Assignment 2

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Example # 17

Use the summation notation to express the sum of the first 100 terms of sequences $\{a_j\}$, where $a_j = 1/j$ for $j = 1, 2, 3, \ldots$

Solution:-

The lower limit for the index of summation is 1, and the upper limit is 100. We write this sum as

$$\sum_{j=1}^{100} 1/j$$

Example 18:

What is the value of $\sum\limits_{j=1}^{15} j^2$?

Solution:-
We have:
$$\sum_{j=1}^{5} j^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2$$

$$= 1 + 4 + 9 + 16 + 25$$
$$= 55.$$

Example 19
What is the value
of $\sum\limits_{K=4}^{8} (-1)^r$?

Solution:
We have

$$\sum_{K=4}^{8} (-1)^K = (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$$

$$= 1 + (-1) + 1 + (-1) + 1$$

$$= 1$$

Theorem 1

If a and r are real number $r \neq 0$ then

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1} - a}{r-1} & \text{if } r \neq 1 \\ (n+1)a & \text{if } r = 1 \end{cases}$$

Proof :-

let
$$S_n = \sum_{j=0}^{n} ar^j$$

$$r S_n = r \sum_{j=0}^{n} ar^j$$

$$= \sum_{j=0}^{n} ar^{j+1}$$

$$= \sum_{k=1}^{n+1} ar^k$$

$$= \left(\sum_{k=0}^{n} ar^k \right) + \left(ar^{n+1} - a \right)$$

$$= S_n + (ab^{n+1} - a)$$

From these quantities we see that

$$SS_n = S_n + (ab^{n+1} - a)$$

Solving for S_n shows that if $b \neq 1$, then

$$S_n = \frac{ab^{n+1} - a}{b - 1}$$

if $b = 1$, then the $S_n = \sum_{j=0}^{n} ab^j$

$$= \sum_{j=0}^{n} a = (n+1)a.$$

Q no 29

What are the values of these sums?

(a)
$$\sum_{k=1}^{6} (k+1)$$

$$= (1+1) + (2+1) + (3+1) + (4+1) + (5+1)$$

$$= 2 + 3 + 4 + 5 + 6$$

$$= 20.$$

Qno: 30

What are the values of these things sums where

$$S = \{1, 3, 5, 7\}$$

(a) $\displaystyle\sum_{j \in S} (j)$

$$= 1 + 3 + 5 + 7$$
$$= 16$$

Qno: 31

What is the value of each of these sums of terms of a Geometric Progression?

(a) $\displaystyle\sum_{j=0}^{8} 3 \cdot 2^{j}$

Note that $a = 3(2)^0 = 3$ and $r = 2$

$$a \frac{\left[r^{n+1} - 1 \right]}{r - 1}$$

$$= 3 \cdot \left[\frac{2^9 - 1}{2 - 1} \right]$$

$$= 1533$$

Q no. 32

Find the values of each sums?

(b)
$$\sum_{j=1}^{8} 2^{j}$$

$$\sum_{j=0}^{8} [3^{j} - 2^{j}]$$

$$= \sum_{j=0}^{8} (3^{j}) - \sum_{j=0}^{8} 2j$$

$$= a \cdot \left[\frac{\gamma^{n+1} - 1}{\gamma - 1} \right] - a \cdot \left[\frac{\gamma^{n+1} - 1}{2 - 1} \right]$$

$$= a \cdot \left[\frac{\gamma^{n+1} - 1}{\gamma - 1} \right] - 1 \left[\frac{2^{9} - 1}{2 - 1} \right]$$

$$= 9841 - 511$$
$$= 9330$$

Q no. 33

Complete each of these.

(a)

$$\sum_{i=1}^{2} \sum_{j=0}^{3} (i+j)$$

$$= \sum_{i=1}^{2} (i+1 + i+2 + i+3)$$

$$= \sum_{i=1}^{2} (3i+6)$$

$$= 3 \sum_{i=1}^{2} (i+2)$$

$$= 3(1+2+2+2)$$

$$= 21$$

Q no. 34

Complete each of these.

(a)
$$\sum_{i=1}^{3} \sum_{j=1}^{2} (i-j)$$

$$\sum_{i=1}^{3} \sum_{j=1}^{2} (i-j)$$

$$= \sum_{i=1}^{3} (i-1+i-2)$$

$$= \sum_{i=1}^{3} (2i-3)$$

$$= 2(1)-3+2(2)-3+2(3)-3$$

$$= 3.$$