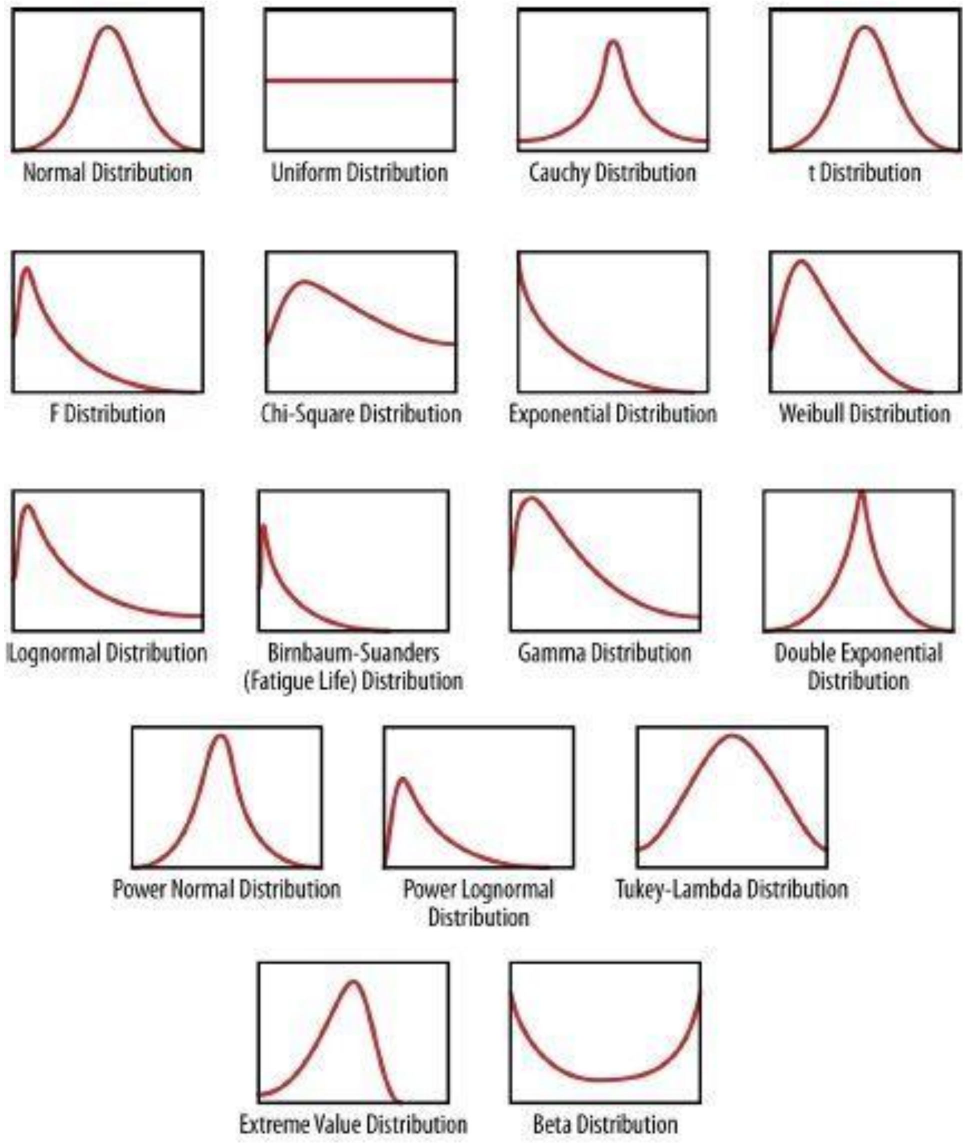
Data Modeling and Probability Distributions

Dr. Ahmad S. Tarawneh



Outline

- Introduction
- Data Collection
- Probability Distributions
- Fit a Distribution on Data
- ExpertFit (Data modeling software)

Introduction

- What we have discussed so far is a set of problems in which the distribution is given
- But from where did we get this table
- If we know the model that fits our data we do not need to use such tables, practically.

Time between Arrivals (Minutes)	*Probability*
1	0.25
2	0.40
3	0.20
4	0.15

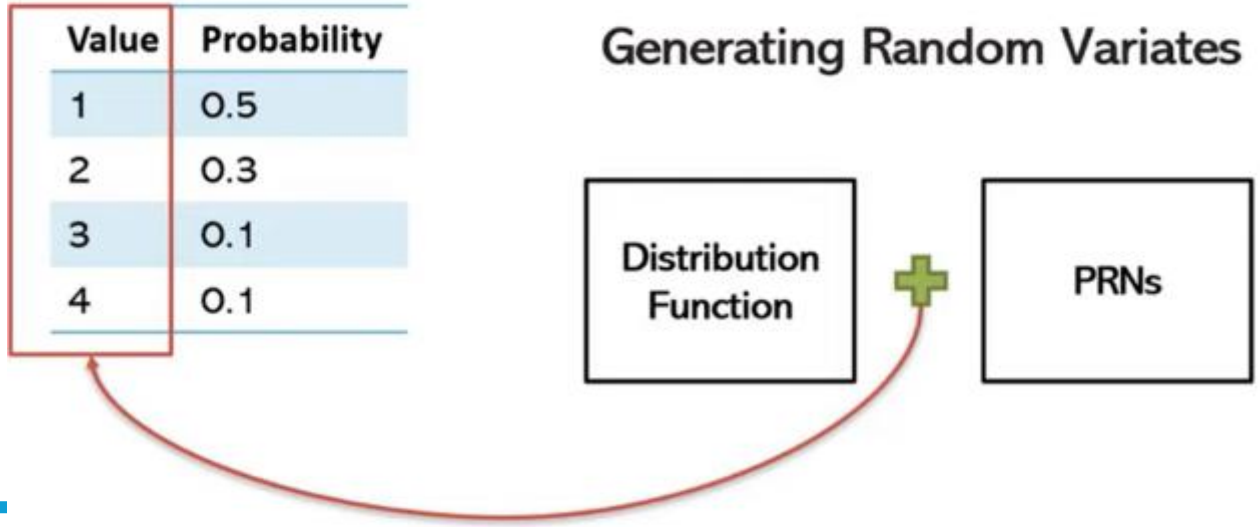
Queueing Sim.

Demand	*Probability*
0	0.10
1	0.25
2	0.35
3	0.21
4	0.09

Inventory Sim.

Cont.

- Knowing the distribution will help us generate the values directly depending on the distribution of the data

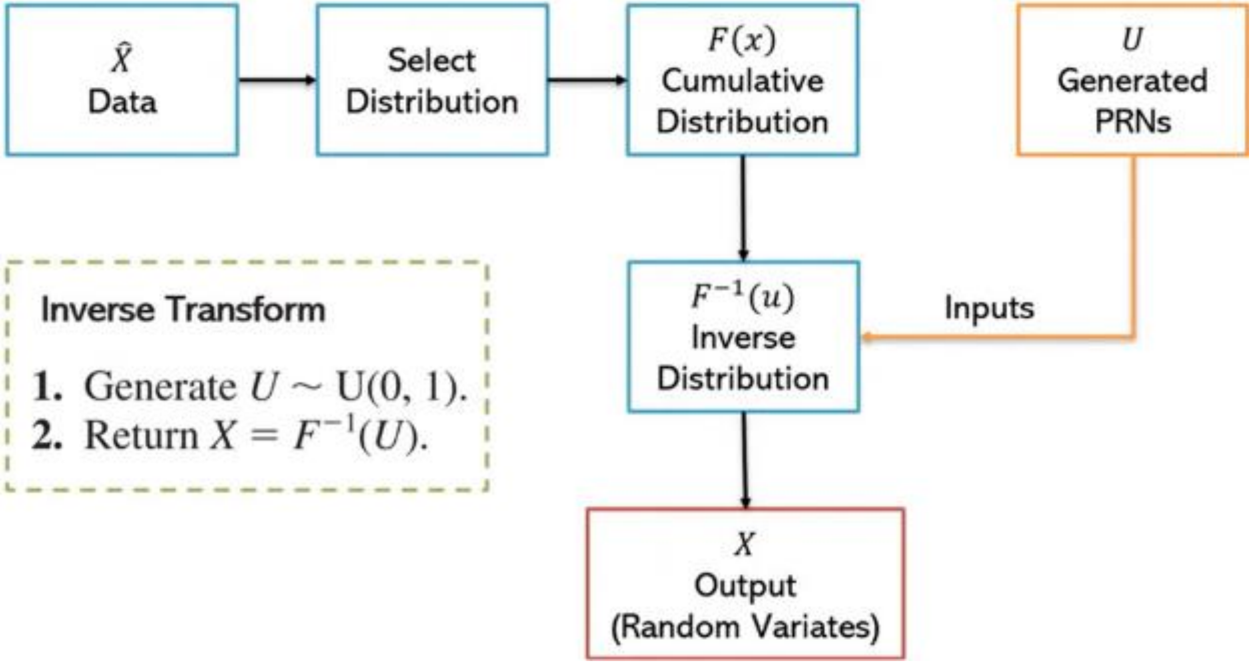


Cont.

- There are several approaches to generate random variables (not numbers) given the distribution:
  1. Inverse Transform
  2. Composition
  3. Convolution
  4. Acceptance-Rejection
  5. Ratio of Uniforms
  6. Many more

Cont.

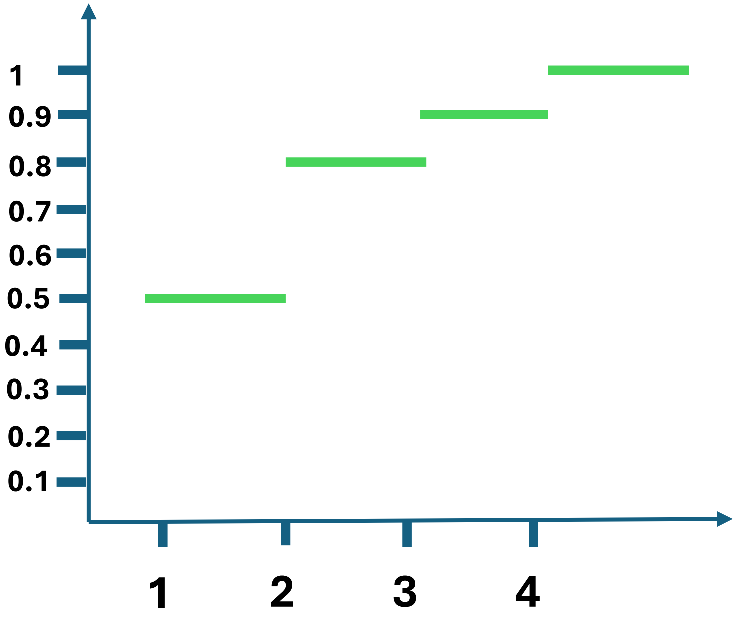
- Inverse Transform is the simplest

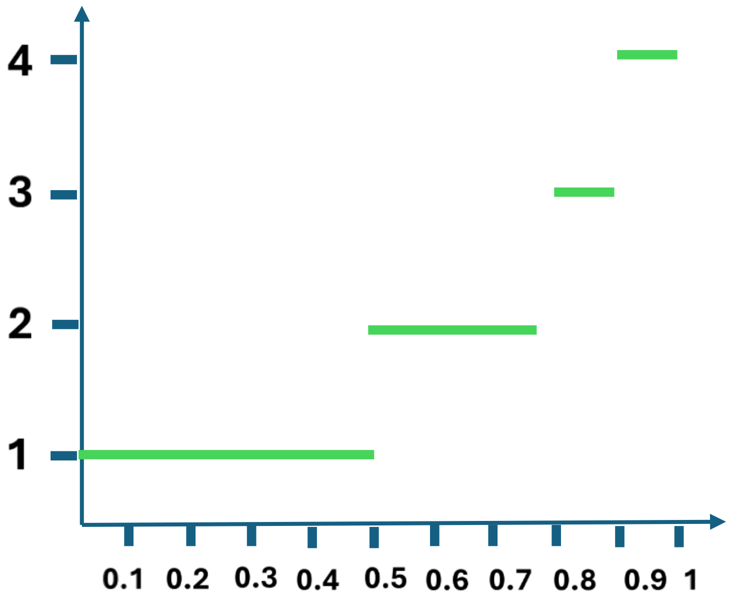


Example

Value	Probability
1	0.5
2	0.3
3	0.1
4	0.1

Com
0.5
0.8
0.9
0.1



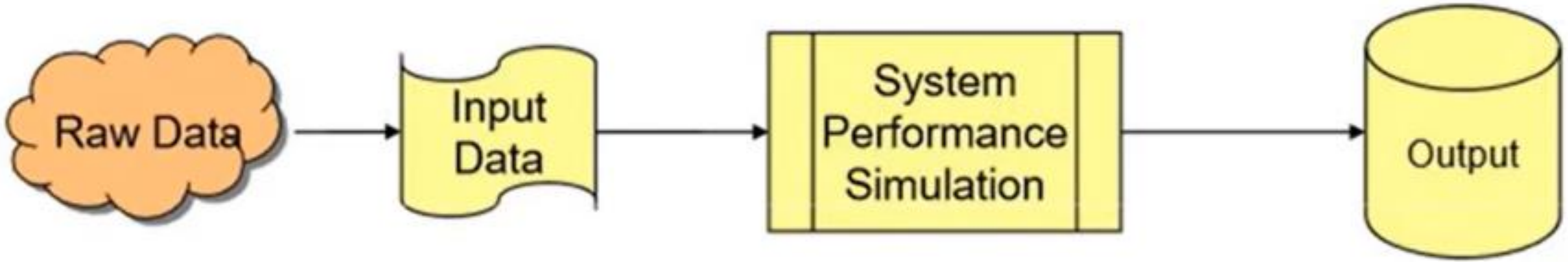


Random variables

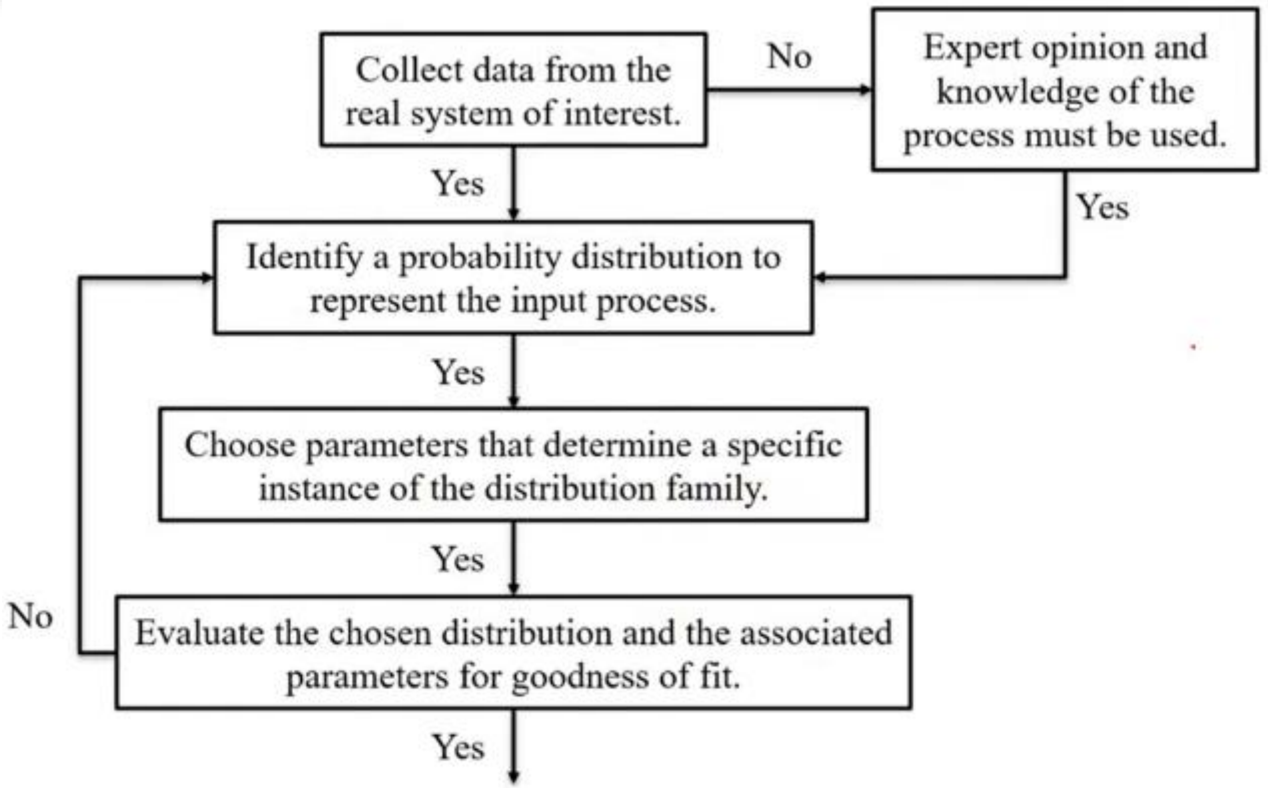
- Inputs are independent variables in the system:
    1. Interarrival time, which is typically a continuous random variable
    2. Amount of demand is a discrete random variable

Data modeling

- Data model (prob distribution of the data) selection is very important for the simulation model, as all the simulation depends upon these models
  - Good data modeling means good simulation performance



Steps of data modeling



Data Collection

- The tool used for simulation should have several features to support building a good performing simulations
  1. Must have a good random number generator
  2. Should be able to create a probability distribution for each source of randomness of the system
  3. Gives the ability to perform several, independent, runs of the simulation, using different random numbers, to validate the stability of the performance

Cont.

- In simulation systems, typically, we try to find the **best standard probability distribution** that fits the source of randomness
  - If it can be found, then it should be used
- If a standard distribution cannot be found, then we can use an **empirical distribution, aka custom distribution or used defined distribution**.
  - This custom distribution is created based on the data
- **Using standard distribution is always preferable over empirical distributions**

Cont.

- The data collected should be:
    1. Accurately collected
    2. Representative of the environment
    3. Analyzed correctly
- Otherwise, the simulation will give a misleading results or wrong conclusions

Cont.

- When the system exists, the data is better to be collected from this system, real system
- **If the system does not exist or no time to collect the data from existed system, an informed guess can be made, by the help of experts.**

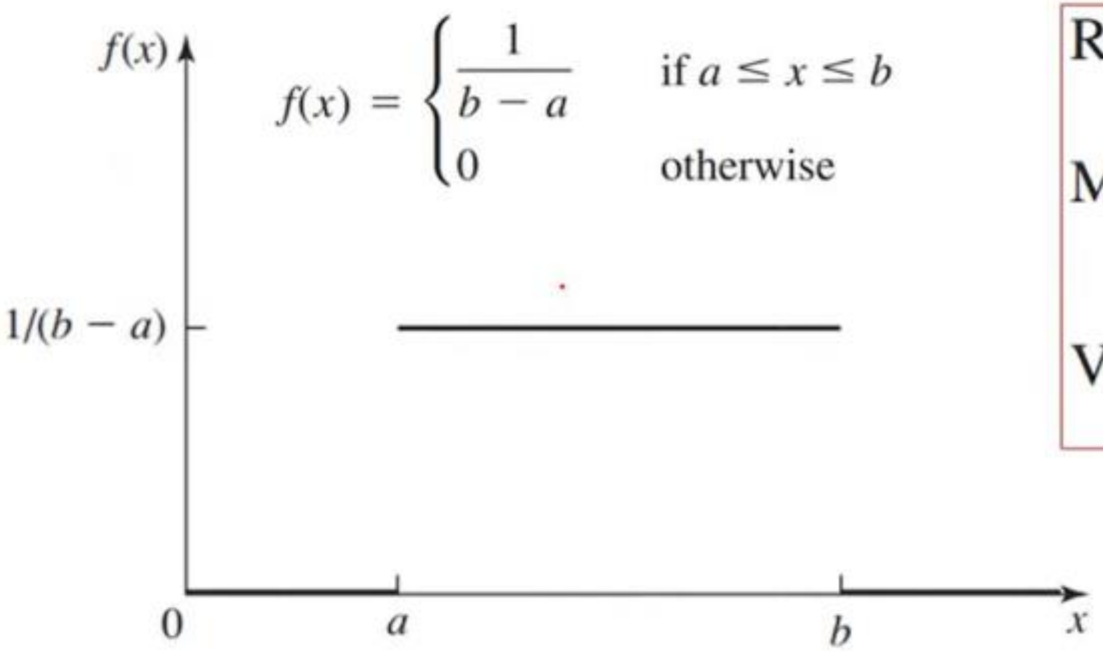
Standard Distributions

- Many standard theoretical distributions are there.
- Some of these distributions work for continuous data and some work for discrete
- **Continuous Distributions include but are not limited to**
  1. Uniform distribution
  2. Exponential distribution
  3. Gamma distribution
  4. Gaussian (Normal) distribution

Uniform Distribution

- a and b real numbers with $a < b$.

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$



Range	$[a, b]$
Mean	$\dfrac{a+b}{2}$
Variance	$\dfrac{(b-a)^2}{12}$

Cont.

Density $\quad f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

Distribution $\quad F(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } b < x \end{cases}$

PRN

$F^{-1}(u) = a + (b-a)u$ Generating Random Variates X

Solution steps

$\dfrac{(x-a)}{b-a} = u$

Multiply both sides by $b-a$

$x - a = u(b-a)$

Move a to the right side

$x = u(b-a) + a; \quad b - a \neq 0$

Exponential Distribution

- Scale parameter $\beta > 0$.

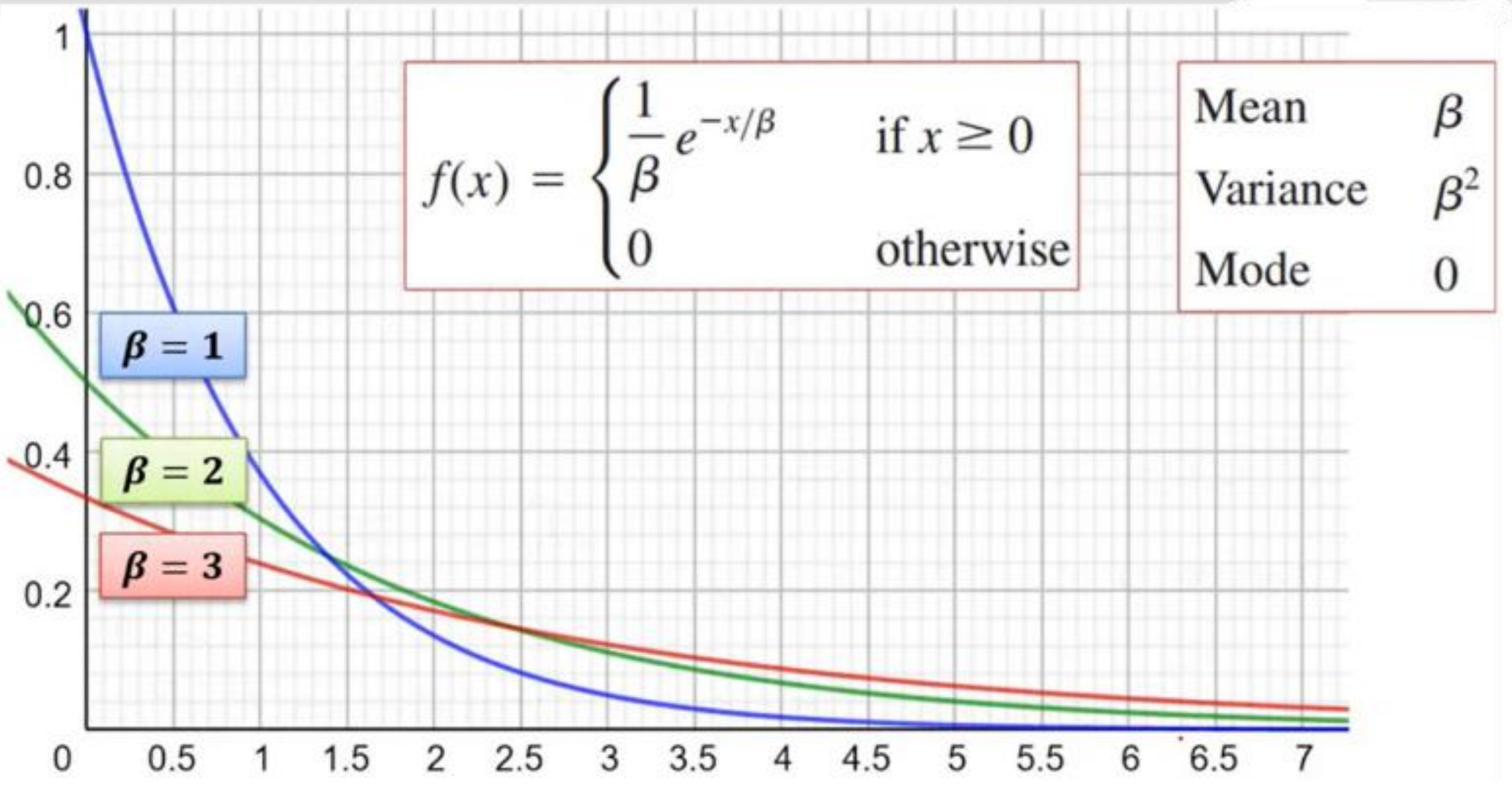
Density $\quad f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Distribution $\quad F(x) = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

PRN

$F^{-1}(u) = -\beta \ln(1 - u)$ Generating Random Variates X's

Cont.



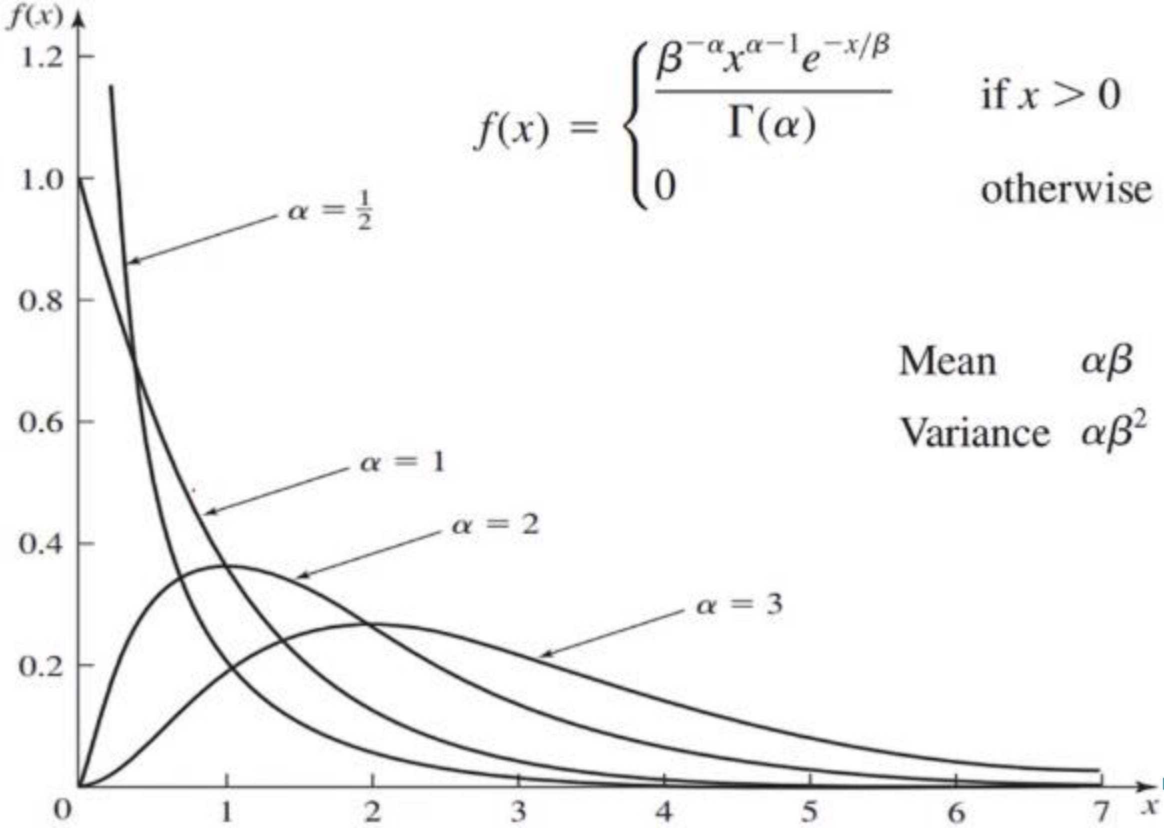
$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
Mean β
Variance β^2
Mode 0
$\beta = 1$
$\beta = 2$
$\beta = 3$
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7

Gamma Distribution

- Shape parameter $\alpha > 0$, Scale parameter $\beta > 0$.

$$f(x) = \begin{cases} \dfrac{\beta^{-\alpha} x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

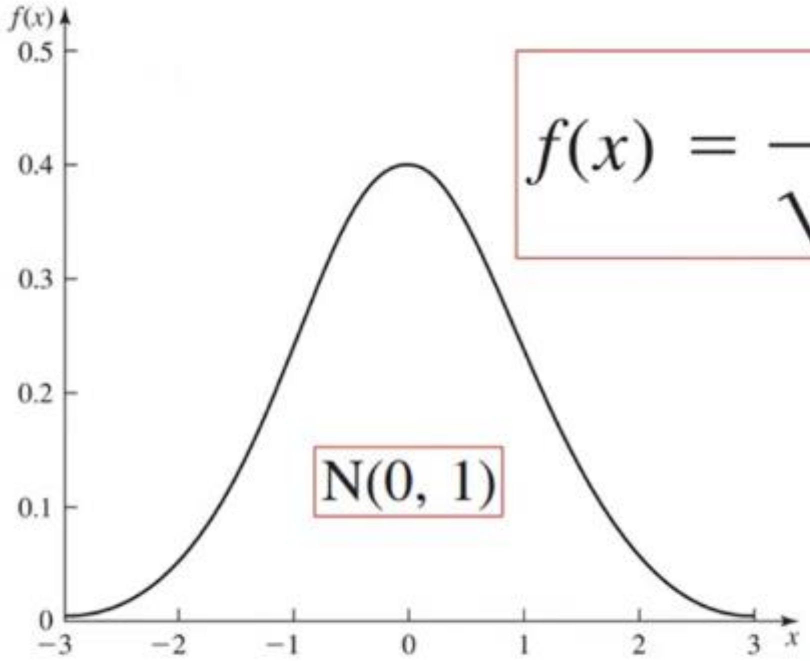
Graph Gamma



gamma(α, 1)

Normal Distribution

- Location parameter $\mu \in \mathbb{R}$, Scale parameter $\sigma > 0$.



$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}}$$

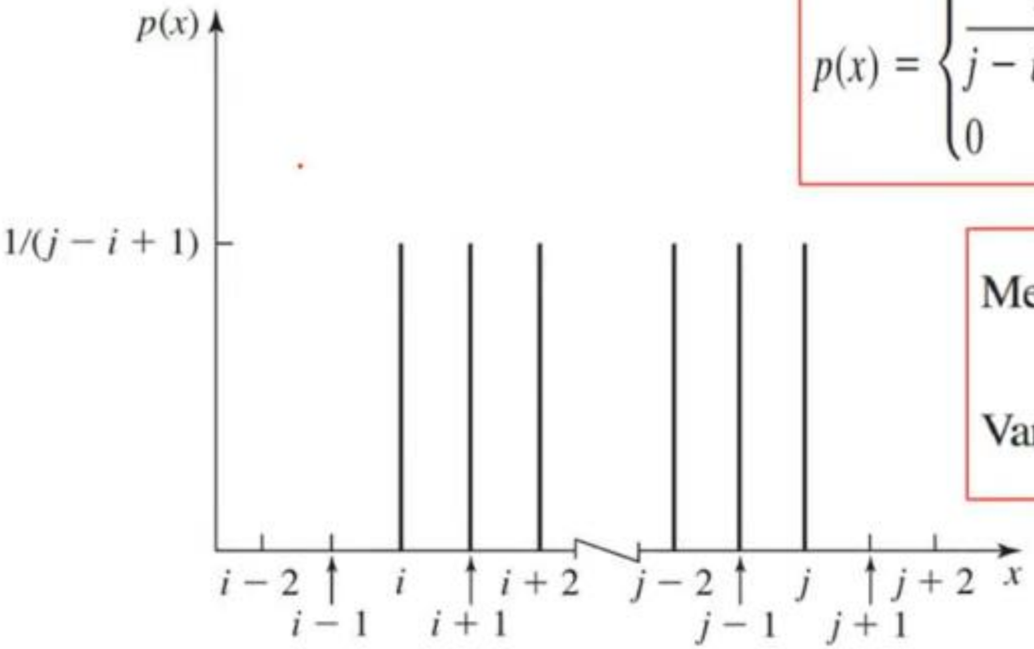
Mean	μ
Variance	σ^2
Mode	μ

Discrete Probability Distributions

- Discrete Uniform Distribution
- Geometric Distribution
- Poisson Distribution

Discrete Uniform Distribution

- i and j integers with $i \leq j$.



$$p(x) = \begin{cases} \dfrac{1}{j-i+1} & \text{if } x \in \{i, i+1, \ldots, j\} \\ 0 & \text{otherwise} \end{cases}$$

Mean	$\dfrac{i+j}{2}$
Variance	$\dfrac{(j-i+1)^2-1}{12}$

Cont.

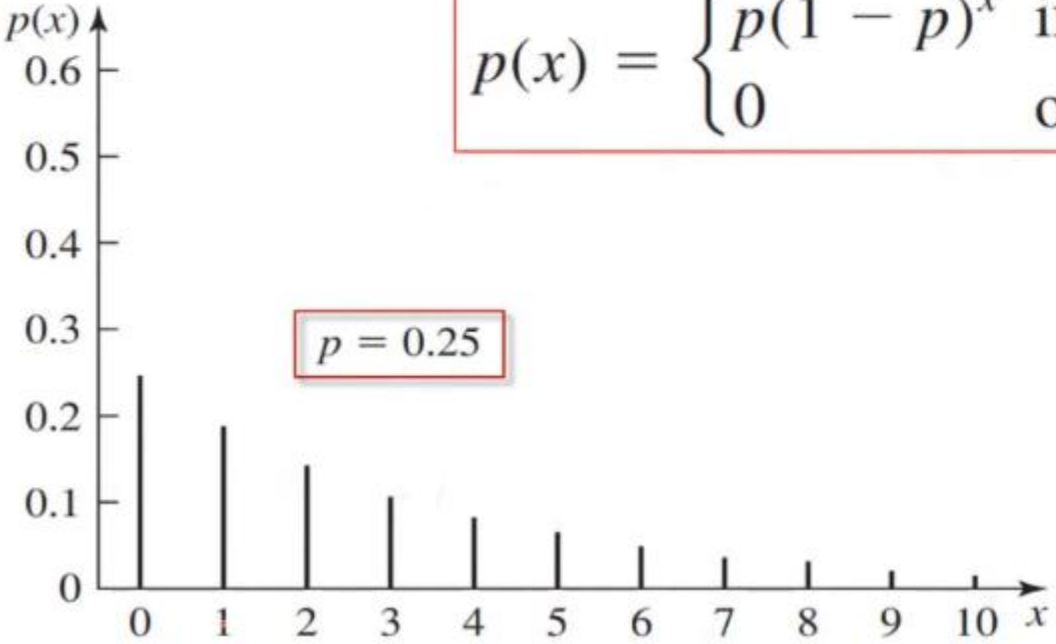
- i and j integers with $i \leq j$.

$$\text{Mass} \qquad p(x) = \begin{cases} \dfrac{1}{j - i + 1} & \text{if } x \in \{i, i+1, \ldots, j\} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Distribution} \quad F(x) = \begin{cases} 0 & \text{if } x < i \\ \dfrac{\lfloor x \rfloor - i + 1}{j - i + 1} & \text{if } i \leq x \leq j \\ 1 & \text{if } j < x \end{cases}$$

Geometric Distribution

- $p \in (0, 1)$.

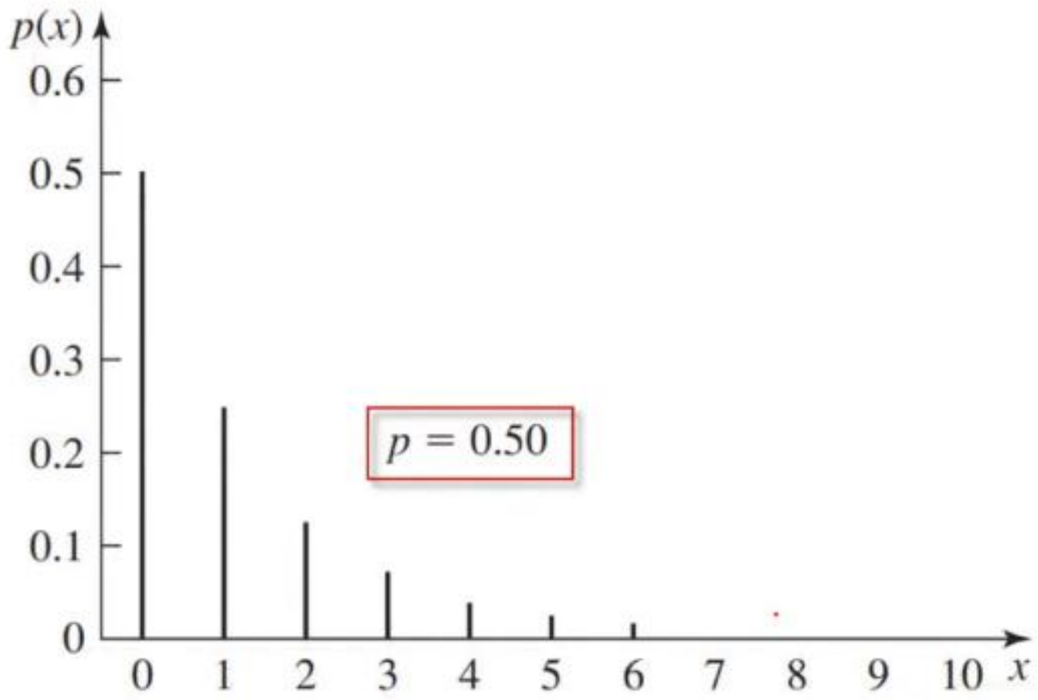


$$p(x) = \begin{cases} p(1-p)^x & \text{if } x \in \{0, 1, \ldots\} \\ 0 & \text{otherwise} \end{cases}$$

Mean	$\dfrac{1-p}{p}$
Variance	$\dfrac{1-p}{p^2}$
Mode	0

Geometric Graph

- $p \in (0, 1)$.



Cont.

- $p \in (0, 1)$.

$$\text{Mass} \qquad p(x) = \begin{cases} p(1-p)^x & \text{if } x \in \{0, 1, \ldots\} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Distribution } F(x) = \begin{cases} 1-(1-p)^{\lfloor x \rfloor + 1} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

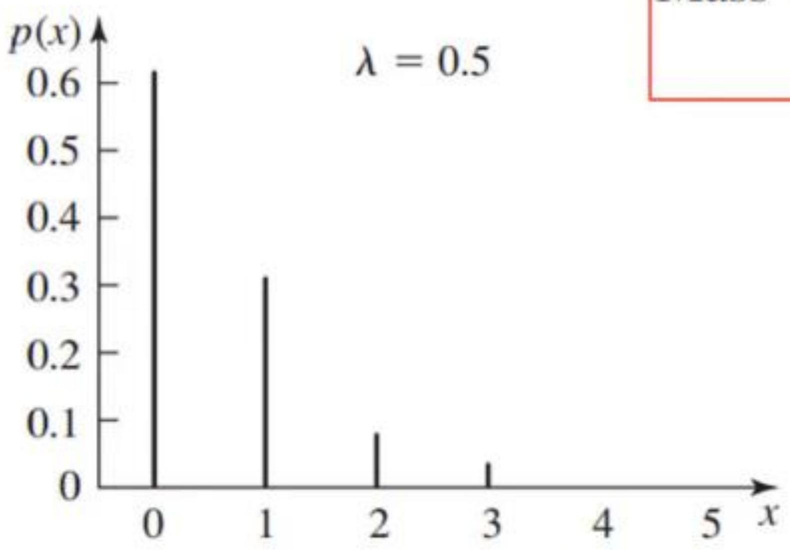
PRN

$$F^{-1}(u) = \lfloor \ln u / \ln(1-p) \rfloor.$$

Generating Random Variates X's

Poisson Distribution

- $\lambda > 0$.

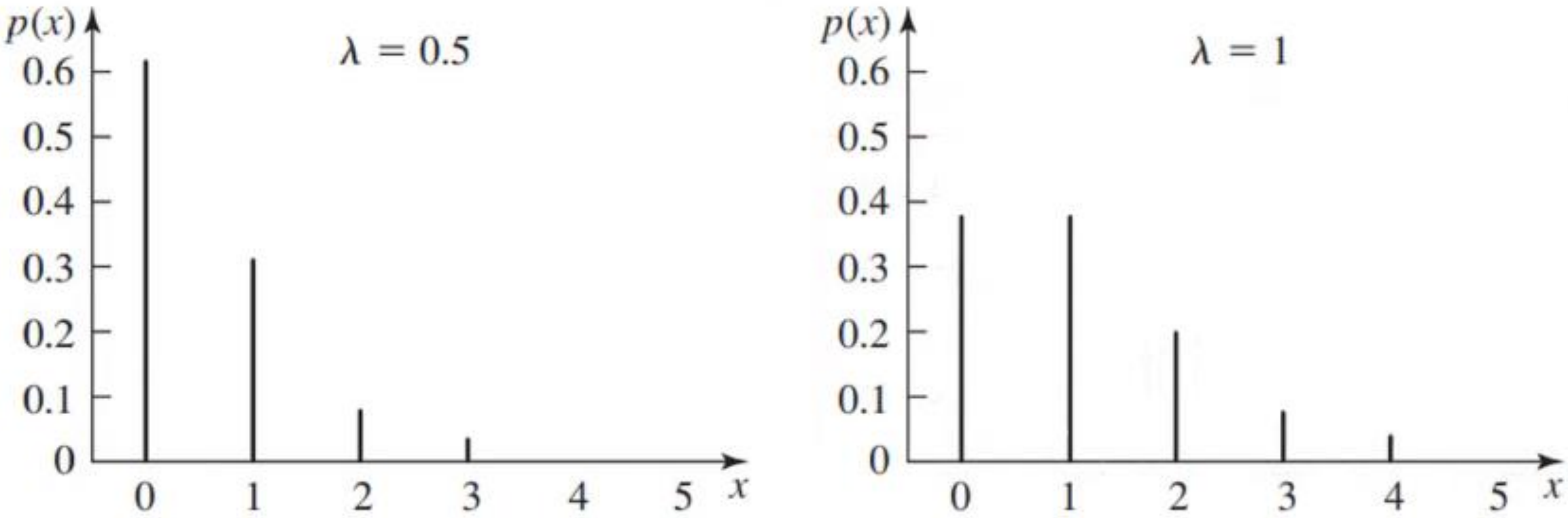


$$\text{Mass} \quad p(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & \text{if } x \in \{0, 1, \ldots\} \\ 0 & \text{otherwise} \end{cases}$$

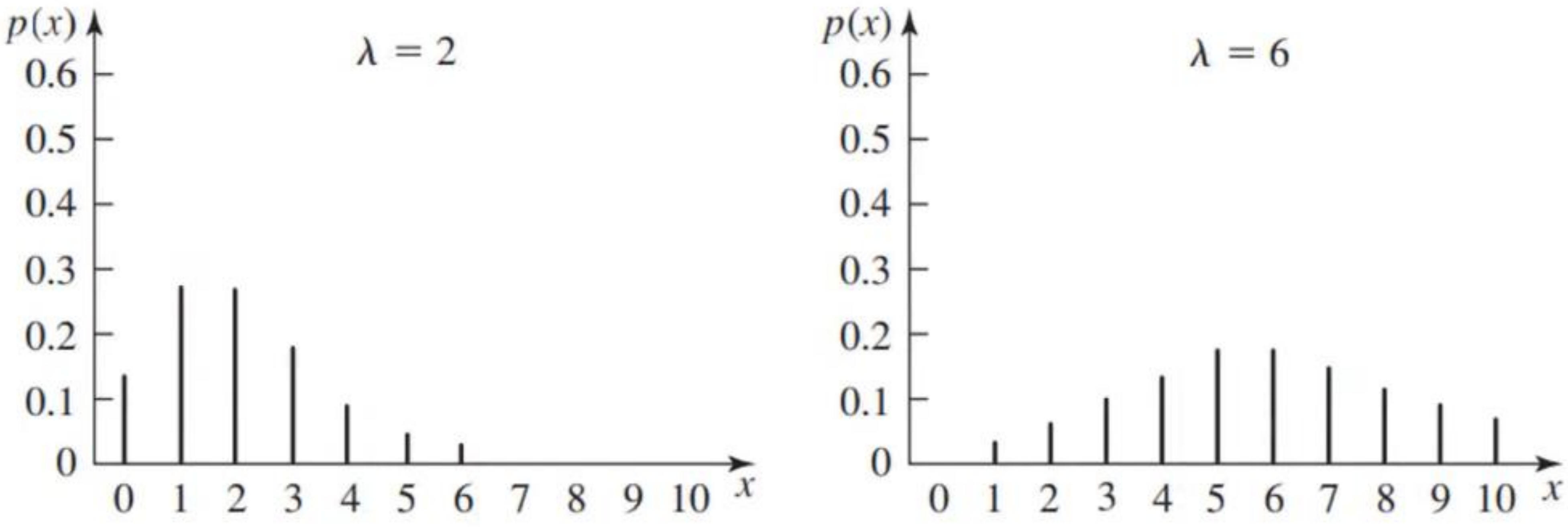
Mean	λ
Variance	λ
Mode	$\begin{cases} \lambda - 1 \text{ and } \lambda & \text{if } \lambda \text{ is an integer} \\ \lfloor \lambda \rfloor & \text{otherwise} \end{cases}$

Poisson Graph 1

- $\lambda > 0$.



Poisson Graph 2



Checking the distribution

- After collecting the data and selecting the standard probability distribution, you need to check if the selected distribution is appropriate and gives good fit of your data.
- This can be doe either:
  1. Graphically using graph comparison
  2. Using statistical tests such as K-S test, Chi-square test, etc.

Selecting The Distribution

- But how to select the most appropriate theoretical, standard distribution for your data?
- There are several methods to use to select a standard distribution for your data:
  1. Summary Statistics
  2. Histograms
  3. Quantile- Quantile Plot
  4. Boxplot
  5. And other methods

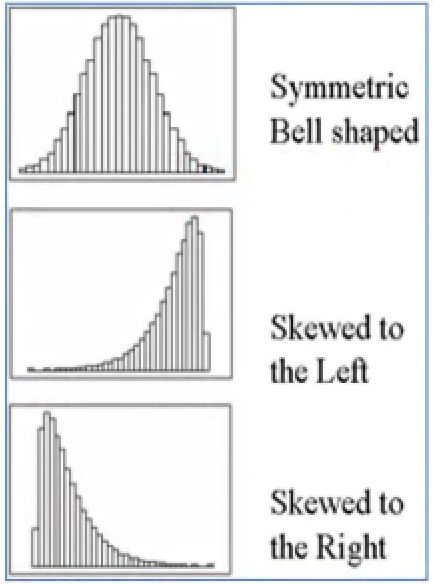
Summary Statistics

Function	Sample estimate (summary statistic)	Continuous (C) or discrete (D)
Minimum, maximum	$X_{(1)}, X_{(n)}$	C, D
Mean μ	$\bar{X}(n)$	C, D
Median $x_{0.5}$	$\hat{x}_{0.5}(n) = \begin{cases} X_{((n+1)/2)} & \text{if } n \text{ is odd} \\ [X_{(n/2)} + X_{((n/2)+1)}]/2 & \text{if } n \text{ is even} \end{cases}$	C, D
Variance σ^2	$S^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$	C, D

Cont.

Function	Sample estimate	Continuous (C) or discrete (D)
Coefficient of variation, cv $= \dfrac{\sqrt{\sigma^2}}{\mu}$	$\widehat{\mathrm{cv}}(n) = \dfrac{\sqrt{S^2(n)}}{\bar{X}(n)}$	C
Lexis ratio, $\tau = \dfrac{\sigma^2}{\mu}$	$\hat{\tau}(n) = \dfrac{S^2(n)}{\bar{X}(n)}$	D
Skewness, $\nu = \dfrac{E[(X-\mu)^3]}{(\sigma^2)^{3/2}}$	$\hat{\nu}(n) = \dfrac{n^2}{(n-1)(n-2)} \dfrac{\sum_{i=1}^{n} [X_i - \bar{X}(n)]^3/n}{[S^2(n)]^{3/2}}$	C, D

Cont.



$$\text{Skewness, } \nu = \frac{E[(X-\mu)^3]}{(\sigma^2)^{3/2}} \quad \hat{\nu}(n) = \frac{n^2}{(n-1)(n-2)} \frac{\sum_{i=1}^{n}[X_i - \bar{X}(n)]^3/n}{[S^2(n)]^{3/2}} \quad \text{C, D}$$

Example

- If the median is equal or near equal to the mean, this indicates symmetric, (e.g., normal) distribution.

- If the coefficient of variation (cv) is close to 1 this indicates exponential distribution because its cv is 1.

Example

- A simulation model was developed for a drive-up banking facility, and data were collected on the arrival pattern for cars. Over a fixed 90-minute interval, 220 cars arrived, and we noted the (continuous) interarrival time X_i (in minutes) between cars i and $i+1$, for $i = 1,2,\ldots,219$.

n = **219 interarrival times (minutes) sorted into increasing order**

0.01	0.06	0.12	0.23	0.38	0.53	0.88
0.01	0.07	0.12	0.23	0.38	0.53	0.88
0.01	0.07	0.12	0.24	0.38	0.54	0.90
0.01	0.07	0.13	0.25	0.39	0.54	0.93
0.01	0.07	0.13	0.25	0.40	0.55	0.93
0.01	0.07	0.14	0.25	0.40	0.55	0.95
0.01	0.07	0.14	0.25	0.41	0.56	0.97
0.01	0.07	0.14	0.25	0.41	0.57	1.03
0.02	0.07	0.14	0.26	0.43	0.57	1.05
0.02	0.07	0.15	0.26	0.43	0.60	1.05
0.03	0.07	0.15	0.26	0.43	0.61	1.06
0.03	0.08	0.15	0.26	0.44	0.61	1.09
0.03	0.08	0.15	0.26	0.45	0.63	1.10
0.04	0.08	0.15	0.27	0.45	0.63	1.11
0.04	0.08	0.15	0.28	0.46	0.64	1.12
0.04	0.09	0.17	0.28	0.47	0.65	1.17
0.04	0.09	0.18	0.29	0.47	0.65	1.18
0.04	0.10	0.19	0.29	0.47	0.65	1.24
0.04	0.10	0.19	0.30	0.48	0.69	1.24
0.05	0.10	0.19	0.31	0.49	0.69	1.28
0.05	0.10	0.20	0.31	0.49	0.70	1.33
0.05	0.10	0.21	0.32	0.49	0.72	1.38
0.05	0.10	0.21	0.35	0.49	0.72	1.44
0.05	0.10	0.21	0.35	0.50	0.72	1.51
0.05	0.10	0.21	0.35	0.50	0.74	1.72
0.05	0.10	0.21	0.36	0.50	0.75	1.83
0.05	0.11	0.22	0.36	0.51	0.76	1.96
0.05	0.11	0.22	0.36	0.51	0.77	
0.05	0.11	0.22	0.37	0.51	0.79	
0.06	0.11	0.23	0.37	0.52	0.84	
0.06	0.11	0.23	0.38	0.52	0.86	
0.06	0.12	0.23	0.38	0.53	0.87	

Cont.

Summary statistics for the interarrival-time data

Summary statistic	Value
Minimum	0.010
Maximum	1.960
Mean	0.399
Median	0.270
Variance	0.144
Coefficient of variation	0.953
Skewness	1.478

Cont.

Since:

$\bar{X}(219) = 0.399 > 0.270 = \hat{x}_{0.5}(219)$ and $\hat{v}(219) = 1.478$, this suggests that the underlying distribution is skewed to the right, rather than symmetric.

Furthermore, $\widehat{cv}(219) = 0.953$, which is close to the theoretical value of 1 for the ***exponential*** distribution.

Summary statistics for the interarrival-time data

Summary statistic	Value
Minimum	0.010
Maximum	1.960
Mean	0.399
Median	0.270
Variance	0.144
Coefficient of variation	0.953
Skewness	1.478

Histograms

- To make a histogram, we break up the range of values covered by the data into k disjoint adjacent intervals $[b_0, b_1), [b_1, b_2), \dots, [D_{k-1}, 1, b_k)$.
- All the intervals should be the same width Δb, which might necessitate throwing out a few extremely large or small X_i's to avoid getting an unwieldy-looking histogram plot.

$$\Delta b = \frac{\max(x) - \min(x)}{k} \qquad k = \frac{\max(x) - \min(x)}{\Delta b}$$

- Selecting the best k or Δb is trial and error process, although there are some rules that helps to approximate the value of k

$$k = \lfloor 1 + \log_2 n \rfloor = \lfloor 1 + 3.322 \log_{10} n \rfloor$$

n = 219 interarrival times (minutes) sorted into increasing order

0.01	0.06	0.12	0.23	0.38	0.53	0.88
0.01	0.07	0.12	0.23	0.38	0.53	0.88
0.01	0.07	0.12	0.24	0.38	0.54	0.90
0.01	0.07	0.13	0.25	0.39	0.54	0.93
0.01	0.07	0.13	0.25	0.40	0.55	0.93
0.01	0.07	0.14	0.25	0.40	0.55	0.95
0.01	0.07	0.14	0.25	0.41	0.56	0.97
0.01	0.07	0.14	0.25	0.41	0.57	1.03
0.02	0.07	0.14	0.26	0.43	0.57	1.05
0.02	0.07	0.15	0.26	0.43	0.60	1.05
0.03	0.07	0.15	0.26	0.43	0.61	1.06
0.03	0.08	0.15	0.26	0.44	0.61	1.09
0.03	0.08	0.15	0.26	0.45	0.63	1.10
0.04	0.08	0.15	0.27	0.45	0.63	1.11
0.04	0.08	0.15	0.28	0.46	0.64	1.12
0.04	0.09	0.17	0.28	0.47	0.65	1.17
0.04	0.09	0.18	0.29	0.47	0.65	1.18
0.04	0.10	0.19	0.29	0.47	0.65	1.24
0.04	0.10	0.19	0.30	0.48	0.69	1.24
0.05	0.10	0.19	0.31	0.49	0.69	1.28
0.05	0.10	0.20	0.31	0.49	0.70	1.33
0.05	0.10	0.21	0.32	0.49	0.72	1.38
0.05	0.10	0.21	0.35	0.49	0.72	1.44
0.05	0.10	0.21	0.35	0.50	0.72	1.51
0.05	0.10	0.21	0.35	0.50	0.74	1.72
0.05	0.10	0.21	0.36	0.50	0.75	1.83
0.05	0.11	0.22	0.36	0.51	0.76	1.96
0.05	0.11	0.22	0.36	0.51	0.77	
0.05	0.11	0.22	0.37	0.51	0.79	
0.06	0.11	0.23	0.37	0.52	0.84	
0.06	0.11	0.23	0.38	0.52	0.86	
0.06	0.12	0.23	0.38	0.53	0.87	

Cont.

- Let us use the previous formula to find k

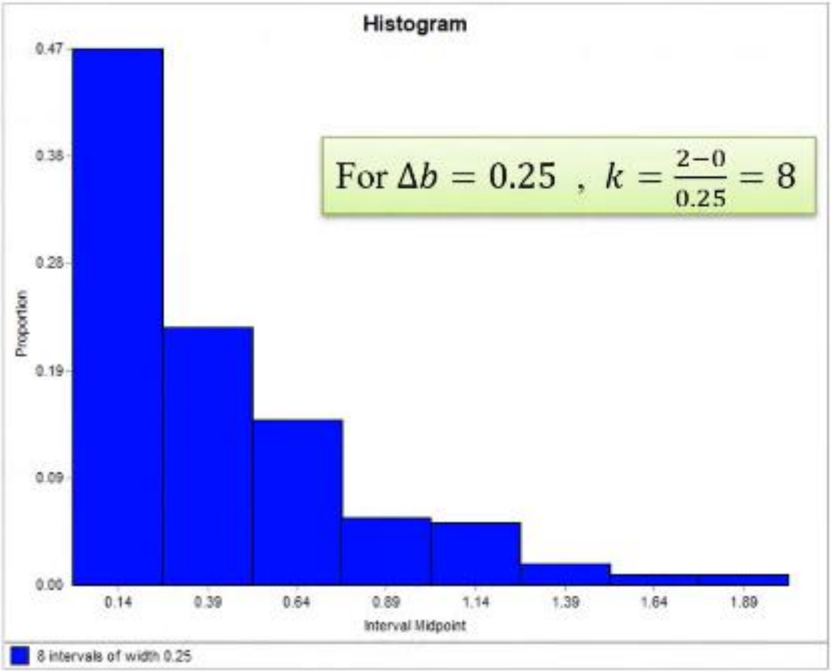
$$k = \lfloor 1 + \log_2 n \rfloor = \lfloor 1 + 3.322 \log_{10} n \rfloor$$

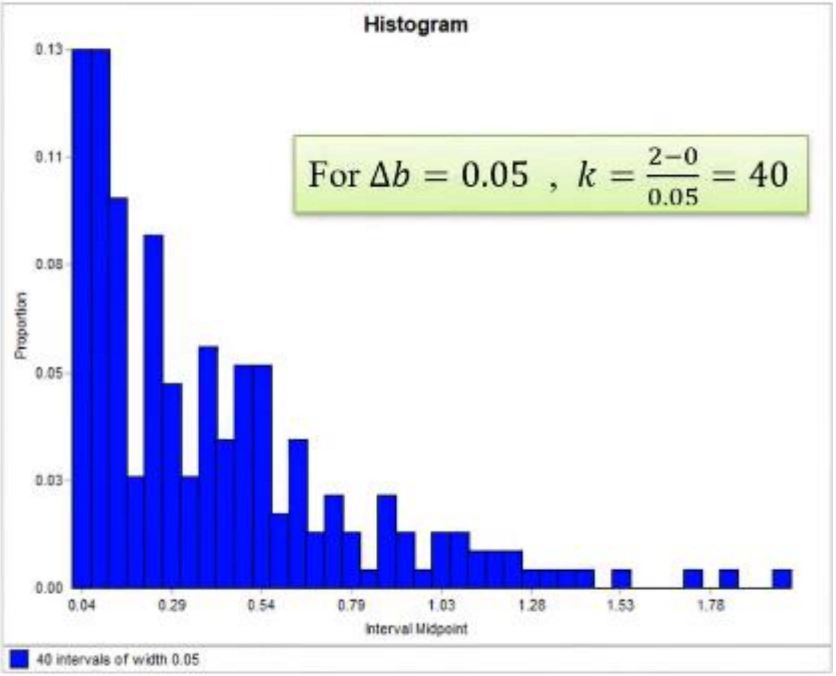
$$k = \lfloor 1 + \log_2 219 \rfloor = 8$$

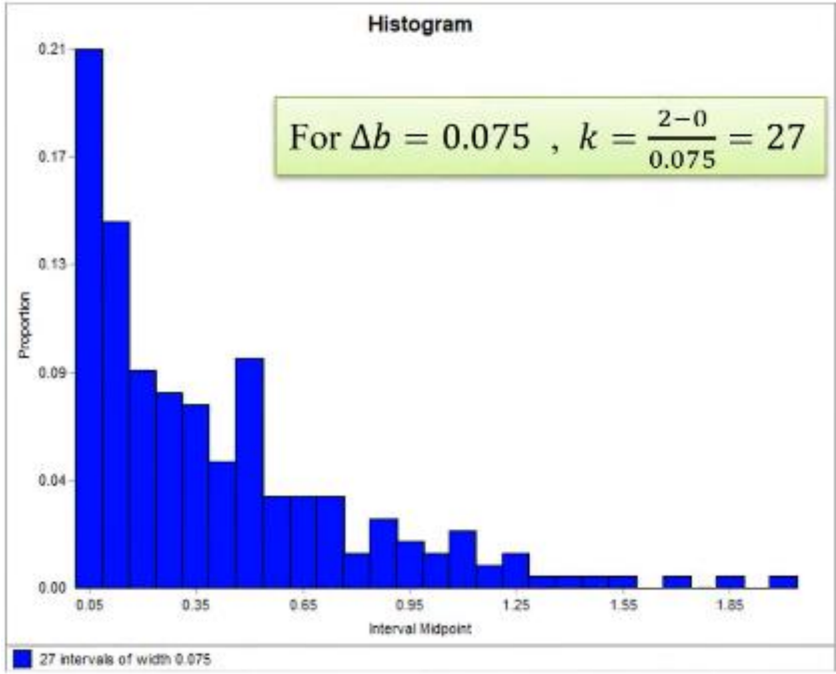
$$\min X = 0.01 \approx 0 \quad , \max X = 1.96 \approx 2 \ , \qquad \Delta b = \frac{2-0}{8} = 0.25$$

$$[0, 0.25), [0.25, 0.5), [0.5, 0.75), [0.75, 1),$$
$$[1, 1.25), [1.25, 1.5), [1.5, 1.75), \ [1.75, 2),$$

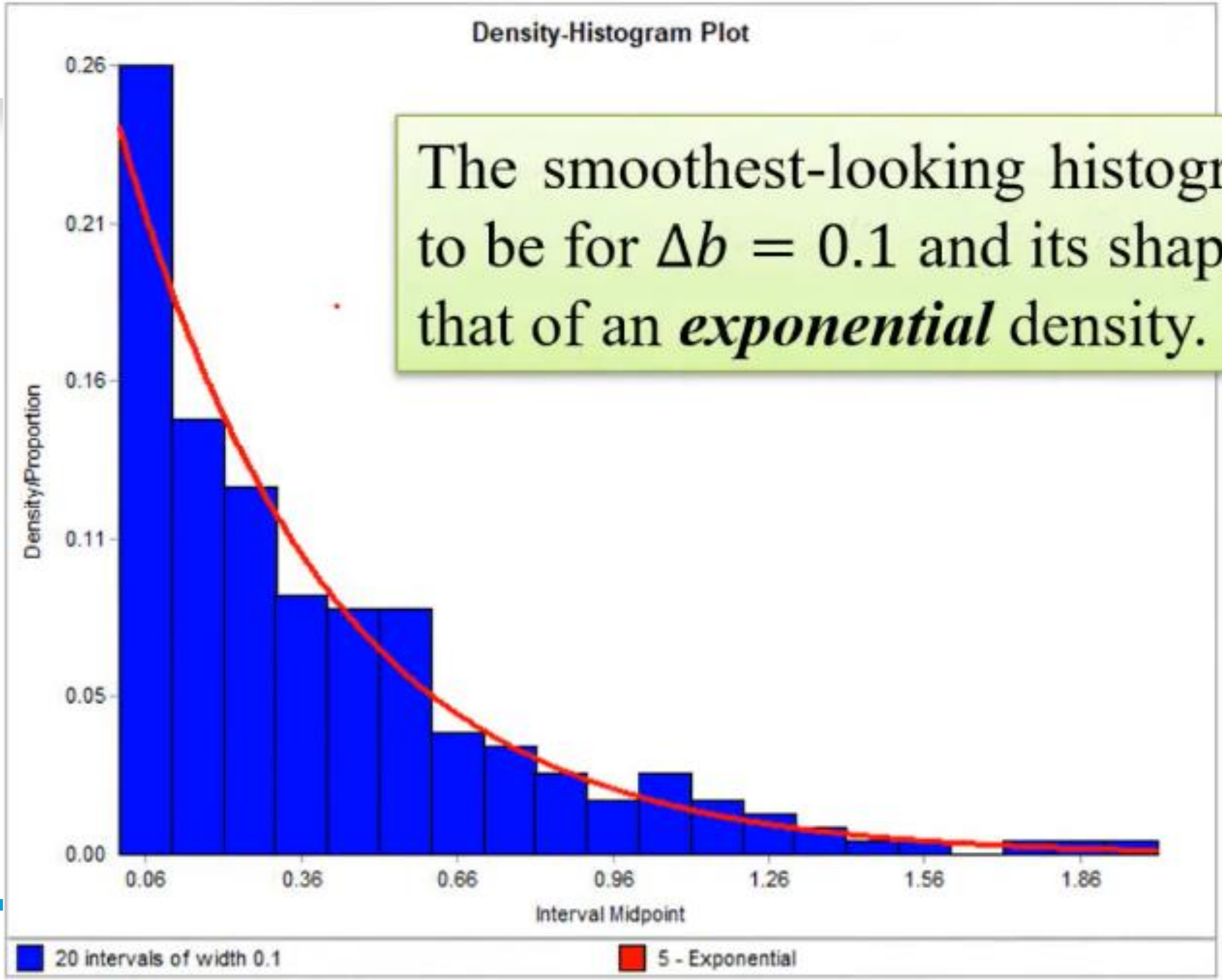
Cont.


Histogram
For $\Delta b = 0.25$, $k = \frac{2-0}{0.25} = 8$
Proportion
Interval Midpoint
8 intervals of width 0.25


Histogram
For $\Delta b = 0.05$, $k = \frac{2-0}{0.05} = 40$
Proportion
Interval Midpoint
40 intervals of width 0.05


Histogram
For $\Delta b = 0.075$, $k = \frac{2-0}{0.075} = 27$
Proportion
Interval Midpoint
27 intervals of width 0.075

Example

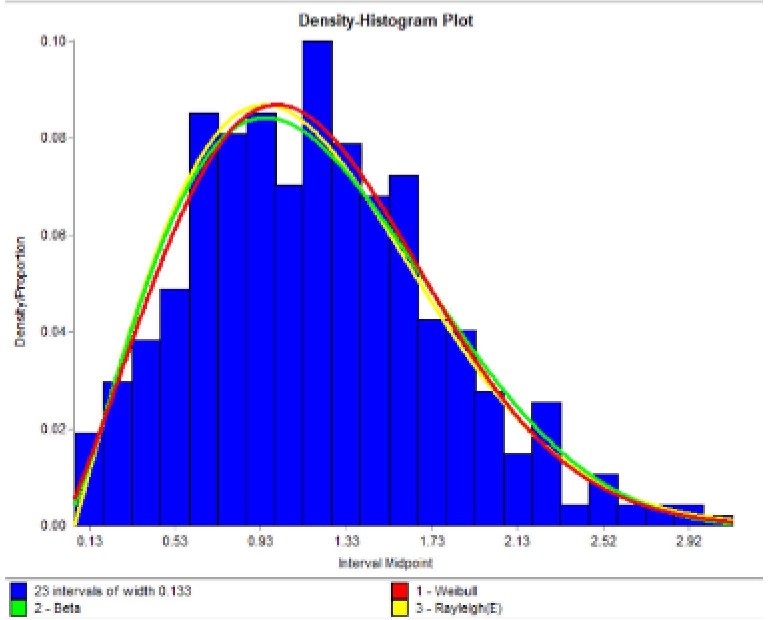


The smoothest-looking histogram appears to be for $\Delta b = 0.1$ and its shape resembles that of an ***exponential*** density.

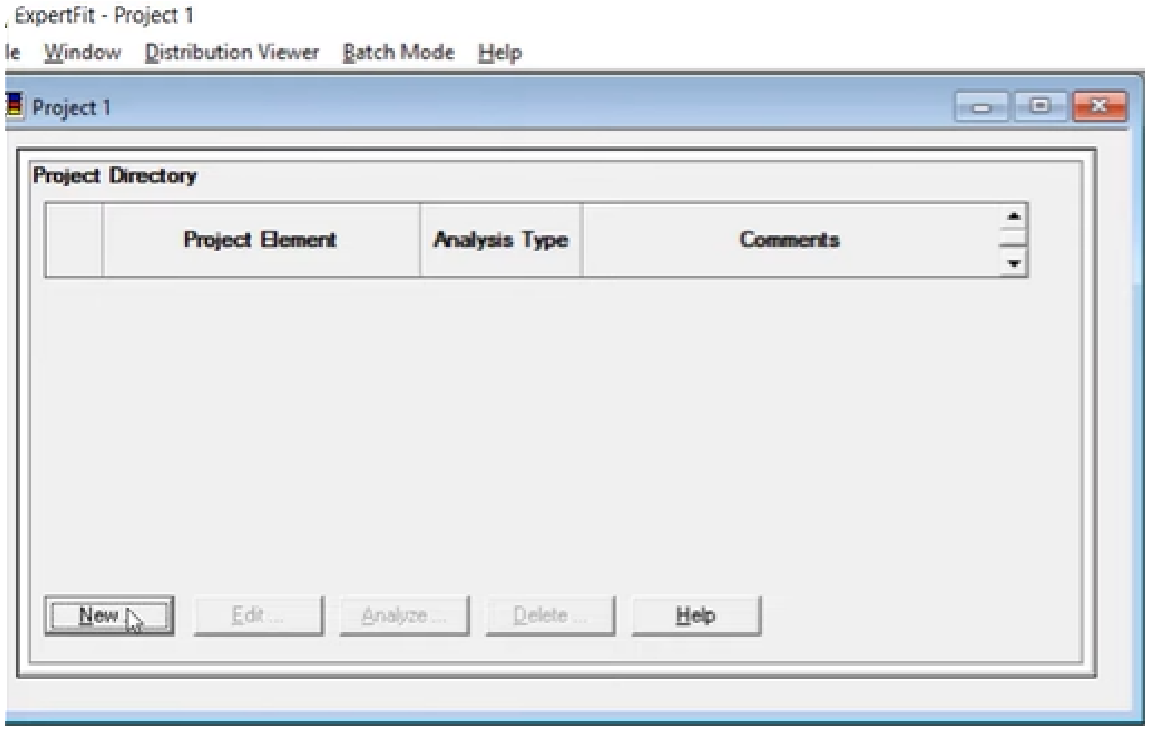
ExpertFit

- There are several software that help in modeling a given data, making the selection of the appropriate distribution an easy task.
- ExpertFit is one of these software

  https://www.averill-law.com/distribution-fitting/



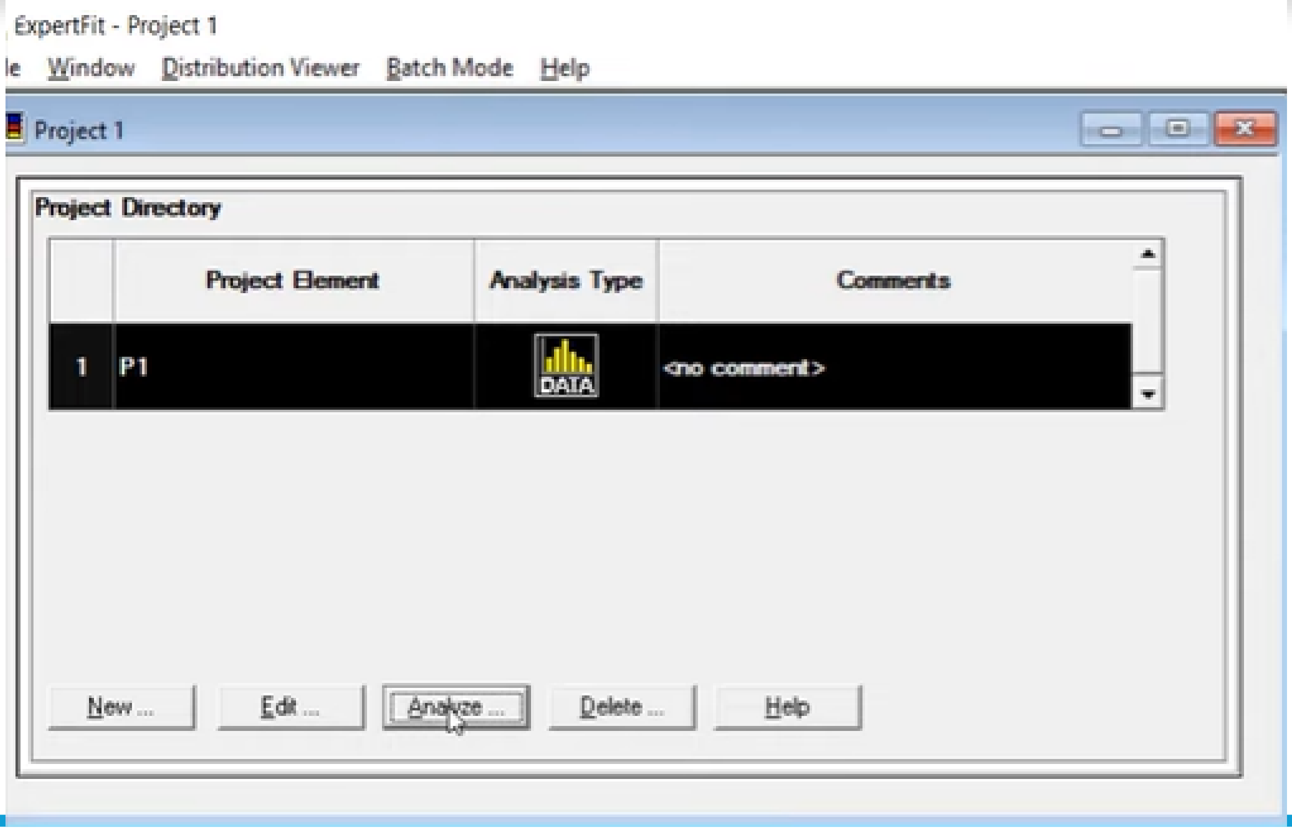
Cont.


ExpertFit - Project 1
le Window Distribution Viewer Batch Mode Help
Project 1
Project Directory
Project Element
Analysis Type
Comments
New
Edit ...
Analyze ...
Delete ...
Help

Cont.

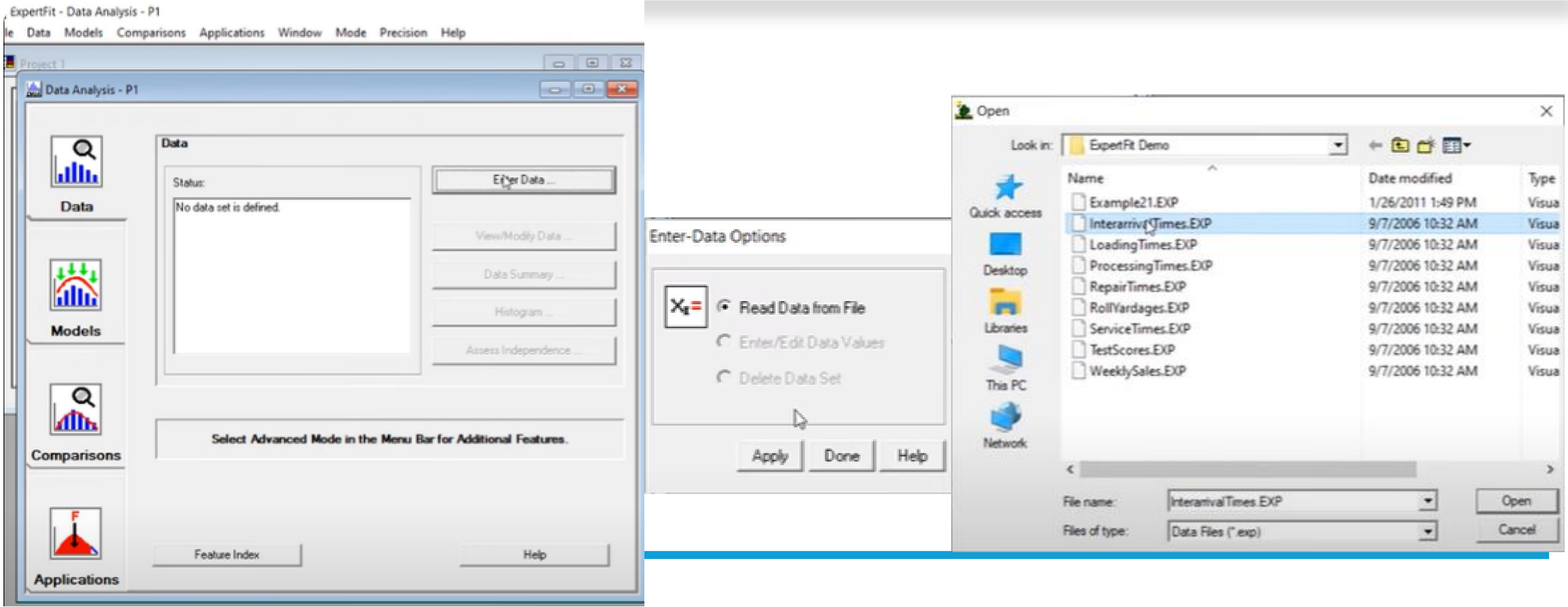
Project-Element Editing
Project-Element Name:
P1
Comments:
<no comment>
Analysis Type
DATA
Fit distributions to data
NO DATA ?
Construct distributions in the absence of data
Help
Cancel
OK

Cont.



ExpertFit - Project 1
le Window Distribution Viewer Batch Mode Help
Project 1
Project Directory
Project Element
Analysis Type
Comments
1
P1
DATA
<no comment>
New ...
Edit ...
Analyze ...
Delete ...
Help

Cont.

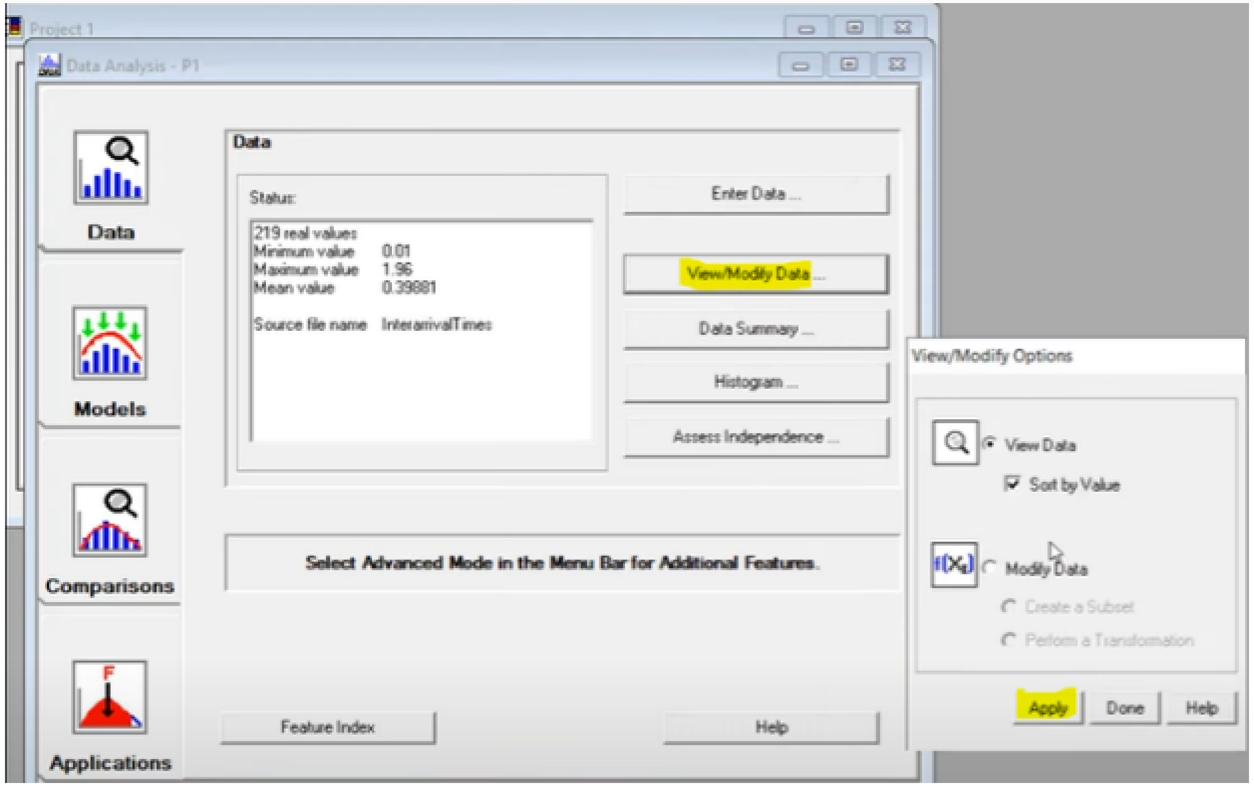

ExpertFit - Data Analysis - P1
le Data Models Comparisons Applications Window Mode Precision Help
Project 1
Data Analysis - P1
Data
Models
Comparisons
Applications
Data
Status:
No data set is defined.
Enter Data ...
View/Modify Data ...
Data Summary ...
Histogram ...
Assess Independence ...
Select Advanced Mode in the Menu Bar for Additional Features.
Feature Index
Help
Enter-Data Options
Read Data from File
Enter/Edit Data Values
Delete Data Set
Apply
Done
Help
Open
Look in: ExpertFit Demo
Quick access
Desktop
Libraries
This PC
Network
Name
Date modified
Type
Example21.EXP
1/26/2011 1:49 PM
InterarrivalTimes.EXP
LoadingTimes.EXP
ProcessingTimes.EXP
RepairTimes.EXP
RollYardages.EXP
ServiceTimes.EXP
TestScores.EXP
WeeklySales.EXP
9/7/2006 10:32 AM
File name: InterarrivalTimes.EXP
Files of type: Data Files (*.exp)
Open
Cancel

Cont.


Data-Summary Table

Data Characteristic	Value
Source file	InterarrivalTimes
Observation type	Real valued
Number of observations	219
Minimum observation	0.01000
Maximum observation	1.96000
Mean	0.39881
Median	0.27000
Variance	0.14439
Coefficient of variation	0.95281
Skewness	1.47845

Copy
Print
Help
Done



Cont.

Listing of Data Values

92	0.22000	202	1.05000
93	0.22000	203	1.06000
94	0.23000	204	1.09000
95	0.23000	205	1.10000
96	0.23000	206	1.11000
97	0.23000	207	1.12000
98	0.23000	208	1.17000
99	0.24000	209	1.18000
100	0.25000	210	1.24000
101	0.25000	211	1.24000
102	0.25000	212	1.28000
103	0.25000	213	1.33000
104	0.25000	214	1.38000
105	0.26000	215	1.44000
106	0.26000	216	1.51000
107	0.26000	217	1.72000
108	0.26000	218	1.83000
109	0.26000	219	1.96000
110	0.27000		

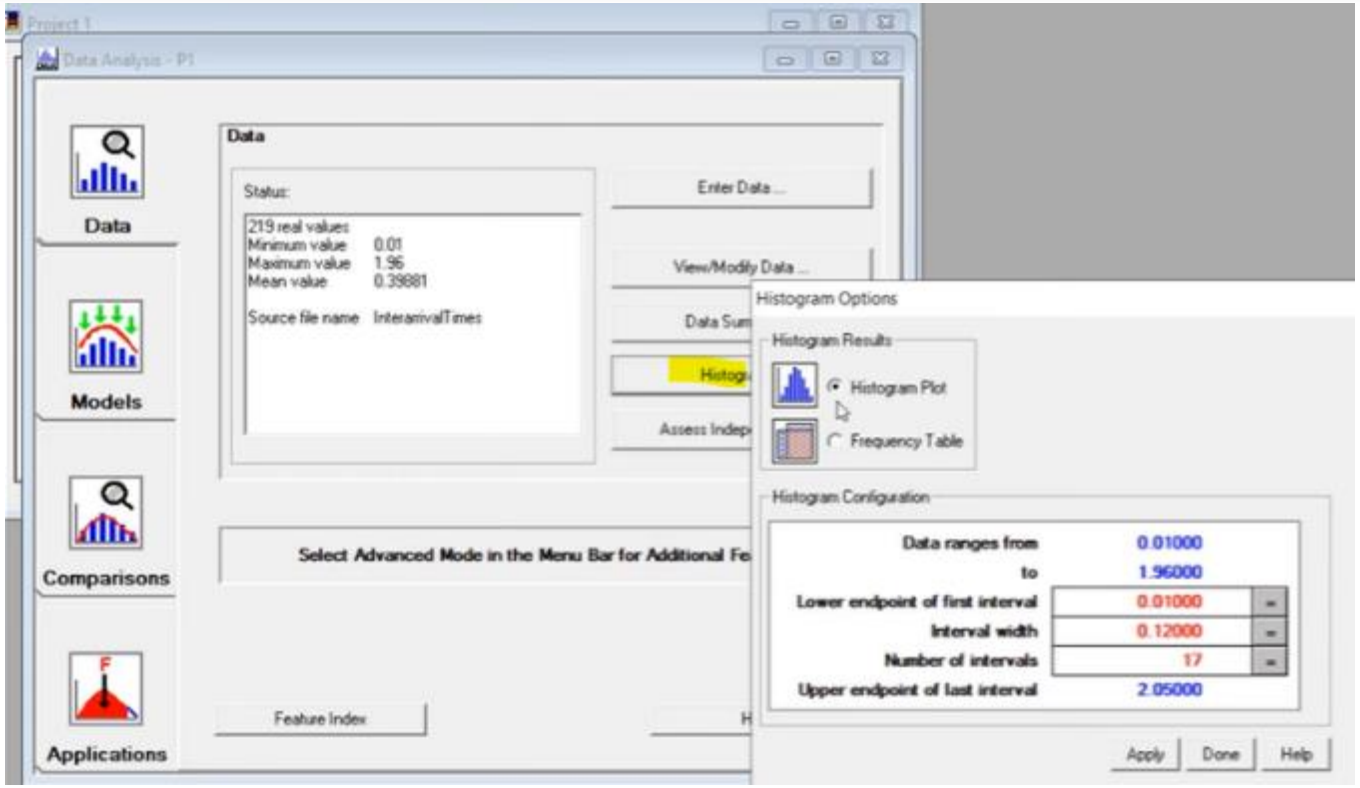
Copy

Print

Help

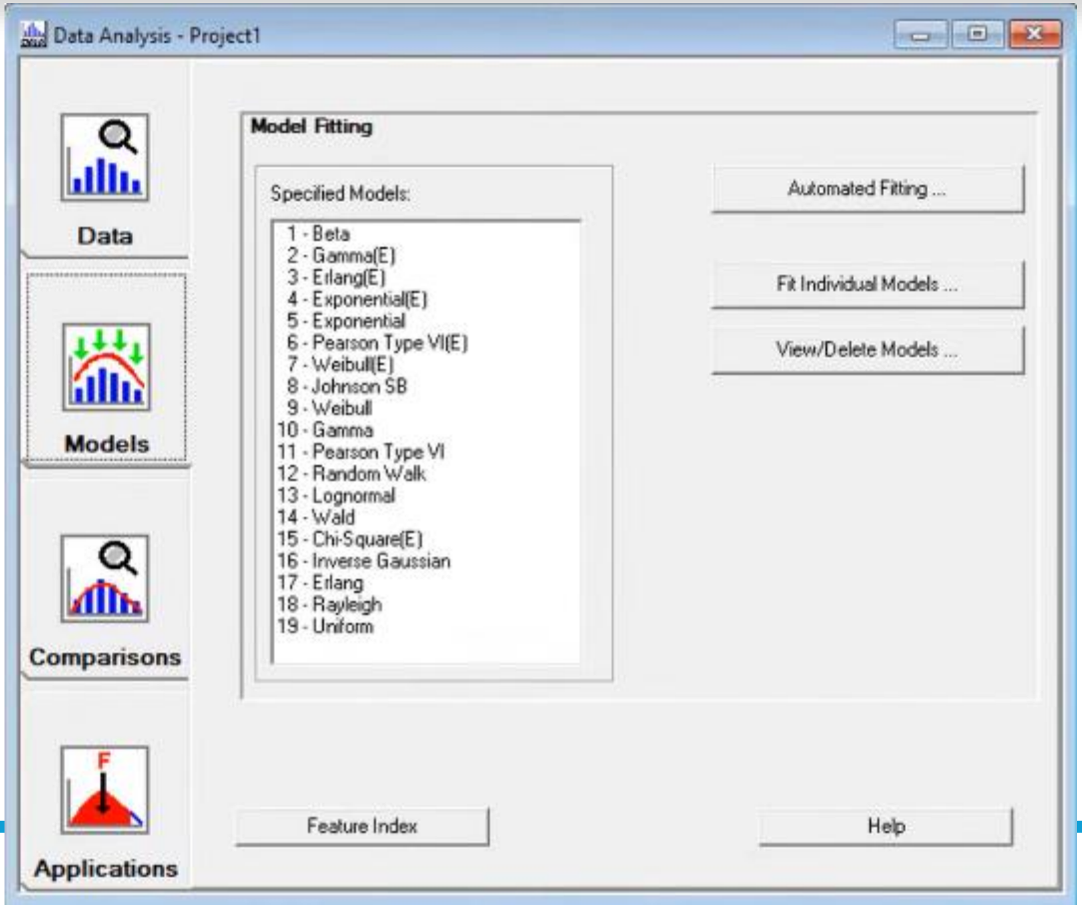
Done

Cont.

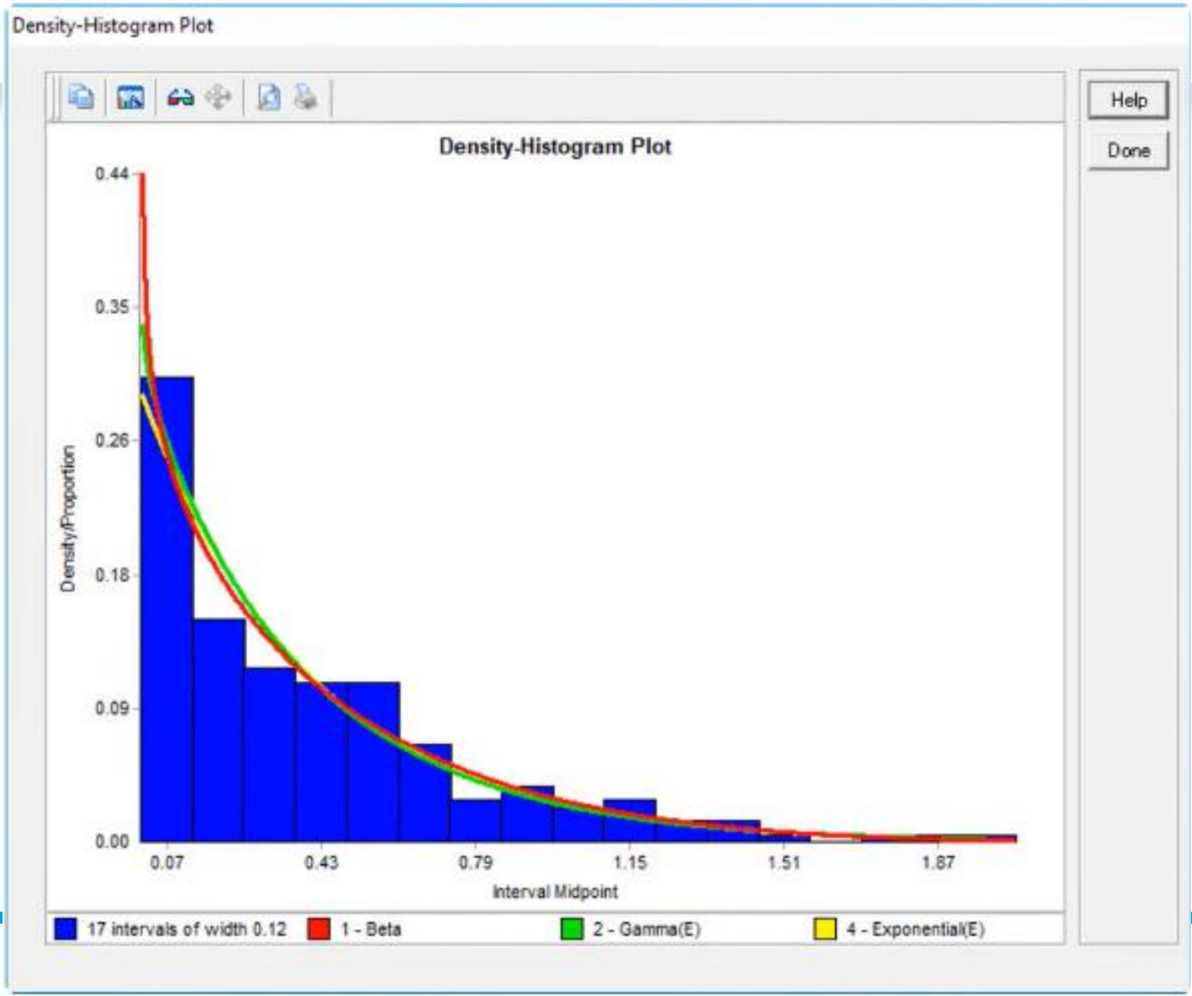


Data Analysis - P1
Data
Models
Comparisons
Applications
Data
Status:
219 real values
Minimum value 0.01
Maximum value 1.96
Mean value 0.39881
Source file name InterarrivalTimes
Enter Data ...
View/Modify Data ...
Select Advanced Mode in the Menu Bar for Additional Fe
Feature Index
Histogram Options
Histogram Results
Histogram Plot
Frequency Table
Histogram Configuration
Data ranges from 0.01000
to 1.96000
Lower endpoint of first interval 0.01000
Interval width 0.12000
Number of intervals 17
Upper endpoint of last interval 2.05000
Apply
Done
Help

Cont.

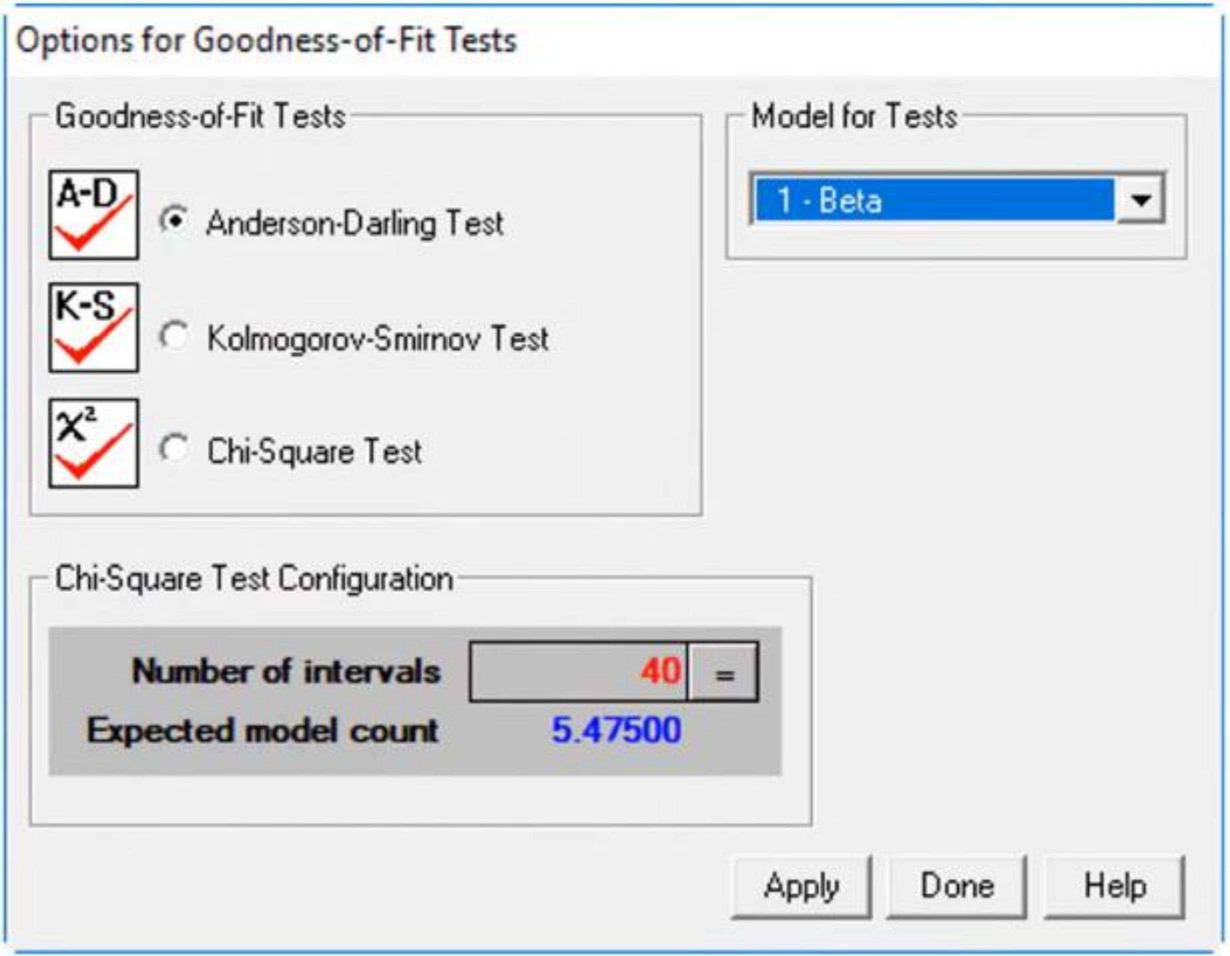

Data Analysis - Project1
Data
Models
Comparisons
Applications
Model Fitting
Specified Models:
1 - Beta
2 - Gamma(E)
3 - Erlang(E)
4 - Exponential(E)
5 - Exponential
6 - Pearson Type VI(E)
7 - Weibull(E)
8 - Johnson SB
9 - Weibull
10 - Gamma
11 - Pearson Type VI
12 - Random Walk
13 - Lognormal
14 - Wald
15 - Chi-Square(E)
16 - Inverse Gaussian
17 - Erlang
18 - Rayleigh
19 - Uniform
Automated Fitting ...
Fit Individual Models ...
View/Delete Models ...
Feature Index
Help

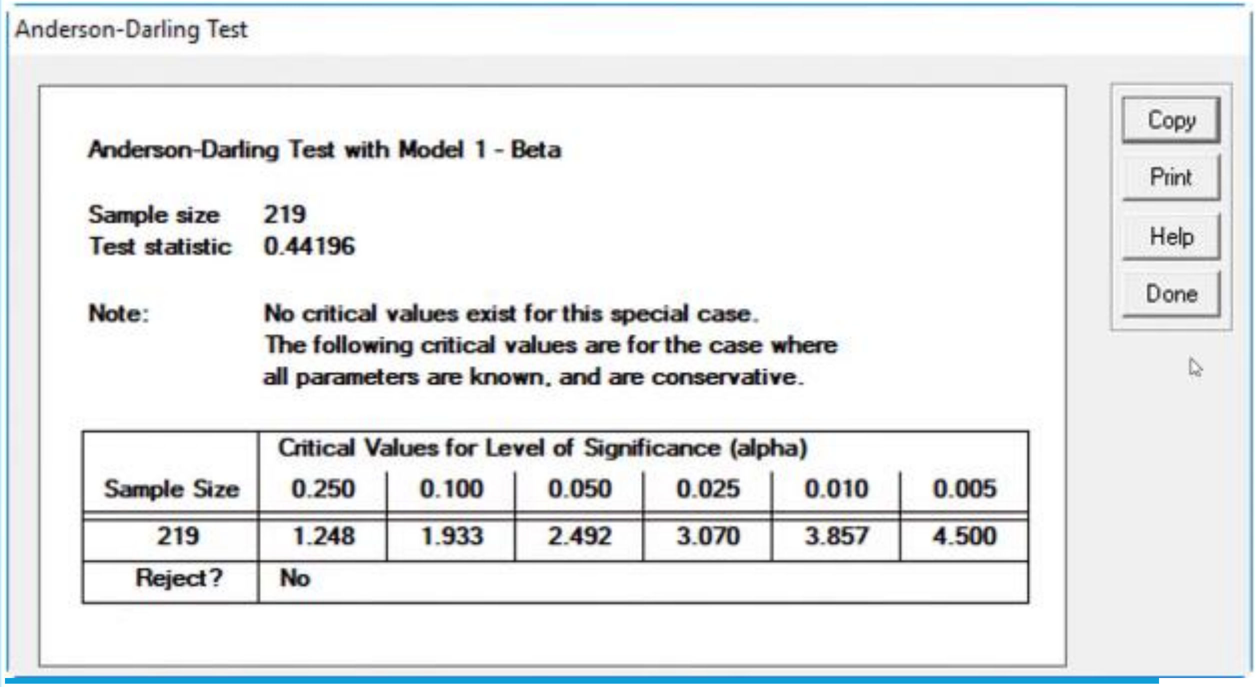
Cont.



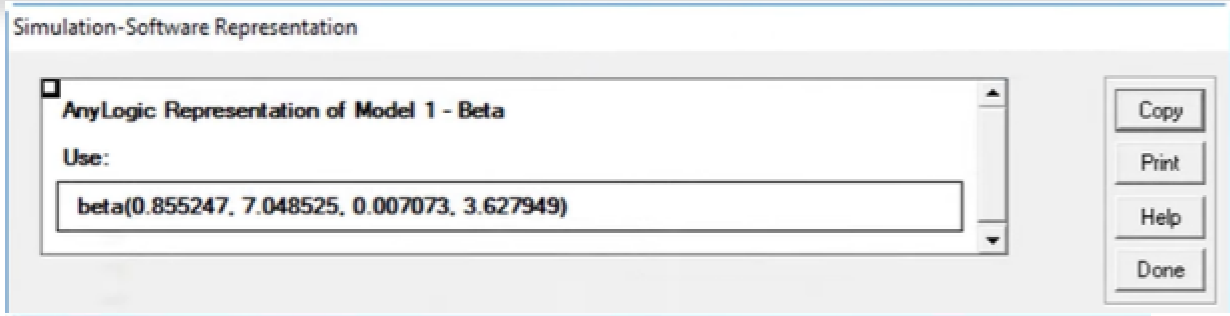
Density-Histogram Plot
Help
Done
Density-Histogram Plot
0.44
0.35
0.26
0.18
0.09
0.00
Density/Proportion
0.07
0.43
0.79
1.15
1.51
1.87
Interval Midpoint
17 intervals of width 0.12
1 - Beta
2 - Gamma(E)
4 - Exponential(E)

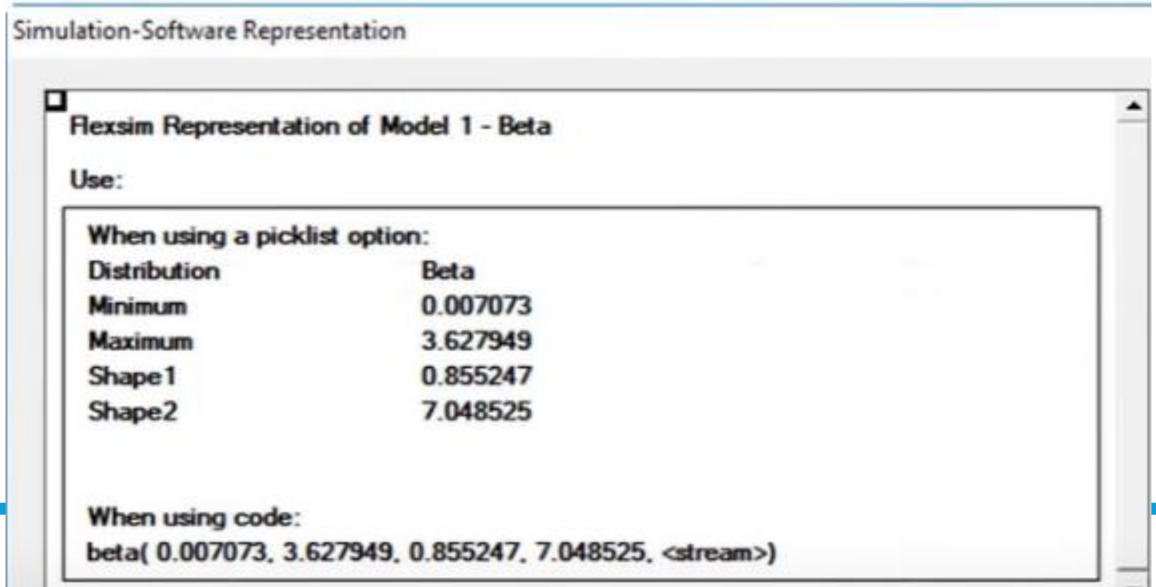
Cont.


Options for Goodness-of-Fit Tests
Goodness-of-Fit Tests
A-D
Anderson-Darling Test
K-S
Kolmogorov-Smirnov Test
Chi-Square Test
Model for Tests
1 - Beta
Chi-Square Test Configuration
Number of intervals
40
=
Expected model count
5.47500
Apply
Done
Help


Anderson-Darling Test
Anderson-Darling Test with Model 1 - Beta
Sample size 219
Test statistic 0.44196
Note: No critical values exist for this special case.
The following critical values are for the case where
all parameters are known, and are conservative.
Critical Values for Level of Significance (alpha)
Sample Size 0.250 0.100 0.050 0.025 0.010 0.005
219 1.248 1.933 2.492 3.070 3.857 4.500
Reject? No
Copy
Print
Help
Done

Cont.


Simulation-Software Representation
AnyLogic Representation of Model 1 - Beta
Use:
beta(0.855247, 7.048525, 0.007073, 3.627949)
Copy
Print
Help
Done


Simulation-Software Representation
Flexsim Representation of Model 1 - Beta
Use:
When using a picklist option:
Distribution Beta
Minimum 0.007073
Maximum 3.627949
Shape1 0.855247
Shape2 7.048525
When using code:
beta(0.007073, 3.627949, 0.855247, 7.048525, <stream>)

Cont.

Thank you!

Hope you Enjoyed the course