Practical no. 1

1)
$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{(a-x)}{(3a-3x)} \frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}} \qquad \frac{1}{3} \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\cancel{4}^{2}\sqrt{a}}{\cancel{2}\sqrt{3a}} = \frac{1}{3}$$

$$= \frac{2}{3\sqrt{3}}$$

(2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$$\lim_{y \to 0} \left(\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$$

$$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$$

(3) $\dfrac{\left(\cos\sqrt{\frac{3}{2}}h - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2}\right) + \left(\cos\sqrt{\frac{3}{2}}h\right)}{6h}$

$$\frac{\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$= \frac{-\sin\frac{4h}{2}}{6h}$$

$$= \frac{\sin\frac{4h}{2}}{6h}$$

$$= \frac{\sin 4h}{\frac{12h}{3}}$$

$$= \frac{1}{3} \times \frac{\sin h}{h}$$

$$= \frac{1}{3} \times 1$$

$$= \frac{1}{3}$$

(4) $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

$\lim_{x\to a}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$

$\lim_{x\to a}\dfrac{(x^2+5-x^2+3)\cdot(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\times(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$\cancel{\dfrac{8}{2}}\,4\left(\dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right)$

$4\,\dfrac{\left(\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}\right)}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

$4\lim_{x\to\infty}\dfrac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+1/x^2)}+\sqrt{x^2(1-3/x^2)}}$

$= 4$

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(5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x < \frac{\pi}{2}$

$= \dfrac{\cos x}{\pi - 2x}$ for $\frac{\pi}{2} < x < \pi$ } at $x = \frac{\pi}{2}$

<u>Solu</u>

$f(\frac{\pi}{2}) = \dfrac{\sin 2(\frac{\pi}{2})}{\sqrt{1-\cos 2(\frac{\pi}{2})}}$

$f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define.

(2) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \dfrac{\cos x}{\pi - 2x}$

a) $x - \frac{\pi}{2} = h$

$x = h + \frac{\pi}{2}$

where $h \to 0$

$\lim_{h \to 0} \dfrac{\cos(h + \frac{\pi}{2})}{\left(\pi - 2(h + \frac{\pi}{2})\right)}$

$\lim_{h \to 0} \dfrac{\cos(h + \frac{\pi}{2})}{\pi - \cancel{2}(\frac{2h + \pi}{\cancel{2}})}$

$\cancel{\pi} - 2h - \cancel{\pi}$

$\lim_{h \to 0} \dfrac{\cos(h + \frac{\pi}{2})}{-2h}$

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$h \to 0^+$ $\qquad -2h$

$$\lim_{x \to 0^+} \frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$= \frac{1}{2}$$

(b) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} = \frac{\sin 2x}{\sqrt{1-(\cos 2x)}}$

$$\lim_{x \to \pi/2} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}.$$

ii) $f(x) = \frac{x^2 - a}{x - 3}$ $\quad 0 < x < 3$

$= x + 3$

$\frac{x^2 - a}{x + 3}$ $\quad 3 \le x < 6$

at $x = 3$

i) (i) $f(3) = \frac{x^2 - a}{x - 3} = 0$

f at $x=3$ define.

(ii)

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+}$

1) $f(3) = x + 3 = 3 + 3 = 6$

is define at $x = 3$

2) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x + 3 = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2 - 9}{x - 3} = \frac{\cancel{(x-3)}(x+3}{\cancel{(x-3)}}$

$= L.H.L = R.H.L$

f is continuous at $x = 3$

for $x = 6$

i) $f(6) = \frac{x^2 a}{x + 3} = \frac{36 - a}{6 + 3}$

036

(2) $\lim_{x \to 6^+} = \frac{x^2 - a}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3$

$\lim_{x \to 6} x+3 = 3+6 = 9$

L·H·L $\neq$ R·H·C

funtion is not continous.

Q(6)

(i) $f(x) = \frac{1 - \cos 4x}{x^2}$ $\quad x < 0$ $\Big\}$ at $x = 0$

$= K$ $\quad x = 0$

Solve f is at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = K$

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$\Rightarrow \lim_{x\to 0} \frac{2\sin^2 x^2}{x^2} = K$

$\Rightarrow \lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = K$

$2(2)^2 = K$

$K = 8$

6(ii) $f(x) = (\sec^2 x)(\cot^2 x$ $\quad x \neq 0$

$= K \quad x = 0$ } at $x = 0$

$\sqrt{3} - 3\tan h - \sqrt{3} - \tan h$

$\frac{1 - \sqrt{3}\tan h}{-3h}$

$= \frac{\frac{-4\tan h}{(-3h)\ (1-\sqrt{3}\tan h)}}{-3h}$

$= \frac{4}{3}\left(\frac{1}{1-0}\right)$

$= 4$

) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \frac{\pi}{3}$

$= K \quad x = \frac{\pi}{2}$ } at $x = \frac{\pi}{3}$

$\pi - \pi - 3h$

$$\frac{\dfrac{\cancel{\sqrt{3}} - \sqrt{3} \times \sqrt{3}\ \tan h - \cancel{\sqrt{3}} - \tan h}{1 - \sqrt{3}\ \tan h}}{-3h}$$

$$\frac{\dfrac{\sqrt{3} - 3\ \tan h - \sqrt{3}\ \tan h}{1 - \sqrt{3}\ \tan h}}{-3h}$$

$$\frac{\dfrac{-4\ \tan h}{1 - \sqrt{3}\ \tan h}}{3h}$$

$$\frac{-4\ \tan h}{-3h(1 - \sqrt{3}\ \tan h)}$$

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$$\frac{4}{3}\lim_{x\to 0} \quad \tan \lim_{h\to 0} \frac{1}{1-\sqrt{3}\tan h}$$

$$= \frac{4}{3} \frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3} \cdot 1 = \frac{4}{3}$$

(7)

(i) $f(x) = \frac{1-\cos 3x}{x\tan x} \quad x \neq 0$
$= a \quad x = 0$ $\}$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\frac{2 \frac{\sin^2 \frac{3x}{2}}{x^2} \times \cancel{x}^2}{\frac{x\tan \cancel{x}}{x^2} \times \cancel{x}^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$\cancel{2}^{1} \times \frac{9}{\cancel{4}} = \frac{9}{2}$$

$\lim_{x\to 0} f(x) = \frac{9}{2}$ $\quad 9 = f(0)$

f is not cts at $x = 0$

Redifine function.

$$f(x) = \begin{cases} \frac{1-\cos 3x}{3\tan^2 x} & x \neq 0 \\ \frac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable dis cts at $x = 0$

ii) $f(x) = \begin{cases} \frac{[e^{3x}-1] \sin x^{\circ}}{x^2} & x = 0 \\ \frac{\pi}{6} & x = 0 \end{cases}$ at $x = 0$

at $x = 0$

Solve

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3 \log e \quad \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

f is cts at $x = 0$

8) $f(x) = \dfrac{e^{x^2} \cos x}{x^2}$ $x = 0$

is continuous at $x=0$

$\therefore f$ is continuous at $x=0$

Solve

$\lim_{x \to 0} f(x) = f(0)$

$= \dfrac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\dfrac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\dfrac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\dfrac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \dfrac{2 \sin x/2}{x^2}$

$= \log e + 2 \left(\dfrac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num + Denom.

$= 1 + \cancel{2} \times \dfrac{1}{\cancel{4}\,2} = \dfrac{3}{2} = f(0)$

$$\overline{\cos^2 x} \qquad \overline{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

practical no 2

Topic = Derivation

Q1 Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i) $\cot x$

$f(x) = \cot x$

$$f(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 0} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan x}}{x-a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{1 x - a + \tan x \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula = $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A + \tan B}$

$\tan A - \tan B = \tan(A-B)(1 + \tan A \tan B)$

$$= \lim_{h \to 0} \frac{(x-a-h) - (1 + \tan a \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore$ f is differentiable $\forall\, a \in R$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

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$$DF(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

Formula :-

$$\sin c - \sin d = 2\cos\left(\frac{c+d}{2}\right)\sin\left(\frac{c-d}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$F(x) = \sec x$

$$DF(a) = \lim_{x\to a} \frac{F(x) - F(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x - a = h$

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~~put~~ $Df(h) = \lim\limits_{h \to 0} \dfrac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$

Formula :-

$$-2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2 \sin\left(\frac{2a+h}{2}\right) \sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}} \times \frac{-1}{2}$$

$$= \frac{-\frac{1}{2} \times -2 \sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \cancel{-}\frac{1}{\cancel{2}} \cancel{\times} \times \cancel{2} \; \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

If $f(x) = 4x + 1, \quad x \leq 2$

$= x^2 + 5 \quad x > 0$ at $x = 2$, then find function is differentiable or not.

<u>Solution</u>:-

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4 \times 2 + 1)}{x - 2}$$

$$= \lim \quad 4x + 1 - 9$$

$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$

$Df(3^+) = 9$

$LHD = Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x \to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$

$Df(3^-) = 4$

$RHD \neq LHD$

f is not differentiable at $x = 3$

04 If $f(x) = 8x - 5 \quad x < 2$

$= 3x^2 - 4x + 7 \quad x > 2$ at $x = 2$ then

find f is differentiable or not.

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD

$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$Df(2^+) = 8$

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$

$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$

$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$

$= 8$

$Df(2^-) = 8$

$LHD = RHD$

f is differentiable at $x = 3$

Practical no 3

Topic - Application of derivation

Q1 Find the interval in which function is increasing or decreasing.

(i) $F(x) = 3^3 - 5x - 11$ (ii) $F(x) = x^2 - 4x$

(iii) $F(x) = 2x^3 + x^2 - 20x + 4$ (iv) $F(x) = x^3 - 27x + 5$

(V) $F(x) = 69 - 24x - 9x^2 + 2x^3$

Q2 Find the interval in which function is concave in upwords and concave downwords

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

Q

i) $f(x) = x^3 - 5x - 11$

$\rightarrow f'(x) = 3x^2 - 5$

f is increasing iff $f'(x) > 0$

$3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm\sqrt{\frac{5}{3}}$

$+ \quad - \quad +$

$-\sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}}$

$x \in (-\infty, -\sqrt{\frac{5}{3}}) \cup (\sqrt{\frac{5}{3}}, \infty)$

$\therefore$ f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \frac{5}{3}$

$x < \pm\sqrt{\frac{5}{3}}$

$+ \quad - \quad +$

$-\sqrt{\frac{5}{3}} \quad \sqrt{\frac{5}{3}}$

$\therefore x \in \left(-\frac{\sqrt{5}}{\sqrt{3}}, \sqrt{\frac{5}{3}}\right)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12 - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x-10)(x+2) < 0$

$+ \quad -2 \quad \frac{10}{6} \quad +$

$\therefore x \in (-2, \frac{10}{6})$

(iv) $f(x) = x^3 - 27x + 5$

$f(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

$\therefore f$ is increasing iff $f(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

f is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

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Q2

(i) $y = 3x^2 - 2x^3$

Let

$f(x) = y = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$\cancel{6x(1-x)}$

$f''(x) = 6 - 12x$

$\therefore = 6(1-2x)$

$f''(x)$ is concave upwoords iff.

$f''(x) > 0$

$6(1-2x) > 0$

$1 - 2x > 0$

$-2x > -1$

$2x < 1$

$x < \frac{1}{2}$

$x \in (-\infty, \frac{1}{2})$

$f''(x)$ is concave downwoords iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1 - 2x < 0$

$-2x < -1$

$2x > 1$

$x > \frac{1}{2}$

$x \in (\frac{1}{2}, \infty)$

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(iii) $y = x^3 - 27x\ x + 5$

let

$F(x) = y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave iff upwards iff.

$f''(x) > 6x\ 0$

$\therefore 6x > 0$

$x > 0$

$x \in (0, \infty)$

$f''(x)$ is concave iff downword iff.

$f''(x) < 0$

$6x < 0$

$x < 0$

$x \in (\infty, 0)$

(iv) $y = 69 - 24x - 9x^2 + 2x^3$

let,

$f'(x) = y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

$f''(x) = -18 + 12x$

$f''(x)$ is concave ~~down~~ upwords iff.

$f''(x) > 0$

$-18 + 12x > 0$

$f''(x)$ is concave downwards iff.

$f''(x) < 0$

$2(6x-1) < 0$

$6x-1 < 0$

$6x < 1$

$x < \frac{1}{6}$

$x \in (-\infty, \frac{1}{6})$

(ii) $F(x) = 3 - 5x^3 + 3x^5$

$F(x) = 15x^2 + 15x^4$

consider

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$F''(x) = -30x + 60x^3$

$F(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore F(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$F''(-1) = -30(-1) + 60(-1)^3$

$- 30 - 60$

$= 30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

(iii) $F(x) = x^3 - 3x^2 + 1$

$f(x) = 3x^2 - 6x$

consider

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at x

$\therefore F(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$f(x) = 15x^2 + 15x^4$

consider

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$- 30 - 60$

$= 30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= \cancel{3+} 5 - 3$

$= 5$

f has the maximom value 5 at $x = -1$
and has the minimum value 1 at $x = 1$

(iii) $f(x) = x^3 - 3x^2 + 1$

$f(x) = 3x^2 - 6x$

consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ and
f has minimum value -3 at $x = 2$

S20

Q2

$x_0 = 0 \rightarrow$ given

(i) $F(x) = x^3 - 3x^2 - 55x + 9.5$

$F'(x) = 3x^2 - 6x - 55$

By Newton's method

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$F(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$

$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$

$$= -0.0829$$

$$F'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$

$$= 0.0895 - 1.0362 - 55$$

$$= 55.9467$$

$$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$$= 0.1712$$

$$x_{n+1} = x_n \quad \frac{F(x_n)}{F'(x_n)}$$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$= 3 - \frac{6}{2^3}$$

$$= 2.7392$$

$$F(x_1) = (2.7392)^3 - 4(2.7392) - 9$$

$$= 20.5528 - 10.9568 - 9$$

$$= 0.596$$

$$F(x_1) = 23(2.7392)^2 - 4$$

$$= 22.5096 - 4$$

$$= 18.5096$$

$$x_2 = x_1 - \frac{F(x_1)}{F'(x_2)}$$

$$= 2.7392 - \frac{0.596}{18.5096}$$

$$= 2.7071$$

$$F(x_2) = (2.7071)^3 - 4(2.7071) - 9$$

$$= 19.8386 - 10.8284 - 9$$

$$= 0.0102$$

$$F'(x_2) = 3(2.7071)^2 - 4$$

$$= 21.9851 - 4$$

$$= 17.9851$$

$$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$$

$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

$x_1 = x_0 - \frac{F(x_2)}{F'(x_2)}$

$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230$

$= 1.577$

$F(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$F'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$\therefore x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$= 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$= 1.6592$

$F(x) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$F'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= 7.7143$

$x_3 = x_2 = \frac{F(x_2)}{F'(x_2)}$

$= 1.6592 + \frac{0.204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618$

$x_4 \quad F(x_3) = (1.6618)^3 - 1.8(1.6618)^2 +$

$= 4.5892 - 4.9708 - 16.618$

$= 0.0004$

$F'(x_3) = 3(1.6615)^2 - 3.6(1.661$

$= 8.2847 - 5.9824 - 1$

$= -7.6977$

$x_4 = x_3 - \frac{F(x_3)}{F'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

$\therefore$ The root of equation is

practical no - 5

Topic = Integration

Q1 Solve the following integration.

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)\,dx$

(iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ (v) $\int t^7 \sin(2t^4)\,dt$

(vi) $\int \sqrt{x}(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ (ix) $\int e^{\cos^2 x}\sin x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$.

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$; $x^2 = (x+1)^2$, $a^2 = (2)^2$

$I = \log |x + \sqrt{x^2-a^2}| + c$

$= \log |x+1 \sqrt{(x+1)^2-(2)^2}| + c$

(ii) $\int (4e^{3x}+1)\,dx$

$I = \int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\frac{e^{3x}}{3} + x + c$

(iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$I = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= 2\int x^2 dx - 3\int \sin x\,dx + 5\int \sqrt{x}\,dx$

$= \frac{2x^3}{3} + 3\cos x + 5 \times \frac{2}{3} x^{3/2} + C$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + C$

(iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$I = \int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right) dx$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right) dx$

$= \int \left(x^3 x^{-1/2} + 3x \cdot x^{-1/2} + 4x^{-1/2}\right) dx$

$= \int x^{5/2} dx + 3\int x^{1/2} dx + 4\int x^{-1/2} dx$

$= \frac{2}{7} x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4x^{1/2} \cdot 2 + C$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

(i) $\int t^7 \sin(2t^4)\,dt$

$I = \int t^7 \sin(2t^4)\,dt$

Let $t^4 = x$

$4t^3 dt = dx$

$I = \frac{1}{4}\int 4t^3 \cdot t^4 \sin(2t^4)\,dt$

$= \frac{1}{4}\int x \cdot \sin(2x)\,dx$

$= \frac{1}{4}\int x \int \sin 2x - \int [\int \sin 2x \cdot \frac{d}{dx}(x)]$

$= \frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{2}\int \cos 2x \cdot 1\right]$

$= \frac{1}{4}\left[-x\frac{\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + C$

$= -\frac{1}{8} x\cos 2x + \frac{1}{16}\sin 2x + C$

Resubstituting $x = t^4$

$\therefore I = -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16}\sin(2$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$= \int (\sqrt{x}\cdot x^2 - \sqrt{x})\,dx$

$= \int (x^{5/2} - \sqrt{x})\,dx$

$= \int x^{5/2}\,dx - \int \sqrt{x}\,dx$

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + C$

(vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

let

$\frac{1}{x^2} = t$

$\therefore\ x^{-2} = t$

$\therefore\ -\frac{2}{x^3}\,dx = dt$

$I = \frac{1}{-2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= \frac{-1}{2} \int \sin t\,dt$

$= \frac{-1}{2} [-\cos t] + C$

$= \frac{1}{2} \cos t + C$

Resubstituting $t = \frac{1}{x^2}$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + C$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

let $\sin x = t$

$\cos x\,dx = dt$

$I = \int \frac{dt}{\sqrt[3]{t^2}}$

$I = \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3}\,dt$

$= \int t^{-2/3}\,dt$

$= 3t^{1/3} + C$

$= 3(\sin x)^{1/3} + C$

$= 3\sqrt[3]{\sin x} + C$

ix) $\int e^{\cos^2 x} \sin 2x\, dx$

$I = \int e^{\cos^2 x} \sin 2x\, dx$

let $\cos^2 x = t$

$-2\cos x \sin x\, dx = dt$

$-2 \sin 2x\, dx = dt$

$I = \int -\sin 2x\, e^{\cos^2 x} dx$

$= -\int e^t dt$

$= -e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

(x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

let ;

$x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x)\, dx = dt$

$3(x^2 - 2x)\, dx = dt$

$(x^2 - 2x)\, dx = \frac{dt}{3}$

$\therefore I = \int \frac{1}{t} \frac{dt}{3}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$

Resubstituting $t = x^3 - 3x^2 + 1$

$\therefore I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

Practical no. 6

Topic: Application of integration & Numerical integration.

Q1 Find the length of the following curve.

$x = t \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

1) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

2) $y = x^{3/2}$ in $[0, 4]$

3) $x = 3 \sin t$, $y = 3 \cos t$ $t \in [0, 2\pi]$

4)

5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2 using simpsons Rule solve the following

① $\int_0^{?} e^{x^2} dx$ with $n = 4$

② $\int_0^4 x^2 dx$ with $n = 4$

③ $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

Q1(i) $= \int_0^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t$ $\quad \therefore \frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$ $\quad \therefore \frac{dy}{dt} = 0 - (-\sin t) = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t + \sin t)^2 + (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \int_0^{2\pi} \sqrt{2(1-\cos t)} = \int_0^{2\pi} \sqrt{2 \times 2 \sin t/2} = \sqrt{4 \sin t/2}$

$= \int_0^{2\pi} 2 \left| \sin \frac{t}{2} \right| dt$ $\quad \therefore \sin \frac{t}{2} = \frac{1 - \cos t}{2}$

$= \int_0^{2\pi} 2 \sin \frac{t}{2}\, dt$

$= \left(-4 \cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos \pi) - (-4 \cos 0)$

$= 4 + 4$

$= 8$

(ii) $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$

$I = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$y = \sqrt{4 - x^2}$ $\quad \therefore \frac{dy}{dx} = 2 \int_0^2 1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2 dx$

$= 2 \int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$

$= 4 \int_0^2 \frac{1}{\sqrt{4 - x^2}}\, dx$

$= 4 \left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

(iii) $y = x^{3/2}$ $\quad$ in $[0, 4]$

$f(x) = \frac{3}{2} x^{1/2}$

$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4} x}\, dx$

$= \int_0^4 \sqrt{\frac{4 + 9x}{4}}\, dx$

$= 1/2 \int_0^4 \sqrt{4 + 9}\, dx$

$= \frac{1}{2} \left[\frac{(4 + 9x)^{1/2 + 1}}{1/2 + 1} \right]_0^4 dx$

$\frac{1}{27} \left[(4 + 9x)^{3/2} \right]_0^4$

$\frac{1}{27} \left[(4 + 0)^{3/2} - (4$

$= \frac{1}{27} (4)^{3/2} - (40)^{3}$

iv) $x = 3\sin t$ $\quad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$

$J = \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$

$= \int_0^{2\pi} \sqrt{9(1)}\, dt$

$= \int_0^{2\pi} 3\, dt = 3\int_0^{2\pi} dt = 3[x]_0^{2\pi}$

$3(2\pi - 0)$

$= 6\pi$

v) $x = \frac{1}{6}y^3 + \frac{1}{2y} \quad y = [1, 2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$

$= \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12}$

Q2 i) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$a = 0 \quad b = 2 \quad n = 4$

$h = \frac{2-0}{4} = \frac{1}{2} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5901
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$\int_0^2 e^{x^2}\, dx = 0.5\left[(1 + 54.5901) + 4(1.2840 + 9.\ldots\right.$

$+ 2(2.7182 + 54.5901)]$

$= \frac{0.5}{3}[55.5901 + 43.0868 + 114.63\ldots$

$= 1.1779$

(ii) $\int_0^4 x^2\, dx$

$l = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{1}{3}[16 + 4(10) + 8]$$

$$= 64/3$$

$$\int_0^4 x^2 dx = 21.533$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\; dx \qquad n = 6$

$l = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.00087	0.8727	0.990

$$\int_0^{\pi/3} \sqrt{\sin} = \pi/54 \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin x}\; dx = 0.7049$$

$P(x) = 2 \qquad Q(x) = e^{x}$

$If = e^{\int p(x)\,dx}$
$= e^{\int 2\,dx}$
$= e^{2x}$

$y = (I.f) = \int \cdot Q(x)\ (I.f)\ dx + c$

$y \cdot e^{2x} \int e^{-x} + \cdot 2x\,dx + c$

$= \int e^{x}\ dx + c$

$y \cdot e^{2x} = e^{x} + c$

(5) $x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$x \dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos x}{x^2}.$

$p(x) = 2(x) \qquad Q(x) = \dfrac{\cos x}{x^2}$

$I.f = e^{\int p(x)\,dx}$
$e^{\int 2/x\,dx}$
$e^{2/x}\ x$
$= \ln x^2$

$I.f = x^2$

$y\,(I.f) = \int Q(x)\,(I.f)\ dx + C$

$\int \dfrac{\cos x}{x^2} - x^2\,dx + C$

$\cdot \int \cos x + C \quad = x^2y = \sin x + C$

(6) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y \tan x \, dy$

$$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

$$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \tan y = e^c$

(7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$x = y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

(8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

AK 10/01/2020

Practical no - 8

Topic - using Euler's method find the following:

1) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

2) $\frac{dy}{dx} = 1 + y^2$ $y(0) = 0$, $h = 0.2$ find $y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$ $h = 0.2$ find $y(1)$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$

for $h = 0.5$ & $h = 0.25$

$\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1)$ find $y(1.2)$ with $h = 0.2$.

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(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $x_0 = 0$ $h = 0.2$

using Eulers iteration formula

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	0	0	0
1	0.2	1		
2	0.4			
3	0.6			
4	0.8			
5	1			

(4) $\frac{dy}{dx} = 3x^2 + 1$ $y_0 = 2,\ x_0 = 1,\ 4 = 0$

for $h = 0.5$

using Euler's itration formula.

$y_{n+1} = y_n + hf(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	20.5
2	2	20.5		

By Eulers formula

$y(2) = 20.5$

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Practical no - 9

Topic = limits and partial order Derivatives

Q1

(i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$, denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{-4(-1)+5}$

$= \dfrac{-64 + 3 + 1 - 1}{4 + 5}$

$= \dfrac{-61}{9}$

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(2,0)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(0+1)((2)^2 + 0 - 4(2))}{2+0}$

$= \dfrac{\cancel{1}(4 + 0 - 8)}{2}$

$= \dfrac{-4}{2}$

$= -2$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$$= 0$$

(4)

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy^2 + 2x^2y)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{\left(x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)\right)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

from (3) & (4);

$f_{xy} = f_{yx}$

050

(ii) $f(x,y)\ x^3+3x^2\ y^2-\log(x^2+1)$

$f_x = \frac{\partial}{\partial x}(x^3+3x^2y^2-\log(x^2+1))$ $\quad$ $f_y = \frac{\partial}{\partial y}(x^3+3x^2y^2-\log(x^2+1))$

$= 3x^2+6xy^2-\frac{2x}{x^2+1}$ $\quad$ $= 0+6x^2y-0$

$= 6x^2y$

$f_{xx} = 6x+6y^2-\left(\frac{x^2+1\frac{\partial(2x)}{\partial x}-2x\frac{\partial(x^2+1)}{\partial x}}{(x^2+1)^2}\right)$

$= 6x+6y^2-\left(\frac{2(x^2+1)-4x^2}{(x^2+1)^2}\right)$ — (1)

$F_{yy} = \frac{\partial}{\partial y}(6x^2y)$

$= 6x^2$ — (2)

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$

$= 0+12xy-0$

$= 12xy$ — (3)

$F_{yx} = \frac{\partial}{\partial x}(6x^2y)$

$= 12xy$

From (3) & (4)

$\therefore f_{xy} = f_{yx}$

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$f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\Rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\qquad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad$ $= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\qquad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

E.S0

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{= 4 + \frac{h}{\sqrt{10}} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

(ii) $f(x) = y^2 - 4x + 1$ $a = (3, 4)$ $u = i + 5j$

then $u = i + 5j$ is not a unit vector

$$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{26}\right)$$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$$f(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(2+\frac{4h}{5}\right)$$

$$= 2+\frac{6h}{5}+6+\frac{12h}{5}$$

$$= \frac{18h}{5}+8$$

$$D_u f(a) = \lim_{h\to 0}\frac{\frac{18h}{5}+8-8}{h}$$

$$= \frac{18}{5}$$

Q. Find gradient vector for the following function at given points.

i) $f(x,y) = x^y + y^x \quad a(1,1)$

$$f_x = y\cdot x^{y-1} + y^x \log y$$

$$f_y = x^y \log x + x y^{x-1}$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (y x^{y-1} + y^x \log y,\ x^y \log x + x y^{x-1})$$

$$f(1,1) = (1+0, 0+1)$$

$$= (1,1)$$

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Q3 Find the equation of tangent & normal to each of the following using curves at given points

$x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \cdot 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0) \therefore x_0 = 1, y_0 = 0$

eqn tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 (-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

it is the required equation of tangent.

eqn. for normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(x) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

(i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2) \therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = 1$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x-2) + (-(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ — it is required eqn of tangent.

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eq^n of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q4 Find the equation of tangent & normal line to each of the following surfaces

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$Fx = 2x - 0 + 0 + z$

$Fx = 2x + z$

$Fy = 0 - 2z + 3 + 0$

$= 2z + 3$

$fz = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0) \therefore x_0 = 2, y_0 = 1, z_0 = 0$

$Fx(x_0, y_0, z_0) = 2(2) + 0 = 4$

$Fy(x_0, y_0, z_0) = 2(0) + 3 = 3$

$Fz(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$Fx(x_0 - x_0) + Fy(y_0 - y_0) + fz(z_0 - z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \rightarrow$ This is required of equation tangent.

Eq^n of normal at $(4,3,-11)$

$= \dfrac{x - x_0}{fx} = \dfrac{y - y_0}{fy} = \dfrac{z - z_0}{fz}$

$\therefore = \dfrac{x-2}{4} = \dfrac{y-1}{3} = \dfrac{z+11}{0}$

(ii) $3xy^2 - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$fx = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$Fy = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$fz = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

077

550

$fx(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1, y_0 = -1, z_0 = 2$

$fy(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$fz(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eqn tangent.

Eqn of normal $(-7, 5, -2)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Find the local maxima & minima for the following function

(1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$fx = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$fy = 0 + 2y - 3x + 0 - y$

$= 2y - 3x - y$

$fx = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ —— (1)

multiply eqn 1 with 2

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = +2$ $y = 2$

Critical point are $(0, 2)$

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$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0.$

$4x^2 + 3y = 0 \rightarrow 1$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow 2$

multiply eqn 1 with 3
2 with 4

$12x^2 + 9y = 0$
$12x^2 - 8y = 0$
$17y = 0$

$y = 0$

Substitute value of y in eqn 1

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$r = 0$

$rt - s^2 = 0(-2) - (s)^2$
$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to says)

079

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= \underline{\underline{0}}$

1) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \cancel{\frac{-2}{2}} \quad \therefore x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{\cancel{-8}\ 4}{\cancel{-2}\ 1}$

$\therefore y = 4$

Critical point is $(-1, 4)$.

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

080

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = \underline{\underline{33}}$

AB

07/01/2020

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