Nondimensionalization

Following Pelesko's Method

$\boxed{u = \frac{u'}{L'+g_0'}}$ $\boxed{x = \frac{x'}{L'+g_0'}}$ $\boxed{\theta = \frac{T - T_\ell}{T_b - T_\ell}}$ $\boxed{\sigma = \frac{\sigma'}{E}}$ $\boxed{\theta_\infty = \frac{T_\infty - T_\ell}{T_\ell - T_\ell}}$

Lame-Navier: $\frac{E}{L}\frac{\partial^2 u}{\partial x^2} - \frac{E}{L}\alpha(T_b - T_\ell)\frac{\partial \theta}{\partial x} = L\rho \frac{\partial^2 u}{\partial t'^2}$

$$\underline{\frac{\partial^2 u}{\partial x} - \alpha(T_b - T_\ell)\frac{\partial \theta}{\partial x} = \frac{L^2 \rho}{E}\frac{\partial^2 u}{\partial t'^2}}$$

Heat Transfer: $\frac{K(T_R - T_L)}{(L+g_0)^2}\frac{\partial^2 \theta}{\partial x^2} - \frac{2h}{W}\left(\theta(T_b - T_\ell) \cancel{+T_\ell} - \theta_\infty(T_b + T_\ell) \cancel{-T_\ell}\right) = \rho c(T_b - T_\ell)\frac{d\theta}{dt'}$

$$\underline{\frac{\partial^2 \theta}{\partial x^2} - \frac{2hL^2}{KW}(\theta - \theta_\infty) = \frac{\rho c L^2}{K}\frac{\partial \theta}{\partial t'}}$$

Hooke's Law: $\underline{\sigma = \frac{\partial u}{\partial t} - \alpha(T_b - T_\ell)\theta}$

Boundary Conditions:

$\boxed{\theta_i = \frac{T_i - T_\ell}{T_b - T_\ell}}$

$\underline{\theta(0,t) = 0}$

$\underline{\theta(x,0) = \theta_i(T_b - T_\ell) + T_\ell}$

$\underline{u(0,t) = 0}$

$\underline{u(x,0) = 0}$

$\underline{\frac{\partial u}{\partial t}(x,0) = 0}$

$$\begin{cases} u(1,t) = 0 & \text{if in contact} \\ \sigma(1,t) = 0 & \text{if not in Contact} \end{cases}$$

$\frac{K}{L+g_0}(T_b - T_\ell)\frac{\partial \theta}{\partial x}(1,t) = \frac{1}{R}\left(T_b - \theta(T_b - T_\ell) - T_e\right)$

$$\begin{cases} \frac{\partial \theta}{\partial x}(1,t) = \frac{L+g_0}{RK}\left(1 - \theta(1,t)\right) & \text{if in contact} \\ \frac{\partial \theta}{\partial x}(1,t) = 0 & \text{if not in contact} \end{cases}$$

Need to nondimensionalize t. How?

$\frac{t'K}{\rho c L^2} = t$?

or

$\frac{t'E}{L^2 \rho} = t$?

Lets take $\boxed{t = \frac{t'K}{\rho c(L+g_0)}}$

$\boxed{\alpha(T_b - T_\ell) = \mu}$

$\boxed{H = \frac{h(L+g_0)^2}{KW}}$ $\boxed{\hat{R} = \frac{RK}{L+g_0}}$

$\boxed{\hat{\rho} = \frac{K^2}{Ec^2L^2\rho}}$

Finalizing Nondimensionalization

$' \equiv$ Dimensionalized

$$u = \frac{u'}{L+g_0} \quad x = \frac{x'}{L+g_0} \quad \theta = \frac{T-T_\ell}{T_b - T_\ell} \quad \sigma = \frac{\sigma'}{E} \quad \theta_\infty = \frac{T_\infty - T_\ell}{T_b - T_\ell} \quad \theta_i = \frac{T_i - T_\ell}{T_b - T_\ell}$$

$$t = \frac{t' K}{\rho c L^2} \quad \mu = \alpha(T_b - T_\ell) \quad H = \frac{h(L+g_0)^2}{KW} \quad \hat{R} = \frac{RK}{L+g_0} \quad \hat{P} = \frac{k^2}{Ec^2(L+g_0)^2 \rho}$$

Lame-Navier: $\dfrac{\partial^2 u}{\partial x^2} - \mu \dfrac{\partial \theta}{\partial x} = \hat{P} \dfrac{\partial^2 u}{\partial t^2}$

Heat Transfer: $\dfrac{\partial^2 \theta}{\partial x^2} - H(\theta - \theta_\infty) = \dfrac{\partial \theta}{\partial t}$

Hooke's Law: $\sigma = \dfrac{\partial u}{\partial x} - \mu \theta$

Boundary Conditions:

i) $\theta(0,t) = 0$ — x

ii) $\theta(x,0) = \theta_i (T_b - T_\ell) + T_\ell$ — t

iii) $u(0,t) = 0$ — x

iv) $u(x,0) = 0$ — t

v) $\dfrac{\partial u}{\partial t}(x,0) = 0$ — t

vi) $\begin{cases} u(1,t) = 0 & \text{if in contact} \\ \sigma(1,t) = 0 & \text{if not in contact} \end{cases}$ — x

vii) $\begin{cases} \dfrac{\partial \theta}{\partial x}(1,t) = \dfrac{1}{\hat{R}}(1 - \theta(1,t)) & \text{if in contact} \\ \dfrac{\partial \theta}{\partial x}(1,t) = 0 & \text{if not in contact} \end{cases}$ — x

Some Comminou Dundurs Stuff

$$\tau_\ell = \frac{\alpha T_\ell \ell}{g_0}$$

If $\tau_\ell < 1$ Not in contact

If $1 < \tau_\ell < 2$ "Difficult Zone"

If $2 < \tau_\ell$ Fully in Contact

$$\tau_0 = \frac{\alpha T_0 \ell}{g_0}$$

$$\rho = \frac{k}{g_0 H}$$

$$\Pi = \frac{P\ell}{E g_0} \equiv \text{Dimensionless Contact Pressure}$$

$$\boxed{\left[\pi - \left(\tfrac{1}{2}\tau_\ell - 1\right)\right] f(\pi) = \rho(\tau_\ell - 1 - \pi)}$$

$$H f(\pi) = \frac{1}{r}$$