ST2004 Applied Probability I

Lab 5

Instructions

- Download the template file *ST2004_Lab5.xlsx* from Blackboard
- Address the points below by adapting what was done in the template.
- Work individually or in small groups.
- If you do not complete the instructions today, you can work on it during the week.

1 Generating from the Normal distribution

See worksheet *histogram*.

1. Read next page, *On the normal distribution*.
2. Set $\mu = 0$ and $\sigma = 1$.
3. Generate 1000 random number from the normal distribution with parameters μ and σ (you can use NORMINV(RAND(),μ, σ)).
3. Considering the sample of generated numbers, compute the average (you can use AVERAGE) and the standard deviation (you can use STDEV.S).
4. Produce a histogram of the generated numbers (follow instructions at page 3).
5. Replicate the same procedure with different values of μ and σ to see how they affect the resulting histogram.

2 System lifetime

See worksheet *system lifetime*.

A system has 3 components A, B and C, with redundancy. It is designed such that it will work if either (C is working) or (both A and B are working). For example, if the lifetimes of A, B and C are 10, 15 and 8 hours, respectively, then it will work for 10 hours. We model the lifetime of each component via the Normal distribution (i.e. NORMINV(RAND(),μ, σ)) with common parameters $\mu = 10$ and $\sigma = 2$

1. What is the average lifetime of such systems?
2. What is this average lifetime if $\sigma = 0.02, 0.2, 2, 4$? Why does the average lifetime change with σ?
3. Large values of σ (try e.g. with $\sigma = 10$) don't always generate 'plausible' lifetimes? Why not?
4. Produce a plot of the average lifetime of the system as a function of σ (consider $\sigma \in \{0.02, 0.2, 2, 4\}$).
5. For a fixed value $\sigma = 2$, produce a plot of the average lifetime of the system as a function of μ (consider $\mu \in \{5, 10, 15, 20\}$).

Replicate the same analysis for a system of 4 components A, B, C and D, designed to work if at least two components are working. Again we model the lifetime of each component via the Normal distribution with common parameters $\mu = 10$ and $\sigma = 2$.

6. What function can you use to compute the system lifetime, once component lifetimes have been generated? (*hint:* observe that the system fails when the third component fails. An option is then provided by the function SMALL()).

7. What is the average lifetime of such system?

Theory - On the normal distribution

- So far we have considered random numbers generated from uniform distributions (on the interval $(0, 1)$ via RAND or on the intergers $\{1, 2, \ldots, M\}$ via RANDBETWEEN).
- Sometimes it is preferable to model an experiment with nonuniform distributions that is distributions that favour some outcomes over other outcomes.
- The normal distribution, characterized by two parameters $\mu \in (-\infty, \infty)$ (mean) and $\sigma > 0$ (standard deviation), is suitable to model many real phenomena.
- For example if $\mu = 0$ and $\sigma = 1$ as in Figure 1 it will be more likely to generate a random number close to 0 than far from 0.

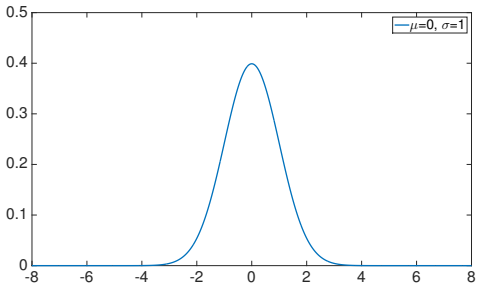


Figure 1: Normal distribution with $\mu = 0$ and $\sigma = 1$.

- For a general μ, it will be more likely to generate numbers close to μ (see Figure 2, left panel).
- The smaller is σ (see Figure 2, right panel) the closer to μ the generated numbers are likely to be.

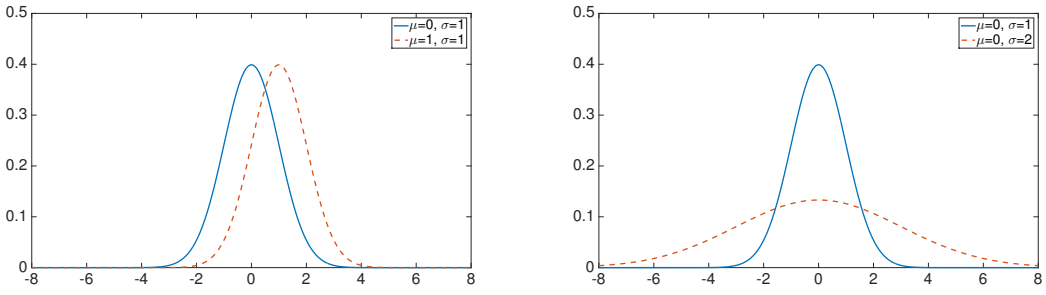
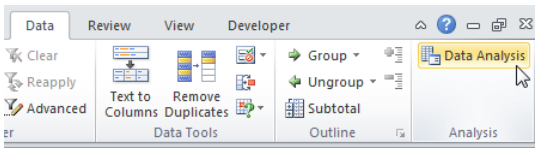


Figure 2: Normal distributions with different values of μ (left panel) and σ (right panel).

- In Excel you can generate a random number from a normal distribution with parameters μ and σ by using NORMINV(RAND(),μ, σ)).

Histogram with Excel

1. On the Data tab, click Data Analysis (if you find it) and move to point 4. If you do not find it you must load the Analysis ToolPak add-in (warning: different versions of Excel might require different procedures for loading the Analysis ToolPak add-in).



2. Click on the Tools tab and choose Excel add-in. Select Analysis ToolPak, click OK and go to point 4. Check that, now, on the Data tab, you can click on Data Analysis.

3. Alternative procedure (for older versions of Excel): click the File tab and choose options. Under Add-ins, select Analysis ToolPak and click on the Go button. Check Analysis ToolPak and click on OK. Check that now, on the Data tab, you can click on Data Analysis.

4. Click on Data Analysis, select Histogram, select the input range and click OK to create a Histogram in Excel. Make sure that Chart Output is checked.