ST2004 Applied Probability I

Lab 4

Lab 4: Simple Queues

Use the file *ST2004_lab4.xlsx* as a template to construct the Excel worksheet needed to address the following points.

Queueing model

We consider a simple model to simulate a queuing system: customers arrive at a single server and are processed. Two simplifying assumptions:

1. costumers can arrive and leave only at discrete times $t = 1, 2, 3, \ldots$;
2. at each time at most one costumer can arrive and at most one costumer can leave.

Arrival process: at each time

- 1 new customer with probability p
- 0 new customer with probability $1 - p$

Departure process: at each time

- 1 customer in service leaves with probability q
- customer does not leave with probability $1 - q$

1. Make sure you understand the simulation in worksheet *Random* where the simple queueing model that we described is implemented. Compare it with the simpler simulations in which costumers arrive every 100 seconds (that is not randomly) and service takes 66 seconds (worksheet *Fixed*).
2. Check how the following quantities were computed and observe the plots that show how they vary as a function of t.
   a. Cumulated number of arrivals and cumulated number of departures at time t.
   b. Number N of in the system at time t.
3. Experiment by changing the values of p and q. What happens when $p < q$, $p = q$, $p > q$?
4. To study the long run behaviour of the queue we have used 5000 seconds. Is this long enough? Focus on the running average and the running variance of N and check the plot showing how they vary as a function of t. Refresh by forcing a re-computation (Shift F9 or 'Calculate now' under Formulas tab). Do you think it has converged?
5. In order to further investigate the convergence of the running average, replicate the whole experiment 10 times: that is compute 10 times the average N over time 5000 by using the Data Table trick. Notice that Excel can be pretty slow. (If the table does not update its values, press F9).
6. Extend the calculation to a run of 20,000. You will need to update the plots of long run average and variance. Is this long enough?
7. You will find that for "large q" you reach the convergence quite easily, provided that $p << q$ (that is p much smaller than q). Otherwise it is difficult. Why do you think this is?