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Roll no:-
002

Programme:~ BS-SE

Operational

Research

Question no #1

Time Period Covered

Time period	1	2	3	4	5	Minimum number of Agent needed
6:00Am to 8Am ✓						48
8:00Am to 10Am ✓	✓					79
1:00Pm to Noon ✓	✓					65
Noon to 2:00Pm ✓	✓	✓				87
2:00Pm to 4 Pm ✓	✓	✓				64
4:00pm to 6 Pm		✓	✓			73
6:00pm to 8 pm			✓	✓		82
8:00pm to 10:00pm				✓	✓	43
10:00pm to Midnight				✓	✓	52
Midnight to 8:00am					✓	15
Daily Cost per Agent						

$$\text{Minimize } Z = 170x_1 + 160x_2 + 175x_3 + 180x_4 + 195x_5$$

Subject to

$$x_1 = 48 \rightarrow (a)$$

$$x_1 + x_2 = 79 \rightarrow (b)$$

$$\begin{aligned}
 x_1 + x_2 + x_3 &= 87 \rightarrow (d) \\
 x_2 + x_3 &= 64 \rightarrow (e) \\
 x_3 + x_4 &= 73 \rightarrow (f) \\
 x_3 + x_4 &= 82 \rightarrow (g) \\
 x_4 &= 43 \rightarrow (h) \\
 x_4 + x_5 &= 52 \rightarrow (i) \\
 x_5 &= 15 \rightarrow (j)
 \end{aligned}$$

from eq (a), (b) & j

$$x_1 = 48$$

$$x_4 = 43$$

$$x_5 = 15$$

as in eq (c), 65 is lesser than 79 in eq (b) so we neglect eq (c) now we have

$$x_1 + x_2 = 79$$

Putting the value of x_1

$$49 + x_2 = 79$$

$$x_2 = 31$$

From x_3 , we take eq (g) as in eq (g), 82 is greater than other eq (t, h, e) so taking equation (g)

$$x_3 + x_4 = 82$$

$$x_3 + 43 = 82$$

$$x_3 = 39$$

Now we have $(x_1, x_2, x_3, x_4, x_5)$ values $(48, 31, 39, 43, 15)$ respectively.

Now the minimize equation:

$$Z = 170x_1 + 160x_2 + 175x_3 + 180x_4 + 195x_5$$
$$= 170(48) + 160(31) + 175(39) + 180(43) + 195(15)$$

$$Z = 30650$$

Question no#2

1)

Maximize Subject to:

$$\rightarrow Z = x_1 - 3x_2 + 3x_3$$

to

$$3x_1 - x_2 + 2x_3 = 7$$

$$-2x_1 - 4x_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

Solution:

$$7 = x_1 - 3x_2 + 3x_3$$

$$3x_1 - x_2 + 2x_3 + S_1 = 7$$

$$-2x_1 - 4x_2 + 0.5S_1 + S_2 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + 0.5S_1$$

$$+ 0.5S_2 + S_3 = 10$$

	C_j	1	-3	3	0	0	0	B	θ
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	7	
0	S_1	3	-1	2	1	0	0	12	
0	S_2	-2	-4	0	0	1	0	10	
0	S_3	-4	3	8	0	0	2	0	
	$Z_j = C_j$	0	0	0	0	0	0		
	$C_j - Z_j$	1	-3	3	0	0	0		
				\downarrow					
				Key					

column

(Not Possible)

2)

$$Z = 5x_1 + 3x_2$$

Minimize to:

$$x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$$

$$x_1 + x_2 + s_1 + 0.5s_2 + 0.5s_3 = 2$$

$$5x_1 + 2x_2 + 0s_1 + s_2 + 0.5s_3 = 10$$

$$3x_1 + 8x_2 + 0s_1 + 0s_2 + s_3 = 12$$

C_j	5	3	0	0	0		
Basic	x_1	x_2	s_1	s_2	s_3	b	θ
s_1	1	1	1	0	0	2	$2/1 = 2$
s_2	5	2	0	1	0	10	$10/2 = 5$
s_3	3	8	0	0	1	12	$12/8 = 1.5$
$Z_j - C_j$	0	0	0	0	0	0	
$C_j - Z_j$	5	3	0	0	0		

x_2 is Key Column as

3 is minimum value in C_j

s_2 is Key row as 5 is max value

2 is Key element.

$C_j = 0$ for Minimization function
 There is no negative value
 in C_j
 $x_1 = 0$ $x_2 = 0$ $\text{Min}(Z) = 0$

So

$$Z = 5(0) + 3(0) = 0$$

3) $Z = 2x_1 - x_2 + 2x_3$
 Maximize subject to

$$2x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 - 2x_3 \leq 20$$

$$x_1 + 2x_3 \leq 5$$

$$L = 2x_1 - x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$$

$$2x_1 + x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 10$$

$$x_1 + 2x_2 - 2x_3 + 0s_1 + s_2 + s_3 = 20$$

$$x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + s_3 = 5$$

	C_j								
OB	Basic	x_1	x_2	x_3	S_1	S_2	S_3	B	b
	S_1	2	1	0	1	0	0	10	$\frac{10}{2} = 5$
	S_2	1	2	-2	0	1	0	20	$\frac{20}{2} = 10$
	S_3	1	0	2	0	0	1	5	$\frac{5}{1} = 5$
		1							
	$Z_j = \sum C_j \cdot a_{ij}$	0	0	0	0	0	0	0	0
	$C_j - Z_j$	2	-1	-1	2	0	0	0	
		Key Column							

I is the key element

$$R_1 \Rightarrow R_1 - 2R_3 = 0$$

$$R_2 \Rightarrow R_2 - R_3 = 0$$

$$C_j - Z_j = 0$$

	C_j	2	-1	0	0	0	0	
CB	Basic	x_1	x_2	x_3	S_1	S_2	S_3	B
0	S_1	0	1	-4	1	0	-2	0
0	S_2	0	2	-4	0	1	-1	15
2	x_1	1	0	2	0	0	1	5
	$Z_j = \sum C_j \cdot a_{ij}$	2	0	-4	0	0	-2	10
	$C_j - Z_j$	0	-1	-2	0	0	-2	

Optimal solution

$$x_1 = 5 \quad x_2 = 0 \quad x_3 = 0$$
$$Z (\text{maximum value}) = 10$$

Check :-

$$Z = 2x_1 - x_2 + 2x_3$$

$$= 2(5) - 0 + 2(0)$$

$$Z = 10 (\text{confirmed})$$
