Hw6

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1a. The Markovian assumption / topological semantics = a node is conditionally independent of its non-descendants given its parents.

1) Since A's only non-descendants are B and E:
    - A is conditionally independent of B and E
2) Since B's only non-descendants are A and C:
    - B is conditionally independent of A and C
3) Since C's non-descendants are D, B, and E (with parent A):
    - C is conditionally independent of D, B, and E, given A
4) Since D's non-descendants are C and E (with parents A and B):
    - D is conditionally independent of C and E, given A and B
5) Since E's non-descendants are C, D, F, G and A (with parent B):
    - E is conditionally independent of C, D, F, G, and A, given B
6) Since F's non-descendants are A, B, and E (with parents C and D):
    - F is conditionally independent of A, B, and E, given C and D
7) Since G's non-descendants are H, C, D, A, B, and E (with parent F):
    - G is conditionally independent of H, C, D, A, B, and E, given F
8) Since H's non-descendants are G, C, D, A, and B (with parents E and F):
    - H is conditionally independent of G, C, D, A, and B, given E and F

1b. The Markov blanket consists of the node's parents, children, and children's parents.

Since D's parents are A and B, D's child is F, and F's parents are C and D, then this means:

- D's Markov blanket consists of nodes A, B, F, and C.

1c. We can express $Pr(A, B, C, D, E, F, G, H)$ as a multiplication of conditional and marginal probabilities, using the chain rule for Bayesian networks as follows:

$Pr(A, B, C, D, E, F, G, H) = Pr(A) * P(B) * P(C \mid A) * P(D \mid A, B) * P(E \mid B) * P(F \mid C, D) * P(G \mid F) * P(H \mid F, E)$

1d, e, f → Not required, as per the TA email!

1g. We can express $Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$ in terms of the parameters in the Conditional Probability Table as follows:

$Pr(a, \neg b, c, d, \neg e, f, \neg g, h) =$

$$= P(a) * P(\neg b) * P(c \mid a) * P(d \mid a, \neg b) * P(\neg e \mid \neg b) * P(f \mid c, d) * P(\neg g \mid f) * P(h \mid f, \neg e)$$

$= 0.2 * 0.3 * P(c \mid a) * 0.6 * 0.1 * P(f \mid c, d) * P(\neg g \mid f) * P(h \mid f, \neg e)$

$= 0.0036 * P(c \mid a) * P(f \mid c, d) * P(\neg g \mid f) * P(h \mid f, \neg e)$

1h. We compute $Pr(\neg a, b)$ with the knowledge that *a* and *b* are independent, so:

$Pr(\neg a, b) = Pr(\neg a) * Pr(b)$

$Pr(\neg a) * Pr(b) = 0.8 * 0.7 = 0.56$

1i. We compute $Pr(\neg e \mid a)$:

Here, we have the intuition that *e* is independent of *a* because *a* is not a parent of *e*. Therefore, *e* is only dependent upon its parent *b*. Since *b* can have two values, *b* and ¬*b*, we do as follows:

$Pr(\neg e \mid a) = Pr(\neg e) = Pr(\neg e \mid b) * Pr(b) + Pr(\neg e \mid \neg b) * Pr(\neg b)$

$= 0.9 * 0.7 + 0.1 * 0.3$

$= 0.66$

2a.

i. $\forall x(Food(x) \rightarrow Likes(John, x))$
ii. $Food(Apples)$
iii. $Food(Chicken)$
iv. $\forall i,j((Eats(j, i) \land \neg MakeSick(i, j)) \rightarrow Food(i)$
v. $\forall n,m(MakeSick(n, m) \rightarrow \neg Well(m))$
vi. $Eats(Bill, Peanuts) \land Well(Bill)$
vii. $\forall z(Eats(Bill, z) \rightarrow Eats(Sue, z))$