

# Wavefront Reconstruction

SC4045 CONTROL FOR HIGH RESOLUTION IMAGING

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## 1 ZONAL AND MODAL RECONSTRUCTION

Implement a function that reconstructs the wavefront based on a *zonal* and *modal reconstruction method*. The main task is to generate the matrices  $G$  and  $B$ , presented in Eqs. (2.8) and (2.11), respectively, and to generate the covariance matrix of the wavefront,  $C_\phi$ . The following subtasks should therefore be accomplished:

### ZONAL

1. For the *Zonal* reconstruction method, the generation of the  $G$  matrix using the Fried geometry can be done as follows:
  - a) Vectorize the phase samples by performing a map from their 2D spatial coordinate  $(i, j)$  to a single coordinate  $m$ . Example:  $(1, 1) \rightarrow 1, (1, 2) \rightarrow 2, \dots$
  - b) Fill the matrix  $G$  for the slopes in the  $x$  direction and then in the  $y$  direction (or viceversa) one row at a time.
  - c) Using the mapping created in *a)* fill each row of  $G$  with the phase points that are related to the slope.
2. Once the matrix  $G$  is generated you may have problems inverting it. Find out the reason behind the rank deficiencies upon consulting the seminal publications [1] and [2].
3. Try the Southwell [3] and Hudgin [4, 3] geometries and compare the obtained reconstruction with the Fried geometry.

### MODAL

1. The matlab code for matrix  $B$  will be provided as it is hard to derive it. The only thing to be done is to port it to Python.

When both the reconstructor matrices have been generated, work together with the team assigned to generate the turbulent wavefront and compute the covariance matrix of the phase for Kolmogorov or von Kármán turbulence. Afterwards, compute the *weighted least squares* [5, § 4] estimate and compare it with the “normal” reconstruction when the wavefront is aberrated by atmospheric turbulence.

When comparing both the methods under similar aberrations, can you think of any reasons (in terms of computational power, accuracy,...) why one could be preferred to the other?

## REFERENCES

- [1] D. L. Fried, "Least-square fitting a wave-front distortion estimate to an array of phase-difference measurements," *J. Opt. Soc. Am.*, vol. 67, pp. 370–375, Mar. 1977.
- [2] J. Herrmann, "Least-squares wave front errors of minimum norm," *J. Opt. Soc. Am.*, vol. 70, pp. 28–35, Jan. 1980.
- [3] W. H. Southwell, "Wavefront estimation from wavefront slope measurements," *J. Opt. Soc. Am.*, vol. 70, pp. 998–1006, Aug. 1980.
- [4] R. H. Hudgin, "Wave-front reconstruction for compensated imaging," *J. Opt. Soc. Am.*, vol. 67, pp. 375–378, Mar. 1977.
- [5] M. Verhaegen and V. Verdult, *Filtering and System Identification: A Least Squares Approach*. New York, NY, USA: Cambridge University Press, 1st ed., 2007.