Quiz # 2

faizan Nadeem
002
BS_SE

Operation research

a) Optimal solution:-

$$Max\ Z = 3x1 + 5x2 + 3x3$$

Subject to

$$3x1 + 2x2 <= 18$$
$$x1 + x3 <= 4$$
$$x2 + 2x3 <= 6$$

$$x1, x2, x3 >= 0$$

Solution:

$$Z = 3x1 + 5x2 + 3x3$$
$$3x1 + 2x2 + S1 = 18$$
$$x1 + x3 + 0 \cdot S1 + S2 = 4$$
$$x2 + 2x3 + 0 \cdot S1 + 0 \cdot S2 + S3 = 6$$

Initial Simplex Method

CB	Cj Basic	3 x1	5 x2	3 x3	0 S1	0 S2	0 S3	B
0	S1	3	2	0	1	0	0	18
0	S2	1	0	1	0	1	0	4
0	S3	0	1	2	0	0	1	6
	Zj= ΣCB aij	0	0	0	0	0	0	0
	Cj= Cj-Zj	3	5	3	0	0	0	

Key column, Keyrow, Keyelement

CB	Cj Basis	3 x1	5 x2	3 x3	0 S1	0 S2	0 S3	B	θ
0	S1	3	2	0	1	0	0	18	9
0	S2	1	0	1	0	1	0	4	∞
0	S3	0	1	2	0	0	1	6	6 → keyrow
	Zj= ΣCB aij	0	0	0	0	0	0	0	
	Cj= Cj-Zj	3	5↓	3	0	0	0		

Key column

For maximum function

$C_j <= 0$

$R_1 \Rightarrow R_1 - 2R_3 = = 0$

CB	C_j / Basis	3	5	3	0	0	0			
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	B	θ	
0	S_1	3	0	-4	1	0	-2	6	2	→ key row
0	S_2	1	~~1~~0	1	0	1	0	4	4	
5	x_2	0	~~5~~1	2	0	0	1	6	∞	
	$Z_j = \Sigma CB \cdot a_{ij}$	0	5	10	0	0	5	30		
	$C_j = c_j - Z_j$	3	0	-7	0	0	-5			

Key column

key element is 3. First we make key element 1 than simplify.

$R_2 \Rightarrow R_2 - R_1$

CB	C_j / Basis	3	5	3	0	0	0	
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	B
3	x_1	~~3~~1	0	-4/3	1/3	0	-2/3	2
0	S_2	0	0	7/3	-1/3	1	2/3	2
5	x_2	~~3~~0	1	2	0	0	1	6
	$Z_j = \Sigma CB \cdot a_{ij}$	3	5	6	1	0	3	36
	$C_j = c_j - Z_j$	0	0	-3	-1	0	-3	

Optimal Solution

$x_1 = 2$, $x_2 = 6$, $x_3 = 0$

$(max) z = 36$

Change in C_j

Here,

$C = (C_1, C_2, C_3, C_4, C_5, C_6)$

Cost co-efficients associated with basic variable x_1, x_2, S_2 are

$C_\beta = (C_1, C_2, C_5) = (3, 5, 0)$

Change in co-efficient C_j (C_3, C_4, C_6) of non-basic variable x_3, S_1, S_3

ΔC_3, ΔC_4, ΔC_6, New objective function will become

$C_3' = C_3 + \Delta C_3$

$C_3 = 3 \Rightarrow C_3' = 3 + \Delta C_3$

$C_4' = C_4 + \Delta C_4$

$C_4 = 0 \Rightarrow C_4' = 0 + \Delta C_4 \Rightarrow \Delta C_4$

$C_6' = C_6 + \Delta C_6$

$C_6 = 0 \Rightarrow C_6' = 0 + \Delta C_6 = \Delta C_6$

New values of

$\Delta C_3 + (C_3 - Z_3) \Rightarrow \Delta C_3 - 3$

$\Delta C_4 + (C_4 - Z_4) \Rightarrow \Delta C_4 - 1$

$\Delta C_6 + (C_6 - Z_6) \Rightarrow \Delta C_6 - 3$

In order to maintain optimal we must have

$\Delta C_3 - 3 <= 0$

$\Delta C_3 <= 3$

$\Delta C_4 - 1 <= 0$

$\Delta C_4 <= 1$

$\Delta C_6 - 3 <= 0$

$\Delta C_6 <= 3$

(b)

Change in co-efficient (C_1, C_2, C_5) of basic variables (x_1, x_2, S_2)

For $K=1$ for x_1 basic variable in Row 1, we have

$$\min\left\{\frac{C_j - Z_j}{Y_{kj} < 0}\right\} >= \Delta CB_k >= \max\left\{\frac{C_j - Z_j}{Y_{kj} > 0}\right\}$$

$$\min\left\{\frac{-3}{-4/3}\right\} >= \Delta C_1 >= \max\left\{\frac{-1}{1/3}\right\}$$

$$\frac{9}{4} >= \Delta C_1 >= -3$$

$$\frac{9}{4} + 3 >= C_1 >= -3+3$$

$$\frac{21}{4} >= C_1 >= 0 \quad \text{— eq (i)}$$

Scanned with CamScanner

The current optimal solution will not change unless C_1 is in the above eq. 1 range

For $K=2$ S_2 will not affect the objective function because It is not a part of objective function.

For $K=3$ basic variable x_2

$$\text{Min}\left\{\frac{-3}{0}\right\} >= \Delta C_2 >= \text{Max}\left\{\frac{-1}{2}\right\}$$

$$\infty >= \Delta C_2 >= \frac{-1}{2}$$

$$\infty + 5 >= \Delta C_2 >= \frac{-1}{2} + 5$$

$$\infty >= \Delta C_2 >= \frac{9}{2}$$

Current O.S will not change unless C_2 in above eq 2.

Qno 2 :- Senstivity Analysis (Change in RHS constraints)

Max $Z = 3x_1 + 5x_2 + 3x_3$

Subject to :-

$$3x_1 + 2x_2 <= 18$$
$$x_1 + x_3 <= 4$$
$$x_2 + 2x_3 <= 6$$
$$x_1, x_2, x_3 >= 0$$

Solution :-

$$3x_1 + 2x_2 + S_1 = 18$$
$$x_1 + x_3 + 0 \cdot S_1 + S_2 = 4$$
$$x_2 + 2x_3 + 0 \cdot S_1 + 0 \cdot S_2 + S_3 = 6$$

As it is same as Question #1 so, I put values directly

	Cj	3	5	3	0	0	0	
CB	Basis	x1	x2	x3	S1	S2	S3	B
03	x1	1	0	-4/3	1/3	0	-2/3	2
0	S2	0	0	7/3	-1/3	1	2/3	2
5	x2	0	1	2	0	0	1	6
	$z_j = \sum c_j \cdot a_{ij}$	3	5	6	1	0	3	30
	$c_j = c_j - z_j$	0	0	-3	-1	0	-3	

Change in RHS values.

$3x_1 + 2x_2 \leq 20$

$x_1 + x_3 \leq 5$

$x_2 + 2x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

Step 1 :- Find optimal Inverse.

$$\begin{bmatrix} 1/3 & 0 & -2/3 \\ -1/3 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 8/3 \\ 7/3 \\ 6 \end{bmatrix}$$

So, our new optimal solution is equal to :-

	C_j	3	5	3	0	0	0	
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	B
3	x_1	1	0	$-4/3$	$1/3$	0	$-2/3$	$8/3$
0	S_2	0	0	$1/3$	$-1/3$	1	1	$7/3$
5	x_2	0	1	2	0	0	1	6
	$Z_j = \Sigma C_j \cdot a_{ij}$	3	5	6	1	0	3	38
	$C_j = C_j - Z_j$	0	0	-3	-1	0	-3	

Optimal solution:-

$Z(max) = 38 \quad x_1 = 8/3 \quad x_2 = 7/3 \quad x_3 = 0$