ST2004 Applied Probability I

Lab 3

Lab 3: Dice game

Use the file *ST2004_lab3.xlsx* as a template to construct the Excel worksheet needed to address the following points. The template can be downloaded from Blackboard.

1 Monte Carlo replications for dice

Your first task is to simulate the roll a single die. Then with replications your task is to simulate the summaries of the variation. The task here is to examine in fine detail some of the possible summaries. We meet the average, the variance and the standard deviation (see Section 2).

1. The worksheet demonstrates three methods to simulate the roll of a die, and 4 replications. Extend the calculations to compute 1000 replications. When you have confirmed, by the following, that all three methods are equivalent, there is no purpose to work with three methods and any of them can be chosen.

2. Summarise the variability by using COUNTIF to compute frequencies for each of the possible scores, whence you can compute relative frequencies; see the template. Are the relative frequencies as you would expect?

3. Confirm that you can compute the average of the 1000 directly from the relative frequencies and the possible scores using SUMPRODUCT(possible scores, rel. frequencies).

4. Compute also the square of each of the 1000 scores, and the relative frequencies for the possible values of the squares. Confirm that you can compute the average of the squares directly from the relative frequencies and the possible (squared) scores using SUMPRODUCT.

5. Compute, for each score, its difference from the average and the squared values of such difference. Compute the average of these squared differences. How could you compute this average using SUMPRODUCT? Confirm that

$$\text{Avg(Sq. Diffs)} = \text{Avg(Squares)} - (\text{Avg})^2.$$

6. Confirm that Excels function =VARP() (or VAR.P), applied to your 1000 dice scores, computes exactly the same as the values in point 5. Confirm that the function STDEVP() (or STDEV.P) computes exactly the square root of the values in point 5.

7. You play a dice game in which you win €2 for odd numbers and €6 for a 'six'. But otherwise you lose €3. Simulate 1000 games, and compute the average and the variance of your winnings.

2 Measures of variability

Consider N replications of the same random experiment. Let us denote the outcomes of these experiments by $X_1, X_2, \ldots, X_N$. The average outcome is defined as the arithmetic mean of $X_1, X_2, \ldots, X_N$, that is

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i.$$

Two useful measures to summarise the variability of the experiment are the variance S^2 and the standard deviation S of the outcomes $X_1, X_2, \ldots, X_N$, that is

$$S^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2, \qquad S = \sqrt{S^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2}.$$

Notice that the variance S^2 coincides with the average of the squares of the outcomes X_i minus the square of average $\bar{X}$, that is the variance can be alternatively computed as

$$S^2 = \frac{1}{N}\sum_{i=1}^{N} X_i^2 - \bar{X}^2.$$

Indeed

$$\begin{aligned} S^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^2 &= \frac{1}{N}\sum_{i=1}^{N}\left(X_i^2 + \bar{X}^2 - 2X_i\bar{X}\right) = \frac{1}{N}\sum_{i=1}^{N} X_i^2 + \frac{1}{N}\sum_{i=1}^{N}\bar{X}^2 - 2\bar{X}\frac{1}{N}\sum_{i=1}^{N} X_i \\ &= \frac{1}{N}\sum_{i=1}^{N} X_i^2 + \frac{1}{N}N\bar{X}^2 - 2\bar{X}\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i^2 + \bar{X}^2 - 2\bar{X}^2 = \frac{1}{N}\sum_{i=1}^{N} X_i^2 - \bar{X}^2. \end{aligned}$$