

$$k \frac{\partial T(0,t)}{\partial x} = \frac{1}{r} T(0,t)$$

$$\frac{1}{r} = h(p) = H f(\pi)$$

$$\Delta l(t) = \alpha \int_0^l T(x,t) dx$$

$$g(t) = g_0 - \alpha \int_0^l T(x,t) dx > 0$$

$$p(t) = \frac{E}{l} \left\{ \alpha \int_0^l T(x,t) dx - g_0 \right\} \geq 0$$

$$\sigma = E \epsilon$$

$$\epsilon = \frac{\partial u}{\partial x}$$

Nondimensionalize

$$\theta_l = \frac{\alpha T_l l}{g_0}$$

$$\pi = \frac{p l}{E g_0}$$

$$\theta_0 = \frac{\alpha T_0 l}{g_0}$$

$$\rho = \frac{K}{g_0 H}$$

Low Applied Temperatures

$$\tau_l < 1$$

$$T(x,t) = T_l$$

High Applied Temperature

$$\tau_l \geq \lambda$$

$$T(x,t) = T_l \frac{x}{l}$$

$$\boxed{\frac{1}{\lambda} \tau_l - 1 \leq \pi < \tau_l - 1}$$

$$\boxed{\rho \frac{d\tau_0}{dx} = \tau_0 f(\pi)}$$

$$\tau_l + \tau_0 - \lambda \geq 0$$

$$\tau_l \geq 1$$

$$0 \leq \tau_0 \leq \tau_l$$

$$\boxed{f(\pi) \geq 0}$$

$$\boxed{f(0) = 0}$$

$$\left[\pi - \left(\frac{1}{\lambda} \tau_l - 1 \right) \right] f(\pi) = \rho (\tau_l - 1 - \pi)$$

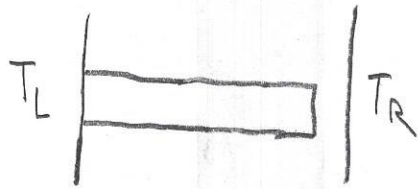
$$\alpha(\pi) f(\pi) = \rho b(\pi)$$

$$\frac{d}{d\pi} [\alpha(\pi) f(\pi)] > \frac{d}{d\pi} [\rho b(\pi)]$$

$$\boxed{\left[\pi - \left(\frac{1}{\lambda} \tau_l - 1 \right) \right] f'(\pi) + f(\pi) + \rho > 0}$$

Comninou & Dunders 6

Pelesko



Nondimensionalizing

$$\theta = \frac{T - T_L}{T_R - T_L}$$

$$t = \frac{k}{\rho C_p L^2} t'$$

$$x = \frac{x'}{L}$$

$$u = \frac{u'}{L}$$

$$\sigma = \frac{\sigma'}{E}$$

Equations

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \quad 0 < x < 1$$

$$\frac{\partial u}{\partial x^2} = \mu \frac{\partial \theta}{\partial x} \quad 0 < x < 1$$

$$\sigma = \frac{\partial u}{\partial x} - \mu \theta \quad 0 < x < 1$$

BC's

$$\theta(0, t) = 0$$

$$u(0, t) = 0$$

$$\begin{cases} u \leq 0 \\ \sigma \leq 0 \\ u\sigma = 0 \end{cases} \text{ at } x=1$$

$$R(\eta) \frac{\partial \theta}{\partial x}(1, t) = 1 - \theta(1, t)$$

Defining

$$\mu = \alpha (T_R - T_L)$$

$$R(\eta) = \frac{k \hat{R}(\eta)}{L}$$

$$\eta = \sigma(1, t) - u(1, t)$$

$$E(x, t) = \frac{\partial u(x, t)}{\partial x}$$

$$\frac{\partial N(x, t)}{\partial x} = -\bar{p}(x, t)$$

$$\bar{p}(x, t) = p(x, t) - \rho \frac{\partial^2 u}{\partial t^2}$$

$$N(x, t) = E(x) \{ \epsilon(x) - \alpha(x) T(x, t) \}$$

$$\epsilon^T = \alpha(x) T(x, t)$$

$$\epsilon(x, t) - \epsilon^T(x, t) = \epsilon^M(x, t)$$

$$u(0, t) = \bar{u}_0(t)$$

$$u(l, t) = u_l(t)$$

$$N(0, t) = \bar{N}_0(t)$$

$$N(l, t) = \bar{N}_l(t)$$

$$\frac{\partial}{\partial x} \left\{ E(x) A(x) \left[\frac{\partial u(x, t)}{\partial x} - \alpha(x) T(x, t) \right] \right\} = -p(x, t) + \rho \frac{\partial^2 u(x, t)}{\partial t^2}$$

$$E = \text{constant} \quad A = \text{constant} \quad \alpha = \text{const}$$

$$\boxed{\frac{\partial^2 u}{\partial x} - \alpha \frac{\partial T}{\partial x} = 0}$$

←

$$\boxed{\frac{\partial \sigma}{\partial x} + b = \rho \frac{\partial^2 u}{\partial t^2}}$$

$$\boxed{K \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t}}$$