# vOptSolver – Version 0.2

## Software developed with the support of the ANR/DFG-14-CE35-0034-01

Université de Nantes

https://github.com/vOptSolver
https://voptsolver.github.io/vOptSpecific/
https://voptsolver.github.io/vOptGeneric/

July 5, 2017

# Table of contents



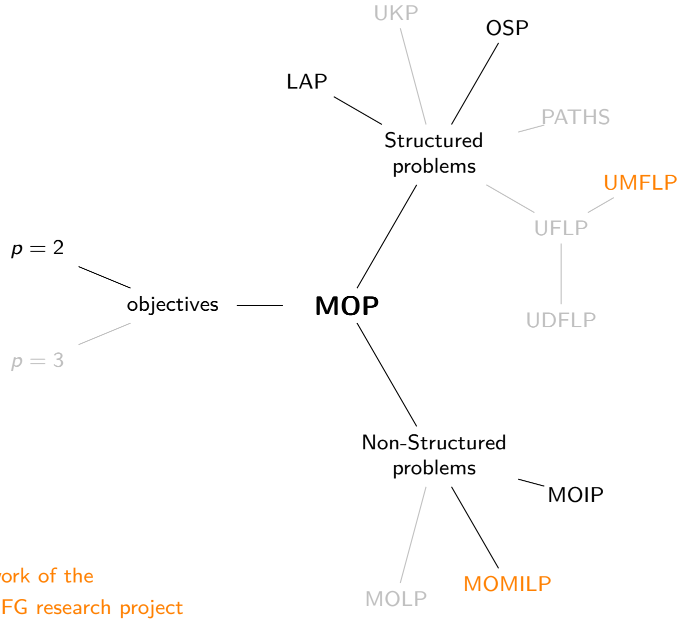
Convention: text in grey $\Rightarrow$ functionality integrated in future releases

# Introduction to vOptSolver

# Multi-objective linear optimization problems targeted

- LP: Linear Program
- MILP: Mixed Integer Linear Program
- IP: Integer Linear program
- CO: Combinatorial Optimization
- LAP: Linear Assignment Problem
- OSP: One machine Scheduling Problem
- UKP: Unidimensional 01 Knapsack Problem
- MKP: Multidimensional 01 Knapsack Problem
- UFLP: Uncapacitated Facility Location Problem
- UDFLP: Discrete Uncapacitated Facility Location Problem
- UMFLP: Mixed Uncapacitated Facility Location Problem
- SSCFLP: Single Source Capacitated Facility Location Problem
- CFLP: Capacitated Facility Location Problem
- PATHS: shortest paths problem

# Multi-objective linear optimization problems targeted




Framework of the
ANR/DFG research project

# Solutions reached

For a given problem, the aim is to compute

**$Y_N$, the set of nondominated "points"**

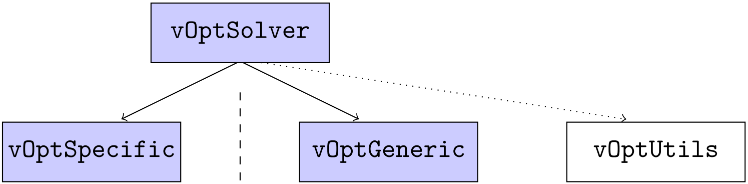
corresponding to

**$X_E$, a complete set of efficient solutions**

More on definitions and notations, refer to this book:

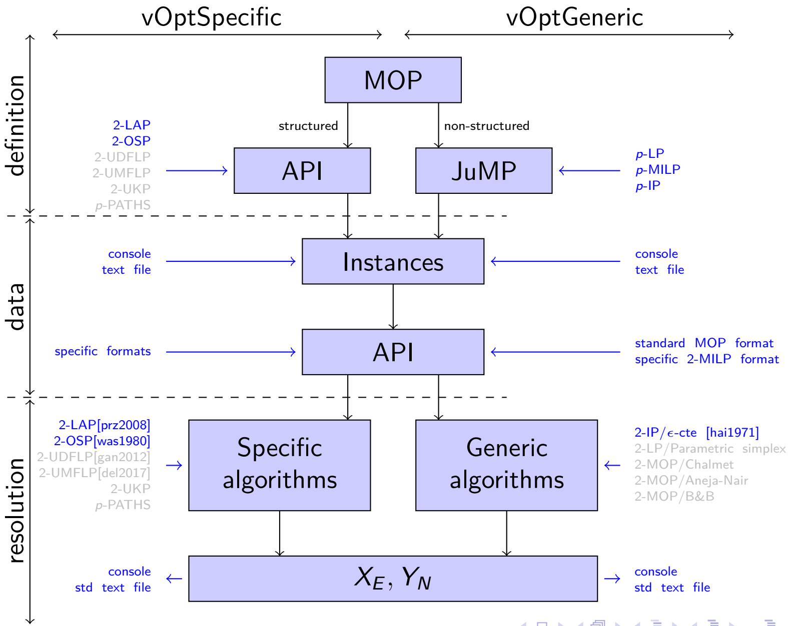

Matthias Ehrgott.
*Multicriteria Optimization.*
Springer-Verlag New York, 2005.

# Design of vOptSolver




| vOptSpecific | vOptGeneric | vOptUtils |
|---|---|---|
| Multiobjective structured problems | Multiobjective non-structured problems | Multiobjective primitives |
| Application Programming Interface (API) | Algebraic Language (JuMP) | Application Programming Interface (API) |
| Collection of specific algorithms | Collection of generic algorithms | Collection of useful algorithms |

# Design of vOptSolver in details



# Design of vOptSolver in details

`vOptUtils`:

a collection of algorithms for managing and analyzing outcome set

- multidimensional datastructure for filtering and storing $Y_N$
- primitives for ploting $Y_N$
- primitives for analyzing $Y_N$

# Integrated specific algorithms (1/2)

- 2-LAP [prz2008]; in C:

  A. Przybylski, X. Gandibleux, and M. Ehrgott. Two phase algorithms for the bi-objective assignment problem. *European Journal of Operational Research*,185(2):509–533, 2008.

  output: $X_E \subseteq \mathbb{N}^n$, $Y_N \subseteq \mathbb{Z}^p$

- 2-OSP with here $1 \mid . \mid (\Sigma C_i, T_{max})$ [was1980]; in Julia:

  L.N. Van Wassenhove and L.F. Gelders. Solving a bicriterion scheduling problem. *European Journal of Operational Research*, 4(1):42–48, 1980.

  output: $X_E \subseteq \mathbb{N}^n$, $Y_N \subseteq \mathbb{Z}^p$

- 2-UKP [jor2010]; in Julia:

  J. Jorge. *Nouvelles propositions pour la résolution exacte du sac à dos multi-objectif unidimensionnel en variables binaires*. Thèse de doctorat, Université de Nantes - France. 2010.

  output: $X_E \subseteq \{0,1\}^n$, $Y_N \subseteq \mathbb{Z}^p$

# Integrated specific algorithms (2/2)

- 2-UDFLP [gan2012]; in C++:

  X. Gandibleux, A. Przybylski, S. Bourougaa, A. Derrien, A. Grimault. Computing the efficient frontier for the 0/1 biobjective uncapacitated facility location problem. *10th International Conference on Multiple Objective Programming and Goal Programming*. June 11-13 2012, Niagara Falls, Canada.

  output: $X_E \subseteq \{0,1\}^n$, $Y_N \subseteq \mathbb{Z}^p$

- 2-UMFLP [del2017]; in C++:

  Q. Delmée, X. Gandibleux, A. Przybylski. Résolution exacte du problème de localisation de services bi-objectif sans contrainte de capacité en variables mixtes. *ROADEF2017 : 18ème congrès annuel de la Société Française de Recherche Opérationnelle et d'Aide à la Décision*, Feb 2017, Metz, France. 2017

  output: $X_E \subseteq \{0,1\}^{n_1} \times \mathbb{R}^{n_2}$, $Y_N$

- PATHS [gan2004]; in C:

  X. Gandibleux, Fr. Beugnies and S. Randriamasy: Martins' algorithm revisited for multi-objective shortest path problems with a MaxMin cost function. *4OR: A Quarterly Journal of Operations Research*. Volume 4, Number 1, pp. 47-59, 2006.

  output: $X_E$, $Y_N$

# Integrated generic algorithms (1/2)

- 2-IP/$\epsilon$-constraint method [hai1971]; in Julia:

  Y.V. Haimes, L.S. Lasdon, D.A. Wismer: On a bicriterion formation of the problems of integrated system identification and system optimization. *IEEE Transactions on Systems, Man and Cybernetics*. Volume SMC-1, Issue 3, Pages 296-297, July 1971.

  output: $X_E \subseteq \mathbb{Z}^n$, $Y_N \subseteq \mathbb{Z}^p$

- 2-IP/dichotomic method:

  Aneja, Y. and K. Nair: Bicriteria transportation problem. *Management Science* 25 (1), 73?78. 1979.

  output: $X_{SE} \subseteq \mathbb{Z}^n$, $Y_{SN} \subseteq \mathbb{Z}^p$

- 2-IP/Chalmet & al., 1986:

  L.G. Chalmet, L. Lemonidis, and D.J. Elzinga. An algorithm for the bi-criterion integer programming problem. *European Journal of Operational Research*, 25:292-300, 1986.

  output: $X_E \subseteq \mathbb{Z}^n$, $Y_N \subseteq \mathbb{Z}^p$

# Integrated generic algorithms (2/2)

- 2-MILP/Vincent & al., 2014:

  Th. Vincent, F. Seipp, S. Ruzika, A. Przybylski, X. Gandibleux. Multiple objective branch and bound for mixed 0-1 linear programming: Corrections and improvements for the biobjective case. *Computers & Operations Research*, Volume 40, Issue 1, pp. 498–509, 2013.

  Fl. Lucas. *Multiobjective branch & cut*. Master Thesis, University of Nantes, France. June 2017.

  output: $X_E \subseteq \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$, $Y_N$

- 2-LP/parametric simplex:

  Description available in: Matthias Ehrgott. *Multicriteria Optimization*. Springer-Verlag New York, 2005.

  output: $X_{SE} \subseteq \mathbb{R}^n$, $Y_{SN} \subseteq \mathbb{R}^p$

# Selectable (MI)LP Engines

- open source:
  - GLPK (GNU Linear Programming Kit)
- commercial:
  - CPLEX
  - GUROBI

NB: CPLEX and GUROBI are currently not available on JuliaBox

# Instructions
# for
# installing and running vOptSolver

# Instructions for installing vOptSolver

vOptSolver has been tested with:

- Julia v0.6
- GLPK v4.60

Choose between:

1. a local use
   - on macOS (tested on v10.12.5)
   - on linux (tested on ubuntu 14.04 LTS)
   - on windows (perhaps later...)

2. a distant use
   - in the cloud with JuliaBox.

# Instructions for a distant use in the cloud

- Local installation
  1. nothing to do
- Run Julia
  1. go to `https://juliabox.com` and sign in to open a session
  2. click on the icon "console"
  3. when the prompt is ready, type in the console
     `julia`
- Before the first use of `vOptSolver`, add the following packages:
  1. when the prompt *julia* is ready, type in the terminal

```
Pkg.add("GLPK")
Pkg.clone("http://github.com/vOptSolver/vOptGeneric.jl")
Pkg.clone("http://github.com/vOptSolver/vOptSpecific.jl")
Pkg.build("vOptSpecific")
```

At this point, vOptSolver is properly installed

# Instructions for a local use on your own computer

- Local installation
  1. install Julia on your computer, instructions here:
     `http://julialang.org/downloads/`
  2. install (e.g.) GLPK on your computer, instructions here:
     `http://jump.readthedocs.io/en/latest/installation.html`

  NB: a standard C/C++ compiler must be installed (GCC is suggested)

  At this point, Julia and GLPK are properly installed

- Run Julia
  1. on linux: open a console on your computer; when the prompt is ready, type in the console `julia`
  2. on macOS: locate the application julia and click on the icon; julia console comes on the screen

- Before the first use of `vOptSolver`, add both following packages:
  1. when the prompt julia is ready, type in the console

```
Pkg.add("GLPK")
Pkg.clone("http://github.com/vOptSolver/vOptGeneric.jl")
Pkg.clone("http://github.com/vOptSolver/vOptSpecific.jl")
Pkg.build("vOptSpecific")
```

  At this point, vOptSolver is properly installed

# Instructions for running vOptSolver

When vOptSolver is properly installed, vOptSpecific and vOptGeneric are ready locally or in the cloud.

- Run Julia
  1. open a console on your computer or in the cloud
  2. when the prompt is ready, type in the console
     `julia`
- when the prompt *julia* is ready, type in the terminal
  1. `using vOptSpecific`
  2. `using vOptGeneric`
  3. `using GLPK`

  Remark 1: you may invoke only `using vOptSpecific` if you are only working with `vOptSpecific`. Same remark for `vOptGeneric`.
  Remark 2: you may invoke CPLEX or GUROBI in the place of GLPK (ps: in local mode, the MILP solver selected must be properly installed)
- vOptSpecific and vOptGeneric are ready. See examples for further informations

# vOptSpecific problem by problem

definition (problem, inputs)
data (console, text file)
resolution (API, outputs, text file)

# LAP | Definition | The linear assignment problem

$$
\left[
\begin{array}{lcll}
\min z^k & = & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^k x_{ij} & k = 1, \ldots, p \\
s/c & & \displaystyle\sum_{i=1}^{n} x_{ij} = 1 & j = 1, \ldots, n \\
& & \displaystyle\sum_{j=1}^{n} x_{ij} = 1 & i = 1, \ldots, n \\
& & x_{ij} = (0, 1) & \begin{array}{l} i = 1, \ldots, n \\ j = 1, \ldots, n \end{array}
\end{array}
\right]
\qquad \text{(p-LAP)}
$$

# LAP | Definition | Inputs

Valid for 2-LAP.

- `n` (integer):
  number $n$ of assignments task-resource
- `C1` (matrix of $n \times n$ of integers):
  coefficients $c_{ij}^1$ of the objective 1
- `C2` (matrix of $n \times n$ of integers):
  coefficients $c_{ij}^2$ of the objective 2

# LAP | Data | Example (console)

```
n  =  5

C1 = [  3   9   0   0   6 ;
       16   0   6  12  19 ;
        2   7  11  15   8 ;
        4  11   7  16   3 ;
        2   5   1   9   0   ]

C2 = [ 16   5   6  19  12 ;
       15   7  13   7   7 ;
        1   2  13   2   3 ;
       14   7   8   1   7 ;
       10  10   1   0   0   ]
```

# LAP | Data | Example (text file)

```
5
 3   9   0   0   6
16   0   6  12  19
 2   7  11  15   8
 4  11   7  16   3
 2   5   1   9   0
16   5   6  19  12
15   7  13   7   7
 1   2  13   2   3
14   7   8   1   7
10  10   1   0   0
```

n

C1

C2

- load2LAP
  Load an instance of a 2-LAP from the file `fname`
  `MOCO = load2LAP( fname )`

# LAP | Resolution | API

- set2LAP
  Create a new instance of a 2-LAP and set up all required values
  `MOCO = set2LAP( n , C1, C2 )`

- LAP_Przybylski2008
  Set up the solver to use for the 2-LAP
  `solver = LAP_Przybylski2008( )`

- vSolve
  Solve the instance provided with the mentioned solver and return the results
  `z1, z2, σ = vSolve( MOCO , solver )`

# LAP | Resolution | Outputs (specification)

Valid for 2-LAP.

vSolve returns:

- z1: vector of $(1, \dots, | Y_N |)$ of integers
- z2: vector of $(1, \dots, | Y_N |)$ of integers
- $\sigma$: matrix of $(1, \dots, | Y_N | ; \sigma_1, \dots, \sigma_n)$ of integers where
  - $\sigma_i$: a permutation coding $(x_{ij} = 1 \Leftrightarrow \sigma_i = j)$

# OSP | Definition | One machine scheduling problem

The general one machine scheduling problem considered here is defined as:

$$1 \mid r_i \mid (f_1, f_2) \tag{2-OSP}$$

and the specific OSP currently considered is:

$$1 \mid . \mid (\Sigma C_i, T_{max})$$

# OSP | Definition | Inputs

Valid for 2-OSP.

- `n` (integer):
  number $n$ of jobs, $i = 1 \dots n$
- `r` (vector of $n$ integers):
  $r_i$, the release date for job $i$
- `p` (vector of $n$ integers):
  $p_i$, the processing time for job $i$
- `d` (vector of $n$ integers):
  $d_i$, the due date for job $i$
- `w` (vector of $n$ integers):
  $w_i$, the weight associated to job $i$

# OSP | Data | Example (console)

```
n = 4
p = [  2    4    3    1 ]
d = [  1    2    4    6 ]
r = [  0    0    0    0 ]
w = [  1    1    1    1 ]
```

# OSP | Data | Example (text file)

| | | | | |
|---|---|---|---|---|
| 4 | | | | ↕ n |
| 2 | 4 | 3 | 1 | ↕ p |
| 1 | 2 | 4 | 6 | ↕ d |
| 0 | 0 | 0 | 0 | ↕ r |
| 1 | 1 | 1 | 1 | ↕ w |

- load2OSP
  Load an instance of a 2-OSP from the file `fname`
  `id = load2OSP( fname )`

# OSP | Resolution | API

- set2OSP
  Create a new instance of a 2-OSP and set up all required values
  `id = set2OSP( n , P, D, R, W )`

- OSP_vanwassenhove1980
  Set up the solver to use for the 2-OSP
  `solver = OSP_vanwassenhove1980( )`

- vSolve
  Solve an instance of a 2OSP with the mentioned solver and return the results
  `status = vSolve( id , solver )`

# OSP | Resolution | Outputs (specification)

Valid for 2-OSP.

vSolve returns:

- z1: vector of $(1, \ldots, \mid Y_N \mid)$ of integers
- z2: vector of $(1, \ldots, \mid Y_N \mid)$ of integers
- $\sigma$: matrix of $(1, \ldots, \mid Y_N \mid ; \sigma_1, \ldots, \sigma_n)$ of integers where
  - $\sigma_i$: a permutation coding ($\sigma_i = j \Leftrightarrow$ job $j$ in position $i$)

# vOptGeneric problem by problem

definition (problem, inputs, JuMP)
data (console, text file)
resolution (solve, outputs, text file)

# MOMILP-MOLP-MOIP | Definition

Three Multi Objective Linear Optimization Problems:

$$\begin{array}{rcl} \min z(x) & = & Cx \\ \text{s/t } Tx & \leqq & d \\ x & \in & \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2} \end{array}$$

Multi Objective Mixed-Integer Linear Problem (MOMILP)

$$\begin{array}{rcl} \min z(x) & = & Cx \\ \text{s/t } Tx & \leqq & d \\ x & \in & \mathbb{R}^{n_1} \end{array}$$

Multi Objective Linear Problem (MOLP)

$$\begin{array}{rcl} \min z(x) & = & Cx \\ \text{s/t } Tx & \leqq & d \\ x & \in & \mathbb{Z}^{n_2} \end{array}$$

Multi Objective Integer Problem (MOIP)

where:

$$\begin{array}{rcl} T \in \mathbb{Z}^{m \times n} & \longrightarrow & m \text{ constraints, } i = 1, \ldots, m \\ C \in \mathbb{Z}^{n \times p} & \longrightarrow & \text{the objective matrix} \\ X = \{x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2} | Tx \leq d\} \subseteq \mathbb{R}^n & \longrightarrow & \text{the set of feasible solutions} \\ Y = z(X) \subseteq \mathbb{R}^p & \longrightarrow & \text{the set of images} \end{array}$$

# MOMILP-MOLP-MOIP | Formulation

Following specifications of JuMP for formulating a model, see:

- http://jump.readthedocs.io/en/latest/quickstart.html#creating-a-model

Plus:

- @addobjective( <model>, <opt>, <function>)

  where

  - model: id of the model concerned
  - opt: function to Min imize or Max imize
  - function: definition of the function

# MOMILP-MOLP-MOIP | Example of formulation

$$
\begin{aligned}
\min \quad & c_1^1 x_1 + c_2^1 x_2 \\
\min \quad & c_1^2 x_1 + c_2^2 x_2 \\
s/t \quad & t_{11} x_1 + t_{12} x_2 \leq d_1 \\
& t_{21} x_1 + t_{22} x_2 \leq d_2 \\
& x_1 \in \mathbb{R}, x_2 \in \mathbb{Z}
\end{aligned}
$$

```
# --- create a MOMILP and set the model ---
  MOMILP = vModel(solver=<working on it>)
  @variable(MOMILP, x1 >= 0)
  @variable(MOMILP, x2 >= 0, Int)
  @addobjective(MOMILP, Min, c11*x1 + c12*x2)
  @addobjective(MOMILP, Min, c21*x1 + c22*x2)
  @constraint(MOMILP, t11*x1 + t12*x2 <= d1)
  @constraint(MOMILP, t21*x1 + t22*x2 <= d2)
```

$$
\begin{aligned}
\min \quad & c_1^1 x_1 + c_2^1 x_2 \\
\min \quad & c_1^2 x_1 + c_2^2 x_2 \\
s/t \quad & t_{11} x_1 + t_{12} x_2 \leq d_1 \\
& t_{21} x_1 + t_{22} x_2 \leq d_2 \\
& x_1, x_2 \in \mathbb{R}
\end{aligned}
$$

```
# --- create a MOLP and set the model ---
  MOLP = vModel(solver=<working on it>)
  @variable(MOMILP, x1 >= 0)
  @variable(MOMILP, x2 >= 0)
  @addobjective(MOLP, Min, c11*x1 + c12*x2)
  @addobjective(MOLP, Min, c21*x1 + c22*x2)
  @constraint(MOLP, t11*x1 + t12*x2 <= d1)
  @constraint(MOLP, t21*x1 + t22*x2 <= d2)
```

$$
\begin{aligned}
\min \quad & c_1^1 x_1 + c_2^1 x_2 \\
\min \quad & c_1^2 x_1 + c_2^2 x_2 \\
s/t \quad & t_{11} x_1 + t_{12} x_2 \leq d_1 \\
& t_{21} x_1 + t_{22} x_2 \leq d_2 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$

```
# --- create a MOIP and set the model ---
  MOIP = vModel(solver=GLPKSolverMIP())
  @variable(MOIP, x1 >= 0, Int)
  @variable(MOIP, x2 >= 0, Int)
  @addobjective(MOIP, Min, c11*x1 + c12*x2)
  @addobjective(MOIP, Min, c21*x1 + c22*x2)
  @constraint(MOIP, t11*x1 + t12*x2 <= d1)
  @constraint(MOIP, t21*x1 + t22*x2 <= d2)
```

# MOMILP-MOLP-MOIP | Data | Example (console)

```
c11 =  3   ;  c12 =  1
c21 = -1   ;  c22 = -2

t11 =  0   ;  t12 =  1   ;  d1 = 3
t21 =  3   ;  t22 = -1   ;  d2 = 6
```

# MOMILP-MOLP-MOIP | Data | Example (format MOP)

parseMOP
Load an instance (MOP format) from the file `fname`
`jumpModel = parseMOP( fname , solver=<solver to invoke> )`

writeMOP
Write a model according to the MOP format to a file `fname`
`writeMOP( vModel , fname )`

NB: the MOP format is an extension of

- the MPS format
  (https://en.wikipedia.org/wiki/MPS_(format))
- to multiple objectives
  (http://moplib.zib.de/format_desc/mop_format.pdf)

# MOMILP-MOLP-MOIP | Example of resolution

MOMILP
Algorithm: branch&bound
Solver: working on it

```
# --- solve the model ---
  status = solve( MOMILP )

# --- Print the results ---
  print_X_E( MOMILP )
```

MOLP
Algorithm: parametric simplex
Solver: working on it

```
# --- solve the model ---
  status = solve( MOLP )

# --- Print the results ---
  print_X_E( MOLP )
```

MOIP
Algorithm: $\epsilon$-constraint
Solver: GLPK or CPLEX

```
# --- solve the model ---
  status = solve( MOIP , method =:epsilon , step = 0.5 )

# --- Print the results ---
  print_X_E( MOIP )

# --- Get the results ---
  getY_N( MOIP )
```

# MOMILP-MOLP-MOIP | Resolution | Outputs (specification)

$Y_N$ of the 2-MOP is defined by

- 2-MILP: a mixed nondominated set (composed of edges that are either closed, half-open, open or reduced to a point)
- 2-LP: a continuous nondominated set (composed of edges)
- 2-IP: a discrete nondominated set (composed of points)

# Technical overview

# Technical overview ....

User application written in Julia

**Declare**

*No-structured MOP*

IP/MIP/LP

*Structured MOP*

LAP

$1 \mid r_i \mid (f_1, f_2)$

**Solve**

*Specific*

2-LAP $\rightarrow Y_N$

$1 \mid . \mid (\Sigma C_i, T_{max}) \rightarrow Y_N$

*Generic*

2-ILP $\rightarrow Y_{SN}$
dicho

2-ILP $\rightarrow Y_N$
Chalmet

2-MILP $\rightarrow Y_N$
B&C

2-ILP $\rightarrow Y_N$
$\epsilon$-cte

2-LP $\rightarrow Y_N$
simplex

**Utils**

*Input/Output*

read/write format MOP
plot $Y_N$

*Misc*

maintain $Y_N$
analyze $Y_N$

Interface:

Library solvers
C/C++ specifics

Modules C

LAP

Interface:

Library solvers
MIP generics

Solvers MIP

*open source*
GLPK
*commercial*
CPLEX, GUROBI

# History

# History

- v1.0: July 2016; first prototype; not opened to the public
- v2.0: June 23, 2017; first stable version; released to the public with
  - a depot on github, webpages, and a tutorial
  - JuMP extended to multiple objectives (vOptGeneric)
  - primitives for loading/printing/writing a non-structured problem in std MOP format
  - $\epsilon$-constraint algorithm for bi-objective integer programming
  - backbone in Julia (vOptSpecific)
  - API and solver for bi-objective linear assignment problem
  - API and solver for bi-objective one machine scheduling problem
  - primitives for I/O of a 2LAP and 2OSP on files

# Taskforce

# Taskforce

Involved in the development of vOptSolver:

Currently:

- GANDIBLEUX Xavier (coordinator)
- SOLEILHAC Gauthier
- PRZYBYLSKI Anthony

Previously:

- CHATELLIER Pauline
- DUMEZ Dorian
- LUCAS Flavien

# Links and contact

# Follow/join us here

Homepage of vOptSolver:
http://voptsolver.github.io/vOptSolver/

Repository of vOptSolver:
http://github.com/vOptSolver
Repository of vOptSpecific:
http://github.com/vOptSolver/vOptSpecific.jl
Repository of vOptGeneric:
http://github.com/vOptSolver/vOptGeneric.jl

Contact concerning vOptSolver:
vopt@univ-nantes.fr

Homepage of the ANR/DFG research project “vOpt”:
http://vopt-anr-dfg.univ-nantes.fr