Name:- Faizan Nadeem
Roll no:- 002
Program:- BSSE

Assignment # 3

Degree Sequence,

Question 36:
Find the degree of sequences for each of the graph in Exercise 21 -25

Exercise 21: 4, 1, 1, 1, 1

Exercise 22: 3, 3, 2, 2, 2

Exercise 23: 4, 3, 3, 2, 2, 2

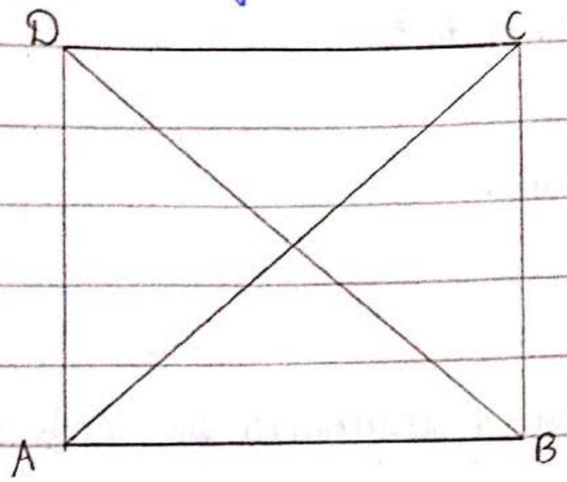
Exercise 24: 4, 4, 2, 2, 2, 2

Exercise 25: 3, 3, 3, 3, 2, 2

Question 37:
Find the degree sequence of each of the following graphs.

(a) K_4

K_4 has 4 vertices and there need to be an edge between every pair of vertices.
The graph is given below.



The degree of vertex is the number of edges that connect to the vertex

deg(A) = 3
deg(B) = 3
deg(C) = 3
deg(D) = 3

The degree sequence is the nonincreasing sequence of the degrees of the vertices.

Degree sequence = 3, 3, 3, 3

(b) C_4

C_4 has 4 vertices A,B,C,D where A and B are connected, B and C are connected, C and D are connected, and D and A are connect.

The graph is given below.

The degree of vertex ~~it~~ is the number of edges that connect to the vertex.

deg(A) = 2

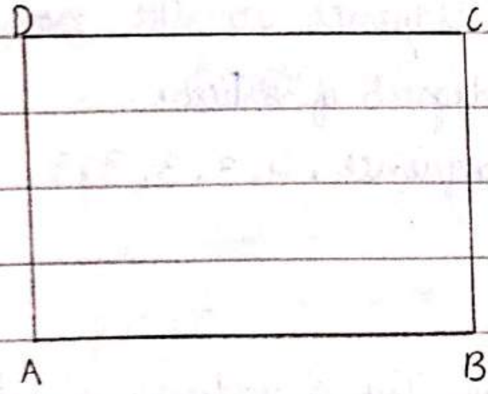
deg(B) = 2

deg(C) = 2

deg(D) = 2

The degree sequence is the nonincreasing sequence of the vertices.

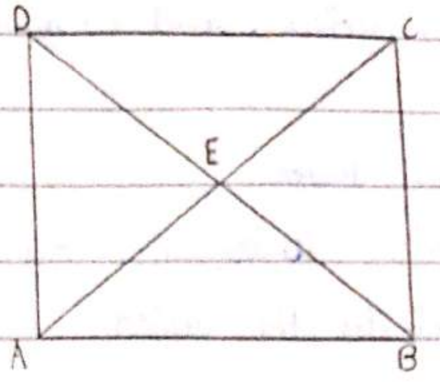
Degree sequence: 2, 2, 2, 2



(c) W_4

W_4 is the graph of C_4 in part (b) to which a vertex E was added and this vertex is connected with all other vertices

This graph is given below.



The degree of vertex is the number of edges that connect to the vertex.

$deg(A) = 3$

$deg(B) = 3$

$deg(C) = 3$

$deg(D) = 3$

$deg(E) = 4$

The degree sequence is the nonincreasing sequence of degrees of vertices.

Degree sequence = 4, 3, 3, 3, 3

(d) $K_{2,3}$

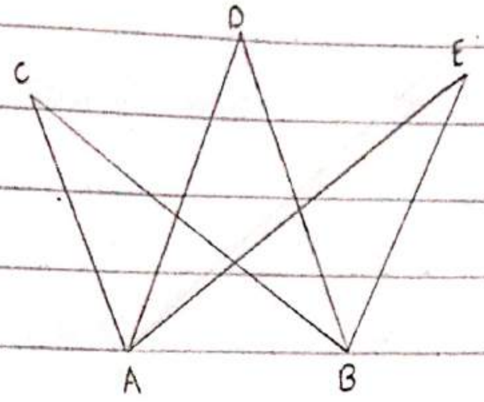
$K_{2,3}$ has two sets of vertices: a set of 2 vertices and a set of 3 vertices.

$M = \{A, B\}$

$N = \{C, D, E\}$

The vertices of M sould be ~~(conno)~~ connected to every vertex in N.

The graph is as follow.



The degree of vertex is the number of edges that connect to the vertex:

deg(A) = 3

deg(B) = 3

deg(C) = 2

deg(D) = 2

deg(E) = 2

The degree of sequence is the non-increasing sequence of the degree of the vertices.

Degree sequences = 3, 3, 2, 2, 2

(c) Q_3

Q_3 has $2^3 = 8$ vertices which are the bit string of length 3: 000, 001, 011, 010, 100, 101, 110, 111

Two vertices of Q_3 are connected, if the two corresponding bit string differ by 1 bit.

deg (000) = 3

deg (001) = 3

deg (010) = 3

deg (011) = 3

deg (100) = 3

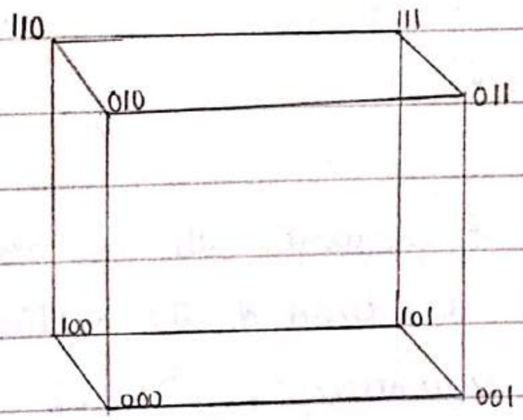
deg (101) = 3

deg (110) = 3

deg (111) = 3

The degree sequence is the nonincreasing sequence of the degree of the vertices.

Degree sequence : 3, 3, 3, 3, 3, 3, 3, 3



Question 38:

What is the degree sequence of the bipartie graph $K_{m,n}$ where m and n are positive integer? Explain your answer.

$K_{m,n}$ has two set of vertices : a set of m vertices M and set of n vertices N.

The vertices in M should be connected to every vertex in N.

Every vertex of M is then connected to the n vertices of N.
Every vertex of N is then connected to the m vertices of M.

The degree of vertex is the number of edges that connect to the vertex
Let $x_1 \in M$ and let $y_1 \in N$

$$deg(x_1) \cdot n$$
$$deg(y_1) \cdot m$$

The degree sequence is the nonincreasing sequence of the degrees of the vertices. The degree sequence then contain n, which is repeated m times, and contain m, which is repeated n times.

Degree sequence if $m \geq n$: $\underbrace{m, m, \cdots m}_{m \text{ repetition}}, \underbrace{n, n, \ldots n}_{n \text{ repetation}}$

Degree sequence if $m < n$: $\underbrace{n, n, \cdots n}_{n \text{ representation}}, \underbrace{m, m \ldots m}_{m \text{ representation}}$

Question 39:

What is the degree sequence of K_n, where n is the positive integers Explain your answer.

K_n has n vertices N and each vertex is connected to each of the other $n-1$ vertices.

The degree of vertices is the number of edges that connect to the vertex

Let $n_i \in N$

$$deg(n_i) = n-1$$

The degree sequence is the nonincreasing sequence of the degrees of the vertices. The degree sequence then contains n, which is repeated m times and contains m, which is repeated n times.

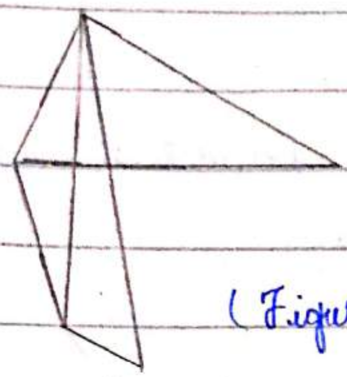
$$\text{Degree sequence} = \underbrace{n-1, n-1, \cdots\cdots n-1}_{n \text{ repetition}}$$

Question 40:

How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2?

Draw such a graph.

$$\text{Edges} = \frac{\text{Sum of degree}}{2} = \frac{14}{2} = 7$$

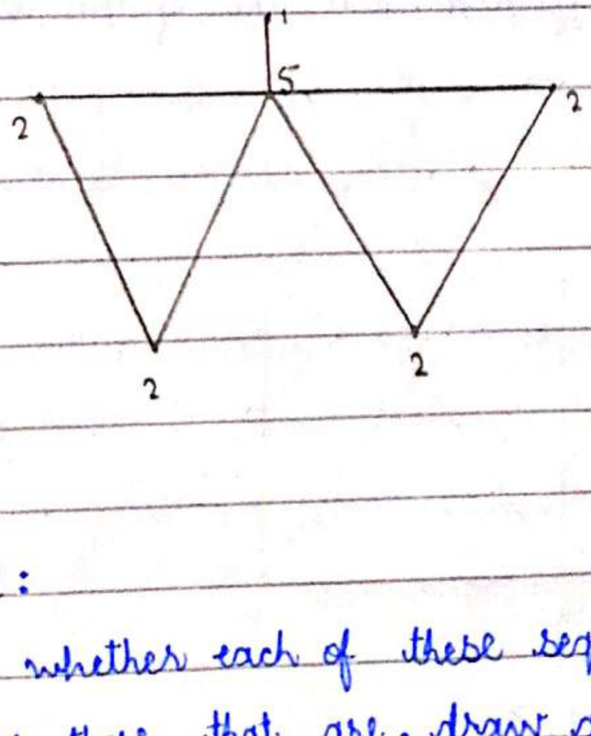


(Figure)

Question 41:

How many edges does a graph have if its degree sequence is 5, 2, 2, 2, 2, 1? Draw such graph.

Edges = $\frac{\text{Sum of degree}}{2} = \frac{14}{2} = 7$



Graphic

Question 42:

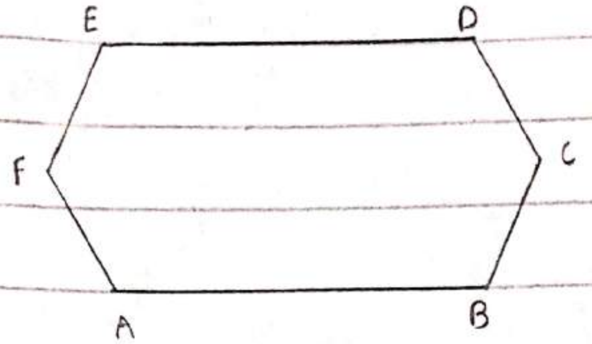
Determine whether each of these sequence is graphics for those that are, draw a graph having the given degree sequence.

(a) 5, 4, 3, 2, 1, 0

It is not graphic because it have three odd degrees (5, 3, 1)

(c) 2, 2, 2, 2, 2, 2

It is graphic. The graph is given as follow.

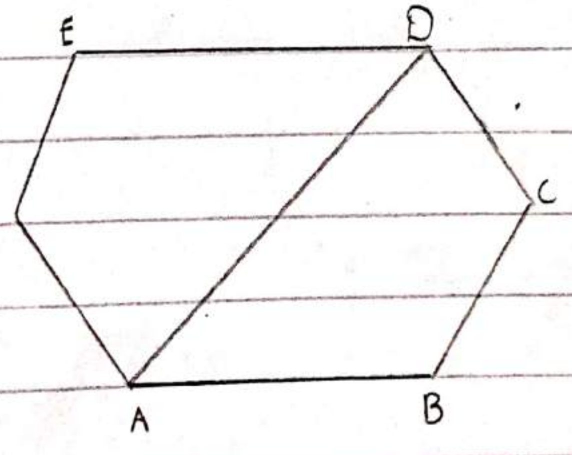


(d) 3, 3, 3, 2, 2, 2

It is not graphic because it have three odd degrees (3, 3, 3)

(e) 3, 3, 2, 2, 2, 2

It is graphic. The graph is given as follow.

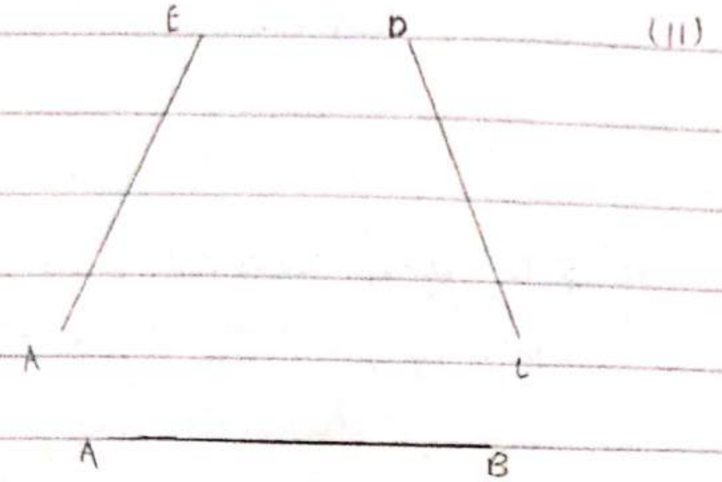


(f) 1, 1, 1, 1, 1, 1

It is graphic. The graph is given as above.

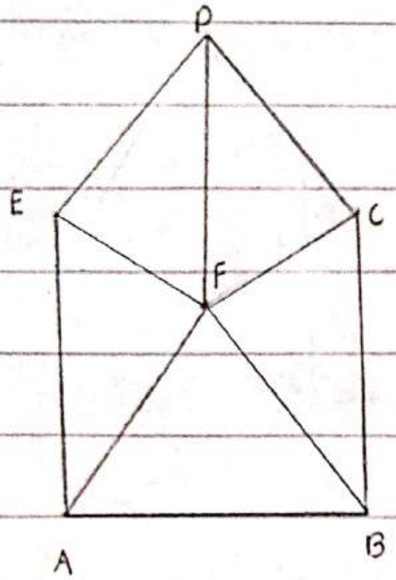
(b) 6, 5, 4, 3, 2, 1, 0

It is not graphic because it have three odd degrees (5, 3, 1)



(g) 5, 3, 3, 3, 3, 3

This is graphic. So the graph is given as follow.



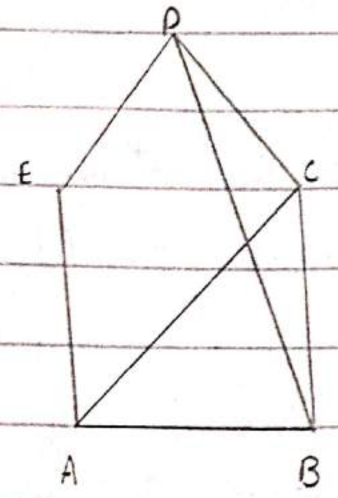
(h) 5, 5, 4, 3, 2, 1

It is not graphic because it have 4 odd degrees (5, 5, 2, 1) and 6 is even.

Question 43:

Determine whether each of these sequence is

graphic. For those that are, draw a graph having the given degree sequence
(a) 3, 3, 3, 3, 2
Yes this sequence is graphic. The graph is given as follow.



(b) 5, 4, 3, 2, 1
No, this sequence is not graphic, as the sum of degree of vertices are odd.
(c) 4, 4, 3, 2, 1
Yes this a graphic sequence apperantly but techniquely there is a complication while the two vertice of degree 4 should be connected to this vertex and thus the vertex of degree 1 should have a degree of at least 2. So, this is not possible.
(d) No, this sequence is not graphic, as the sum of degree of vertices are odd.
(e) 3, 2, 2, 1, 0
Yes this sequence is graphic. The graph is given above.

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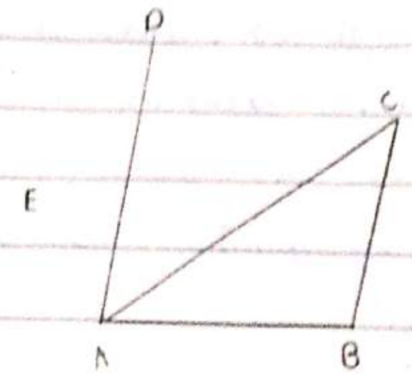

(f) 1, 1, 1, 1, 1

No, this sequence is not graphic, as the sum of degree of vertices are odd.

Regular:

Question 53:

For which values of n are these graphs regular

(a) K_n

In K_n each vertex is connected with other $n-1$ vertices and thus each vertex has degree $n-1$, which means that the graph K_n is regular for all values of n.

$$n \geq 1$$

(b) C_n

In C_n every vertex is connected to exactly two vertices and thus each vertex has degree 2, which mean that the graph C_n is regular for all values of n

$$n \geq 3$$

(c) W_n

In W_n each vertices of C_n have degree 2. Now the addition vertex V_{n+1} will be connected to the n vertices and thus the first n vertices have degree 3 each.

$$\deg(V_i) = 3$$
$$i = 1, 2, \ldots n$$

For additional vertex V_{n+1}

$$\deg(V_{n+1}) = n$$

We then note all vertices have the same degree if n is equal to 3.

$$n = 3$$

(d) Q_n

In Q_n two vertices are connected if two corresponding bit string differ by 1 bit. so

$$n \geq 1$$

Question 54:

For which value of m and n is $K_{m,n}$ regular

$K_{m,n}$ has 2 sets of vertices : a set of vertices M and a set of n vertices N. The vertices in M should be connected to every vertex in N.

$$\deg(x_1) = n$$
$$\deg(y_1) = m$$

Question 55:

How many vertices does a regular graph of degree four with to edges have.

We want to determine a regular graph of degree four with $m = 10$ edges

Let the graph contains n vertices $v_1, v_2 \cdots v_n$ then each of these n vertices have degree 4

$$\deg(v_i) = 4$$

$$i = 1, 2, \cdots n$$

By handshaking theorem

$$20 = 2(10) = 2m = \sum_{v=1}^{n} \deg(v_i) = \sum_{v=1}^{n} 4 = 4n$$

We then obtained the equation $20 = 4n$

$$20 = 4n$$

$$n = \frac{20}{4} = 5$$