Solving the D.E's

Assume: Steady State, no convection, in contact

Start with Conduction Eq.

$$\frac{\partial^2 \theta}{\partial x^2} = 0$$

B.C. $\theta(0,t) = 0$ (i)

$$\frac{\partial \theta(1,t)}{\partial t} = \frac{1}{\hat{R}}\left(1 - \theta(1,t)\right) \quad \text{(ii)}$$

$$\theta(x) = C_1 x + C_2$$

Applying (i)

$$\theta(x) = C_1 x \qquad C_2 = 0$$

$$\frac{\partial \theta}{\partial x} = C_1$$

Applying (ii)

$$C_1 = \frac{1}{\hat{R}}\left(1 - C_1(1)\right) = \frac{1}{\hat{R}} - \frac{C_1}{\hat{R}} \Rightarrow C_1 + \frac{C_1}{\hat{R}} = \frac{1}{\hat{R}} = C_1\left(1 + \frac{1}{\hat{R}}\right) \cdot \frac{1}{\hat{R}}$$

$$C_1 = \frac{1}{\hat{R}\left(1 + \frac{1}{\hat{R}}\right)} = \frac{1}{\hat{R}+1}$$

$$\boxed{\theta(x) = \frac{1}{\hat{R}+1} x}$$

Then look at Lamé Navier

$$\frac{\partial^2 u}{\partial x^2} = \mu \frac{\partial \theta}{\partial x} \xRightarrow{\text{integrate}} \frac{\partial u}{\partial x} = \mu \theta + C_3 \xRightarrow{\text{integrate}} u = \mu \int \theta(x) + C_3 x$$

$$u(x) = \frac{\mu}{\hat{R}+1} \int x + C_3 x = \frac{\mu x^2}{2(\hat{R}+1)} + C_3 x + C_4$$

Applying $u(0,t) = 0 \Rightarrow C_4 = 0$

Applying $u(1,t) = 0 \Rightarrow 0 = \frac{\mu x^2}{2(\hat{R}+1)} + C_3 \Rightarrow C_3 = -\frac{\mu x^2}{2(\hat{R}+1)}$

$$\boxed{u(x) = \frac{\mu}{2(\hat{R}+1)}\left(x^2 - x\right)}$$

Strain-Displacement

$$\boxed{\varepsilon = \frac{\partial u}{\partial x} = \frac{\mu}{2(\hat{R}+1)}\left(2x - 1\right)}$$

Hooke's Law

$$\sigma = \frac{\partial u}{\partial x} - \mu \theta$$

$$\sigma = \frac{\mu}{\lambda(\hat{R}+1)}(\lambda x - 1) - \frac{\mu}{\hat{R}+1} x$$

$$\sigma = \cancel{\frac{\mu}{\hat{R}+1} x} - \frac{\mu}{\lambda(\hat{R}+1)} - \cancel{\frac{\mu}{\hat{R}+1} x}$$

$$\boxed{\sigma = -\frac{\mu}{\lambda(\hat{R}+1)}}$$

Contact Pressure

<u>Note</u> $L \neq 1$, $\boxed{L = \frac{L'}{L'+g_0'}}$

Dimensionalize Contact Pressure

$$P' = \frac{E}{L'}\left(u(L') - g_0\right)$$

$$\boxed{P' = E\left\{-\frac{\alpha(T_b - T_e)}{\lambda(Rk + L' + g_0')} g_0' - \frac{g_0'}{L'}\right\}}$$

Or nondimensionalized

$$\boxed{P = \frac{1}{L}\left(\frac{\mu}{\lambda(\hat{R}+1)}\left(L^{\lambda} - L\right) - \frac{g_0}{L}\right)}$$

where $L = \frac{L'}{L'+g_0'}$ $\quad P = \frac{P'}{E}$

$g_0 = \frac{g_0'}{L'+g_0'}$ $\quad \hat{R} = \frac{Rk}{L'+g_0'}$

$\mu = \alpha(T_b - T_e)$