
Proposition

What is Proposition ???

A declarative sentence which is either true or false but not both. It declared by "P".

Propositions

So why waste time on such matters?

Propositional logic is the study of how simple propositions can come together to make more complicated propositions. If the simple propositions were endowed with some meaning –*and they will be very soon*– then the complicated proposition would have meaning as well, and then finding out the truth value is actually important!

False, True, Statements

False is the opposite to ***Truth***.

A ***statement*** is a description of something.

Examples of statements:

- I’m 31 years old.
- I have 17 children.
- I always tell the truth.
- I’m lying to you.

False, True, Statements

Statement: I'm lying to you.

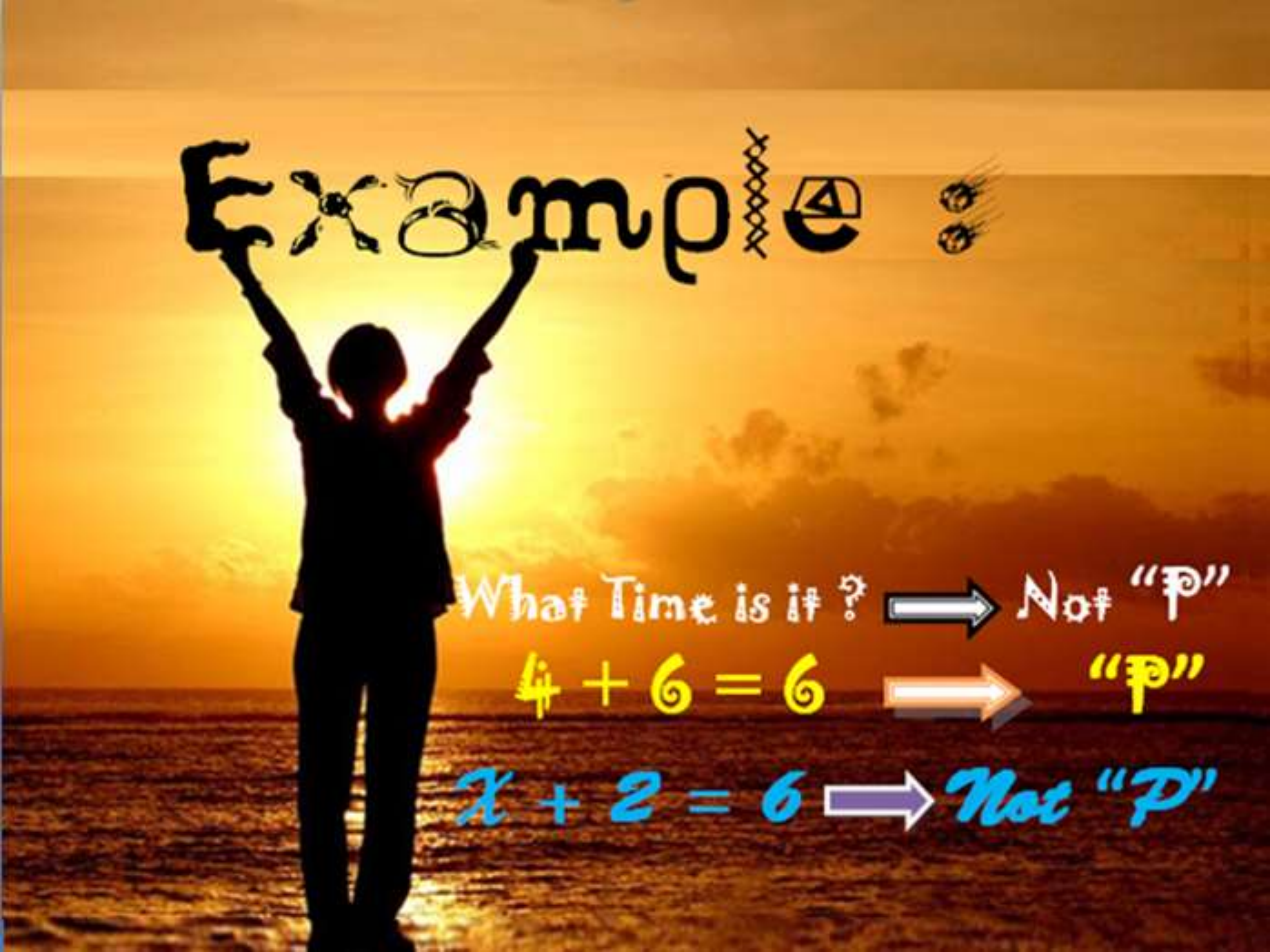
Well suppose that

S = "I'm lying to you."

In particular, I am actually lying, so *S* is false. So it's both true and false, impossible by the Axiom.

Okay, so I guess *S* must be false. But then I must not be lying to you. So the statement is true. Again it's both true and false.

In both cases we get the opposite of our assumption, so *S* is neither true nor false.


Example :
What Time is it ? → Not "P"
4 + 6 = 6 → "P"
X + 2 = 6 → Not "P"

Another example of proposition

- I worked hard or I played the piano $(W \vee P)$.
- If I worked hard, then I will get a bonus $(W \Rightarrow B)$.
- I did not get a bonus $\neg(B)$.

Compound Propositions

In Propositional Logic, we assume a collection of atomic propositions are given: p, q, r, s, t, ….

Then we form compound propositions by using logical connectives (logical operators) to form propositional "molecules".

Logical Connectives

Operator	Symbol	Usage
Negation	$\neg$	not
Conjunction	$\wedge$	and
Disjunction	$\vee$	or
Exclusive or	$\oplus$	xor
Conditional	$\rightarrow$	If , then
Biconditional	$\leftrightarrow$	iff

Compound Propositions: Examples

p = "Cruise ships only go on big rivers."

q = "Cruise ships go on the Hudson."

r = "The Hudson is a big river."

$\neg r$ = "The Hudson is not a big river."

$p \wedge q$ = "Cruise ships only go on big rivers and go on the Hudson."

$p \wedge q \rightarrow r$ = "If cruise ships only go on big rivers and go on the Hudson, then the Hudson is a big river."

Precedence of Logical Operators:

1. Negation (¬)
2. Conjunction (∧)
3. Disjunction (∨)
4. Implication (→)
5. Biconditional (↔)

Negation Of Proposition

- I like Facebook very much.
- $\neg P$ = *I do not like Facebook very much.*

Logical Connective: Logical And

- The logical connective And is true only when both of the propositions are true. It is also called a conjunction
- Examples
  - It is raining and it is warm
  - (2+3=5) and (1<2)

Logical Connective: Logical And

TRUTH TABLE

p	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Connective: Logical OR

- The logical <u>disjunction</u>, or logical OR, is true if one or both of the propositions are true.
- Examples
  - It is raining or it is the second lecture
  - $(2+2=5) \lor (1<2)$
  - You may have cake or ice cream

Logical Connective: Logical OR

Truth table

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Connective: Exclusive Or

- The exclusive OR, or XOR, of two propositions is true when exactly one of the propositions is true and the other one is false
- Example
  - The circuit is either ON or OFF but not both
  - Let $ab<0$, then either $a<0$ or $b<0$ but not both
  - You may have cake or ice cream, but not both

Logical Connective: Exclusive Or

Truth table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Logical Connective: Implication

- **Definition:** Let p and q be two propositions. The implication $p \rightarrow q$ is the proposition that is false when p is true and q is false and true otherwise

  p is called the hypothesis, antecedent, premise

  q is called the conclusion, consequence

Truth Table for Implication

In General
statement: if p then q
converse: if q then p
inverse: if not p then not q
contrapositive: if not q then not p

P	Q	$P \Rightarrow Q$
t	t	t
t	f	f
f	t	t
f	f	t

Logical Connective: Biconditional

- **Definition:** The biconditional $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values. It is false otherwise.

Logical Connective: Biconditional

Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

TAUTOLOGY

A compound proposition that is always true is called tautology .

Example Of Tautology

P	¬P	P v¬P
T	F	T
F	T	T

CONTRADICTION

A compound proposition which is always false is called contradiction.

Table of contradiction

P	¬P	P^P
T	F	F
F	T	F

Example of Tautology by logical equivalence

$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$

$\equiv T \vee T$

$\equiv T$

THANKS TO ALL FOR YOUR KIND ATTENTION AND SUPPORT