HW5

ID#: 105-083-037

1a. Truth Table:

Smoke	Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
T	T	T
T	F	T
F	T	F
F	F	T

This sentence is neither (it is satisfiable), because it is true in some models.

1b. Truth Table:

Smoke	Fire	Heat	Smoke ⇒ Fire	Smoke ∨ Heat	((Smoke ∨ Heat) ⇒ Fire)	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T	T	T	T
T	F	T	F	T	F	T
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	T	F	F
F	T	F	T	T	T	T
F	F	F	T	F	T	T

This sentence is neither (it is satisfiable), because it is true in some models.

1c. Truth Table:

Smoke	Fire	Heat	Smoke ∧ Heat	((Smoke ∧ Heat) ⇒ Fire)	Smoke ⇒ Fire	Heat ⇒ Fire	((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
T	T	T	T	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	T	F	F	T	T	T	T	T

T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	T	F	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

This sentence is valid, because it is true in all models.

2.

Propositional Symbols & Meanings:

U = is a unicorn

My = is mythical

M = is mortal

Mam = is a mammal

H = is horned

Mag = is magical

a.

First Version of Knowledge Base:

$((U \wedge My) \rightarrow (U \wedge \neg M)) \wedge ((U \wedge \neg My) \rightarrow (U \wedge M \wedge Mam))$

$((U \wedge (\neg M \vee Mam)) \rightarrow (U \wedge H)$

$(U \wedge H) \rightarrow (U \wedge Mag)$

We assume that our universe only consists of unicorns, and delete the propositional symbol U. We also simplify our rules, so that our final knowledge base is the following.

Knowledge Base:

S1: $(My \rightarrow \neg M)$

S2: $(\neg My \rightarrow (M \wedge Mam))$

S3: $((\neg M \vee Mam) \rightarrow H)$

S4: $(H \rightarrow Mag)$

b.

S1:

$(My \rightarrow \neg M)$

(¬My ∨ ¬M) Implication Elimination

(¬My ∨ ¬M) Double-Negation Elimination

S2:

(¬My → (M ∧ Mam))

(¬(¬My) ∨ (M ∧ Mam)) Implication Elimination

(My ∨ (M ∧ Mam)) Double-Negation Elimination

(My ∨ M) ∧ (My ∨ Mam) ∨ Distributivity

S3:

((¬M ∨ Mam) → H)

((¬(¬M ∨ Mam)) ∨ H) Implication Elimination

((¬(¬M)) ∧ ¬Mam) ∨ H) De Morgan

((M ∧ ¬Mam) ∨ H) Double-Negation Elimination

(M ∨ H) ∧ (¬Mam ∨ H) ∨ Distributivity

S4:

(H → Mag)

(¬H ∨ Mag) Implication Elimination

Our CNF Knowledge Base is now:

S1: (¬My ∨ ¬M)

S2: (My ∨ M) ∧ (My ∨ Mam)

S3: (M ∨ H) ∧ (¬Mam ∨ H)

S4: (¬H ∨ Mag)

c. We determine whether or not it is true that the unicorn is always magical, horned, or mythical.

To prove if the unicorn is always magical, we begin with α = ¬Mag and resolve for the empty clause:

1. (¬My ∨ ¬M) From S1
2. (My ∨ M) From S2
3. (My ∨ Mam) From S2
4. (M ∨ H) From S3
5. (¬Mam ∨ H) From S3
6. (¬H ∨ Mag) From S4
7. ¬Mag From α

8. ¬H From 6, 7
9. ¬Mam From 5, 8
10. M From 4, 8
11. My From 3, 9
12. ¬My From 1, 10
13. Empty Clause From 11, 12

We have proven that the unicorn is always magical. Next, we try to prove that the unicorn is always horned, beginning with α = ¬H:

1. (¬My ∨ ¬M) From S1
2. (My ∨ M) From S2
3. (My ∨ Mam) From S2
4. (M ∨ H) From S3
5. (¬Mam ∨ H) From S3
6. (¬H ∨ Mag) From S4
7. ¬H From α
8. ¬Mam From 5, 7
9. My From 3, 8
10. ¬M From 1, 9
11. H From 4, 10
12. Empty Clause From 7, 11

We have proven that the unicorn is always horned. Next, we try to prove that the unicorn is always mythical, beginning with α = ¬My:

1. (¬My ∨ ¬M) From S1
2. (My ∨ M) From S2
3. (My ∨ Mam) From S2
4. (M ∨ H) From S3
5. (¬Mam ∨ H) From S3
6. (¬H ∨ Mag) From S4
7. ¬My From α
8. M From 2, 7
9. Mam From 3, 7
10. ¬My From 1, 8
11. H From 5, 9
12. Mag From 6, 11

There is nothing left to resolve and no empty clause, so we cannot prove that the unicorn is always mythical.

3.

Oil	Gas	Positive Test Probability	Presence Probability
T	F	N/A	0.5
F	T	N/A	0.2

F	F	N/A	0.3
T	F	0.9	N/A
F	T	0.3	N/A
F	F	0.1	N/A

We are calculating the probability that oil is present given a positive test result, by the formula:

*((Oil Prob Pos Test) * (Oil Pres Prob)) / (((Oil Prob Pos Test) * (Oil Pres Prob)) + ((Gas Prob Pos Test) * (Gas Pres Prob)) + ((Neither Prob Pos Test) * (Neither Pres Prob)))*

(0.9 * 0.5) / ((0.9 * 0.5) + (0.3 * 0.2) + (0.1 * 0.3)) = 0.45 / (0.45 + 0.06 + 0.03) = 0.45 / 0.54 = 0.8333