From Lydian to Blues to Pentatonic: A Cyclical Enharmonic Scale Transformation

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Abstract

This study introduces a cyclical bijective transformation for generating a family of musically popular and significant scales, including blues and pentatonic scales, and some new scales too, via a process of sequentially flattening forths or sharpening fifths, starting from the Lydian scale. The Lydian to Locrian mode can be generated this way, using the circle of fifths. However, by starting the transformation from a different note, we can generate a total of 154 scales. As we encounter scales with less than 7 notes, we will also introduce the concept of hidden notes, which have their own triads, and scales. We will introduce a way to represent all these scales in staff notation, and finally apply the transfromation to existing sheet music as a proof of concept, transforming a song in G Major to F♭ Blues Lydian, and E Blues Phrygian (they are not the same). Ultimately, the transformation offers a structured yet flexible tool for compositional modulation and theoretical analysis.

The relationship between diatonic modal systems and blues-derived scales has been a topic of enduring interest in both jazz theory and ethnomusicology, especially since they are often treated as discrete. It seems natural that the Lydian scale and Chromatic scale have 7 and 12 tones respectively, because 7 perfect fifth is close to an octave, and 12 fifths is even closer to an octave. However, is there a reason for the blues and pentatonics scales, having 6 and 5 notes, to be so prevalent in music? By sequentially flattenting forths or sharpening fifth, it is indeed possible to genereate blues and pentatonic scales from the Lydian scale. This approach reveals a cyclical structure that not only recovers well-known scales but also introduces novel intermediary scales, expanding the theoretical landscape of scale construction. This approach also suggests that the blues and the pentatonic scales actually do have 7 notes, but that they are hidden behind existing notes. These hidden notes have their their own triads, and reveal interesting chord progression, allowing modulating a song in the major scale to the blues and pentatonic scales.

1 Methodology

1.1 Existing Theory

It is well established that the Lydian scale can be generated by stacking perfect fifths seven times, for example starting on F:

$$F \to C \to G \to D \to A \to E \to B$$

Flattening these fifths sequentially from the last fifth produces the diatonic modes in descending order of brightness:

$$\begin{aligned} F \text{ Lydian} &\to F \text{ Ionian} \to F \text{ Mixolydian} \\ &\to F \text{ Dorian} \to F \text{ Aeolian} \\ &\to F \text{ Phrygian} \to F \text{ Locrian} \\ &\to E \text{ Lydian (full cycle).} \end{aligned}$$

This flattening modulates the scale's modal quality from bright to dark. For ease, we can perceive sequentially flattening the last fifths as flatting consecutive forths, since the last fifth is same as the root note's forth.

Thus by flattening the forths in the Lydian, we can modulate the "darkness" of a scale.

However, we can also interpret this as sharpening the fifths, starting from the darkest note. Let us start from B Locrian, since it shares the same notes as F Lydian. This allows us to avoid altering the root note F which performing the transformation:

$$\begin{aligned} B \text{ Locrian} &\to B \text{ Phrygian} \to B \text{ Aeolian} \\ &\to B \text{ Dorian} \to B \text{ Mixolydian} \\ &\to B \text{ Ionian} \to B \text{ Lydian} \\ &\to C \text{ Locrian (full cycle).} \end{aligned}$$

We can represent this in the circle of fifths below:

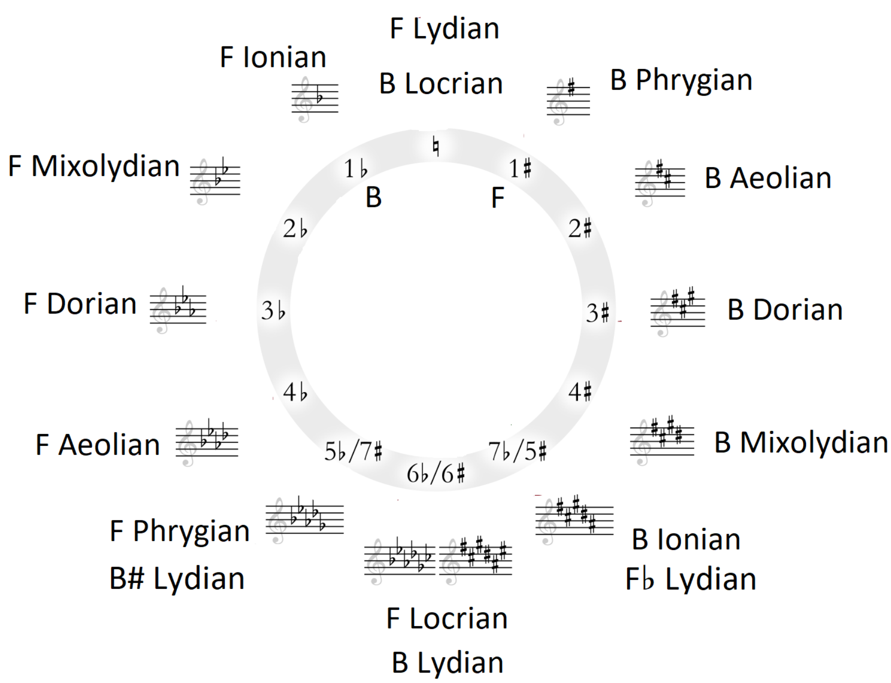

Figure 1: Basic Fifths: Lydian Forths and Locrian Fifths

1.2 Sequential Sharpening Process

Instead of sharpening the fifth starting from the root, we can generate many popular scales if we start sharpening the fifths starting from second degree of the Lydian scale. However, unlike the Lydian-Locrian common scales, the transformation from Lydian to Harmonic Minor through sharpening the second scale degree produces a fundamentally different pitch-class set that cannot be achieved through any rotation or transposition of the original Lydian mode. This is because the operation creates a new interval structure that is not isomorphic to any diatonic mode. In set theory terms, while the diatonic modes all belong to the same set class [7-35], the Harmonic Minor scale belongs to a distinct set class [7-32], demonstrating that our sharpening transformation generates genuinely new scalar structures rather than mere rotations of existing ones. By starting from a different note, we will be able to generate the same scales above, and some new ones too, with their own set classes.

Table 1: Scales generated by sequential sharpening starting from the second fifth

Transformation	**Resulting Scale**	**Enharmonic Equiv.**
Original	F G A B C D E	F Lydian
Sharpen 2nd (G $\rightarrow$ G$\sharp$)	F G$\sharp$ A B C D E	A Harmonic Minor
Sharpen 6th (D $\rightarrow$ D$\sharp$)	F G$\sharp$ A B C D$\sharp$ E	F Double Harmonic
Sharpen 3rd (A $\rightarrow$ A$\sharp$)	F G$\sharp$ A$\sharp$ B C D$\sharp$ E	F Blues + Maj 7
Sharpen 7th (E $\rightarrow$ E$\sharp$)	F G$\sharp$ A$\sharp$ B C D$\sharp$ E$\sharp$	F Blues Minor
Sharpen 4th (B $\rightarrow$ B$\sharp$)	F G$\sharp$ A$\sharp$ B$\sharp$ C D$\sharp$ E$\sharp$	F Pentatonic Minor
Sharpen root (F $\rightarrow$ F$\sharp$)	F$\sharp$ G$\sharp$ A$\sharp$ B$\sharp$ C D$\sharp$ E$\sharp$	F$\sharp$ Lydian $\flat$5
Sharpen 5th (C $\rightarrow$ C$\sharp$)	F$\sharp$ G$\sharp$ A$\sharp$ B$\sharp$ C$\sharp$ D$\sharp$ E$\sharp$	F$\sharp$ Lydian

This sequence generates a rich palette of scales, including canonical modes, blues scales, and lesser-known modal variants. Note that in the Blues Minor, a note is ”lost” because $E\sharp$ is the same as F, and again in the Pentatonic Minor, a note is ”lost” because $B\sharp$ is the same as C. Let us call them hidden notes of their real counterparts. It is important to track the hidden $E\sharp$ and $B\sharp$, as they are still part of the scale, and affect the triads that we will construct. Not only have we generated very famous and popular scales using a very simple process, but we have created 2 new scales, the Blue Minor + Major 7th and the Lydian $\flat$5. The $\flat$5 in F Lydian is a $\flat$C that hides behind a C.

2 Analysis

2.1 Theoretical Context

The sequential sharpening process aligns with transformational theory's emphasis on systematic pitch-class operations [Cohn, 2012] and echoes enharmonic modulation techniques prevalent in Romantic harmony [Tymoczko, 2011]. The cyclical nature of the transformations parallels the circle of fifths but operates through enharmonic equivalences rather than key centers, offering a novel perspective on modal and blues scale interrelations.

2.2 Triad Structures Across the Cycle

Stacking thirds within each altered scale yields characteristic triads that illuminate their harmonic functions and compositional potentials. Table 2 summarizes the triads and their qualities for each step. After passing Blues, rather than representing the chords degrees with sharps, we switch to flats because it takes less flats to reach the notes than sharps.

Table 2: Triads and their analysis across the scale transformation cycle

Step	**Scale**	**Triads**
1: F Lydian	F G A B C D E	Generic triads: IV, V, vi, vii°, I, ii, iii
2: A Harmonic Minor / E Phrygian Dominant	A B C D E F G♯	vi, vii°, I Aug, ii, III, IV, v°
3: E Double Harmonic Major/ Byzantine	E F G♯ A B C D♯	III, IV, ♯v, vi, vii ♭5, I Aug, ♯ii Sus2 ♭5
4: F Blues Minor + Major 7th	F G♯ A♯ B C D♯ E	IV Sus4, ♯v, ♯vi°, vii ♭5, I Aug, ♯ii Sus2, III
5: F Blues Minor	F G♯ A♯ B C D♯ E♯	♭IV Sus4, v, VI Sus2, ♭vii ♭5, ♭I 6, II Sus2, iii°
6: F Pentatonic Minor	F G♯ A♯ B♯ C D♯ E♯	♭IV Sus4, V, VI Sus2, vii ♭♭5, ♭I 6, II Sus2, iii
7: F♯ Lydian ♭5	F♯ G♯ A♯ B♯ C D♯ E♯	IV♭5, V, VI Sus2, vii°, ♭I 6, ii, iii
8: F♯ Lydian	F♯ G♯ A♯ B♯ C♯ D♯ E♯	Generic triads: IV, V, vi, vii, I, ii, iii

We can see from the table, why the hidden notes are so important. They have their own triads. In F Pentatonic Minor, E♯ min is a different chord than FSus4 even though E♯ and F are the same note. Switching between the 2 will indicate to the listener not only that the scale is pentatonic minor, or also indicate a change in chord.

2.3 Mathematical Representation of the Transformation

Let us represent all the transformations mathematically. This will help us especially with the next section, where we will be representing the new scales in staff notation. Let us represent the scale as a starting note, and a vector of integers. The integer represents the number of semitones away from the starting note to the next note generated in the scale. Let us describe Lydian as $(0, 7, 14, 21, 28, 35, 42)$. F Lydian is therefore:

$$\text{F Lydian} = \text{F} : (0, 7, 14, 21, 28, 35, 42)$$

This is because a perfect fifth is 7 semitones apart, which is also 3 times the previous interval's frequency. If we then mod 12 on all the numbers, we get:

$$\text{F Lydian} = \text{F} : (0, 7, 2, 9, 4, 11, 6)$$

$$\text{F Lydian} = \text{F} : (0, 2, 4, 6, 7, 9, 11)$$

This makes the scale more obviously a Lydian scale. However, we will not keep this sorted, because we can generate all the other scales from F: (0, 7, 2, 9, 4, 11, 6).

Transposition is just changing the starting note of the scale:

$$\text{F} : (0, 7, 2, 9, 4, 11, 6) \xrightarrow{\text{T1}} \text{F}\sharp : (0, 7, 2, 9, 4, 11, 6)$$

Modal shift is then just a matter of shifting the starting note to the next lowest number in the vector (which will always be 2 numbers away from the previous starting note because a fifth's fifth is a second). Then we have to lower every number by the first number in the vector and mod 12, to represent them as semitones away from the new starting note.

$$\begin{aligned} \mathrm{F} : (0,7,2,9,4,11,6) &\xrightarrow{\text{Rotate vector by 2}} \mathrm{F} : (2,9,4,11,6,0,7) \\ &\xrightarrow{\text{Lower by 2}} \mathrm{G} : (0,7,2,9,4,10,5) \end{aligned}$$

We can represent this as MS (Modal Shift):

$$\mathrm{F} : (0,7,2,9,4,11,6) \xrightarrow{\mathrm{MS}} \mathrm{G} : (0,7,2,9,4,10,5)$$

$$\mathrm{F} : \text{Lydian} \xrightarrow{\mathrm{MS}} \mathrm{G} : \text{Mixolydian}$$

We can represent Raising a scale with RN by sharpening the Nth note in our vector by 1 semitone, then mod 12 on all the numbers. The inverse of R(N) is R(-N).

$$\begin{aligned} \mathrm{E} : (0,7,2,9,4,11,6) &\xrightarrow{\mathrm{R0}} \mathrm{F} : (0,6,1,8,3,10,5) \\ &\quad \text{(Lydian to Locrian, note key shift)} \\ \mathrm{F} : (0,6,1,8,3,10,5) &\xrightarrow{\mathrm{R1}} \mathrm{F} : (0,7,1,8,3,10,5) \\ &\quad \text{(Locrian to Phrygian)} \\ \mathrm{F} : (0,7,1,8,3,10,5) &\xrightarrow{\mathrm{R2}} \mathrm{F} : (0,7,2,8,3,10,5) \\ &\quad \text{(Phrygian to Aeolian)} \end{aligned}$$

Note that the reason all the numbers decreased in the first operation is because we sharpened the starting note, which is the same as flattening everything else. (Like walking to the right is the same as pulling everyone else to the left.) This is also where the transformation becomes cyclic. Therefore, the operation of R0 will always require a Transposition of T-1 to preserve key note. Also note that performing R2 on F Lydian is the same as peforming R0 on G Mixolydian, and then the R3 that is performed on F Lydian afterwards is the same as performing R1 on G Mixolydian afterwards. Althought one could suggest simply calling it R without numbers, it is important to note that the behavior of raising the fifth if different each time it is repeatedly performed.

Now we can produce the following cycle:

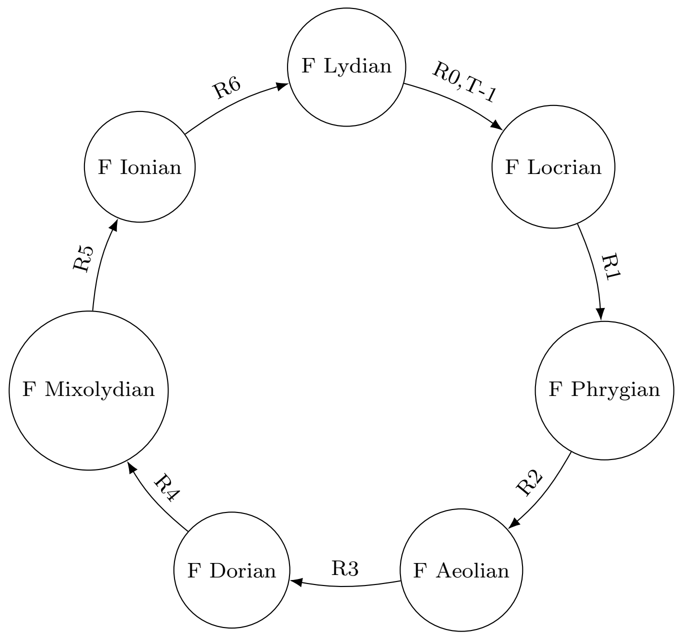

Figure 2: Cyclic transformation of Common modes through sequential sharpening

However, Rather than performing R0 on Lydian, if we perform R2 instead, we get the scales that are the key discussion in this paper.

If we start from the root note, we can represent a Modal Shift as a Raising a scale twice, and then transposing it. This will be prepresented as MST (Modal Shift then Transposition).

Although the common modes can be reached either through sharpening or flattening fifths, or by modal shifts, the new scales generated cannot be reached by modal shifts. For example, it is not possible to from A Harmonic Minor to E Phrygian Dominant with sharpening fifths or transposition. Let us describe a scale that after undergoing the transformation, can also be reached by modal shifts, as an *isomodal* scale, because the generated modes have not changed. Let us describe a *kernel* as a set of scales whose modes do not overlap.

Therefore, applying R0-R6 on Lydian will an *isomodal* transformation that only generates

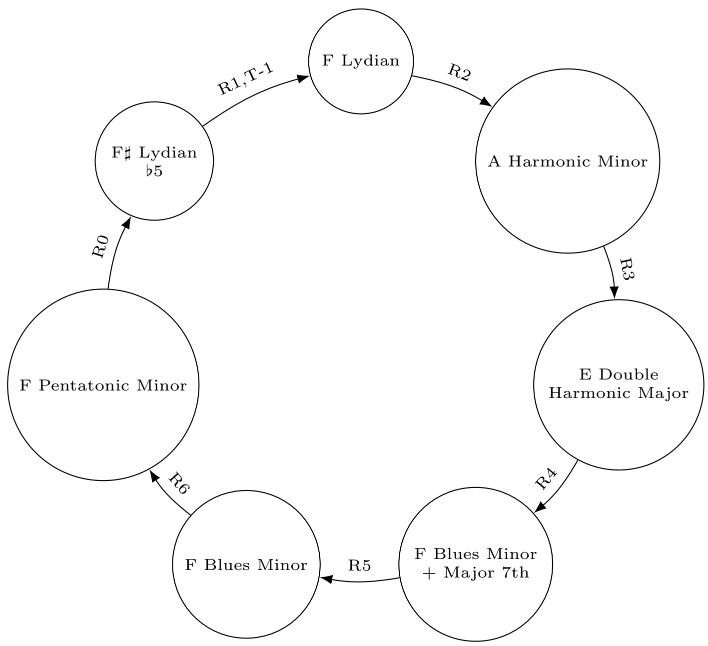

Figure 3: Cyclic transformation through sequential sharpening starting from second.

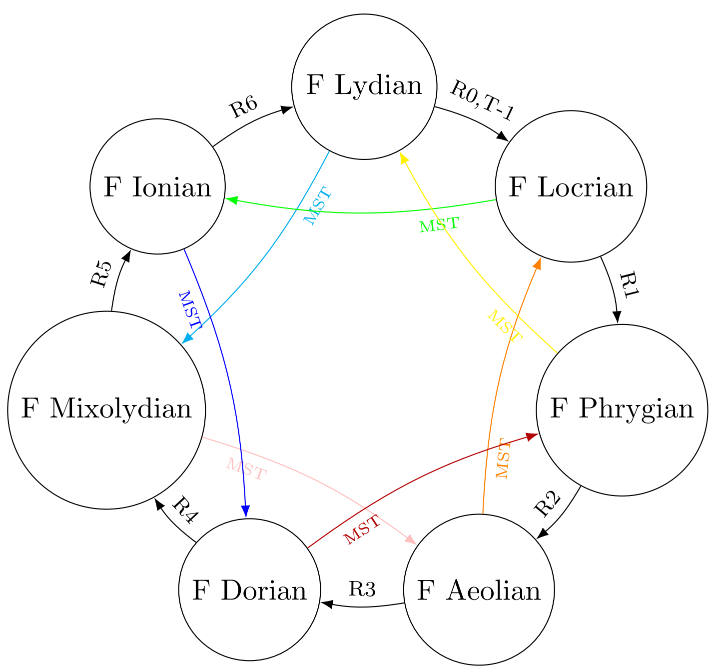

Figure 4: Common modes can be reached though both raising the Lydian from root note, or through model shifts.

1 kernel. However, applying R2-R1 on Lydian will generate 6 *kernel* scales. In fact, we will see that by applying this transformation on any root note, we will generate 6 kernels, except when applyed on the root note. Only Lydian tranformation of R0 is *isomodal*, and the rest are not.

Let us represent all possible scales that can be generated from Lydian through our transformations:

From figure 6, we can see there are repeated kernels. Note the the center circle with the common modes is all just one kernel. However, the every node in the remaiing 6 petals are all kernels.The numbering on unfamiliar scales is (Starting Root, number of transformations applied). If you see an inconsistency, it is becuase another cycle produces the same kernel. Infact, there are quite a lot of overlapping kernels. In total we have could have hoped for 37 kernels (each petal has 6 node excluding the center, and there are 6 petals, and the center circle is just one kernel), but due to the overlapping, we only get 22 unique kernels.

We can thus make a graph showing the relationships between the kernels:

From this graph, we can see the closeness of the kernels. Every kernel is only one semitone apart from another. What is fascinating is that there is a line of sysmettry going grom common to melodic to double harmonic, and on one side is Harmonic Minor and on the the other Harmonic Major. Also interesting is that the pentatonic is one of the only 6 scales that has only 2 neighbors, so it is quite impressive that it became populat despite being "far" from common. Every kernel is 2 transformations away from common, except 3,3 3,2 and 4,2. I would be interesting to see what the music sounds like. Since there are now 22 unique kernels, each one with 7 modes of their own, we may want to develop a conventiion for them. Key, Transformation, Mode is complete, for example, A (4,2) Dorian.

A Harmonic Minor for examle is technically A Harmonic Aeolian, and E Phrygian Dominant is technically E Harmonic Phygian. The actual scale that is generated is technically F Harmonic Lydian. Also important to note that there is both the Harmonic Major kernel and the Harmonic Kernel. Saying Harmonic Major Aeolian is confusing, so when both kernel

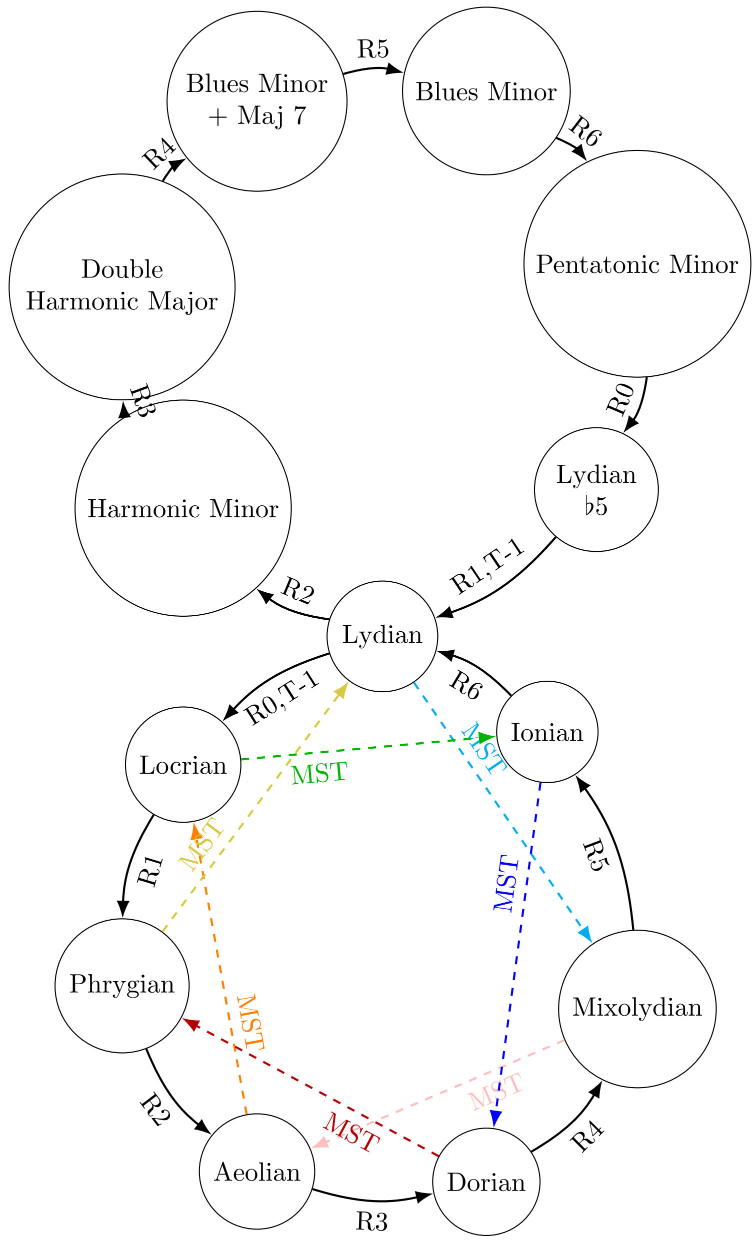

Figure 5: Scales transformations through sharpening the fifths. Modal shifts only map to existing scales in the bottom circle. Minor has been omitted in ”Harmonic Minor”, because the generated scale is technically ”Harmonic Lydian”. Phrygian is in quotes because the scale does not have an actual name. Also note that R2 on Lydian is the same as R0 on Mixolydian. Arrow points from Lydian because the generated scales are most familiar when the root note of the new scales is the same as the root note of the Lydian scale.

Figure 6: Complete hexadic modal system with a central common mode and 6 petals. Each petal represents a transformational cycle starting from the common mode, following R0-R6 operations. Connections between petals create secondary cycles through shared nodes.

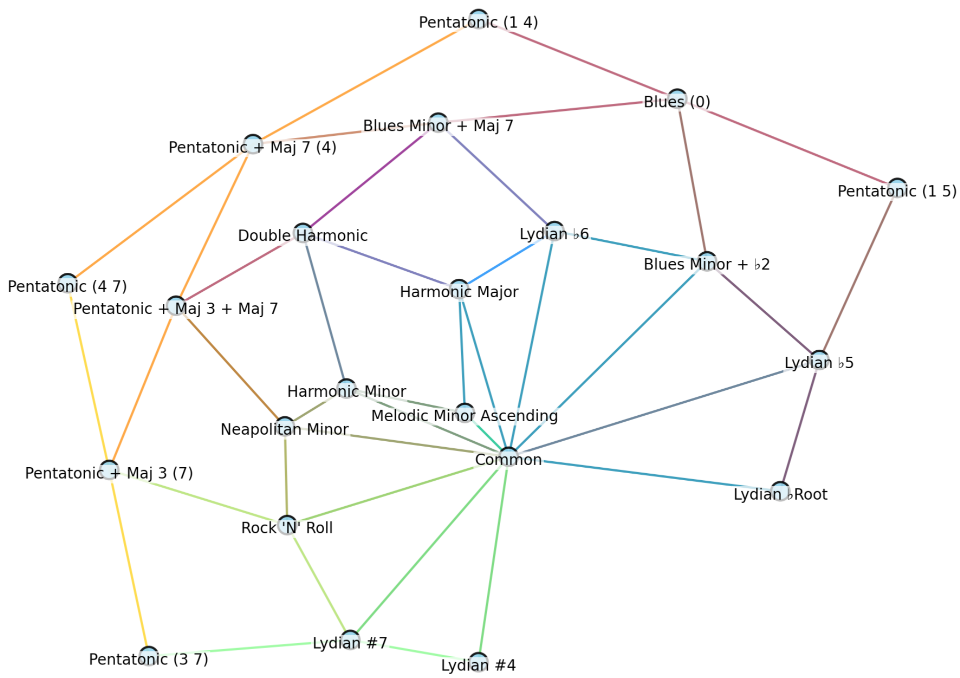

Figure 7: Graph showing the relationships between the kernels.

and mode are present we will refer to Harmonic Major as Harmajic, and Harmonic Minor and Harminic. Often the pentatonic minor scales are just filtered out version of an exisiting scale. For example, a pentatonic minor scale will riff over an song in Aeolian, or Phygian or Dorian, so calling it Pentatoic Minor was sufficent. However, now it is beneficial to have a song be specifically in Pentatonic Aeolian or Pentatonic Phygian, etc. When a song in C major is transformed to A Pentatonic Phygian or A Pentatonic Aeolian, the results are actually quite different. Futheremore we have hidden scales, like E♯ Blues Phygian hiding behind F Blues Lydian, which, although both are modes of the same scale, and also share the same root note, they produce different results when a song is transformed into them.

2.4 Pentatonic Multiplicities

Notice that there are multiple pentatonic scales. However, they are differnt kernels because their hidden notes are behind different notes. The parenthesis denote which degrees in the minor pentatonic scale are repeated, because there is a hidden note behind that degree. For example, (1 4) means that the root note and the forth notes are repeated. Pentantonic (1 4) has intervals 0, 0, 3, 5, 5, 7, 10 which is different than Pentatonic (1 5) which has intervals 0,0, 3, 5, 7, 7, 10, which is the one we orignally discovered. This is very easy to verify. If we take the F lydian scale 0, 2, 4, 6, 7, 9, 11. If we flatten the F and the C, we get the E Pentatonic (1 5) scale. However, If we sharpen the E and the B, we get the D Pentatonic (3, 7) scale. If we flatten the F but sharpen the B, we get the A Pentatonic (3, 5), which interestingly, is not even a kernel that can be generated from the scales above. The (1 4) and (4 7) are possible because sharpening the third, or flattening the root note of the major pentantonic scale also produces a pentatonic scale, the same way sharpening the root note or flattening the forth note of the Lydian scale produces a common scale (Lydian-¿Locrian). The transformation is thus isomodular. To clarify, B Pentatonic Minor (1 4) has the same non-hidden notes of E Pentatonic Minor (1 5) with a G♭. Although the hidden notes are in the same place, since the root note has moved, the degree of the hidden notes have change.

2.5 Representing new Key Signatures in Staff Notation

The in common Lydian to Locrian scales, it is possible to represent all of them with 6 sharps or 6 flats for any key signature due to the isomodal nature of the Lydian-based transformations. We can see that in the figure Basic Fifths.

However, let us compare that with the Mixolydian-based transformations, as seen in Harmonic Fifths.

(Small note: Fladian is the name given to the scale shape of a Lydian with a flattened fifth that hides behind the it's forth.) Notice now that although C Ionian is the same as D Mixolydian, they do not end up being equal with 6 sharps on C, or 6 flats on D. This is because the Mixolydian-based transformations are not isomodal, and the number of sharps or flats needed to represent the scale is inherently different.

It can be complex to represent all of the the scales, however, is is possible to represent

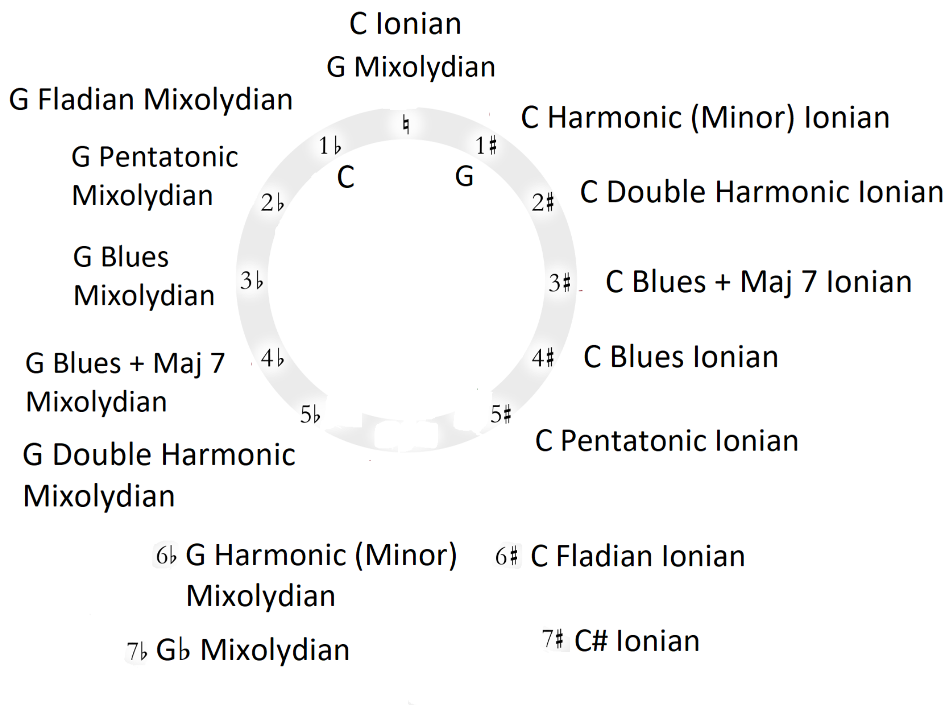

Figure 8: Second Degree Fifths: Mixolydian Forths and Ionian Fifths.

all the kernels with either 3 sharps or 3 flats. It is easiest to see in the figure below, which we will now refer to as the Flower of Fifths.

The center of the pentals are denoted with the most conveinient root note and mode to choose for the cycle of transformations. This is helpful because it is possible that the roote note can change in the middle of the cycle if we are not careful. For example, in the second degree of transformations, F Lydian becomes F Fladian Lydian after flattening the C, and then F flat Pentatnoic Lydian after flattening the F. However, if we started chose G Mixolydian, the G would not be flattened mid cycle, and C Ionian would remain unsharpened mid cycle.

Now, we can represent all the kernels with either 3 sharps or 3 flats. Note that we there are also multiple ways to represent the same kernel. Now to represent any key signature, we would first add the kernel signature, and then add the key signature on top of that to transpose the scale. We may then even want to indicate which mode the song is in by

Figure 9: Flower of Fifths help us identify the kernel signature. The note names describe the scale shape, and not necessarily the root note.

indicating the root note.

For example, if a song were in A Blues Dorian, we could first get G Blues Mixolydian kernel signature from the Flower of Fifths. G Mixolydian is the same scale as D Dorian, and since D has not been flattened, G Blues Mixolydian has the same notes as D Blues Dorian. Finally, since A is D's fifth, we know we can transpose the scale by a fifth by adding a single sharp in the key signature. We can see this in the figure of A Blues Dorian below.

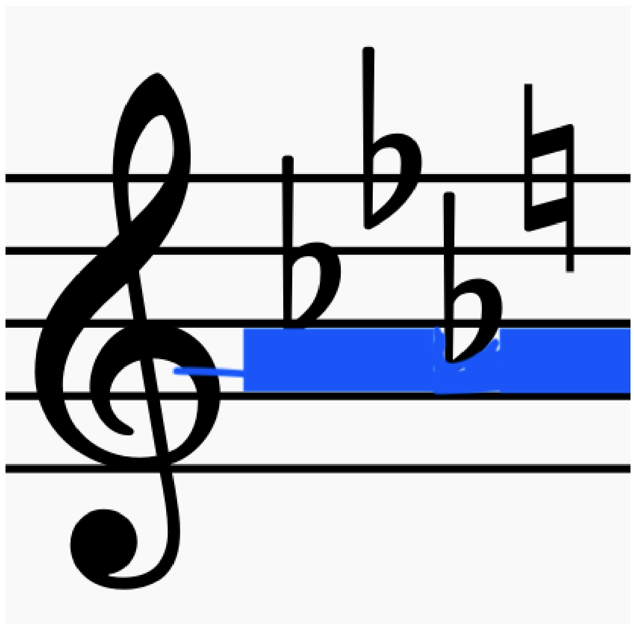

Figure 10: A Blues Dorian: G Blues Mixolydian kernel signature, indentical to D Blues Dorian, and the add A sharp to transpose to D's fifth. Since the F not is already flattened from the kernel signature, the key signature makes the F return back to F natural. The A space is colored Blue to denote that A is the root note, and Blue because it is our arbitrary color designated to Mixolydian. From this alone, we can determine that the song is in A Blues Dorian.

Below, we can see Minuet in G, a famous song, transformed to F♭ Blues Lydian, and E Blues Phrygian. Not that although the root note is the same, the songs are quite different. This expresses the hidden scales.

2.6 Known Works

Several well-known songs demonstrate the use of these overlapping triads:

Figure 11: Minuet in G Transfromed to F♭ Blues Lydian.

Figure 12: Minuet in G Transfromed to E Blues Phrygian.

- “All Blues” by Miles Davis uses both the F minor triad (F-A♭-C) and the E♯ major triad (E♯-G♯-B♯) in its progression, particularly during the bridge section
- “Footprints” by Wayne Shorter employs both B♯ minor (B♯-D♯-F♯) and C minor (C-E♭-G) triads in its characteristic minor blues progression
- “Blue in Green” by Miles Davis/Bill Evans makes use of the duality between F minor and E♯ major sonorities in its modal framework
- “So What” by Miles Davis, while primarily Dorian, explores the tension between the B♯ and C triads in its characteristic two-chord vamp

These examples demonstrate how master jazz composers intuitively understood and exploited the hidden notes inherent in these scales, even if they didn’t explicitly theorize them in these terms.

3 Discussion

The results demonstrate a compelling cyclical relationship among modal, harmonic, and blues-derived scales, mediated by enharmonic reinterpretation and sequential sharpening of scale degrees. Notably, the emergence of augmented and diminished triads in intermediate steps (e.g., Double Harmonic Major, Blues Minor + Major 7th) reveals a harmonic tension that bridges the bright Lydian mode and the characteristic blues tonality.

This framework offers a novel lens for understanding scale construction and modulation, with potential applications in jazz improvisation, composition, and computational music theory. The concept of *hidden notes*—enharmonically equivalent but functionally distinct pitch classes—adds further depth to harmonic analysis and voice-leading considerations. This method enables:

- Seamless modulation between Lydian brightness and blues minor grit.
- Hybrid scales (e.g., F blues minor + major 7th) for avant-garde improvisation.
- Creation of hidden notes and degrees (IV$\sharp$ and VII$\sharp$) that hide beneath existing notes, and yet which are still part of the scale.
- Hidden notes have different triads than their real counterparts.

4 Conclusion

This study presents a systematic method for generating a rich family of scales through sequential sharpening and enharmonic reinterpretation, revealing a cyclical pathway that connects Lydian, harmonic minor, blues, and pentatonic minor scales. By situating this method within transformational and enharmonic theory, it contributes a new perspective on scale relationships and modulation.

Future research may explore microtonal generalizations, algorithmic implementations, and empirical studies of listener perception to further validate and extend these findings.

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