Inventories simulation

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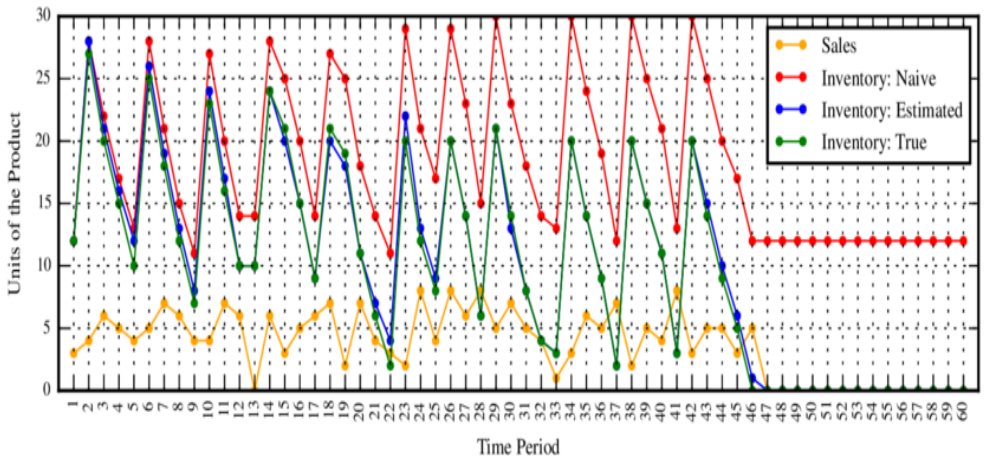
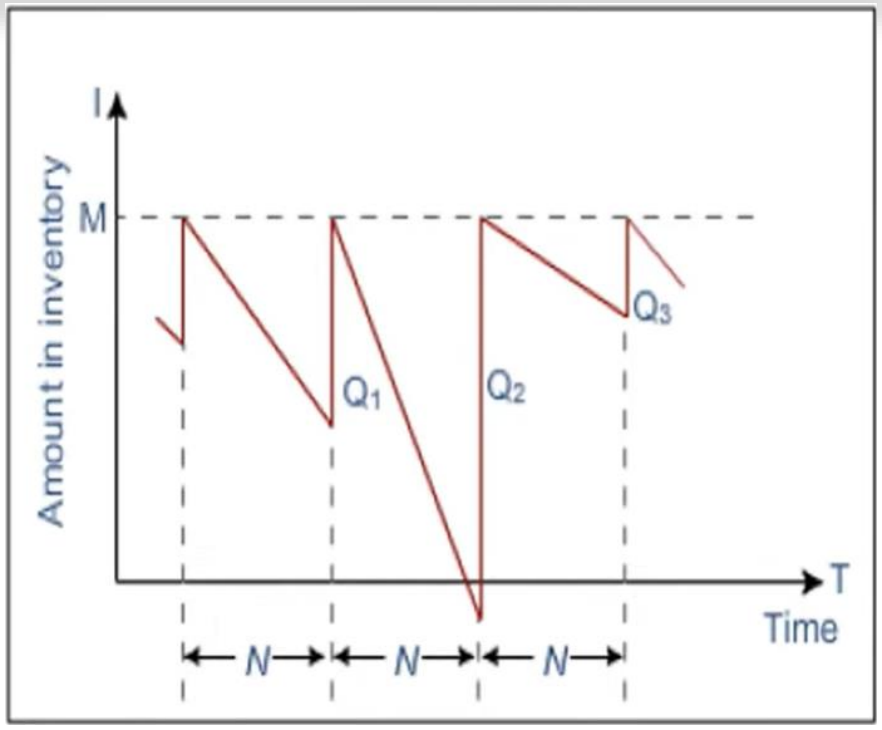


Outline

- Introduction
- Simulation and inventory system
- Examples

Introduction

- An inventory mainly consists of the following attributes:
  - N: Periodic review length: The time needed before the inventory is reviewed
  - M: The capacity (inventory standard level)
  - Q_i: The quantity needed to fill the inventory up to M
  - Lead time: the time needed before the new order arrives
- The demand from customers is not known, therefore we take it from probability distributions.



What is the lead time in the graph?

Cont.

- Usually, inventory owners try to avoid a shortage
  - Customers demand and their demand is not available in the inventory
- This can be avoided by renting a bigger inventory and fill it with items
  - But is this a helpful solution?
  - More rental fees are needed
  - more items means more cash is needed to fill the inventory
  - Bigger inventories needs more security
  - etc.

Cont.

- Another solution is to make the review intervals shorter
- But this also has additional costs
  - orders require fees for the delivery

- The main **performance** metric for an inventory system is the total profits (or costs)
- The goal is to tune the parameters **M**, inventory level, and **N**, the **review cycle**, such that we get the best **performance**

Events in (M, N) inventory system

- The inventory system has several events:
  1. Demand for an item or items
  2. Review of the inventory
  3. Place an order at the end of the review cycle

- We need to find out **the average ending units**, and **the number of days in which a shortage occurs**
- Given:
  1. **The number of units demanded per day (distribution)**
  2. **The lead time (distribution)**

Example

- Assume that we have an inventory with standard level, M, equal to 11 and review period length, N, equal to 5
  - M = 11 units
  - N = 5 days

- And we collected the data and calculated the distributions of **daily demand** and **lead time**
  - following slide

Cont.

Demand	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
0	0.10		
1	0.25		
2	0.35		
3	0.21		
4	0.09		

Lead Time (Days)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.6		
2	0.3		
3	0.1		

Cont.

Demand	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
0	0.10		
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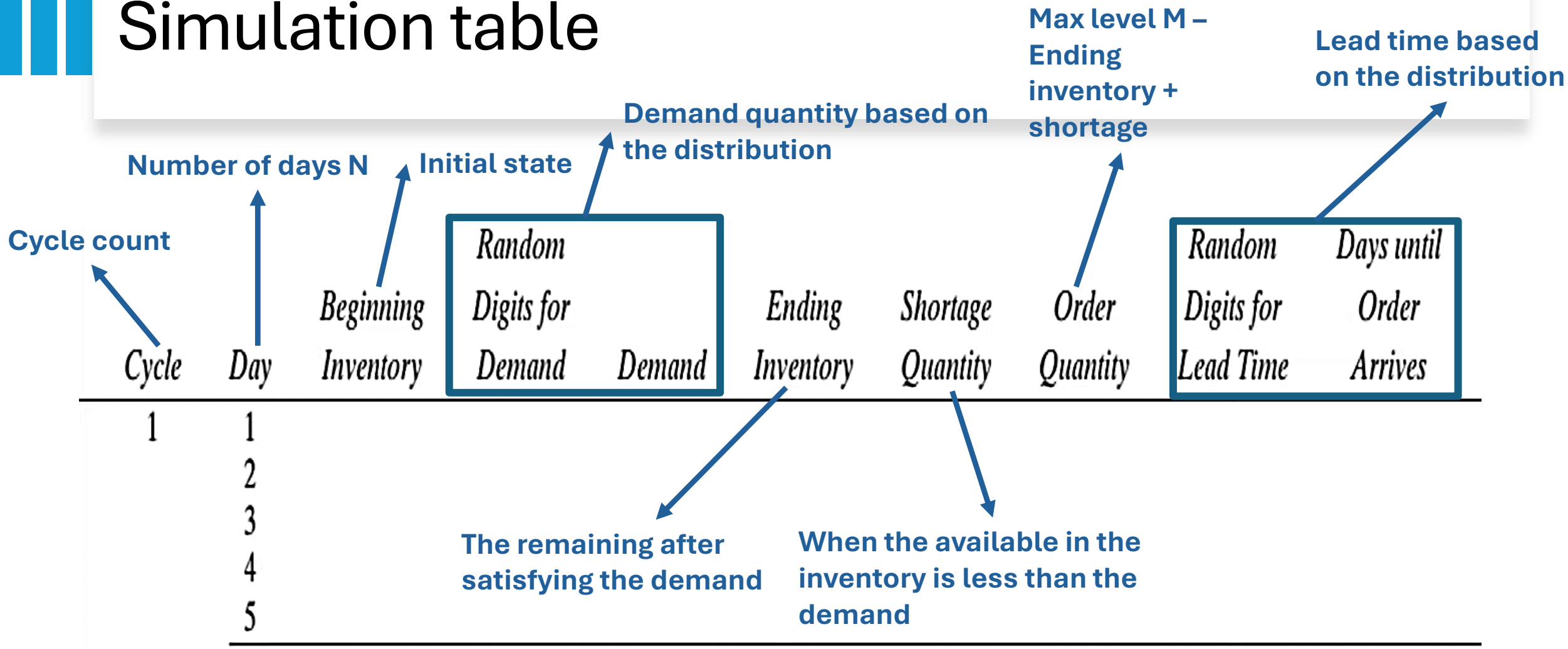
Lead Time (Days)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.6		
2	0.3		
3	0.1		

Starting the simulation

- Recall
  - M = 11
  - N = 5 days
- Review length = N => once each 5 days
- Simulate for 5 cycles

- Initial state, at the beginning of the simulation:
  - The inventory has 3 units
  - An order is placed, 8 units, which will arrive in 2 days.

Simulation table



Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3							
	2								
	3								
	4								
	5								

3 is defined in the scenario

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24						
	2		35						
	3		65						
	4		81						
	5		54						

Cont.

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24						
	2		35						
	3		65						
	4		81						
	5		54						

Demand	Probability	Cumulative Probability	Random-Digit Assignment
0	0.10	0.10	01–10
1	0.25	0.35	11–35
2	0.35	0.70	36–70
3	0.21	0.91	71–91
4	0.09	1.00	92–00

`Cont.

- Remember the initial state says we ordered 8 units to be arrived in 2 days, day 1 is done. So, the units will arrive after **1** day

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1					
	2		35						
	3		65						
	4		81						
	5		54						

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2		35				–	–	
	3		65						
	4		81						
	5		54						

- Now, the 8 units arrived so they will be added to the inventory. therefore, we start the new day with 9: 1+8

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3		65						
	4		81						
	5		54						

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3	9	65						
	4		81						
	5		54						

- At the end of the cycle, we should order to fill the inventory. If there is a shortage, it will be added to the order as an extra

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3	9	65	2	7	0	–	–	–
	4	7	81	3	4	0	–	–	–
	5	4	54	2	2	0			

In our case we need to order 9 to make the inventory full: 11 units

- Now we need to generate a number for the lead time and identify when the order is expected to arrive.

Lead Time (Days)	Probability	Cumulative Probability	Random-Digit Assignment
1	0.6	0.6	1–6
2	0.3	0.9	7–9
3	0.1	1.0	0

		Random				Order	Random Digits for	Days until Order
						Quantity	Lead Time	Arrives
						–	–	1
2	2	35	1	1	0	–	–	0
3	9	65	2	7	0	–	–	–
4	7	81	3	4	0	–	–	–
5	4	54	2	2	0	9	5	

According to the distribution, the new units, 9, are assumed to arrive after 1 day

Cycle 2

- The first row is the last row from the past cycle

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	54	2	2	0	9	5	1
2	1		03					–	
	2		87					–	
	3		27					–	
	4		73					–	
	5		70					0	

Cycle 2

- we start the new cycle with what the past cycle was ended

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	54	2	2	0	9	5	1
2	1		03					–	
	2		87					–	
	3		27					–	
	4		73					–	
	5		70					0	

Cycle 2

- **Now the units we ordered from the previous cycle will arrive**
  - **We start the new day with 2: from the previous day + 9 ordered in the previous cycle:11**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	–	–	0
	2		87					–	
	3		27					–	
	4		73					–	
	5		70					0	

Cycle 2

- **Now the units we ordered from the previous cycle will arrive**
  - **We start the new day with 2: from the previous day + 9 ordered in the previous cycle:11**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	–	–	0
	2	11	87	3	8	0	–	–	–
	3		27					–	
	4		73					–	
	5		70					0	

Cycle 2

- **We finish this cycle by placing an order to fill the inventory. The order will arrive after 3 days**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	–	–	0
	2	11	87	3	8	0	–	–	–
	3	8	27	1	7	0	–	–	–
	4	7	73	3	4	0	–	–	–
	5	4	70	2	2	0	9	0	3

Cycle 3

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1		47				–	–	2
	2		45					–	1
	3		48					–	0
	4		17					–	–
	5		09						

Cycle 3

- **A shortage occurs at day 2, where a demand is placed, 2 units, while the inventory is empty.**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2		45				–	–	1
	3		48					–	0
	4		17					–	–
	5		09						

Cycle 3

- **Another shortage at day 3**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4		17					–	–
	5		09						

Cycle 3

- **The 9 units arrived, but we already have a shortage of 4 units.**
  - **This is similar to starting the day with 5 units.**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4	9	17					–	–
	5		09						

Cycle 3

- **A new demand, 1 unit, comes at day 4, and we had 4 units shortage, then we end the inventory for day 4 with 4 units**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4	9	17	1	4	0	–	–	–
	5		09						

Cycle 3

- **We finish cycle 3 by placing an order to fill the inventory**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4	9	17	1	4	0	–	–	–

9-4

Etc.

- **We complete the same process for the rest of the cycles**

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3	9	65	2	7	0	–	–	–
	4	7	81	3	4	0	–	–	–
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	–	–	0
	2	11	87	3	8	0	–	–	–
	3	8	27	1	7	0	–	–	–
	4	7	73	3	4	0	–	–	–
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4	9	17	1	4	0	–	–	–
	5	4	09	0	4	0	7	3	1
4	1	4	42	2	2	0	–	–	0
	2	9	87	3	6	0	–	–	–
	3	6	26	1	5	0	–	–	–
	4	5	36	2	3	0	–	–	–
	5	3	40	2	1	0	10	4	1
5	1	1	07	0	1	0	–	–	0
	2	11	63	2	9	0	–	–	–
	3	9	19	1	8	0	–	–	–
	4	8	88	3	5	0	–	–	–
	5	5	94	4	1	0	10	8	2

88

Performance analysis

- The average ending inventory during the simulated cycles is

$$AEI = \frac{88}{25} = 3.52$$

- The percentage of shortage is

$$SP = \frac{2}{25} = 0.08$$

- This process can be done for large number of cycles

Graph the simulation

Cycle	Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity	Order Quantity	Random Digits for Lead Time	Days until Order Arrives
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3	9	65	2	7	0			–
	4	7	81	3	4	0			–
	5	4	54	2	2	0			1
2	1	2	03	0	2	0	–	–	0
	2	11	87	3	8	0	–	–	–
	3	8	27	1	7	0	–	–	–
	4	7	73	3	4	0	–	–	–
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0			
	2	0	45	2	0	2			
	3	0	48	2	0	4			
	4	9	17	1	4	0			
	5	4	09	0	4	0			
4	1			2	2	0			
	2			3	6	0			
	3	6	26	1	5	0			
	4	5	36	2	3	0			
	5	3	40	2	1	0			
5	1	1	07	0	1	0			
	2	11	63	2	9	0			
	3	9	19	1	8	0			
	4	8	88	3	5	0			
	5	5	94	4	1	0			

88

Amount: y-Axis

Time: x-Axis