**Convex Optimizations – Homework 1**

Q1.

1. Given:

is strongly Convex and twice differentiable with bounded Hessian

Since, is convex and M-strongly smooth,

For gradient descent optimization,

Using the property (1) of strong smoothness, and , we get

)

Since, , we get

Since is m-strongly convex,

We have,

Using (2) and (3), we have,

The function will be sub-optimal after at most iterations of the algorithm, where is

Since the step size here, which is a constant, we don’t have to find the best step size in every iteration. Hence, no function evaluations are required at this point.

Once we have the step size, we need to perform the gradient evaluation once for every step.

We need to perform gradient evaluations and 0 function evaluations for this algorithm.

1. Consider the function

with and

and direction

Assume and we stop when

;

, we perform the next iteration

;

We know , since the algorithm directly jumped from one side of the minima to the other side of minima and is constant at , we can conclude that it won’t converge at .

We know that if a function is M-Strongly smooth, and the best step size is ­­­­­­

If we want to decide a fixed step size for all steps, it depends on the Hessian of the function.

2. Newton’s Method

1. To Prove:

Given

, and are the Newton Steps of and .

We know that, the Newton step for is given by,

from (1)

Hence, proved.

1. **To Prove:** For any , the exit condition for backtracking linesearch on in direction of will hold if and only if the exit condition holds for for

Consider the exit condition for

… [given] and … [from ]

The exit condition for for holds if and only if the exit condition holds for for .

1. Given

We need to prove that and

Let’s look at it as an Induction problem,

Let’s assume hypothesis,

When we run the Newton Algorithm on starting at , we get next positions of as

From definition of Newton’s step,

… [from ]

1. The Newton decrement for is defined as

Similarly, for we have,

… [given]

… [from ]

Since the stopping criterion for the Newton’s method is,

The stopping conditions are also identical.