# Strassen's Matrix Multiplication

SUBJECT-CODE : KCS-503

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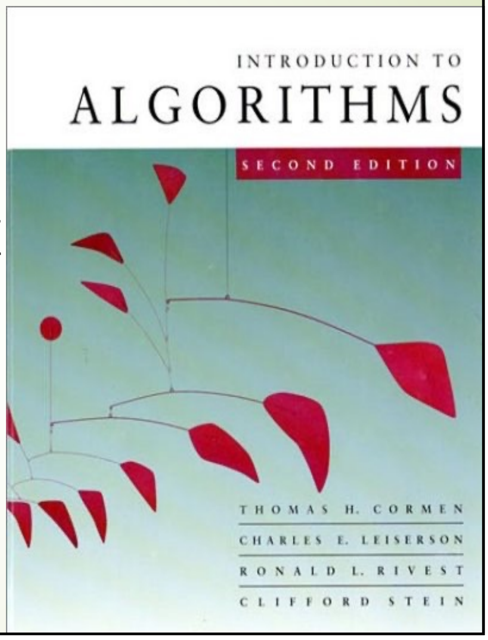


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## Strassen's Matrix Multiplication

- Strassen's algorithm can be viewed as an application of a familiar design technique:

  Divide And Conquer

- Suppose we wish to compute the product C=AB, where each of A, B, and C are n*n matrices.
- Assuming that n is an exact power of 2, we divide each of A, B, and C into four n/2 * n/2 matrices, rewriting the equation C=AB as follows:

$$\begin{bmatrix} r & s \\ t & u \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

- There are 4 equations corresponding to the above i.e. :

  r = ae + bg
  s = af + bh
  t = ce + dg
  u = cf + dh



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# Strassen's Matrix Multiplication

- Each of these four equations specifies two multiplications of n/2 x n/2 matrices and the addition of their n/2 x n/2 products.
- Using these equation to define a straightforward divide-and-conquer strategy, we derive the following recurrence for the time T(n) to multiply two n x n matrices:

$$T(n) = 8T(n/2) + \Theta(n^2) \quad = \Theta(n^3)$$

- Unfortunately, recurrence has the solution $T(n) = \Theta(n^3)$, and thus this method is no faster than the ordinary one.
- Strassen discovered a different recursive approach that requires only 7 recursive multiplications of n/2 x n/2 matrices and $\Theta(n^2)$ scalar additions and subtractions, yielding the recurrence

$$T(n) = 7\, T(n/2) + \Theta(n^2) = \Theta(n^{\lg 7}) = O(n^{2.81})$$





# Strassen's Matrix Multiplication

- **Strassen's method has four steps:**

**1. Divide the input matrices A and B into n/2 * n/2 submatrices.**

**2. Using $\Theta(n^2)$ scalar additions and subtractions, compute 14 n/2 * n/2 matrices $A_1, B_1, A_2, B_2, \ldots, A_7, B_7$.**

**3. Recursively compute the seven matrix products $P_i = A_iB_i$ for i =1, 2,………, 7.**

**4. Compute the desired submatrices r, s, t, u of the result matrix C by adding and/or subtracting various combinations of the $P_i$ matrices, using only $\Theta(n^2)$ scalar additions and subtractions.**

# Strassen's Matrix Multiplication

- We know that C = A*B

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix}$$

**In stressen we calculate 7, n/2 x n/2 matrices, we get**

**$P = (A_{11} + A_{22})(B_{11} + B_{22})$**
**$Q = (A_{21} + A_{22}) * B_{11}$**
**$R = A_{11} * (B_{12} - B_{22})$**
**$S = A_{22} * (B_{21} - B_{11})$**
**$T = (A_{11} + A_{12}) * B_{22}$**
**$U = (A_{21} - A_{11}) * (B_{11} + B_{12})$**
**$V = (A_{12} - A_{22}) * (B_{21} + B_{22})$**

**We find Cij's using the formula -**

**$C_{11} = P+S-T+V$**
**$C_{12} = R+T$**
**$C_{21} = Q+S$**
**$C_{22} = P+R-Q+U$**



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# Strassen's Matrix Multiplication

- **Compute the matrix product and show the calculations :**

| | | | | |
|---|---|---|---|---|
| **1** | **5** | **and** | **7** | **4** |
| **3** | **8** | | **6** | **2** |

**$P = (A_{11} + A_{22})(B_{11} + B_{22})$**
**$Q = (A_{21} + A_{22}) * B_{11}$**
**$R = A_{11} * (B_{12} - B_{22})$**
**$S = A_{22} * (B_{21} - B_{11})$**
**$T = (A_{11} + A_{12}) * B_{22}$**
**$U = (A_{21} - A_{11}) * (B_{11} + B_{12})$**
**$V = (A_{12} - A_{22}) * (B_{21} + B_{22})$**

**$C_{11} = P+S-T+V$**
**$C_{12} = R+T$**
**$C_{21} = Q+S$**
**$C_{22} = P+R-Q+U$**



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