**Convex Optimizations – Homework 2**

Q1.

1. Given:

The objective function is

s.t

Writing the above in Lagragian form, we get

Writing the second term in vector form, we get,

To find the dual function we need to find

Consider, the part, , this is the fenchel conjugate of Z

To find infimum w.r.t we can take derivative of and equate it to 0

From

* 1. In the current problem we don’t have any inequality constraints, therefore, we will have to satisfy less number of KKT conditions. The KKT conditions for the current problem are

are the KKT conditions for the pair of primal and dual optimal solutions for the current problem.

When the primal problem is convex, the KKT conditions are also sufficient for the points to be primal and dual optimal. To see this, note that the condition notes that are primal feasible. In the case where we also have p inequality constraints in the problem, we need to satisfy the following extra KKT conditions.

Since, is convex in , the conditions state that its gradient with respect to W and Z vanish when , so it follows that minimize

From this we can conclude that, where is the objective function.

Where in the last line we use and . This shows that and have zero duality gap, and therefore the primal and dual optimal. In summary, for any convex optimization problem with differentiable objective and constraint functions, any points that satisfy the KKT conditions are primal and dual optimal, and have zero duality gap.

1.3

1. We need to derive the Fenchel conjugate, first we consider

Using the formula for Fenchel conjugate, we get

To find the sup, we take the derivate of (7) w.r.t Z and equate it to 0.

Substitute this in the equation (7),

Hence, we found the fenchel conjugate for the function.

1. Consider the function

Using the formula for Fenchel conjugate, we get

To find the sup, we take the derivate of (8) w.r.t Z and equate it to 0.

Consider the case,

Taking derivative and equate to 0 we get,

Consider the case,

Taking derivative and equate to 0 we get,

Case:

Taking derivative and equate to 0 we get,

Therefore the Fenchel conjugate of the original function is,

Where,

1.4

1.4.1 Consider

1.4.2 Consider