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Operation Research Project

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Transportation of EGGs:

A case study using multi-objective
Transportation Problem
(MOTP)

Table 1 :- Distance Matrix (d_{ij})

Origin	D_1	D_2	D_3	Supply
S_1	551	314	280	8
S_2	521	267	341	5
S_3	396	142	193	3
Demand	5	3	2	

The 1st matrix is not balanced Problem. To make it balanced add a column and we considered as a dummy variable that is called D_4

Table :- 2

Origin	D_1	D_2	D_3	D_4	Supply
S_1	551	314	280	0	8
S_2	521	267	341	0	5
S_3	396	142	193	0	3
Demand	5	3	2	6	16

Solve it with one of the transportation method that is called approximation

Origin	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	² 551	² 314	⁶ 280	0	820	280 34 34 211 -
S ₂	² 521	³ 267	341	0	520	267 74 74 180 51
S ₃	³ 396	142	193	0	30	42 51 - - -
Supply	520	30	280	60	16	
	125	125	87	0		
	125	125	87	-		
Penalty	30	47	61	-		
	30	-	61	-		
	521	-	-	-		
	-	-	-	-		

	D ₁	D ₂	D ₃	D ₄
S ₁	² 551	² 314	⁶ 280	0
S ₂	² 521	³ 267	341	0
S ₃	³ 396	142	193	0

The optimal solution select the allocated cells & numbers that is

$$S_1 D_3 = x_{13} = 2, \quad S_1 D_4 = x_{14} = 6$$

$$S_2 D_1 = x_{21} = 2, \quad S_2 D_2 = x_{22} = 3$$

$$S_3 D_1 = x_{31} = 3$$

Origin	D ₁	D ₂	D ₃	D ₄	Supply	Penalty				
S ₁	551	314	² 280	⁶ 0	820	280	34	34	211	-
S ₂	² 521	³ 267	341	0	820	267	74	74	126	520
S ₃	³ 396	142	193	0	30	42	51	-	-	-
Supply	520	30	20	60	16					
	125	125	87	0						
	125	125	87	-						
Penalty	30	47	61	-						
	30	-	61	-						
	521	-	-	-						
	-	-	-	-						

	D ₁	D ₂	D ₃	D ₄
S ₁	551	314	² 280	⁶ 0
S ₂	² 521	³ 267	341	0
S ₃	³ 396	142	193	0

The optimal solution select the allocated cells & numbers that is

$S_1 D_3 = x_{13} = 2$, $S_1 D_4 = x_{14} = 6$

$S_2 D_1 = x_{21} = 2$, $S_2 D_2 = x_{22} = 3$

$$S_3 D_1 = x_{31} = 3$$

Minimum distance = ?

Minimum Cost = ?

$$x_{13} = 2 = 280, x_{14} = 6 = 0 \text{ (dummy)}$$

$$x_{21} = 2 = 521, x_{22} = 3 = 2.67$$

$$x_{31} = 3 = 396$$

So

$$\text{Minimum distance} = 280 + 521 + 2.67 + 396$$

$$= 1464$$

Minimum Transportation Cost =

$$\cancel{2 \times 521} \quad 2 \times 280 + 6 \times 0 + 2 \times 521 + 2.67 \times 3 + 396 \times 3$$

$$= 560 + 0 + 1042 + 801 + 1188$$

$$= 3591$$

Table 3- Time Matrix (t_{ij}) Speed 30 km/h

Destination/ origin	D ₁	D ₂	D ₃	Supply in Lac
S ₁	18.36	10.46	9.33	8
S ₂	17.5	8.9	11.36	5
S ₃	13.25	4.73	6.43	3
Demand in Lac	5	3	2	

The Proposed Matrix is not balanced Problem to make it a balanced Problem we consider a dummy variable that is called D₄

Destination / origin	D ₁	D ₂	D ₃	D ₄	Supply in Lac
S ₁	18.36	10.46	9.33	0	8
S ₂	17.5	8.9	11.36	0	5
S ₃	13.25	4.73	6.43	0	3
Demand in Lac	5	3	2	6	

Now Solve it with the transportation Method of Vogel approximation.

	D_1	D_2	D_3	D_4	supply	Penalty			
S_1	2.36	10.46	² 9.33	⁶ 0	820	7.33	1.13	1.13	9.03
S_2	² 17.5	³ 8.9	11.36	0	820	8.9	2.46	2.46	6.14
S_3	³ 13.25	4.73	6.43	0	80	4.73	1.7	-	-
Demand in Lacks	250	30	20	80					
	4.25	4.17	2.9	0					
	4.25	4.17	2.9	-					
Penalty	0.86	1.56	2.03	-					
	0.86	-	2.03	-					

	D_1	D_2	D_3	D_4
S_1	18.36	10.46	² 9.33	⁶ 0
S_2	² 17.5	³ 8.9	11.36	0
S_3	³ 13.25	4.73	6.43	0

Optimum solution of travelling time (Speed 30 km/h) is

$$x_{13} = 2 \quad x_{14} = 0 \quad x_{21} = 2 \quad x_{22} = 3 \quad x_{33} = 3$$

Therefore minimum travelling time (Speed 30 km/h)

$$x_{13} = 2 = 9.33 \quad x_{14} = 0 = 6 \quad x_{21} = 2 = 17.5$$

$$x_{22} = 3 = 8.9 \quad x_{33} = 3 = 13.25$$

$$9.33 + 17.5 + 8.9 + 13.25$$

$$= 48.98 \text{ hrs}$$

Table 4 Time Matrix (tis) Speed 35 kmph

	D_1	D_2	D_3	Supply
S_1	15.74	8.97	8	8
S_2	14.89	7.63	9.74	5
S_3	11.31	4.06	5.51	3
Demand	5	3	2	

The Proposed Matrix is not balanced Problem to make it a balanced Problem we considered a dummy variable that is called D_4

Destination	D_1	D_2	D_3	D_4	Supply
S_1	15.74	8.97	8	0	8
S_2	14.89	7.63	9.74	0	5
S_3	11.31	4.06	5.51	0	3
Demand	5	3	2	6	

Now Solve it with the transportation Method of Vogel Approximation

Mon ☐ Tues ☐ Wed ☐ Thurs ☐ Fri ☐ Sat ☐

Date: _____

Penalty 8

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	15.74	8.97	² 8	⁶ 0	8 2 0	8 0.97 0.97 2.74 - -
S ₂	² 14.89	³ 7.63	9.74	0	8 2 0	7.63 2.11 2.11 5.15 - -
S ₃	³ 11.31	4.06	5.51	0	8 0	4.06 1.45 - - - -
Demand	8 2 0	8 0	2 0	6 0		
	2.88	3.57	2.49	0		
	3.58	3.57	2.49	-		
Penalty	0.85	1.34	1.74	-		
	0.85	-	1.74	-		
	-	-	-	-		
	-	-	-	-		

	D ₁	D ₂	D ₃	D ₄
S ₁	15.74	8.97	² 8	⁶ 0
S ₂	² 14.89	³ 7.63	9.74	0
S ₃	³ 11.31	4.06	5.51	0
				0

Optimum Solution of travelling time (Speed 35 km/h is

$$x_{13} = 2 \quad x_{14} = 0 \quad x_{21} = 2$$

$$x_{22} = 3 \quad x_{31} = 3$$

Therefore Minimum travelling time (Speed 30 km/h)

$$x_{13} = 2 = 8 \quad x_{14} = 0 = 6 \text{ (dummy)}$$

$$x_{21} = 2 = 14.89 \quad x_{22} = 7.63 = 3$$

$$x_{31} = 3 = 11.31$$

$$= 8 + 14.89 + 7.63 + 11.31 = 41.83 \text{ hours}$$

$$= 41.83 \text{ hours}$$

Table: 5 Time Matrix (t_{ij}) Speed
40 km/h

origin/destination	D ₁	D ₂	D ₃	Supply
S ₁	13.78	7.85	7	8
S ₂	13.03	6.68	8.53	5
S ₃	9.9	3.55	4.83	3
Demand	5	3	2	

Make it balanced by adding column that is considered as dummy variable called D₄

origin	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	13.78	7.85	7	0	8
S ₂	13.03	6.68	8.53	0	5
S ₃	9.9	3.55	4.83	0	3
Demand	5	3	2	6	16

Solve it with one of the transportation method called Vogel Approximation.

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	13.78	7.85	² 7	⁶ 0	8.20	7 0.25 0.25 6.70 -
S ₂	² 13.03	³ 6.68	8.53	0	8.20	6.68 1.85 1.85 4.51 3.03
S ₃	³ 9.9	3.55	4.83	0	3.0	3.55 1.24 - - -
Demand	8.20	8.0	2.0	6.0	16	
	3.13	3.13	2.17	0		
	3.13	3.13	2.17	-		
	0.75	1.17	1.53	-		
	0.75	-	1.53	-		
	13.03	-		-		

	D ₁	D ₂	D ₃	D ₄
S ₁	13.78	7.85	² 7	⁶ 0
S ₂	² 13.03	³ 6.68	8.53	0
S ₃	³ 9.9	3.55	4.83	0

The optimal solution for time Matrix (Volume/h)

$$S_1 D_3 = x_{13} = 2, S_1 D_4 = x_{14} = 6, S_2 D_1 = x_{21} = 2,$$

$$S_2 D_2 = x_{22} = 3, S_3 D_1 = x_{31} = 3$$

Therefore Minimum travelling time (Speed 4 km/h)

$$x_{13} = 2 = 7 \quad x_{14} = 6 = 0 \text{ (dummy)}$$

$$x_{21} = 2 = 13.03 \quad x_{22} = 3 = 6.68 \quad x_{31} = 3 = 9.9$$

$$= 7 + 13.03 + 6.68 + 9.9$$

$$= 36.61 \text{ hrs}$$

Table 6: Time Matrix(t_{ij}) Speed 45km/h

Destination/origin	D ₁	D ₂	D ₃	Demand in lacs
S ₁	12.24	6.98	6.22	8
S ₂	11.58	5.93	7.58	5
S ₃	8.8	3.16	4.29	3
Demand in lacs	5	3	2	

The prepared matrix is not balanced problem to make it a balanced problem we considered a dummy variable that is called D₄

Destination/origin	D ₁	D ₂	D ₃	D ₄	Demands in lacs
S ₁	12.24	6.98	6.22	0	8
S ₂	11.58	5.93	7.58	0	5
S ₃	8.8	3.16	4.29	0	3
Demands in lacs	5	3	2	6	

Use the vogel approximate method for this transportation problem.

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	12.24	6.98	6.22	0	8	2.78
S ₂	11.58	5.93	7.58	0	5	2.77
S ₃	8.8	3.16	4.29	0	3	1.93
Demand	8	3	2	6		
Penalty	2.78	2.77	1.93	0		
	2.78	2.77	1.93	-		
	0.66	1.05	1.36	-		
	0.66	-	1.36	-		
	-	-	-	-		
	-	-	-	-		

	D_1	D_2	D_3	D_4
S_1	12.24	6.98	6.22	0
S_2	11.58	5.93	7.58	0
S_3	8.8	3.16	4.29	0

For optimum solution select the allocation numbers. From the table above.

$$S_1 D_3 = x_{13} = 2, S_1 D_4 = x_{14} = 0$$

$$S_2 D_1 = x_{21} = 2, S_2 D_2 = x_{22} = 3$$

$$S_3 D_1 = x_{31} = 3$$

Now minimum travelling time (speed 45 km/h)
For this select the cell of allocation cell.

$$= 6.22 + 11.58 + 5.93 + 8.8$$

$$= 32.53 \text{ hrs}$$

Table 7: Percent Breakages Matrix (P_{ij})
Speed 30 km/h

Destination/origin	D_1	D_2	D_3	Supply in lack
S_1	5	2.85	2.54	8
S_2	4.77	2.42	3.1	5
S_3	3.61	1.29	1.75	3
Demand in lacks	5	3	2	

The proposed Model is not balanced problem to make it a balanced problem we considered a dummy variable that is called D_4

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Destination/origin	D ₁	D ₂	D ₃	D ₄	Supply in lacs
S ₁	5	2.85	2.54	0	8
S ₂	4.77	2.42	3.1	0	5
S ₃	3.61	1.29	1.75	0	3
Demand in lacs	5	3	2	6	16

Now we solve it with the transportation method of Vogel approximate method.

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	5	2.85	2.54	0	8	2.54 0.31 0.31 2.46
S ₂	4.77	2.42	3.1	0	5	2.42 0.68 0.68 1.67
S ₃	3.61	1.29	1.75	0	3	1.29 0.48 - -
Demand	5	3	2	6		
	1.16	1.13	0.79	0		
Penalty	1.16	1.13	0.79	-		
	0.23	0.43	0.56	-		
	0.23	-	0.56	-		
	4.77	-	-	-		

	D ₁	D ₂	D ₃	D ₄
S ₁	5	2.85	2.54	0
S ₂	4.77	2.42	3.1	0
S ₃	3.61	1.29	1.75	0

Optimum solution of percent breakage of eggs is given as.

$$x_{13} = 2 \quad x_{14} = 6 \quad x_{24} = 2 \quad x_{22} = 3 \\ x_{31} = 3$$

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Minimum Breakages with speed 30km/h

$$x_{13} = 2 = 2.54$$

$$x_{14} = 6 = 0$$

$$x_{21} = 2 = 4.77$$

$$x_{22} = 3 = 2.42$$

$$x_{21} = 3 = 3.61$$

$$= (2.54 + 4.77 + 2.42 + 3.61) / 4$$

$$= 3.34\% \text{ Breakages.}$$

Table 8:- Percent Breakage (p_{ij}) Matrix
Speed 35km/h

Destination/Origin	D ₁	D ₂	D ₃	Supply in lacks
S ₁	5.5	3.13	2.8	8
S ₂	5.2	2.66	3.4	5
S ₃	3.95	1.42	1.93	3
Demand in lacks	5	3	2	

The proposed model is not balanced problem to make it a balanced we considered a dummy variable that is called D₄

Destination/Origin	D ₁	D ₂	D ₃	D ₄	Supply in lacks
S ₁	5.5	3.13	2.8	0	8
S ₂	5.2	2.66	3.4	0	5
S ₃	3.95	1.42	1.93	0	3
Demand in lacks	5	3	2	6	16

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Now solve it with the transportation method of Vogel approximation.

	D ₁	D ₂	D ₃	D ₄	Supply	Penalty
S ₁	5.5	3.13	2.8	0	820	
S ₂	5.2	2.66	3.4	0	820	
S ₃	3.95	1.42	1.93	0	30	
Demand	820	30	20	60	16	
	1.25	1.24	0.87	0		
Penalty	1.25	1.24	0.87	-		
	0.3	0.47	0.6	-		
	0.3	-	0.6	-		
	0.3	-	-	-		
	-	-	-	-		

Optimum solution of percent breakage of Eggs is given as
 $x_{13} = 2$ $x_{14} = 6$ $x_{21} = 2, x_{22} = 3$
 $x_{31} = 3$

Minimum Breakage with speed 35km/h
 $x_{13} = 2.8 = 2$ $x_{14} = 6 = 0$
 $x_{21} = 2 = 5.2$ $x_{22} = 3 = 2.66$
 $x_{31} = 3 = 3.95$

$$= (2.8 + 5.2 + 2.66 + 3.95) / 4$$

= 3.65% Breakages.

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Table 9: Percent Breakage Matrix (Pij)
Speed 40 km/h

Destination/origin	D ₁	D ₂	D ₃	Supply in lacks
S ₁	6	3.42	3.05	8
S ₂	5.67	2.91	3.71	5
S ₃	4.13	1.55	2.1	3
Demand in lacks	5	3	2	

The Proposed Model is not balanced problem to make it a balanced problem we considered a dummy variable.

Destination/origin	D ₁	D ₂	D ₃	D ₄	Supply in lacks
S ₁	6	3.42	3.05	0	8
S ₂	5.67	2.91	3.71	0	5
S ₃	4.13	1.55	2.1	0	3
Demand in lacks	5	3	2	6	16

Now solve it with the transportation method of vogel approximation

	D ₁	D ₂	D ₃	D ₄	Supply	
S ₁	6	3.42	3.05	0	8	3.42 3.71 3.05 0
S ₂	5.67	2.91	3.71	0	5	5.67 2.91 3.71 0
S ₃	4.13	1.55	2.1	0	3	4.13 1.55 2.1 0
Demand in lacks	5	3	2	6	16	5 3 2 6
	1.54	1.36	0.95	0		
	1.54	1.36	0.95	-		
	0.33	0.8	0.66	-		
	0.33	-	0.66	-		
	5.67	-	-	-		

Optimum Solution of Percent breakage of eggs is given as

$$x_{13} = 2 \quad x_{14} = 6 \quad x_{21} = 2 \quad x_{22} = 3 \\ x_{31} = 3$$

Minimum Breakage with speed 40 km/h

$$x_{13} = 2 = 3.05$$

$$x_{14} = 6 = 0$$

$$x_{21} = 2 = 5.67$$

$$x_{22} = 3 = 2.91$$

$$x_{31} = 3 = 4.13$$

$$= (3.05 + 5.67 + 2.91 + 4.13) / 4 \\ = 3.94\% \text{ Breakages}$$

Table 10: Percent Breakage Matrix (P_{ij})
Speed 45 km/h

Destination/Origin	D_1	D_2	D_3	Supply in lack
S_1	6.5	3.71	3.3	8
S_2	6.15	3.15	4.02	5
S_3	4.67	1.68	2.28	3
Demand in lack	5	3	2	

The proposed Model is not balanced problem to make it a balanced we considered a dummy variable that is called D_4

Destination/origin	D_1	D_2	D_3	D_4	Supply in lack
S_1	6.5	3.7	3.3	0	8
S_2	6.15	3.15	4.02	0	5
S_3	4.67	1.68	2.28	0	3
Demand in lack	5	3	2	6	16

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	D_1	D_2	D_3	D_4	Supply	Penalty
						5.3 0.4 0.4 3.2
S_1	6.5	3.7	3.3	0	80	3.15 0.8 0.8 2.2
S_2	6.15	3.15	4.02	0	80	1.68 0.6 - -
S_3	4.62	1.68	2.28	0	80	
Demand	80	80	20	60	16	
	1.48	1.47	1.02	0		
Penalty	1.48	1.47	1.02	-		
	0.35	0.55	0.72	-		
	0.85	-	0.72	-		
	-	-	-	-		

Optimum solution of percent of eggs is given as

$$x_{13} = 2 \quad x_{23} = 2 \quad x_{22} = 3 \quad x_{31} = 3 \\ x_{14} = 0$$

Minimum Breakages with speed 45 km/h

$$x_{13} = 2 = 3.3 \quad x_{21} = 2 = 6.15$$

$$x_{14} = 0.6 \text{ (Dummy Value)}$$

$$x_{22} = 3 = 3.15 \quad x_{31} = 4.67$$

$$= (3.3 + 6.15 + 3.15 + 4.67) / 4$$

$$= 4.32\% \text{ Breakage}$$

The End