University of Management & Technology

Knowledge unit of Science and Technology

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PROGRAM :BS : SE

SUBJECT : DISCRETE STRUCTRURE

Symmetric Definition:

The symmetric difference of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

Q no 32:

Find the symmetric differenc of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

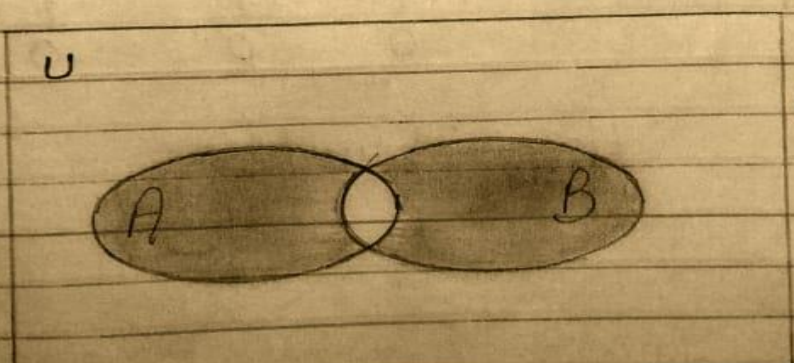
Solution:

Let $A = \{1, 3, 5\}$
$B = \{1, 2, 3\}$

$A \oplus B = \{2, 5\}$

Q no 34:

Draw a Venn Diagram for the symmetric difference of the sets A and B.



(shaded area shows $A \oplus B$)

Q no 35:

Show that $A \oplus B = (A \cup B) - (A \cap B)$

Using membership table to show the statement.

A	B	$A \oplus B$	$A \cup B$	$A \cap B$	$(A \cup B) - (A \cap B)$
1	1	0	1	1	0
1	0	1	1	0	1
0	1	1	1	0	1
0	0	0	0	0	0

Hence Proved!

Q no 36:

Show that $A \oplus B = (A - B) \cup (B - A)$

Using membership table to show the statement.

A	B	$A \oplus B$	$A - B$	$B - A$	$(A - B) \cup (B - A)$
1	1	0	0	0	0
1	0	1	1	1	1
0	1	1	1	1	1
0	0	0	0	0	0

Hence Proved!

Q no 37:

Show that if A is a subset of a universal set U, then

(a) $A \oplus A = \emptyset$

Let universal set

$U = \{1, 2, 3, 4 \ldots\}$

Let A is its subset

$A = \{1, 2, 3, 4, 5\}$

(a) $A \oplus A = \emptyset$

$= \{1, 2, 3, 4, 5\} \oplus \{1, 2, 3, 4, 5\}$

$= \emptyset$

Hence Proved!

(b) $A \oplus \emptyset = A$

$= \{1, 2, 3, 4, 5\} \oplus \emptyset$

$= \{1, 2, 3, 4, 5\} \quad \therefore (\{1, 2, 3, 4, 5\} = A)$

$= A$

Hence Proved!

(C). $A \oplus U = \bar{A}$

Left Hand Side:

$= \{1,2,3,4,5\} \oplus \{1,2,3,4,5,6 \ldots\}$

$= \{6,7,8,9 \ldots\}$

Right Hand Side:

$\bar{A} = U - A$

$= \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},6 \ldots\} - \{\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5}\}$

$= \{6,7,8,9 \ldots\}$

Hence Proved!

(d) $A \oplus \bar{A} = U$

As $\bar{A} = U - A = \{6,7,8,9 \ldots\}$

So $\Rightarrow \{1,2,3,4,5\} \oplus \{6,7,8,9 \ldots\}$

$= \{1,2,3,4,5,6,7 \ldots\} \ldots (i)$

The term (i) is equals to Universal set

$= U$

Hence Proved!

Q38:

Show that if A and B are sets, then

Solution:

we consider that

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{3, 4, 5, 6, 7\}$$

Because it is not specified in the question so we considered the set by self.

(a) $A \oplus B = B \oplus A$

Left Hand Side:

$A \oplus B$

$= \{1, 2, 3, 4, 5\} \oplus \{3, 4, 5, 6, 7\}$

$= \{1, 2, 6, 7\}$

Right Hand side:

$B \oplus A$

$= \{3, 4, 5, 6, 7\} \oplus \{1, 2, 3, 4, 5\}$

$= \{1, 2, 6, 7\}$

Hence Proved!

(b) $(A \oplus B) \oplus B = A$

Left Hand Side:

$(A \oplus B) \oplus B$

$A \oplus B = \{1, 2, 3, 4, 5\} \oplus \{3, 4, 5, 6, 7\}$

$A \oplus B = \{1, 2, 6, 7\}$

$(A \oplus B) \oplus B = \{1, 2, 6, 7\} \oplus \{3, 4, 5, 6, 7\}$

$(A \oplus B) \oplus B = \{1, 2, 3, 4, 5\}$

It is equals to A

Hence Proved!

Q no 398

What can you say about the sets A and B if $A \oplus B = A$?

Ans: This condition is only fulfill when the set B is empty. Because symmetric means the set which contains the elements of Both A and B other then that which is present in both.

So the symmetric of two set can only be equals to the set of A or B when one set is the empty set.