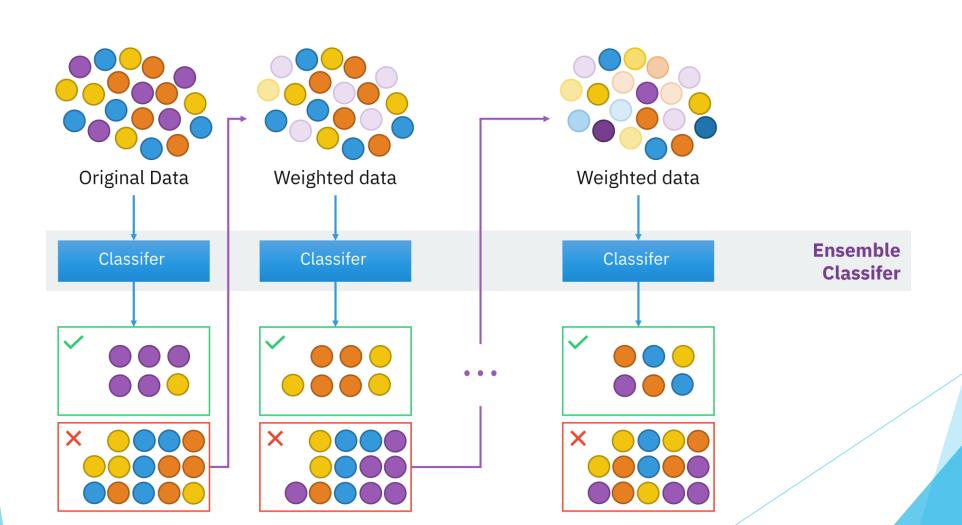
Important notes Cont.

- max_samples is another parameter to play with using these methods
- its default value is between 0-1, 1 means all original datasets will be used for the bootstrap sampling, and the size of each subset will be the same as the original dataset
 - Although they are not the same
 - the subset might include some samples multiple times, while other samples do not appear
- ▶ If max_samples is assigned an integer value, e.g., 100, Each base learner will be trained on exactly 100 samples, regardless of the total number of samples in the original dataset

Boosting

- Boosting is a powerful ensemble learning technique aimed at improving model performance by combining several weak learners to form a strong learner.
- Boosting focuses on sequentially correcting errors made by previous models.
- Pmethods in this approach:
 - AdaBoost
 - Gradient Boosting
 - XGboost

Boosting Cont.



AdaBoost

- AdaBoost (Adaptive Boosting) is a widely used boosting algorithm that combines multiple weak learners to form a strong classifier.
- How It Works:
- 1. **Weak Learners:** Typically uses **decision stumps** (single-level decision trees) as weak learners.
- Sequential Learning: Each weak learner is trained on the dataset, focusing on misclassified instances from the previous model.
- 3. Weighting: After each round, AdaBoost:
 - 1. **Increases weights** of misclassified points, so the next learner pays more attention to them.
 - 2. Decreases weights of correctly classified points, reducing their influence.
- 4. Final Prediction: A weighted vote of all weak learners' predictions forms the final output.

Adaboost Procedure

Steps of Adaboost learning

- 1- Create w vector that determine each sample to be considered in the classification
 - Set $w_i = \frac{1}{n}$ for each sample i, where n is the total number of training samples
- 2- Create a weak classifier C_t , In AdaBoost the weak learner is a 1-depth decision tree (decision stamp)
- 3- Classify the data using this stamp
- 4- Calculate the error

The error is given by $\epsilon_t = \sum_{i=1}^n w_i \cdot I(\overline{y}_i \neq y_i) => I(\cdot) = 1 \text{ if } \overline{y}_i \neq y_i \text{ otherwise } 0$

5- Calculate a_t (the amount of say)

$$\alpha_t = \frac{1}{2} \log(\frac{(1 - \epsilon_t)}{\epsilon_t})$$

6- Update the weights

 $w_i = w_i. e^{-\alpha_t}$ if the sample is correctly classified, and $w_i = w_i. e^{-\alpha_t}$ otherwise

7- Normalize the weights so that they sum to 1

$$w_i = \frac{w_i}{\sum_{i=1}^n w_i}$$

8- Create the new dataset for the next stamp (resampling based on the weights)

Adaboost Example

Assume you have the following data

#	X1	X2	Х3	Class	
1	83	0.3	73	+	
2	91	0.06	7	+	
3	98	0.41	42	+	
4	95	0.16	29	+	
5	89	0.71	99	+	
6	73	0.81	37	-	
7	58	0.66	82	-	
8	32	0.65	36	-	
9	13	0.11	91	-	
10	82	0.28	91	-	

Adaboost Initialize the weights

Assign an initial weights for each sample in the dataset

$$\frac{1}{10} = 0.1$$

#	X1	X2	Х3	Class	weight
1	83	0.3	73	+	0.1
2	91	0.06	7	+	0.1
3	98	0.41	42	+	0.1
4	95	0.16	29	+	0.1
5	89	0.71	99	+	0.1
6	73	0.81	37	-	0.1
7	58	0.66	82	-	0.1
8	32	0.65	36	-	0.1
9	13	0.11	91	-	0.1
10	82	0.28	91	-	0.1

Adaboost Predict

Now use the decision stamp to classify the data and find the predictions

#	X1	X2	X3	Class	weight	P
1	83	0.3	73	+	0.1	+
2	91	0.06	7	+	0.1	+
3	98	0.41	42	+	0.1	-
4	95	0.16	29	+	0.1	-
5	89	0.71	99	+	0.1	-
6	73	0.81	37	-	0.1	-
7	58	0.66	82	-	0.1	-
8	32	0.65	36	-	0.1	-
9	13	0.11	91	-	0.1	-
10	82	0.28	91	-	0.1	-

Adaboost Calculate the error

Calculate the errors

#	X1	X2	X3	Class	weight	P	Ε
1	83	0.3	73	+	0.1	+	0
2	91	0.06	7	+	0.1	+	0
3	98	0.41	42	+	0.1	-	1
4	95	0.16	29	+	0.1	-	1
5	89	0.71	99	+	0.1	-	1
6	73	0.81	37	-	0.1	-	0
7	58	0.66	82	-	0.1	-	0
8	32	0.65	36	-	0.1	-	0
9	13	0.11	91	-	0.1	-	0
10	82	0.28	91	-	0.1	-	0

Adaboost Calculate the error

The error is given by $\epsilon_t = \sum_{i=1}^n w_i \cdot I(\overline{y}_i \neq y_i) => I(\cdot) = 1 \text{ if } \overline{y}_i \neq y_i \text{ otherwise } 0$

#	X1	X2	X3	Class	weight	P	E
1	83	0.3	73	+	0.1	+	0
2	91	0.06	7	+	0.1	+	0
3	98	0.41	42	+	0.1	-	1
4	95	0.16	29	+	0.1	-	1
5	89	0.71	99	+	0.1	-	1
6	73	0.81	37	-	0.1	-	0
7	58	0.66	82	-	0.1	-	0
8	32	0.65	36	-	0.1	-	0
9	13	0.11	91	-	0.1	-	0
10	82	0.28	91	-	0.1	-	0
Error	= 1*0.	1 + 1*0.1	+ 1*0.	.1			0.3

Adaboost Amount to say

Calculate α_t (the amount of say) $\alpha_t = \frac{1}{2} log(\frac{(1-\epsilon_t)}{\epsilon_t})$

#	X1	X2	X3	Class	weight	Р	E		
1	83	0.3	73	+	0.1	+	0		
2	91	0.06	7	+	0.1	+	0		
3	98	0.41	42	+	0.1	-	1		
4	95	0.16	29	+	0.1	-	1		
5	89	0.71	99	+	0.1	-	1		
6	73	0.81	37	-	0.1	-	0		
7	58	0.66	82	-	0.1	-	0		
8	32	0.65	36	-	0.1	-	0		
9	13	0.11	91	-	0.1	-	0		
10	82	0.28	91	-	0.1	-	0		
Error	Error = 1*0.1 + 1*0.1 + 1*0.1								
α =							0.42		

To avoid log 0, which is undefined, we add a very small number called epsilon (EPS). E.g., EPS=0.0001

Adaboost Update the weights

 $w_i = w_i. e^{-\alpha_t}$ if the sample is correctly classified, and $w_i = w_i. e^{-\alpha_t}$ otherwise

a_t	0.42
Correct w	0.65
Incorrect w	1.53

Adaboost Update the weights

#	X1	X2	Х3	Class	weight	Р	Ε	New w
1	83	0.3	73	+	0.1	+	0	0.065
2	91	0.06	7	+	0.1	+	0	0.065
3	98	0.41	42	+	0.1	-	1	0.153
4	95	0.16	29	+	0.1	-	1	0.153
5	89	0.71	99	+	0.1	-	1	0.153
6	73	0.81	37	-	0.1	-	0	0.065
7	58	0.66	82	-	0.1	-	0	0.065
8	32	0.65	36	-	0.1	-	0	0.065
9	13	0.11	91	-	0.1	-	0	0.065
10	82	0.28	91	-	0.1	-	0	0.065

Adaboost Normalize the weights

Normalize the weights so that they sum to 1 $w_i = \frac{w_i}{\sum_{i=1}^n w_i}$

#	X1	X2	Х3	Class	weight	Р	Ε	New w	Nor. w
1	83	0.3	73	+	0.1	+	0	0.065	0.071
2	91	0.06	7	+	0.1	+	0	0.065	0.071
3	98	0.41	42	+	0.1	-	1	0.153	0.167
4	95	0.16	29	+	0.1	-	1	0.153	0.167
5	89	0.71	99	+	0.1	-	1	0.153	0.167
6	73	0.81	37	-	0.1	-	0	0.065	0.071
7	58	0.66	82	-	0.1	-	0	0.065	0.071
8	32	0.65	36	-	0.1	-	0	0.065	0.071
9	13	0.11	91	-	0.1	-	0	0.065	0.071
10	82	0.28	91	-	0.1	-	0	0.065	0.071

- Create a new resampled dataset.
- In the new data the wrongly classified samples are more likely to appear
- You can do this by performing a cumulative sum and picking uniformly distribution random values in the range [0-1]

Cumulative distribution

#	X1	X2	Х3	Class	weight	P	E	New w	Nor. w	Low	Up
1	83	0.3	73	+	0.1	+	0	0.065	0.071	0	0.071
2	91	0.06	7	+	0.1	+	0	0.065	0.071	0.071	0.142
3	98	0.41	42	+	0.1	-	1	0.153	0.167	0.142	0.307
4	95	0.16	29	+	0.1	-	1	0.153	0.167	0.307	0.467
5	89	0.71	99	+	0.1	-	1	0.153	0.167	0.467	0.634
6	73	0.81	37	-	0.1	-	0	0.065	0.071	0.634	0.705
7	58	0.66	82	-	0.1	-	0	0.065	0.071	0.705	0.776
8	32	0.65	36	-	0.1	-	0	0.065	0.071	0.776	0.847
9	13	0.11	91	-	0.1	-	0	0.065	0.071	0.847	0.918
10	82	0.28	91	-	0.1	-	0	0.065	0.071	0.918	0.99

- Now pick Uniformly distributed random number [0-1]
- As the wrong examples have bigger interval, they are more likely to present in the new data

UDRNumber

0.5	0.2	0.3	0.2	0.1	0.9	0.47	0.1	0.95
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#	X1	X2	Х3	Class	weight	P	E	New w	Nor. w	Low	Up
1	83	0.3	73	+	0.1	+	0	0.065	0.071	0	0.071
2	91	0.06	7	+	0.1	+	0	0.065	0.071	0.071	0.142
3	98	0.41	42	+	0.1	-	1	0.153	0.167	0.142	0.307
4	95	0.16	29	+	0.1	-	1	0.153	0.167	0.307	0.467
5	89	0.71	99	+	0.1	-	1	0.153	0.167	0.467	0.634
6	73	0.81	37	-	0.1	-	0	0.065	0.071	0.634	0.705
7	58	0.66	82	-	0.1	-	0	0.065	0.071	0.705	0.776
8	32	0.65	36	-	0.1	-	0	0.065	0.071	0.776	0.847
9	13	0.11	91	-	0.1	-	0	0.065	0.071	0.847	0.918
10	82	0.28	91	-	0.1	-	0	0.065	0.071	0.918	0.99

UDRNumber

#	X1	X2	Х3	Class	weight	P	Ε	New w	Nor. w	Low	Up
1	83	0.3	73	+	0.1	+	0	0.065	0.071	0	0.071
2	91	0.06	7	+	0.1	+	0	0.065	0.071	0.071	0.142
3	98	0.41	42	+	0.1	-	1	0.153	0.167	0.142	0.307
4	95	0.16	29	+	0.1	-	1	0.153	0.167	0.307	0.467
5	89	0.71	99	+	0.1	-	1	0.153	0.167	0.467	0.634
6	73	0.81	37	-	0.1	-	0	0.065	0.071	0.634	0.705
7	58	0.66	82	-	0.1	-	0	0.065	0.071	0.705	0.776
8	32	0.65	36	-	0.1	-	0	0.065	0.071	0.776	0.847
9	13	0.11	91	-	0.1	-	0	0.065	0.071	0.847	0.918
10	82	0.28	91	-	0.1	-	0	0.065	0.071	0.918	0.99

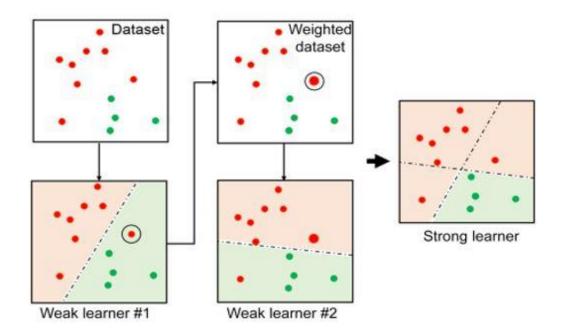
Add this sample, 5, to the new resampled dataset

Adaboost resampled dataset

#	X1	X2	Х3	Class
5	89	0.71	99	+
3	98	0.41	42	+
3	98	0.41	42	+
4	95	0.16	29	+
5	89	0.71	99	+
2	91	0.06	7	+
9	13	0.11	91	-
5	89	0.71	99	+
2	91	0.06	7	+
10	82	0.28	91	-

Repeat and prediction

- Repeat the previous steps for the number of defined classifiers.
- The final prediction is achieved using the following formula.
 - \triangleright prediction = $Sign(\sum \alpha_t. y_t)$
 - \triangleright y_t is the prediction of the t weak classifier (estimator)



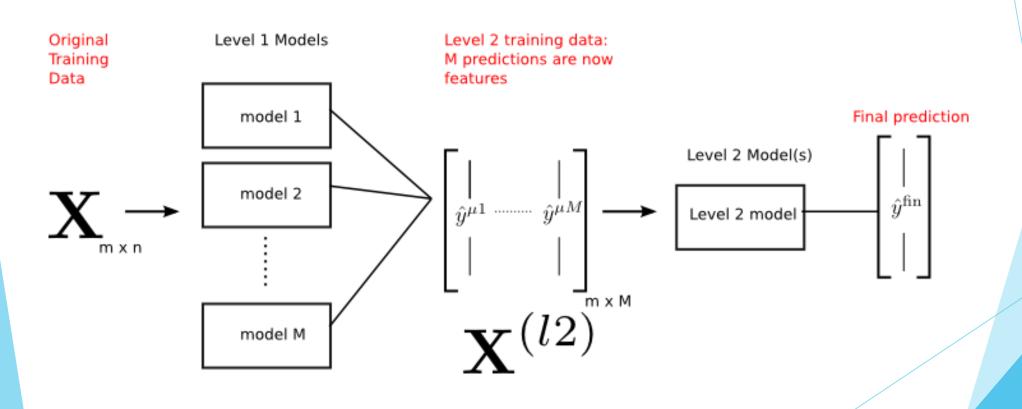
Stacking

- Stacking is an ensemble learning technique that combines multiple machine learning models to achieve better predictive performance than any individual model
- How it works
 - Multiple Base Learners: Several models (e.g., decision trees, logistic regression, SVM, etc.) are trained on the same dataset.
 - Meta-Learner (Blender): The predictions from base learners are then passed to a meta-learner, trained to make the final prediction.
 - Diverse Models: Unlike other ensembles like bagging or boosting, stacking typically uses models of different types to capture different patterns in the data

Stacking Cont.

- After the base classifiers make their predictions, their outputs become the meta-learner's input features.
- Meta-Learner Input: Each base classifier outputs either:
 - 1. Class predictions (for classification tasks).
 - 2. Probability estimates (e.g., the probability of belonging to each class).
 - 3. Regression outputs (for regression tasks).
- These outputs form a new dataset.
- Each row in this dataset consists of the predictions made by the base classifiers for a particular sample.

Stacking Cont.



Notes

- Ensemble learning provides powerful learning abilities by combining the outputs of multiple models
 - This combination allows the model to divide complex tasks between several learners, each focusing on different aspects of the data.
- Bagging trains independent classifiers in parallel because each base learner is trained on a different subset of the data, often sampled with replacement.
 - This independence makes it easy to parallelize bagging across multiple machines or nodes in a cluster, enabling faster training.
- Boosting, on the other hand, cannot be parallelized easily because it trains sequential classifiers.
 - Each subsequent model is built to correct the errors made by the previous models, making it inherently sequential.
 - Limits the ability to parallelize the process effectively.
- Ensemble learning increases computational requirements
 - This is because you are training multiple models, which can increase memory and processing time requirements.
 - However, this tradeoff is often justified by the significant improvement in performance for complex problems where a single model might underperform.
 - Inevitable for some tasks.