ST2004 Applied Probability I

Lab 2

Instructions

- Download the template file *ST2004_lab2.xlsx* that you can find on Blackboard.
- Address the points below by adapting what was done in the template.
- Work individually or in small groups.
- The last page of this file contains a short summary of the functions used in the template.

Lab 2: Does skill win the league?

Three teams, A, B and C play a mini-league. In the simplest version each team plays the other two, i.e., three games in total, and there are no draws. Suppose that teams B and C are of equal skill, but that A is α times more (or less if $\alpha < 1$) likely to win than to lose when playing B and C. There are four possible outcomes of the league:

i. "A wins the league" by winning two games

ii. "B wins the league" by winning two games

iii. "C wins the league" by winning two games

iv. "The league ends with a tie": A, B and C win one game each.

1. **What is the probability that A wins the league?**
2. **What is the probability of the league ending with a tie?**

In order to answer these questions run a simulation study by replicating for a generic value of α what was done in the template only for the case $\alpha = 1$ (see *Simulation* worksheet). Namely,

- Fix some value for α.
- Simulate the outcome of each game by using RAND(). Observe that the probability that team A wins in a given game is equal to $\alpha/(\alpha + 1)$. Why? (see next page).
- Count how many wins per each team in the league by using COUNTIF.
- Replicate this 1000 times (you might use the TABLE trick in Excel, see last page).
- Count the relative frequency of the outcome "A wins the league"
- Count the relative frequency of the outcome "the league ends with a tie"

3. **How is the relative frequency of the outcome "A wins the league" when $\alpha = 1$?**
4. **How is the relative frequency of the outcome "A wins the league" affected by the choice of α?**

You can follow the steps:

- Try the same experiment (each time with 1000 replicates) for several values of α (e.g. $\alpha = 0.1, 0.2, 0.4, 0.8, 1.6, 3.2, 6.4, 12.8$)

- Make a graph of the relative frequency of the outcome "A wins the league" as a function of α.
- Compare your findings with what explained in the next section and with the worksheet *Theory* in the template.

5. If you finish early, read the discussion on the role of luck and skills in determining the Premier League table, by Prof. David Spiegelhalter, that you can find at

understandinguncertainty.org/node/56

For further reading, give a look to

ig-legacy.ft.com/content/9d732642-4579-11e4-9b71-00144feabdc0#axzz4uxRnzWkn

Theory

We have that

$$\mathrm{P}(\text{"A beats B"}) = \alpha \mathrm{P}(\text{"B beats A"})$$

and at the same time

$$\mathrm{P}(\text{"A beats B"}) + \mathrm{P}(\text{"B beats A"}) = 1.$$

Therefore

$$\mathrm{P}(\text{"A beats B"}) = \frac{\alpha}{\alpha + 1}$$

and

$$\mathrm{P}(\text{"B beats A"}) = \frac{1}{\alpha + 1}.$$

Similarly

$$\mathrm{P}(\text{"A beats C"}) = \frac{\alpha}{\alpha + 1} \quad \text{and} \quad \mathrm{P}(\text{"C beats A"}) = \frac{1}{\alpha + 1}.$$

Observe that

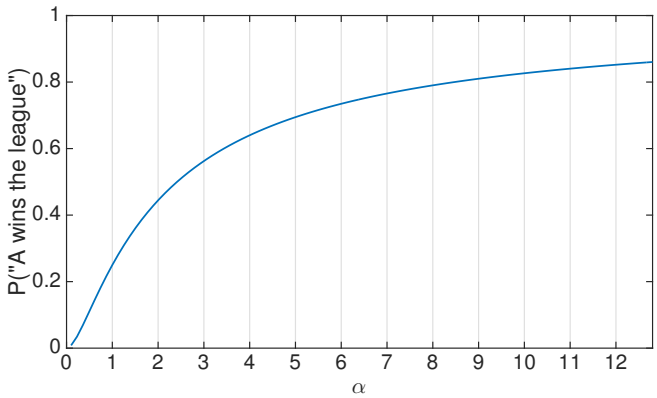
$$\text{"A wins the league"} = \text{"A beats B"} \cap \text{"A beats C"}.$$

Since the events "A beats B" and "A beats C" are assumed independent (the outcome of one match does not affect the outcome of the other one) we can write

$$\begin{aligned} \mathrm{P}(\text{"A wins the league"}) &= \mathrm{P}(\text{"A beats B"} \cap \text{"A beats C"}) \\ &= \mathrm{P}(\text{"A beats B"})\mathrm{P}(\text{"A beats C"}) \\ &= \frac{\alpha}{\alpha + 1} \frac{\alpha}{\alpha + 1} \\ &= \frac{\alpha^2}{(\alpha + 1)^2} \end{aligned}$$

Thus we have the following table

α	0.1	0.2	0.4	0.8	1.6	3.2	6.4	12.8	1
P("A wins the league")	0.0083	0.0278	0.0816	0.1975	0.3787	0.5805	0.7480	0.8603	0.25



List of functions used in the template:

- RAND() returns an evenly distributed random real number in $(0, 1)$
- COUNTIF counts the number of cells that meet a criterion
- IF makes logical comparison between two values. E.g. IF(C2 $= 1$, "Yes", "No") returns "yes" if the value at cell C2 is equal to 1, "no" otherwise
- INDEX returns a value from within a table or range
- MATCH searches for a specified item in a range of cells. E.g. MATCH(2, B8:D8, 0) finds the first value that is exactly equal to 2 in the range B8:D8.
- Shortcut: how to make many replicates (say 1000) of the same random experiment by using a Data Table.
  - Write numbers from 1 to 1000 in a range of 1000 cells on the same column (range G3:G1002 in *Simulation* worksheet). You can do this by writing 1 and 2 in the first two cells, selecting them and dragging the fill handle across the range that you want to fill.
  - Write in the cell beside number 1 (cell H3 in *Simulation* worksheet) the operation you want to replicate.
  - Select both columns (columns G and H in *Simulation* worksheet).
  - Use Data tab and select "What-If Analysis", "Data Table".
  - Choose as a "Column input cell" any empty cell (F2 in *Simulation* worksheet).