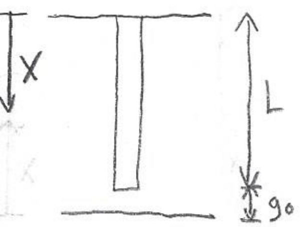
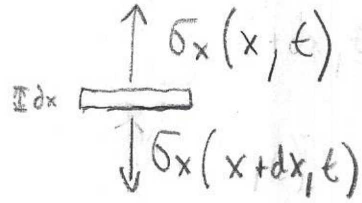
1D Thermoelasticity





$$\sigma_x(x+dx,t) - \sigma_x(x,t) + b_x\,dx = \rho\,dx\,\frac{\partial^2 u_x}{\partial t^2}$$

$$\sigma_x(x+dx,t) = \sigma_x(x,t) + \frac{\partial \sigma_x}{\partial x}dx$$

So

Equilibrium : $$\boxed{\frac{\partial \sigma_x}{\partial x} + b_x = \rho \frac{\partial^2 u_x}{\partial t^2}}$$

Hooke's Law : $$\boxed{\sigma(x,t) = E(x)\left\{\epsilon(x,t) - \underbrace{\alpha(x)\,\Delta T(x,t)}_{\text{Thermal Strain}}\right\}}$$

Strain-Displacement : $$\boxed{\epsilon(x,t) = \frac{\partial u(x,t)}{\partial x}}$$

Assuming: $E \equiv$ Constant , $\alpha \equiv$ Constant

Subbing in to get the Lamé-Navier Equation

$$\sigma(x,t) = E\left\{\frac{\partial u}{\partial x} - \alpha\,\Delta T\right\}$$

$$\frac{\partial}{\partial x}\left\{E\left[\frac{\partial u(x,t)}{\partial x} - \alpha\,\Delta T(x,t)\right]\right\} + b_x = \rho\frac{\partial^2 u}{\partial t^2}$$

Lamé-Navier : $$\boxed{E\frac{\partial^2 u(x,t)}{\partial x^2} - E\alpha\frac{\partial T}{\partial x}(x,t) + b_x = \rho\frac{\partial^2 u}{\partial t^2}}$$

Typically assume $b_x = 0$

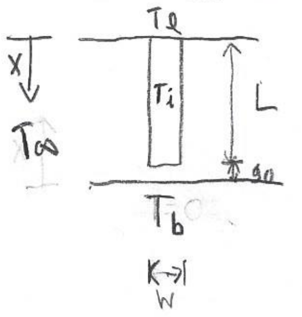
And if we want steady-state solution $\frac{\partial^2 u}{\partial t^2} = 0$

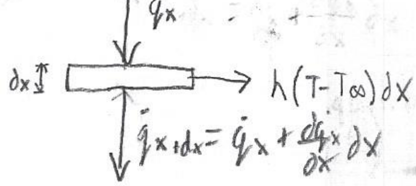
Steady State 1D Lamé-Navier : $$\boxed{\frac{\partial^2 u(x,t)}{\partial x^2} = \alpha\frac{\partial T(x,t)}{\partial x}}$$

Gap Distance : $$\boxed{g(t) = g_0 - \alpha\int_0^l T(x,t)\,dx}$$ if not in contact

Contact Pressure : $$\boxed{P(t) = E\frac{d}{dx}\left\{\alpha\int_0^l T(x,t)\,dx - g_0\right\}}$$ in contact

1D Heat Transfer





$$\cancel{\dot q_x W} - W(\cancel{\dot q_x} + \frac{\partial \dot q_x}{\partial x}dx) - 2h\,dx(T-T_\infty) = \rho c_p \frac{dT}{dt} W dx$$

$$\frac{\partial \dot q_x}{\partial x} W \cancel{dx} - 2h \cancel{dx}(T-T_\infty) = \rho c \frac{dT}{dt} W \cancel{dx}$$

$$\dot q_x = K\frac{\partial T}{\partial x}$$

Heat Conduction Equation

$$\boxed{K\frac{\partial^2 T}{\partial x^2} - \frac{2h}{W}(T-T_\infty) = \rho c \frac{\partial T}{\partial t}}$$

Boundary Conditions for HCE + LNE (Need 7)

$T(0,t) = T_l$ (i)

$u(0,t) = 0$ (ii)

$T(x,0) = T_i$ (iii)

$u(x,0) = 0$ (iv)

$\frac{\partial u}{\partial t}(x,0) = 0$ (v)

$K\frac{\partial T}{\partial x}(L+g_0, t) = \frac{1}{R}\left(T_b - T(L+g_0, t)\right)$ (vi)

$\begin{cases} u(L+g_0, t) = 0 & \text{if in Contact} \\ \sigma(L+g_0, t) = 0 & \text{if not in Contact} \end{cases}$ (vii)

We can define $T_i = T_b = 0$ if we want

or

$\boxed{\frac{\partial T}{\partial x}(L+g_0, t) = 0}$ (not in contact)

This condition says it can either be in or out of contact, Not both

$u(1,t)\,\sigma(1,t) = 0$

$L \gg g_0 \quad L+g_0 \approx L$? → Is this an assumption we can make?

Pelesko actually formulates R such that it is nonzero even if out of contact, so it is a function of both contact pressure and gap distance. $R(\sigma(1,t) - u(1,t))$. We could do this, or assume there is no heat transfer out of contact. (Less correct but easier)

Thermal Resistance

We need to find a function $R(P)$. Comninou Dundurs 6 outlines some requirements for the function in nondimensionalized parameters.

$\frac{1}{R} = H f(\pi)$, $H \equiv$ constant, $\rho = \frac{k}{g_0 H}$, $\theta_l = \frac{\alpha T_l l}{g_0}$, $\pi = \frac{P l}{E g_0}$

$\boxed{\frac{1}{2}\theta_l - 1 \le \pi < \theta_l - 1}$ $\boxed{f(\pi) \ge 0}$ $\boxed{f(0) = 0}$ $\boxed{\left[\pi - \left(\frac{1}{2}\tau_l - 1\right)\right]\frac{\partial f}{\partial \pi} + f(\pi) + \rho > 0}$