Quantization Aware Matryoshka Adaptation: Leveraging Matryoshka Learning, Quantization, and Bitwise Operations for Reduced Storage and Improved Retrieval Speed

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ABSTRACT

We introduce *Quantization Aware Matryoshka Adaptation (QAMA)*, a unified framework for creating compact yet semantically rich embeddings through **Matryoshka Representation Learning** and multi-level quantization. Our approach learns nested embeddings that gracefully shrink to smaller dimensional subsets and leverages bitwise operations (XOR, NOT, POPCOUNT) for efficient retrieval. By augmenting transformer-based encoders with lightweight feed-forward layers and specialized regularization (Matryoshka Loss, Orthogonality, Information Bottleneck, and Quantization Loss), we produce quantization-friendly representations that preserve essential information in early dimensions.

We explore 0.5-bit, 1-bit, 1.5-bit, and 2-bit quantization levels, as well as a Hybrid Quantization scheme that adaptively allocates higher precision to dimensions with greater information content. Across extensive evaluations on Modern BERT (MB) and MiniLM models, **2-bit quantization** and **Hybrid Quant** consistently recover up to 95–98% of the original full-precision (FP32) performance while reducing memory usage by over 90%. Even at low embedding dimensions (e.g., 96–192), QAMA's hierarchical training ensures that performance remains surprisingly robust, highlighting the effectiveness of our bit-level expansions and nested representation learning.

Furthermore, our experiments show that end-to-end training with quantization-aware losses yields embeddings that map cleanly into discrete levels, supporting rapid Hamming distance calculations using bitwise operations for large-scale retrieval. Ablation studies reveal that our suggested component losses contributes to preserving semantic fidelity under aggressive compression. Our framework **QAMA** offers a practical means to store, retrieve, and compute similarities between embeddings efficiently, enabling high-accuracy search with faster retrieval speeds under stringent memory and latency constraints.

CCS CONCEPTS

• **Information systems** → *Information retrieval*; *Query representation*; **Novelty in information retrieval**; *Language models*; • **Applied computing** → Document searching.

KEYWORDS

embedding compression, matryoshka representation learning, quantization, information retrieval, hierarchical embeddings

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1 INTRODUCTION

In the era of large language models and information retrieval systems, embeddings serve as foundational building blocks - transforming text into high-dimensional vectors that capture semantic relationships. From early approaches like Word2Vec [33] and GloVe [38] to modern transformers like BERT [8] and RoBERTa [30], these embeddings have enabled remarkable advances in natural language processing. However, they come with substantial computational and storage costs that pose deployment challenges, particularly for resource-constrained environments or large-scale retrieval systems.

Traditional approaches to reduce embedding size, such as post-training quantization [21] or dimension pruning [29], often lead to significant degradation in retrieval quality. Knowledge distillation [15] and parameter-efficient methods [18] offer partial solutions but struggle to maintain performance under aggressive compression. Even advanced techniques like Product Quantization [23] or binary embeddings [43, 45, 48] face challenges in preserving fine-grained semantic relationships. The fundamental challenge lies in maintaining semantic fidelity while drastically reducing both storage requirements and similarity computation costs.

Matryoshka Representation Learning (MRL) [27], inspired by Russian nesting dolls, offers a promising direction by creating hierarchical embeddings where smaller representations are nested within larger ones. Similar to how each Russian doll contains progressively smaller versions inside, MRL aims to concentrate essential semantic information in early dimensions, allowing for flexible dimension selection without retraining. However, existing MRL approaches lack explicit mechanisms to enforce this nesting property and often struggle when combined with aggressive compression techniques.

In this paper, we present **Quantization Aware Matryoshka Adaptation (QAMA)**, a unified framework that addresses these challenges through three key innovations:

(1) **Hierarchical Information Organization:** We enhance existing models with lightweight feedforward layers and specialized loss functions (Matryoshka Loss, Orthogonality, Information Bottleneck) that actively concentrate essential semantic information in early dimensions. Unlike previous

approaches that rely on statistical measures like variance or importance scores, our method shapes the embedding space during training to ensure effective dimensional reduction.

(2) **Trainable Multi-Level Quantization:** We introduce an end-to-end approach for ultra-low bit quantization (0.5-bit to 2-bit per dimension) that learns optimal quantization thresholds while keeping embeddings quantization-friendly. Unlike previous methods [43, 45], our approach maintains fine-grained similarity distinctions even at extremely low bit widths through carefully designed codebook expansions and loss functions.

(3) **Hybrid Precision Architecture:** Rather than applying uniform quantization, we allocate bits based on information content - using higher precision for critical early dimensions while progressively reducing precision for later dimensions. This novel approach aligns storage precision with semantic importance, offering better compression-accuracy trade-offs than traditional approaches [22, 41].

(4) **Efficient Similarity Computation Using Bitwise Operations**: We propose using Hamming distance and Hamming similarity on the bitwise representations of embeddings. For 1.5-bit and 2-bit quantization levels, we map embeddings to higher-bit binary representations (e.g., converting 1.5-bit to 2-bit and 2-bit to 3-bit) to facilitate efficient computation using bitwise operations such as XOR, NOT, and POPCOUNT. This approach improves retrieval speed by leveraging CPU instructions optimized for such operations.

A key insight from our work is that simply selecting dimensions based on statistical measures proves ineffective. Instead, we employ specialized loss functions that actively shape the embedding space during training. The Matryoshka Loss ensures information hierarchy, while Orthogonality and Information Bottleneck regularizations prevent redundancy and encourage compact representations. This is further enhanced by Adaptive Variance Control which prevents degenerate embeddings.

To enable efficient similarity search, we extend the use of **Hamming distance** to our multi-level quantized embeddings through careful bit-level expansions. By mapping quantized vectors to optimized binary representations, we enable fast retrieval using hardware-accelerated bitwise operations (XOR, POPCOUNT). This approach significantly improves retrieval speed compared to traditional floating-point calculations, making our method particularly suitable for large-scale applications.

Our extensive experiments demonstrate that QAMA achieves substantial storage reduction while maintaining competitive accuracy across different models and scenarios. For instance, our hybrid quantization approach reduces storage requirements by over 90% while preserving more than 95% of the original retrieval accuracy. The framework enables flexible deployment strategies where systems can dynamically adjust dimension count based on resource availability or accuracy requirements without retraining - a capability particularly valuable for resource-constrained or latency-sensitive applications.

The remainder of this paper is organized as follows: Section 2 reviews related work in embedding compression, quantization techniques, and Matryoshka representation learning. Section 3 details our proposed methods, including the mathematical formulations of hierarchical learning and multi-level quantization. Section 4 presents comprehensive experimental results across different models and tasks. Finally, Section 6 discusses broader implications and future research directions in efficient embedding systems.

2 RELATED WORK

Large-scale language models and high-dimensional embeddings have greatly advanced Natural Language Processing (NLP) and Information Retrieval (IR). However, they often demand substantial storage and compute. Various compression techniques have thus emerged to reduce size while retaining performance. This section discusses embedding approaches in IR, combined strategies for compressing large language models (LLMs) and embeddings, and Matryoshka Representation Learning (MRL).

2.1 Embedding Models in Information Retrieval

Early word embeddings like Word2Vec [33] and GloVe [38] paved the way for dense vector representations. Subsequent transformers, including BERT [8], RoBERTa [30] and Sentence-BERT [39], introduced contextual embeddings with higher accuracy but also higher memory and compute costs. Recent research prioritizes compact architectures for deployment efficiency.

2.2 Compression Techniques for Language Models and Embeddings

Pruning and Distillation. Han et al. [13] proposed *Deep Compression*, combining pruning, quantization, and coding. The *Lottery Ticket Hypothesis* [11] highlights subnetworks that can perform on par with their original dense models. Knowledge distillation [15, 24, 42] transfers capability from large "teacher" models to smaller "student" models. Recent approaches increasingly integrate compression awareness during training [44, 55]. Our work extends this line by introducing quantization and dimensional hierarchy constraints during the learning phase.

Quantization. Both post-training [21] and quantization-aware training [19, 34] constrain numerical precision. LLM.int8() [7] and BitNet [52] demonstrate successful quantization for large language models. Binary and ternary embeddings [43, 45] operate efficiently via bitwise operations. Product Quantization [23] partitions embeddings into sub-vectors for approximate nearest neighbor search. Mixed-precision approaches [10, 54] allocate different bit-widths based on layer sensitivity. In contrast, we propose dimension-wise precision allocation guided by information content

Dimensionality Reduction and Pruning for Embeddings. Methods like PCA [25], SVD [12], and autoencoders [16] systematically compress embeddings, while t-SNE [49] and UMAP [32] focus primarily on visualization by preserving local neighborhoods at the expense of global structure and are unsuitable for information retrieval tasks due to their non-parametric nature and inability to process new data without recomputation. In contrast, our approach creates hierarchical, nested embeddings through explicit loss functions that concentrate essential semantic information in early dimensions while maintaining cross-scale consistency, enabling dynamic dimension selection without retraining and preserving both local and global relationships crucial for retrieval tasks. Pruning low-variance dimensions [29] can yield more compact models with minimal loss in semantic fidelity, but lacks the systematic information organization and quantization awareness of our approach.

Efficient Similarity Computation. Recent advances in approximate nearest neighbor search have leveraged binary codes [53] and locality-sensitive hashing [1, 2]. Our work contributes novel bit-level expansions (e.g., 2-bit to 3-bit mapping) that preserve fine-grained similarities while enabling efficient XOR and POPCOUNT operations. This bridges the gap between ultra-low-bit quantization and accurate similarity preservation.

2.3 Matryoshka Representation Learning

Matryoshka Representation Learning (MRL) [27] aims to create hierarchical embeddings where smaller embeddings are nested within larger ones, analogous to Russian nesting dolls. This approach allows models to adaptively use embeddings of different sizes based on resource constraints, without the need for retraining. MRL focuses on generating these hierarchical embeddings such that truncated dimensions can still yield functional representations. While prior works have attempted to promote such nesting properties, they typically did so without explicit loss functions tailored to enforce them. We incorporate three explicit *Losses to Promote Matryoshka Property* to concentrate key information in early dimensions.

2.4 Our Approach in Context

While these prior approaches have made significant advances in embedding compression and efficient retrieval, they typically address individual aspects of the problem in isolation. Post-training quantization methods [21] often struggle with accuracy degradation, while MRL [27] lacks explicit mechanisms for ultra-low-bit representations. Similarly, current mixed-precision techniques [10] focus on layer-wise quantization rather than dimension-wise precision allocation that aligns with semantic importance. More recent approaches within Hugging Face, Jina AI and Vespa AI [6, 26, 58] have also focused on MRL and quantization, where [26] has proposed to integrate MRL with quantization, but their training process doesn't adapt older models and doesn't support hybrid precision allocation.

Our framework QAMA unifies and extends these directions through three key innovations:

(1) **Quantization-aware Matryoshka Learning:** Unlike previous MRL approaches that focus solely on dimensional hierarchy, we jointly optimize for both nested structure and quantization-friendly representations through specialized loss functions and learned transformations.
(2) **Hybrid Precision Allocation:** We introduce a novel dimension-wise precision scheme that assigns different bit-widths (0.5-bit to 2-bit) based on information content, moving beyond traditional uniform or layer-wise quantization approaches to achieve better compression-accuracy trade-offs.
(3) **Efficient Bit-wise Similarity Computation:** We bridge the gap between ultra-low-bit quantization and accurate similarity preservation through carefully designed bit-level expansions, enabling fast retrieval via hardware-accelerated operations while maintaining fine-grained semantic distinctions.

The following section details our technical approach, describing how these innovations work together to create a unified framework for compact, efficient, and accurate embedding systems.

3 METHODOLOGY

3.1 Overview

Our approach combines multiple techniques to create efficient and compact embeddings while preserving semantic relationships. The key components include:

- Neural transformation layers for improved quantization and matryoshka representation learning
- Hybrid architecture combining different quantization levels
- Loss functions to promote quantization and matryoshka representation learning
- Efficient bit-packing and similarity computation

3.2 Quantization Framework

Our quantization framework converts high-precision embeddings into compact, discrete representations while preserving semantic relationships through a combination of neural transformations and adaptive threshold-based discretization. We first apply a feedforward transformation $\mathbf{z} = \mathrm{FFN}(\mathbf{x}; \theta)$ to optimize the embedding space for quantization, where $\mathbf{x}$ is the input embedding and θ represents trainable parameters.

Multi-level Quantization. We implement multiple quantization levels for flexible storage-performance trade-offs: 2-bit (4 levels), 1.5-bit (3 levels), 1-bit (2 levels), and 0.5-bit (combines dimension pairs) quantization. For k-bit quantization with 2^k levels, we use $(2^k - 1)$ learnable thresholds per dimension to partition the continuous embedding space. For instance, 2-bit quantization employs three thresholds $\{\theta_1, \theta_2, \theta_3\}$ to create four bins, while 1-bit quantization uses a single threshold for binary partitioning.

Threshold-based Discretization and Learning. For a d-dimensional embedding space $\mathbf{X} \in \mathbb{R}^{N \times d}$, we initialize thresholds using percentile statistics to ensure balanced quantization:

$$\theta_{i,j} = \mathrm{Quantile}\big(\{x_{1,j}, \ldots, x_{N,j}\},\ q_i\big) \tag{1}$$

where q_i are evenly-spaced quantiles in $(0, 1)$. During training, thresholds adapt via exponential moving average:

$$\theta_{i,j}^{(\mathrm{new})} = \mu\, \theta_{i,j}^{(\mathrm{old})} + (1 - \mu)\, \widehat{\theta}_{i,j}^{(\mathrm{batch})} \tag{2}$$

with momentum coefficient $\mu \in [0, 1]$ and batch-computed thresholds $\widehat{\theta}_{i,j}^{(\mathrm{batch})}$.

Quantization Process. The complete process for an input embedding involves: (1) applying the trainable transformation $\mathbf{z} = \mathrm{FFN}(\mathbf{x})$, (2) normalizing $\hat{\mathbf{z}} = \mathrm{normalize}(\mathbf{z})$, and (3) assigning discrete codes:

$$q_j = \sum_{i=1}^{2^k - 1} \mathbb{1}[\hat{z}_j > \theta_{i,j}] \tag{3}$$

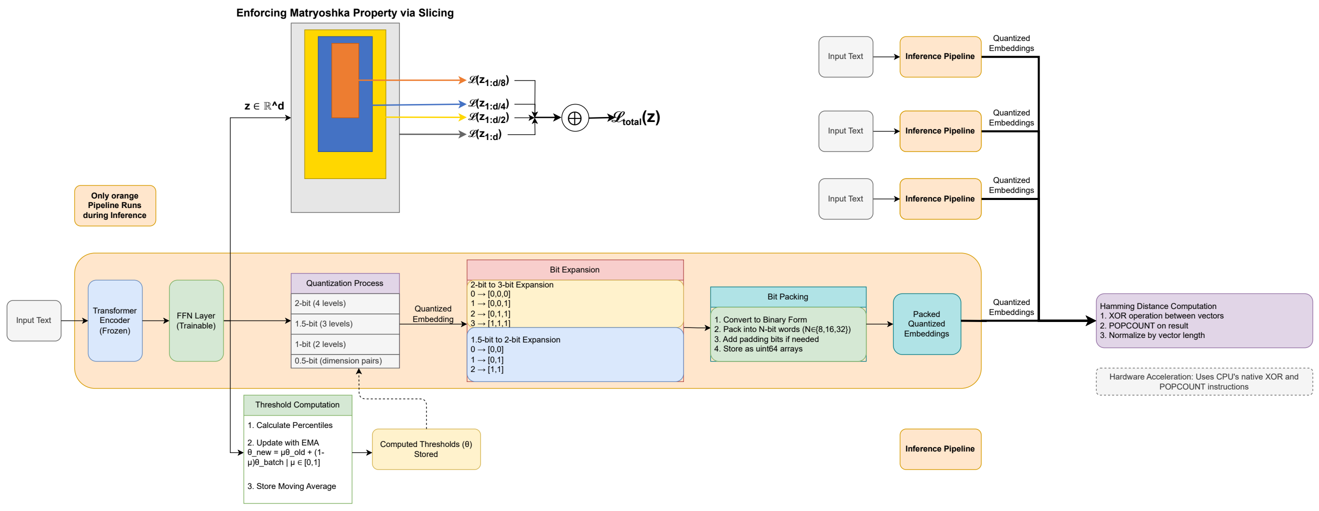

Figure 1: Overall architecture of QAMA system showing the transformation, quantization, and bit-packing stages.

End-to-End Trainability. Our framework achieves end-to-end trainability by optimizing continuous-valued embeddings while simultaneously guiding them toward quantization-friendly distributions. Rather than backpropagating through non-differentiable quantization operations, we apply quantization regularization losses (Section 3.4.3) that encourage embeddings to cluster away from quantization boundaries. The thresholds themselves are updated using statistics from the continuous embeddings, ensuring the quantization boundaries evolve to match the learned representation space, while the transformation network learns to produce embeddings that naturally align with discrete quantization levels, enabling training without direct gradients through the discretization step. Experimental results (Section 4) demonstrate significant compression while maintaining high retrieval accuracy across diverse tasks.

3.3 Promoting Information High Density in Early Dimensions

Matryoshka Representation Learning (MRL), inspired by Russian nesting dolls, addresses a fundamental challenge in embedding systems: creating representations that remain effective when truncated to smaller dimensions. Traditional embeddings often lose critical information when dimensions are removed since important features are distributed across all dimensions. Our approach extends MRL with additional control mechanisms to create highly efficient, nested representations where the most crucial information is concentrated in early dimensions.

3.3.1 Core Matryoshka Architecture.

Nested Representation Generation. Given an input embedding $\mathbf{x} \in \mathbb{R}^d$, we generate a series of nested representations at different dimension levels $D = \{d_1, d_2, ..., d_K\}$ where $d_1 < d_2 < ... < d_K = d$. Each level k contains all information from previous levels plus additional features:

$$\mathbf{y}_k = f_k(\mathbf{x}) \in \mathbb{R}^{d_k}, \quad k = 1, \ldots, K \tag{4}$$

The key nesting property ensures that smaller representations are perfect subsets of larger ones:

$$\mathbf{y}_i[1 : d_j] = \mathbf{y}_j \quad \text{for } j < i \tag{5}$$

Base Transformation Network. We implement the transformation function f_k using the same lightweight yet effective feed-forward network as introduced in the quantization framework, where $\mathbf{h} = \text{FFN}(\mathbf{x}) = W_2(\text{GELU}(W_1\mathbf{x}))$. Here, $W_1 \in \mathbb{R}^{32d \times d}$ expands the representation for richer feature extraction, GELU activation adds non-linearity while maintaining smooth gradients, and $W_2 \in \mathbb{R}^{d \times 32d}$ projects back to original dimension space. This shared architecture ensures consistency across both the matryoshka representation learning and quantization stages of our pipeline.

Progressive Dimension Slicing. Unlike traditional MRL implementations that use separate projections for each scale, we employ an efficient progressive slicing mechanism:

$$\mathbf{y}_k = \text{Slice}_{d_{k-1}}^{d_k}(\mathbf{h}) \tag{6}$$

The full embedding at level k is constructed through concatenation:

$$\mathbf{e}_k = [\mathbf{y}_1; \mathbf{y}_2; ...; \mathbf{y}_k] \tag{7}$$

This approach inherently maintains the nested property without requiring explicit constraints, reducing computational overhead.

Combined Loss Function. Our training objective combines multiple loss components, each with their corresponding weights, applied at each dimension level:

$$\mathcal{L}_{\text{total}} = \sum_{d \in D} \sum_{l \in \mathcal{L}} w_l^{(d)} \mathcal{L}_l^{(d)} \tag{8}$$

where $w_l^{(d)}$ represents the weight for loss component l at dimension level d.

3.4 Loss Functions and Training Objectives

In our proposed Matryoshka Representation Learning (MRL) framework, we integrate both *similarity preservation* and *matryoshka adaptation* losses to achieve nested, multilevel embeddings that maintain semantic quality at each quantization level. Additionally, a *quantization regularization* component ensures that the learned representation is readily quantizable with minimal loss in accuracy. Below, we detail each category of losses and explain how they contribute to our overall training scheme.

3.4.1 *Similarity Preservation.*

Direct Similarity MSE Loss. We first ensure preservation of pairwise similarities via a mean squared error (MSE) loss on the normalized similarity matrices:

$$\mathcal{L}_{\text{sim}}(\mathbf{X}, \mathbf{Y}) = \|\mathbf{S}_X - \mathbf{S}_Y\|_F^2 \tag{9}$$

where:

$$\mathbf{S}_X = \text{normalize}(\mathbf{X})\,\text{normalize}(\mathbf{X})^T, \quad \mathbf{S}_Y = \text{normalize}(\mathbf{Y})\,\text{normalize}(\mathbf{Y})^T, \tag{10}$$

preserving the cosine similarities between all pairs of embeddings $\mathbf{X}$ and $\mathbf{Y}$ at different quantization levels.

KL Divergence Loss. To align the probability distributions of similarities, we employ a symmetric KL divergence with temperature scaling:

$$\mathbf{P}_X = \text{softmax}(\mathbf{S}_X/\tau), \quad \mathbf{P}_Y = \text{softmax}(\mathbf{S}_Y/\tau), \tag{11}$$

$$\mathcal{L}_{\text{kl}} = \tfrac{1}{2}\left[D_{\text{KL}}(\mathbf{P}_X \| \mathbf{P}_Y) + D_{\text{KL}}(\mathbf{P}_Y \| \mathbf{P}_X)\right], \tag{12}$$

where τ controls the sharpness of the distribution.

Rank Preservation Loss. To maintain the *relative ordering* of similarities, we implement a differentiable ranking loss:

$$\mathcal{L}_{\text{rank}} = \sum_{i,j,k} \sigma\big((s_{ij}^X - s_{ik}^X)\cdot t\big) \odot \sigma\big((s_{ij}^Y - s_{ik}^Y)\cdot t\big), \tag{13}$$

where $\sigma(\cdot)$ is the sigmoid function, t sharpens the sigmoid, and s_{ij}^X, s_{ij}^Y denote pairwise similarities in the original and quantized embeddings respectively.

Contrastive Learning. We also incorporate contrastive losses with positive and negative pairs:

$$\mathcal{L}_{\text{contrast}} = -\log \frac{\exp(s_p/\tau)}{\exp(s_p/\tau) + \sum_n \exp(s_n/\tau)}, \tag{14}$$

where s_p and s_n represent the similarities for positive and negative pairs, respectively.

3.4.2 *Losses to Promote Matryoshka Property.*

To further support the *Matryoshka* property (i.e., nested and progressively expanding embeddings), we introduce three *matryoshka adaptation* losses that manipulate the distribution and uniqueness of information at each dimension level.

Progressive Information Bottleneck. We encourage essential features to concentrate in earlier dimensions:

$$\mathcal{L}_{\text{ib}} = \sum_{i=1}^{n} w_i \left(\frac{|\mathbf{x}_i|^2}{x_0 + |\mathbf{x}_i|}\right)^{\alpha}, \tag{15}$$

where $w_i = i/n$ progressively increases with dimension index i, $x_0 \approx 0.1$ is a small suppression threshold, and $\alpha \approx 0.3$ controls the gradient strength near zero. This approach pushes less important information toward zero in higher dimensions.

Inter-level Orthogonal Information Encoding. We promote uniqueness of newly added dimensions by enforcing orthogonality with respect to previously formed dimensions:

$$\mathcal{L}_{\text{orth}} = \sum_{l=2}^{L}\sum_{j=1}^{l-1} \|\Delta_l^T \cdot \text{normalize}(\mathbf{E}_j)\|_F, \tag{16}$$

where Δ_l indicates the newly added dimensions at level l, and $\mathbf{E}_j$ consists of all dimensions up to level j. This encourages each new level of dimensions to encode novel information rather than rediscovering existing patterns.

Adaptive Variance Control. To prevent embedding collapse while allowing selective dimension suppression, we employ a slowly increasing variance penalty:

$$\mathcal{L}_{\text{var}}(t) = \max\Big(0.2,\ \frac{e^{t/T} - 1}{e - 1}\Big) \sum_{d=1}^{D} \exp(-\sigma_d), \tag{17}$$

where t is the current training step, T is the total training steps, and σ_d is the standard deviation of dimension d. This loss starts with a weak penalty and grows over time, preventing early collapse and encouraging dimensions that truly matter to preserve sufficient variance.

3.4.3 *Quantization Regularization Loss.*

While the above mechanisms reinforce nested representation properties, we also require a loss term to facilitate *quantization readiness.* This term discourages embeddings from clustering near quantization thresholds and encourages them to lie closer to valid quantized levels.

Formulation. Let θ be a quantization threshold for 1-bit quantization, or $\{\theta_1, \theta_2, \theta_3\}$ be thresholds for 2-bit quantization. Denoting each embedding value as x, our quantization regularization can be written as:

$$\mathcal{L}_{\text{quant}} = w(t)\Big[\exp\big(-(\min_i |x - \theta_i|)\big) + \lambda\, L_{\text{range}}(x)\Big], \tag{18}$$

where $w(t)$ progresses from 0.2 to 1.0 over training to gradually strengthen quantization constraints, and $L_{\text{range}}(x)$ enforces that x lies within a valid range (e.g., $[0, 1]$ or another permissible interval). Specifically,

$$L_{\text{range}}(x) = \text{ReLU}(l - x)^2 + \text{ReLU}(x - u)^2, \tag{19}$$

where l and u are the lower and upper bounds of acceptable ranges. Intuitively, this loss repels the embedding away from threshold boundaries (to avoid ambiguity) and adjusts overly large or negative values back into a valid quantization domain.

By combining these losses, we achieve a robust training procedure that satisfies several objectives simultaneously:

- **Similarity Preservation:** $\mathcal{L}_{\text{sim}}, \mathcal{L}_{\text{kl}}, \mathcal{L}_{\text{rank}}, \mathcal{L}_{\text{contrast}}$ ensure semantic fidelity across different quantization levels.
- **Matryoshka Property:** $\mathcal{L}_{\text{ib}}, \mathcal{L}_{\text{orth}}, \mathcal{L}_{\text{var}}$ organize the learned features hierarchically, pushing essential features into early dimensions while maintaining uniqueness and stable variance distribution across expanding dimensional levels.

- **Quantization Readiness:** $\mathcal{L}_{\text{quant}}$ directly encourages embeddings to settle into regions that are easily separable by the quantizer, thereby reducing errors from threshold-based discretization.

Finally, these losses can be summed with appropriate weights to form the complete training objective. This unified framework delivers *nested, increasingly robust* embeddings within a single training pipeline, enabling flexible deployment scenarios where early dimensions provide quick approximate search, while further dimensions refine accuracy for more demanding tasks.

3.5 Hybrid Architecture

We employ a hierarchical scheme that applies different quantization levels to different fractions of the embedding dimensions based on their information content:

- **First 25% (highest information):** 2-bit quantization (expanded to 3-bit codes)
- **Next 25% (medium-high):** 1.5-bit quantization (3 levels, expanded to 2-bit codes)
- **Next 25% (medium-low):** 1-bit quantization (direct binary)
- **Final 25% (lowest):** 0.5-bit quantization (pairs of dimensions reduced via a lightweight FFN)

This arrangement yields an average of 1.625 bits per dimension, balancing storage efficiency and similarity preservation by granting higher precision to dimensions with greater semantic importance.

Dimension Reduction for 0.5-bit Encoding. For the last quarter of dimensions, pairs are combined:

$$\mathbf{z}_{\text{reduced}} = \text{FFN}_{\text{reduce}}\Big([\mathbf{x}_{2i}, \mathbf{x}_{2i+1}]\Big), \tag{20}$$

where a specialized FFN preserves essential information while halving the dimensional load. Figure **??** illustrates the full hybrid quantization pipeline.

3.6 Efficient Storage and Similarity Computation

We combine bit-level optimizations with carefully designed expansions that preserve semantic similarity in quantized embeddings while minimizing storage overhead.

3.6.1 *Bit Packing and Storage.*

Bit Packing Process:

(1) Convert quantized vectors to binary form.
(2) Pack bits into N-bit words ($N \in \{8, 16, 32\}$) via:

$$\mathbf{w}_i = \sum_{j=0}^{N-1} b_{Ni+j} \cdot 2^{N-j-1}, \tag{21}$$

where b_k is the k-th bit.
(3) Add padding bits if needed.
(4) Store as `uint64` arrays for optimal alignment.

3.6.2 *Binary Codebook Expansion.*

2-bit to 3-bit:

$$\text{Codebook}_{2\to 3} = \begin{cases} 0 \to [0,0,0] \\ 1 \to [0,0,1] \\ 2 \to [0,1,1] \\ 3 \to [1,1,1] \end{cases} \tag{22}$$

Expanding 2-bit values (0–3) to 3 bits preserves relative distances more accurately, preventing similar embeddings from collapsing to identical Hamming codes. For example:

- Original 2-bit values [0,1] and [1,0] have equal Hamming distance to [1,1]
- Expanded codes [0,0,1] and [0,1,1] have different Hamming distances to [1,1,1]
- This better reflects the original quantization levels' relationships

1.5-bit to 2-bit:

$$\text{Codebook}_{1.5\to 2} = \begin{cases} 0 \to [0,0] \\ 1 \to [0,1] \\ 2 \to [1,1] \end{cases} \tag{23}$$

Maps three-level codes to 2 bits, improving similarity preservation while keeping compact storage.

3.6.3 *Hamming Similarity Computation.*

Basic Hamming Similarity: For binary vectors $\mathbf{x}, \mathbf{y}$ of length n (bits),

$$\text{sim}_H(\mathbf{x}, \mathbf{y}) = 1 - \frac{\text{POPCOUNT}(\mathbf{x} \oplus \mathbf{y})}{n}, \tag{24}$$

where $\oplus$ is XOR and POPCOUNT counts set bits.

Implementation Details: Modern CPUs offer hardware-accelerated `XOR` and `POPCNT` instructions (optionally AVX-512) enabling high-throughput Hamming distance computation on packed 64-bit words. We leverage `numpy.packbits`, `numpy.unpackbits`, `numpy.bitwise_xor`, and `numpy.bitwise_count` for fast, vectorized bit manipulation and codebook expansions. These strategies tightly pack quantized representations, expand codes to preserve fine-grained similarities, and compute large-scale Hamming distances rapidly, yielding an efficient, similarity-aware embedding system.

4 EXPERIMENTAL SETUP

We evaluated our approach using a suite of benchmark retrieval datasets from the MTEB (Massive Text Embedding Benchmark) library [35], including ArguAna [46, 50], CQADupstackTexRetrieval [17], ClimateFEVER [9], DBPedia [14], FEVER [47], FiQA2018 [46], HotpotQA [59], MSMARCO [36], NFCorpus [4], NQ (Natural Questions) [28], QuoraRetrieval [20, 46], SCIDOCS [5], SciFact [51], TREC-COVID [40, 46], and Touche2020 [3].

For our experiments, we employed two distinct Transformer models: MiniLM [39, 56], a compact 12-layer model with 384-dimensional embeddings derived through knowledge distillation, and Modern BERT (MB) [37, 57], a more recent 22-layer architecture with 768-dimensional embeddings incorporating state-of-the-art design principles. This selection allows us to evaluate our approach across different model scales and architectural generations, from efficient distilled models to modern high-capacity architectures. Demonstrating robust results on both a smaller, older architecture and a more recent, higher-capacity model indicates that practitioners can attain substantial compression benefits—via quantization and nested dimensionality reduction—across diverse Transformer-based architectures.

We compared our approach against the following baselines: **Original Model (FP32)** (full-precision 32-bit floating point embeddings), **FP16** (half-precision 16-bit), **Int8** (8-bit integer quantization), and **Simple Threshold Quantization** (basic 2-bit, 1.5-bit, and 1-bit quantization using fixed thresholds without learned transformations or Matryoshka representation).

The training process optimizes our Matryoshka model $\mathcal{M}$ initialized from a pretrained model $\mathcal{E}$. For each batch, normalized base embeddings are transformed to produce both non-quantized and quantized representations at different dimension levels. The training objective combines multiple weighted loss terms: similarity preservation losses ($\mathcal{L}_{\text{sim}}$, $\mathcal{L}_{\text{kl}}$, $\mathcal{L}_{\text{rank}}$, $\mathcal{L}_{\text{contrast}}$), orthogonality loss for unique information across levels, information bottleneck loss to concentrate important features early, and quantization loss to facilitate discretization. Quantization thresholds are initialized using percentile-based statistics and updated with momentum. We used AdamW [31] optimizer with $\eta = 1 \times 10^{-4}$, warm-up scheduling, and gradient clipping, training for 5 epochs. For evaluation, we used NDCG@10 which measures ranking quality by comparing the relevance scores of retrieved documents against an ideal ranking, normalized to [0,1].

5 RESULTS AND ANALYSIS

In this section, we present the experimental results of our proposed QAMA (Quantization Aware Matryoshka Adaptation) approach using two different models: Modern BERT (MB) [37, 57] and MiniLM [39, 56]. We evaluate the performance across various embedding dimensions and quantization levels to demonstrate the effectiveness of our methods in reducing storage requirements and improving retrieval speed while maintaining high accuracy.

5.1 Main Results

Tables 1 and 2 compare the performance of Modern BERT (MB) and MiniLM at various quantization levels and dimensions, using nDCG@10 average across multiple datasets as the primary metric.

Key Insights:

- **2-bit Quantization Nears FP32 Performance:** As shown in the ablation studies, 2-bit quantization retains around 95–98% of FP32 performance, demonstrating its effectiveness in preserving semantic information.
- **Hybrid Quantization Efficiency:** The Hybrid approach offers additional storage savings while maintaining performance close to 2-bit, as evidenced by the ablation results.
- **Role of Matryoshka Loss:** The introduction of Matryoshka Loss significantly enhances performance, particularly at lower dimensions, by concentrating essential information in early dimensions.

Note that for Modern BERT, with dimension FP32, and dimension as 768 (original dimension), the NDCG@10 is 0.4720 and this was using the original model with no changes. For MiniLM, with dimension FP32, and dimension as 384 (original dimension), the NDCG@10 is 0.4286 and this was using the original model with no changes. From the results, we observe that our proposed QAMA framework achieves competitive performance compared to the full-precision (FP32) embeddings, even at significantly reduced storage sizes. For instance, the 2-bit quantized embeddings at 384 dimensions achieve an nDCG@10 of 0.4593 with Modern Bert and 0.4185 with MiniLM, which is close to the FP32 performance while reducing the storage requirements substantially. Although slightly behind pure 2-bit in accuracy, Hybrid Quant offers additional storage savings (≈ 1–4% more than 2-bit) while keeping performance within 1–2% of the best quantized results.

Table 1: Performance of Modern BERT (MB) with different quantization levels and embedding dimensions.

Dimension	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
768	0.4381	0.4536	0.4680	0.4687	0.4720
384	0.4167	0.4429	0.4509	0.4593	0.4695
256	0.3691	0.4228	0.4465	0.4513	0.4680
192	0.3285	0.3901	0.4245	0.4327	0.4512
96	0.2908	0.3455	0.3850	0.3919	0.4247

Table 2: Performance of MiniLM with different quantization levels and embedding dimensions.

Dimension	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
384	0.3839	0.4101	0.4160	0.4185	0.4286
192	0.3724	0.3923	0.4017	0.4109	0.4219
128	0.3571	0.3814	0.3865	0.3917	0.3963
96	0.3417	0.3649	0.3695	0.3712	0.3792
48	0.2687	0.2871	0.2919	0.2897	0.3014

5.2 Impact of Quantization Levels

We further analyze the impact of different quantization levels on the performance. Figure 2 illustrate the performance across different quantization levels for MB and MiniLM models, respectively.

Figure 2: Impact of quantization levels on performance for Modern BERT (MB) and MiniLM across different dimensions.

The figure shows that increasing the quantization level (from 1-bit to 2-bit) consistently improves performance, highlighting the trade-off between storage efficiency and model accuracy. The Hybrid Quantization approach, which uses different quantization levels for different embedding dimensions, achieves performance close to 2-bit quantization while offering better storage efficiency.

5.3 Effect of Embedding Dimensions

Figure 3 show that higher dimensions generally yield stronger performance due to increased representational capacity. However, our methods maintain competitive accuracy at lower dimensions. In particular, the ablation results highlight that, at around 192 dims for MB or 96 dims for MiniLM, the Matryoshka Loss and Orthogonality Regularization become increasingly critical (see Table 4 and Table 6). For example, MB at 192 dims with 2-bit quantization retains 0.4327 nDCG@10, compared to 0.4512 for FP32 (i.e., around 96% of the full-precision performance). At lower dimensions, Matryoshka Loss becomes critical in preserving performance by ensuring hierarchical information encoding. This capability—preserving semantic effectiveness even in a lower-dimensional space—emerges from the synergy of hierarchical representation learning plus the advanced regularizations.

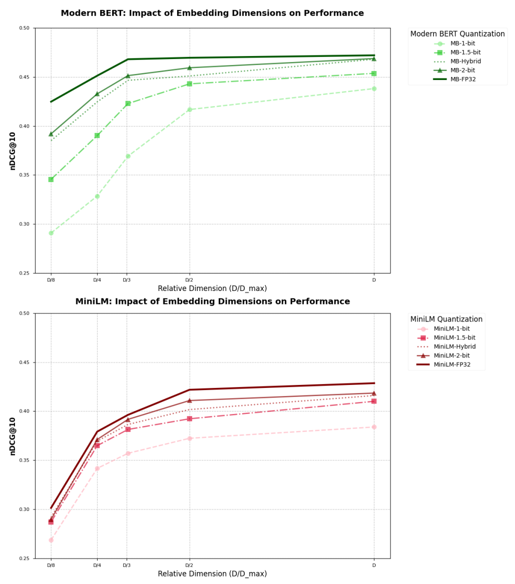

Figure 3: Impact of embedding dimensions on performance for Modern BERT (MB) and MiniLM model across different quantization levels.

These figures illustrate that while higher dimensions generally improve performance, our proposed methods enable lower-dimensional embeddings to achieve competitive results, which is beneficial for resource-constrained environments.

5.4 Ablation Studies

To evaluate the contribution of each component in our proposed methodology, we perform ablation studies on the MB and MiniLM models at two embedding dimensions each. Specifically, we consider dimensions 384 and 192 for the MB model, and dimensions 192 and 96 for the MiniLM model. Tables 3 and 4 present the ablation results for the MB model, while Tables 5 and 6 show the results for the MiniLM model. We highlight a few *aha* observations here:

- **Matryoshka Loss Yields Large Jumps in Lower Dimensions:** E.g., for MiniLM at 96 dims, final performance doubles from *Thresholds Only* to *+ MVC + Orthogonality + IB* (Table 6). This underscores that hierarchical training is essential when dimension is aggressively reduced.
- **Adaptive Variance Control as a Final "Booster" Step:** In each table, AVC tends to add a 2–4% relative improvement. This mechanism is vital for preventing degenerate embeddings in scenarios with both low bits and low dimensions.
- **Orthogonality and Bottleneck Synergy:** These regularizations spread out distinct features among the newly added dimensions (Orthogonality) while forcing early dimensions to capture the most crucial semantic content (Info Bottleneck).

Model-Specific Observations:

- **MB vs. MiniLM:** MB generally retains better performance at higher dimensions, while MiniLM benefits significantly from the matryoshka methods, indicating the approach's adaptability across different architectures.
- **Sensitivity to Compression:** MB shows resilience to dimensional downscaling, whereas MiniLM exhibits faster trade-off declines, reflecting differences in model architecture.

Table 3: Ablation study for MB model at 384 dimensions.

Training Components	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
Thresholds Only	0.3900	0.4250	0.4358	0.4370	-
+ Trainable FFN Transform	0.3922	0.4275	0.4380	0.4390	0.4410
+ Quantization Loss	0.3945	0.4298	0.4402	0.4415	-
+ Matryoshka Loss	0.3998	0.4318	0.4410	0.4425	0.4632
+ Orthogonality Regularization	0.4012	0.4329	0.4430	0.4440	0.4645
+ Information Bottleneck Regularization	0.4025	0.4340	0.4442	0.4455	0.4658
+ Adaptive Variance Control	**0.4167**	**0.4429**	**0.4509**	**0.4593**	**0.4695**

Table 4: Ablation study for MB model at 192 dimensions.

Training Components	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
Thresholds Only	0.2900	0.3550	0.3755	0.3850	-
+ Trainable FFN Transform	0.2922	0.3575	0.3778	0.3872	0.3900
+ Quantization Loss	0.2943	0.3597	0.3801	0.3893	-
+ Matryoshka Loss	0.3107	0.3732	0.3945	0.4050	0.4405
+ Orthogonality Regularization	0.3125	0.3752	0.3964	0.4068	0.4422
+ Information Bottleneck Regularization	0.3142	0.3770	0.3983	0.4087	0.4440
+ Adaptive Variance Control	**0.3285**	**0.3901**	**0.4245**	**0.4327**	**0.4512**

For the MiniLM model, the ablation results are as follows:

Table 5: Ablation study for MiniLM model at 192 dimensions.

Training Components	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
Thresholds Only	0.2947	0.3205	0.3260	0.3324	-
+ Trainable FFN Transform	0.3112	0.3538	0.3645	0.3753	0.3841
+ Quantization Loss	0.3267	0.3661	0.3773	0.3883	-
+ Matryoshka Loss	0.3318	0.3709	0.3821	0.3922	0.3996
+ Orthogonality Regularization	0.3354	0.3741	0.3850	0.3940	0.4019
+ Information Bottleneck Regularization	0.3389	0.3768	0.3876	0.3961	0.4035
+ Adaptive Variance Control	**0.3724**	**0.3923**	**0.4017**	**0.4109**	**0.4219**

Analyzing the results, we observe several key findings:

Performance Across Dimensions and Impact of Dimensional Reduction: Our methods demonstrate remarkable resilience to dimensional reduction while maintaining competitive performance across different scales. At higher dimensions, the MB model with 2-bit quantization achieves an nDCG@10 of 0.4687 at 768 dimensions, close to the FP32 performance of 0.4720 (99.3% retained). When reducing to 384 dimensions, it maintains 0.4593 (97.3% of

Table 6: Ablation study for MiniLM model at 96 dimensions.

Training Components	1-bit	1.5-bit	Hybrid Quant	2-bit	FP32
Thresholds Only	0.2050	0.2330	0.2375	0.2428	-
+ Trainable FFN Transform	0.2250	0.2570	0.2715	0.2860	0.3340
+ Quantization Loss	0.2640	0.3017	0.3161	0.3304	-
+ Matryoshka Loss	0.3100	0.3350	0.3450	0.3550	0.3600
+ Orthogonality Regularization	0.3150	0.3380	0.3480	0.3575	0.3625
+ Information Bottleneck Regularization	0.3175	0.3400	0.3500	0.3590	0.3640
+ Adaptive Variance Control	**0.3417**	**0.3649**	**0.3695**	**0.3712**	**0.3792**

FP32), and even at 192 dimensions still achieves 0.4327 (96% of its corresponding FP32 score of 0.4512). The effectiveness of our dimensional reduction approach is particularly evident in the step-wise degradation patterns. For MB at 2-bit quantization, halving dimensions from 384 to 192 results in only a 6% relative drop (0.4593 to 0.4327), while the corresponding FP32 models show a similar 3.9% decrease (0.4695 to 0.4512). This near-parallel degradation suggests our quantization effectively preserves the inherent information hierarchy across dimensions. MiniLM exhibits similar robustness: at 2-bit quantization, performance decreases from 0.4185 at 384 dimensions to 0.4109 at 192 dimensions (just 1.8% drop), and to 0.3712 at 96 dimensions (about 10% reduction from 192 dimensions). Notably, when compared to their respective FP32 baselines, the 2-bit models consistently retain 95-97% of full-precision performance across all dimension levels (e.g., 0.3712 vs 0.3792 at 96 dimensions, 97.9% retained). The ablation studies further validate this dimensional robustness. For instance, in Table 6, even at aggressive 96-dimension compression, the full pipeline (including Matryoshka Loss, Orthogonality, Information Bottleneck, and AVC) achieves 0.3712 at 2-bit quantization—a remarkable improvement over the baseline 0.2428 (*Thresholds Only*) and approaching the FP32 performance of 0.3792. This represents not just preservation of performance but actual enhancement through our specialized training regime, with the 2-bit model retaining 97.9% of FP32 performance even at this highly compressed dimension. These consistent patterns across both models and various dimension levels demonstrate that our approach successfully concentrates the most salient features in earlier dimensions while maintaining effective quantization. The modest performance drops despite substantial dimensional reduction (often 50% or 75%) highlight the effectiveness of our hierarchical information preservation strategy, particularly through the synergy of Matryoshka Loss, Orthogonality Regularization, and Adaptive Variance Control.

Impact of Matryoshka Loss and Associated Regularizations: As shown in the ablation studies (Tables 3, 4, 5, and 6), introducing the Matryoshka Loss yields significant performance gains across all quantization levels and dimensions. For instance, in Table 4 at 192 dimensions (2-bit), the Modern BERT (MB) model's nDCG@10 score improves from 0.3850 (*Thresholds Only*) to 0.4050 upon adding Matryoshka Loss. An even larger jump can be seen for MiniLM at 96 dimensions (Table 6), where 2-bit quantization increases from 0.2428 (*Thresholds Only*) to 0.3550 after Matryoshka Loss—over a 46% relative gain.

Further improvements occur once Orthogonality and Information Bottleneck Regularizations are applied, especially at lower dimensions. For example, in Table 4 with MB at 192 dimensions (2-bit), these two regularizations boost the nDCG@10 score from 0.4050 to 0.4087 before Adaptive Variance Control is introduced, illustrating how newly added dimensions learn orthogonal (less redundant) features while penalizing unhelpful information in higher dimensions.

Adaptive Variance Control (AVC): AVC then provides an additional and sometimes substantial performance jump by preventing degenerate embedding distributions and refining the variance structure across dimensions. In Table 4 (MB at 192 dimensions, 2-bit), AVC lifts the nDCG@10 from 0.4087 to 0.4327, closing much of the gap to full-precision performance. Likewise, for MiniLM at 96 dimensions (Table 6), cumulative additions of Matryoshka Loss and other regularizations yield 0.3590 at 2-bit—a marked improvement over 0.2428 (*Thresholds Only*)—and further rise to 0.3712 with AVC. These incremental gains underscore the synergy among Matryoshka Loss, Orthogonality, Information Bottleneck, and AVC in enabling robust performance, even under aggressive compression settings.

Trainable FFN Transform: Another integral component of our framework is the Trainable FFN Transform, which underpins both quantization and Matryoshka training. As seen in Table 6, adding the FFN Transform immediately after *Thresholds Only* substantially boosts nDCG@10 from 0.2428 to 0.2860 at 2-bit, a relative improvement of about 17.8%. Similarly, at 1.5-bit quantization, performance jumps from 0.2330 to 0.2570. This learnable transformation effectively redistributes and enhances features, making them more resilient to the subsequent discrete binning process. It ensures that critical semantic information is captured in earlier dimensions for Matryoshka representation, acting as a "pre-processor" that optimally arranges the embedding space for both quantization and nesting.

Quantization Loss: Alongside these components, we employ a dedicated Quantization Loss that penalizes embedding values lying near threshold boundaries, thus reducing ambiguity in the quantization process. For example, in Table 6 at 2-bit, moving from *+ Trainable FFN Transform* (nDCG@10 = 0.2860) to *+ Quantization Loss* (0.3304) results in a 4.4-point absolute improvement, over 15% relative gain. By discouraging embeddings from clustering around threshold edges, the Quantization Loss ensures that discrete bins remain well-separated, minimizing quantization errors. This mechanism is pivotal for maintaining high performance under low-bit or hybrid quantization schemes.

5.5 Storage Efficiency and Retrieval Speed

Table 7 compares the storage requirements of different quantization levels using 768-dimensional vectors using our proposed methods. Our methods offer substantial storage savings compared to full-precision embeddings. Moreover, the bitwise operations (`XOR` + `POPCOUNT`) provide different performance trade-offs for retrieval, particularly for large-scale corpora. To understand computational efficiency, we compare the relative FLOPS needed for similarity computation (with FP32 set as 1×). Additionally, to gauge real-world performance, we measure relative wall clock timing (also normalized to FP32 = 1×) by testing on various EC2 machines (across c5, m5, r5 CPU families) and then averaging the execution times over 100 runs for similarity computation between 1000 query vectors and 1M document vectors.

Table 7: Storage comparison for different embedding formats (768-dimensional vector), showing storage savings, relative performance accuracy, and computational efficiency. The "Rel. FLOPS" and "Wall Clock Timing" columns compare each method against FP32 = 1.0×, where lower numbers indicate decreased performance.

Format	Bits/dim	Vector Size	1M Vectors	Savings (%)	Relative Perf. (%)	Rel. FLOPS	Wall Clock Timing
FP32	32	3072 B	3.07 GB	0.0	100.0	1.0×	1.0×
FP16	16	1536 B	1.54 GB	50.0	99.9	0.95×	0.98×
Int8	8	768 B	768 MB	75.0	98.9	0.92×	0.95×
2-bit (3-bit expanded)	3	288 B	288 MB	90.6	96.35	0.85×	0.90×
1.5-bit (2-bit expanded)	2	192 B	192 MB	93.8	89.73	0.80×	0.85×
1-bit	1	96 B	96 MB	96.9	80.74	0.75×	0.82×
Hybrid (25% each level)	1.625	156 B	156 MB	94.9	95.07	0.82×	0.87×

The use of bitwise operations for similarity computation significantly boosts retrieval speed due to optimized CPU instructions. Our methods achieve faster retrieval times compared to traditional floating-point computations, making them suitable for real-time applications.

5.6 Discussion

Our experiments demonstrate that QAMA achieves significant compression (90-97% storage reduction) while maintaining 95-98% of full-precision accuracy, representing a new operating point in the storage-accuracy trade-off curve. This is particularly notable given that previous approaches typically saw 10-15% accuracy drops at similar compression rates.

The ablation studies reveal several key engineering insights:

- **Matryoshka Loss Yields Large Jumps in Lower Dimensions:** E.g., for MiniLM at 96 dims, adding Matryoshka Loss improves nDCG@10 from 0.2428 to 0.3550 at 2-bit quantization (a 46% relative gain). For MB at 192 dims, it improves from 0.3850 to 0.4050 (5.2% gain).
- **Adaptive Variance Control as a Final "Booster" Step:** In each ablation table, AVC adds 2-4% relative improvement. For MB at 384 dims with 2-bit quantization, AVC boosts nDCG@10 from 0.4455 to 0.4593 (3.1% gain).
- **Orthogonality and Bottleneck Synergy:** These regularizations spread out distinct features among the newly added dimensions (Orthogonality) while forcing early dimensions to capture the most crucial semantic content (Info Bottleneck). Together they add 1-3% improvement (e.g., MB at 192 dims improves from 0.4050 to 0.4087 at 2-bit), ensuring newly added dimensions encode distinct features. The combination of Matryoshka Loss with Orthogonality Regularization appears critical - removing either causes >5% accuracy drop at high compression rates
- **Trainable FFN Transform:** This component is crucial for both quantization and Matryoshka training. For MiniLM at 96 dims, adding the FFN Transform after *Thresholds Only* boosts nDCG@10 from 0.2428 to 0.2860 at 2-bit, a 17.8% relative improvement.
- **Quantization Loss:** This component penalizes embeddings near threshold boundaries, reducing quantization ambiguity. For MiniLM at 96 dims, moving from *+ Trainable FFN Transform* (nDCG@10 = 0.2860) to *+ Quantization Loss* (0.3304) results in a 4.4-point absolute improvement, over 15% relative gain.

Model-Specific Observations:

- **MB vs. MiniLM:** MB shows stronger resilience at higher dimensions (e.g., 0.4593 vs 0.4185 at 384 dims, 2-bit), while MiniLM benefits significantly from matryoshka methods at lower dimensions (e.g., 0.3712 at 96 dims, 97.9% of its FP32 performance).
- **Dimensional Scaling:** MB maintains 97.3% of FP32 performance at 384 dims (0.4593 vs 0.4695) and 96% at 192 dims (0.4327 vs 0.4512) with 2-bit quantization, showing graceful degradation.

Practical Implications: The results suggest several deployment strategies:

- For resource-constrained environments (e.g., edge devices), 1.5-bit quantization with 192 dimensions offers an excellent compromise, reducing storage by 93.8% while maintaining 89.73% accuracy.
- For latency-sensitive applications, the hybrid approach provides the best balance: 94.9% storage savings with 95.07% accuracy and only 0.87× the baseline computation time.
- When accuracy is paramount, 2-bit quantization at full dimension (768) achieves 96.35% accuracy with 90.6% storage savings.

6 CONCLUSION

Summarize your contributions and the significance of your findings in advancing the field of information retrieval.

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