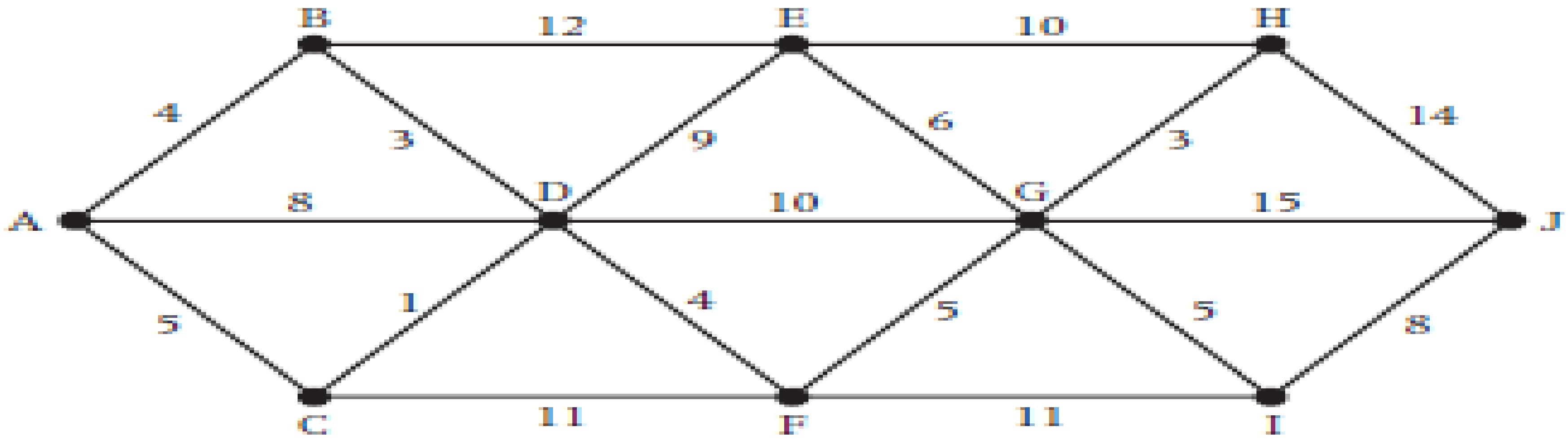
Dijkstra's algorithm

Step 1 Label the start vertex as 0.
Step 2 Box this number (permanent label).
Step 3 Label each vertex that is connected to the start vertex with its distance (temporary label).
Step 4 Box the smallest number.
Step 5 From this vertex, consider the distance to each connected vertex.
Step 6 If a distance is less than a distance already at this vertex, cross out this distance and write in the new distance. If there was no distance at the vertex, write down the new distance.
Step 7 Repeat from step 4 until the destination vertex is boxed.
Note: When a vertex is boxed you do not reconsider it. You need
to show all temporary labels together with their crossings out

Worked Example

Find the shortest distance from A to J on the network below.



Step 1 Label A as 0.

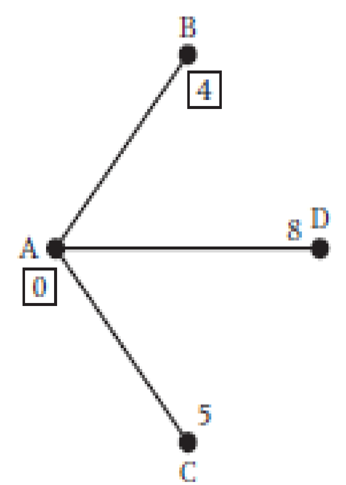
Step 2 Box this number.

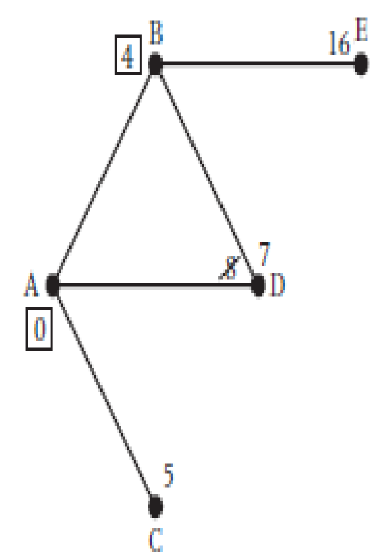
Step 3 Label values of 4 at B, 8 at D and 5 at C.

Step 4 Box the 4 at B.

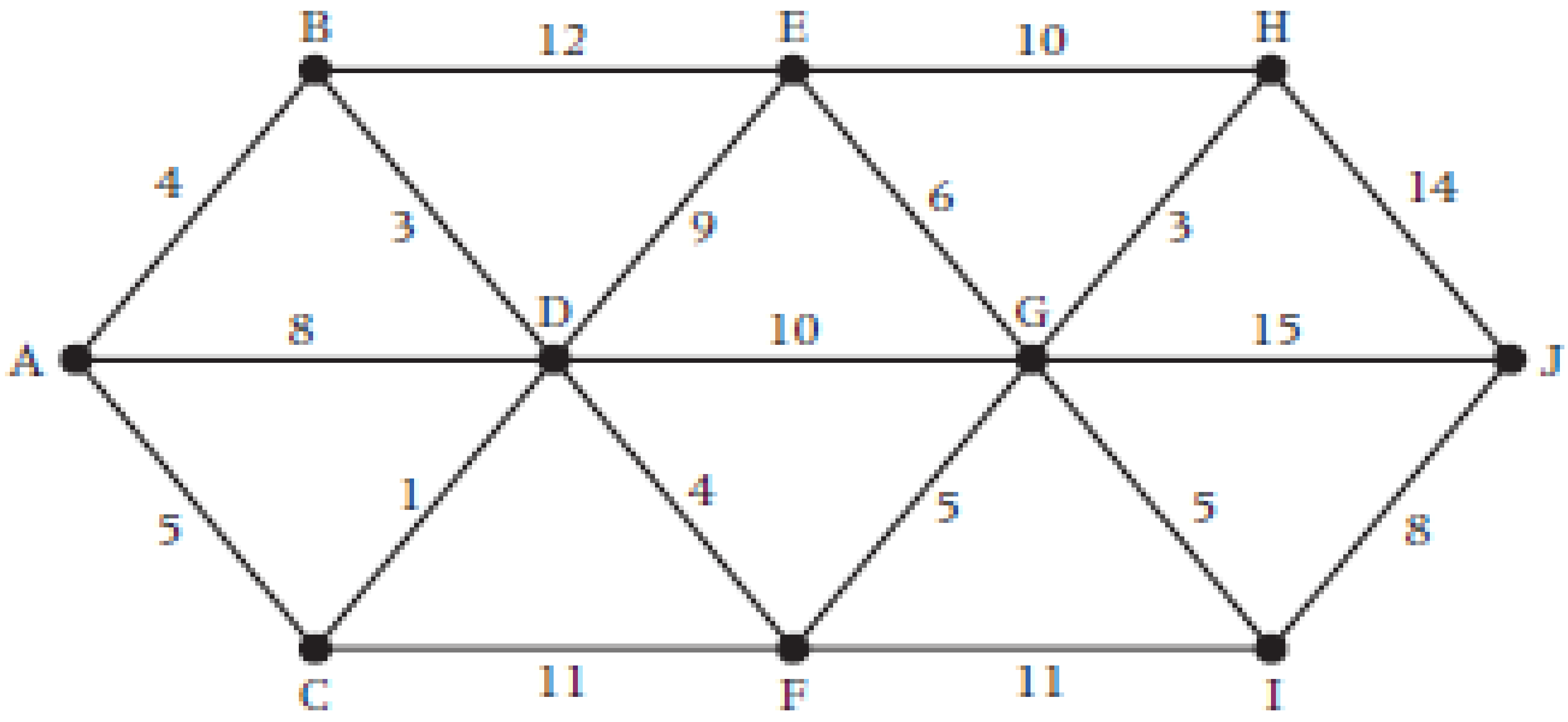
Step 5 From B, the connected vertices are D and E. The distances at these vertices are 7 at D (4 + 3) and 16 at E (4 + 12).

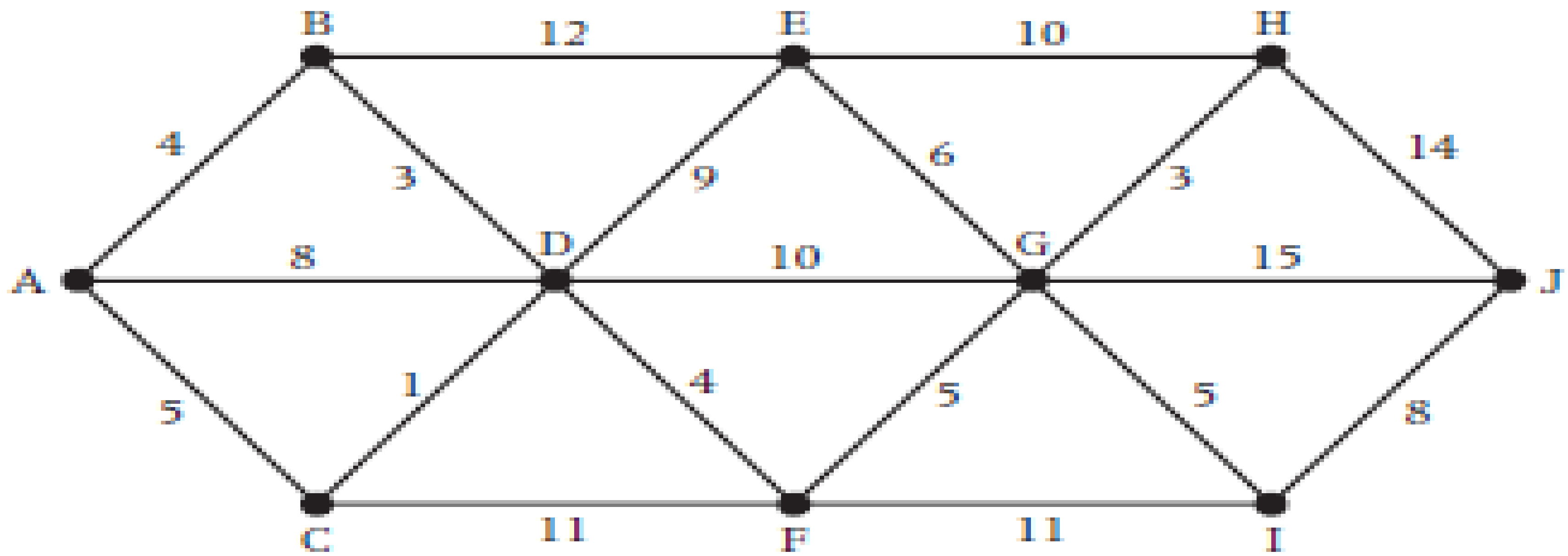
Step 6 As the distance at D is 7, lower than the 8 currently at D, cross out the 8.





Find the shortest distance from A to J on the network below.

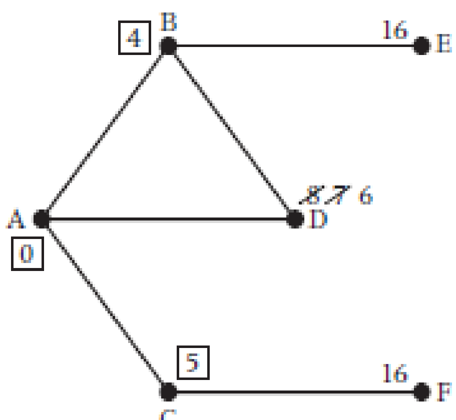




Step 4 Box the smallest number, which is the 5 at C.

Step 5 From C, the connected vertices are D and F. The distances at these vertices are 6 at D (5 + 1) and 16 at F (5 + 11).

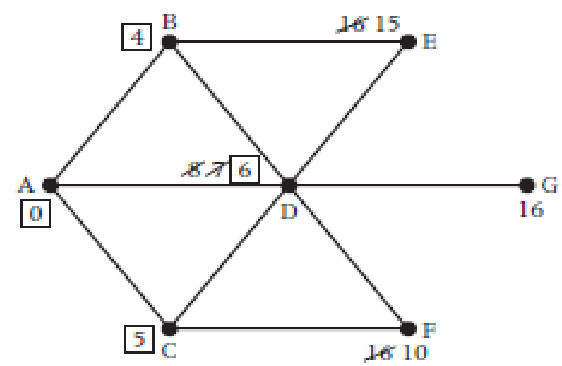
Step 6 As the distance at D is 6, lower than the 7 currently at D, cross out the 7.

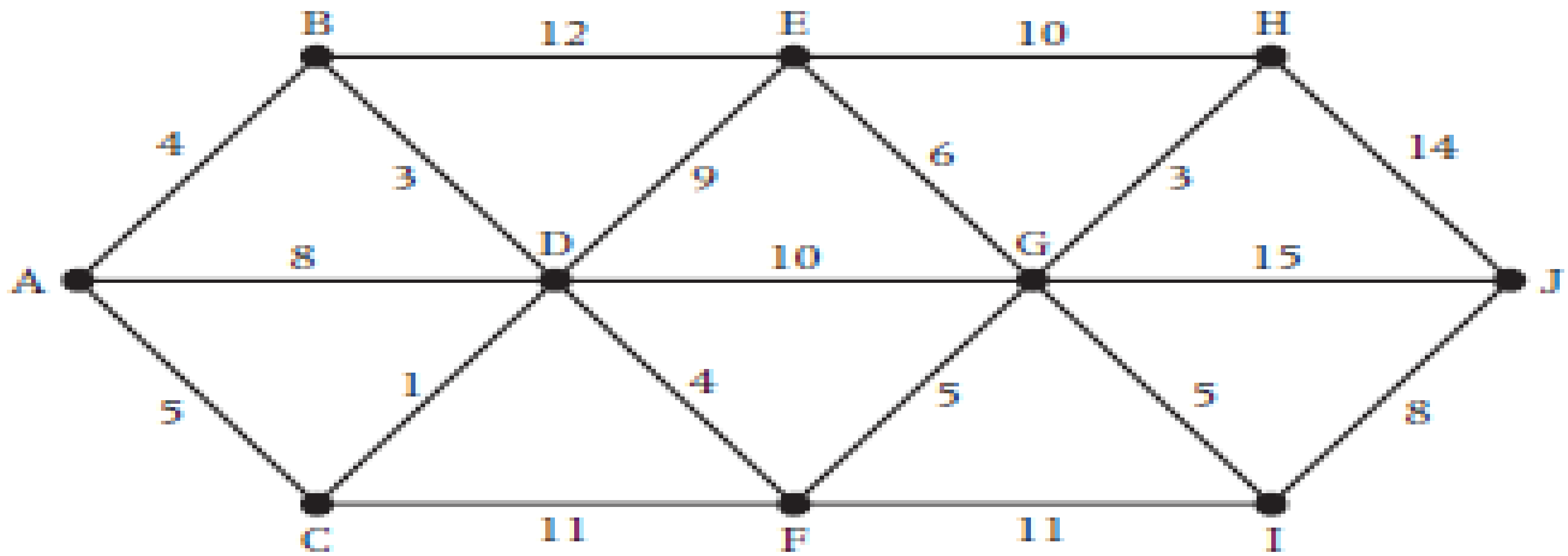


Step 4 Box the smallest number, which is the 6 at D.

Step 5 From D, the connected vertices are E, F and G. The distances at these vertices are 15 at E (6 + 9), 16 at G (6 + 10) and 10 at F (6 + 4).

Step 6 As the distance at E is 15, lower than the 16 currently at E, cross out the 16. As the distance at F is 10, lower than the 16 currently at F, cross out the 16.

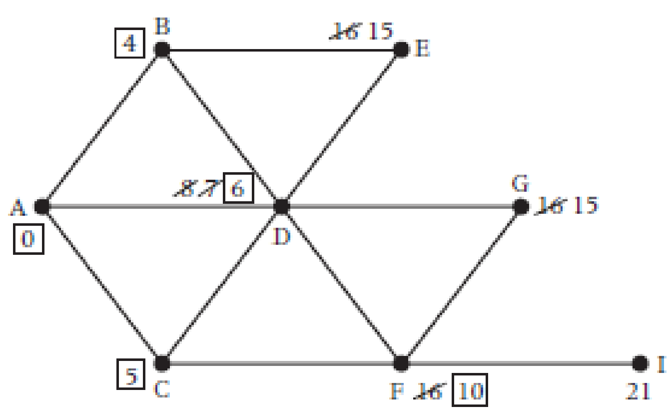




Step 4 Box the smallest number, which is the 10 at F.

Step 5 From F, the connected vertices are G and I. The distances at these vertices are 15 at G (10 + 5) and 21 at I (11 + 10).

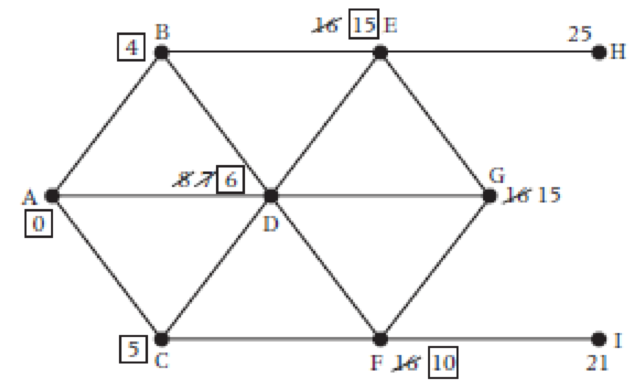
Step 6 As the distance at G is 15, lower than the 16 currently at G, cross out the 16.

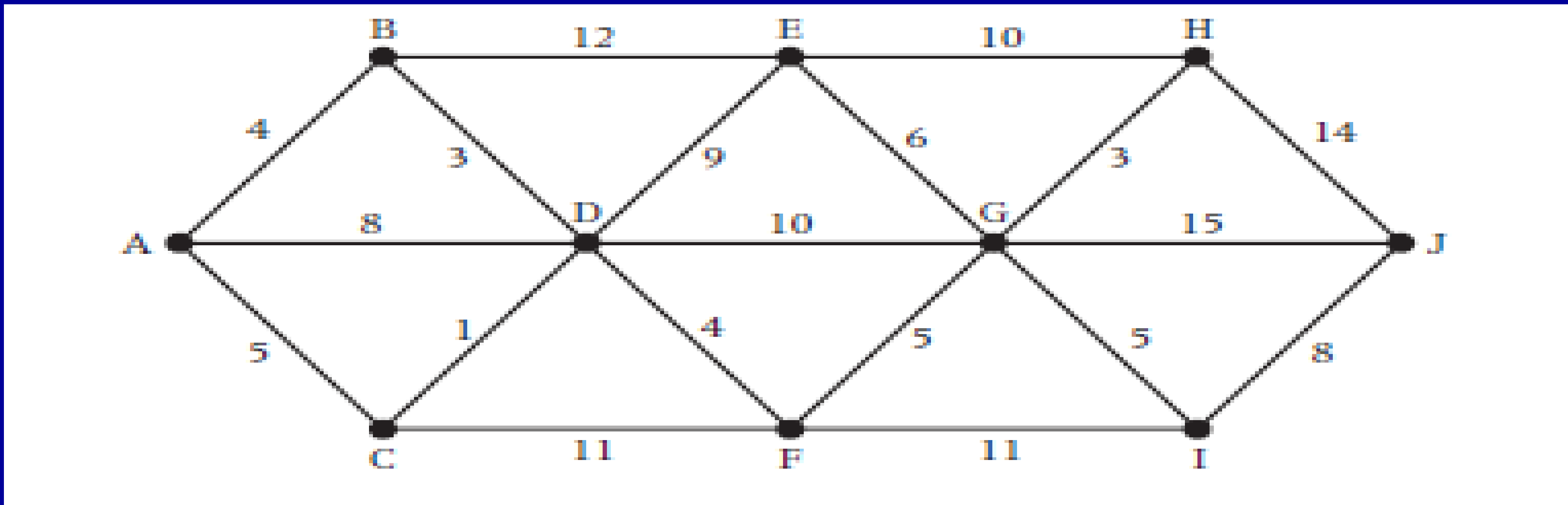


Step 4 Box the smallest number, which the 15 at either E or G (it doesn't matter which you chose).

Step 5 From E, the connected vertices are H and G. The distances at these vertices are 21 at G (15 + 6) and 25 at H (15 + 10). Do not write down the value of 21 at G as this is greater than the number already there.

Step 6 There are no improvements, so there is no crossing out.

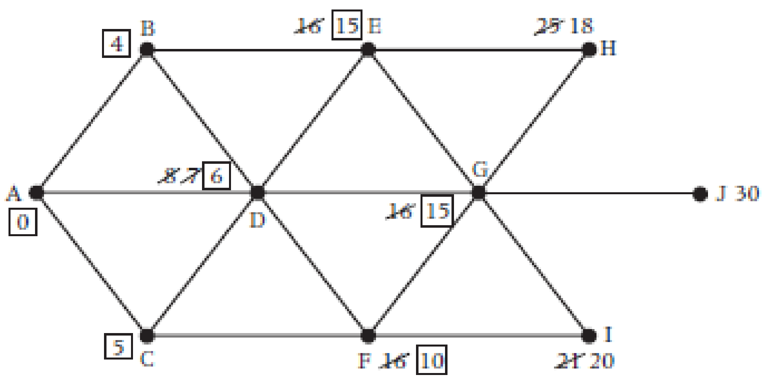




Step 4 Box the smallest number, which is the 15 at G.

Step 5 From G, the connected vertices are H, I and J. The distances at these vertices are 18 at H (15 + 3), 20 at I (15 + 5) and 30 at J (15 + 15).

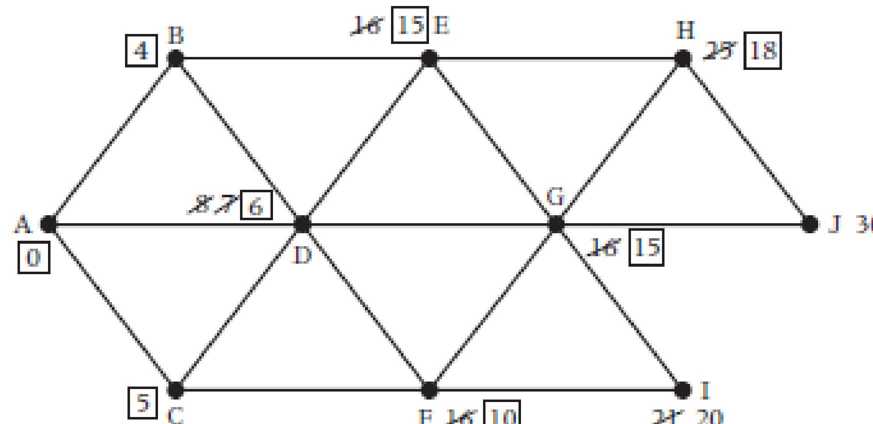
Step 6 As the distance at H is 18, lower than the 25 currently at H, cross out the 25. As the distance at I is 20, lower than the 21 currently at I, cross out the 21.

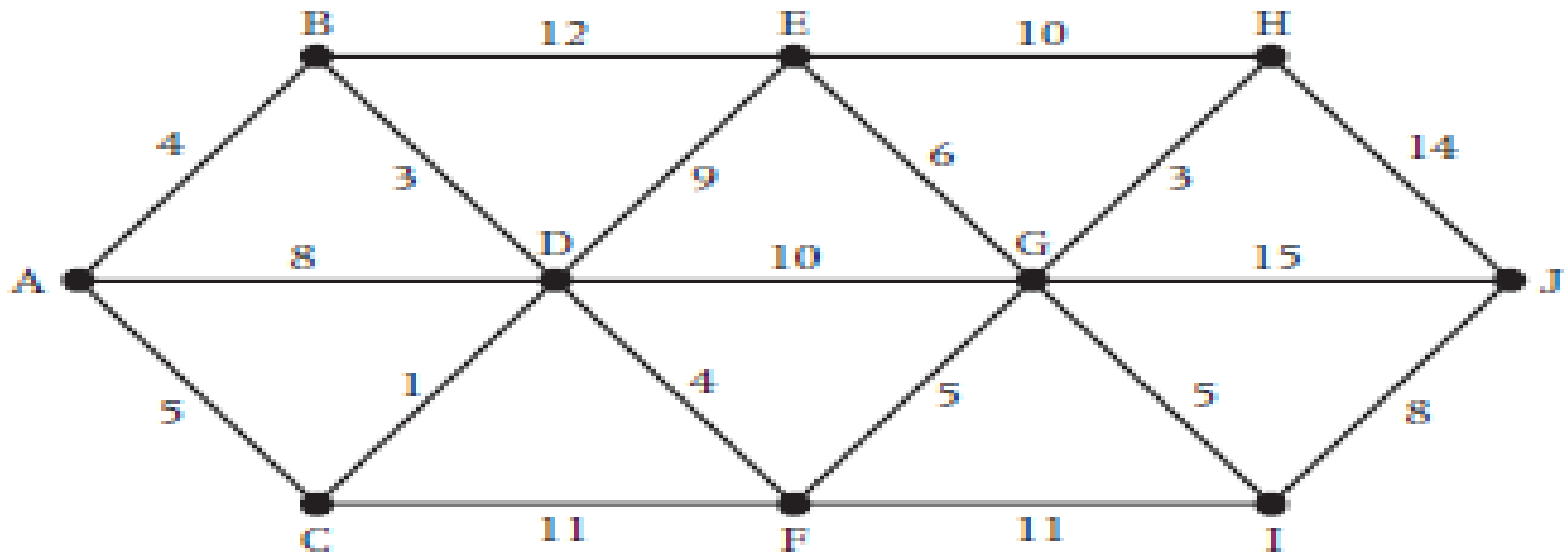


Step 4 Box the smallest number, which is the 18 at H.

Step 5 From H, the connected vertex is J. The distance at this vertex is 32 (18 + 14). Do not write down the value of 32 at J as this is greater than the 30 already there.

Step 6 There are no improvements, so there is no crossing out.

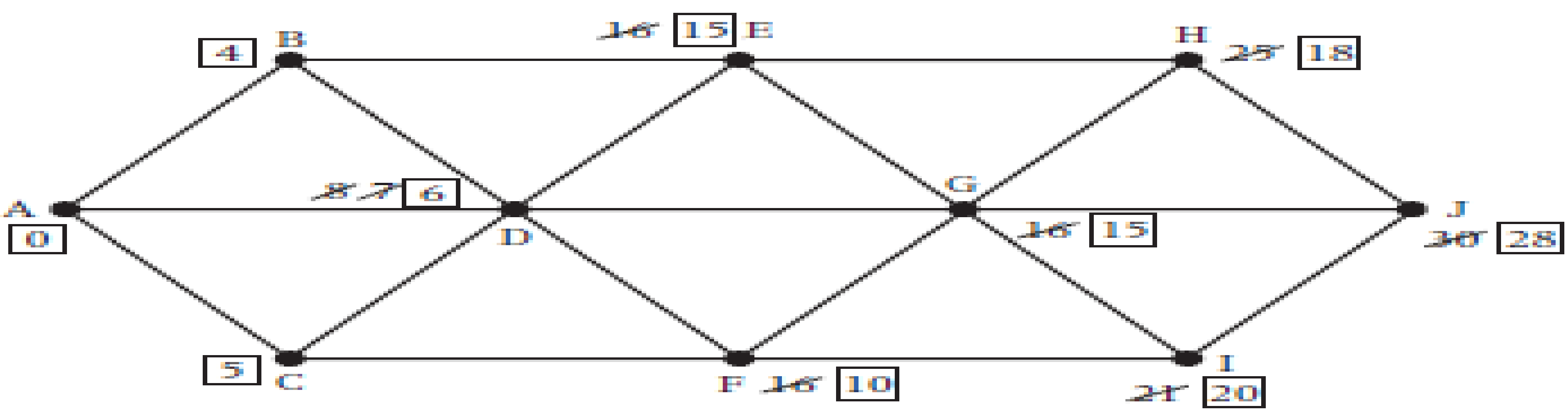


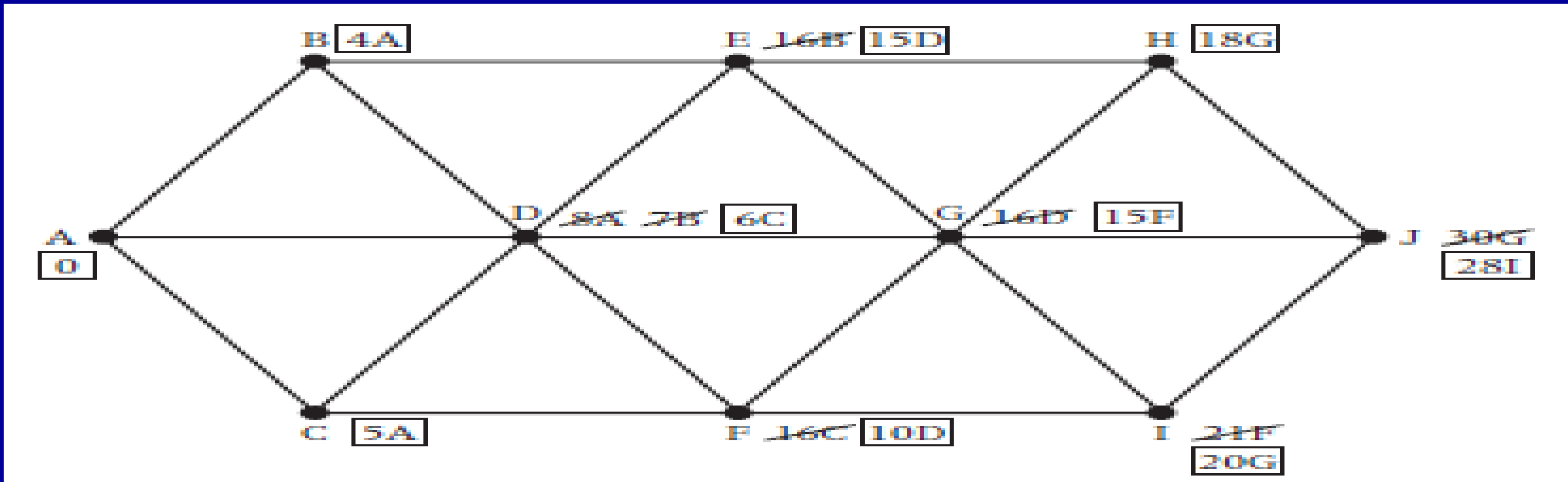


Step 4 Box the smallest number, which is the 20 at I.

Step 5 From I, the connected vertex is J. The distance at this vertex is 28 (20 + 8).

Step 6 As the distance at J is 28, lower than the 30 currently at J, cross out the 30.



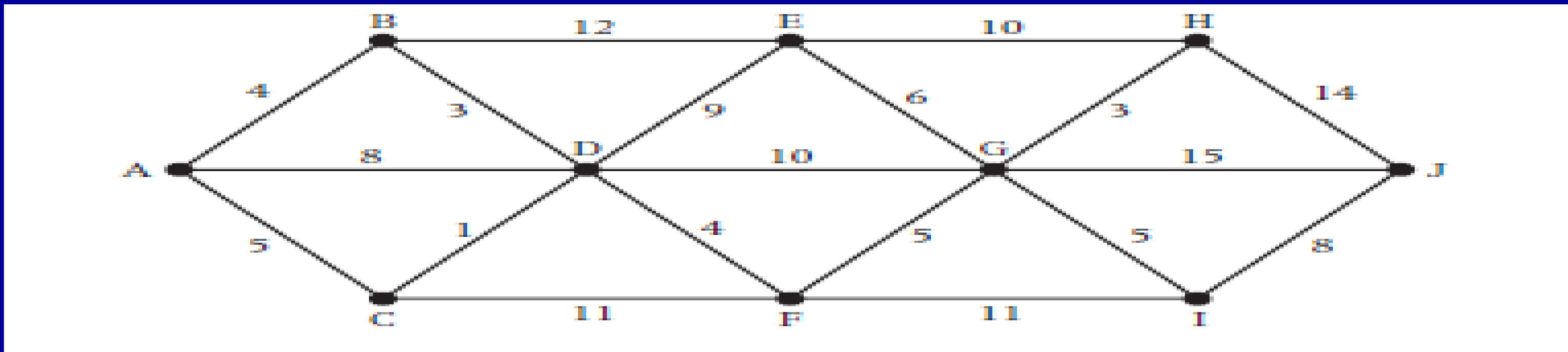


Box A has a value of zero. At B, the distance from A is 4 so we now write 4A. At D, the distance from A is 8 so we write 8A and at C we write 5A.We then box the value of 4 at B, so 4A is now boxed.

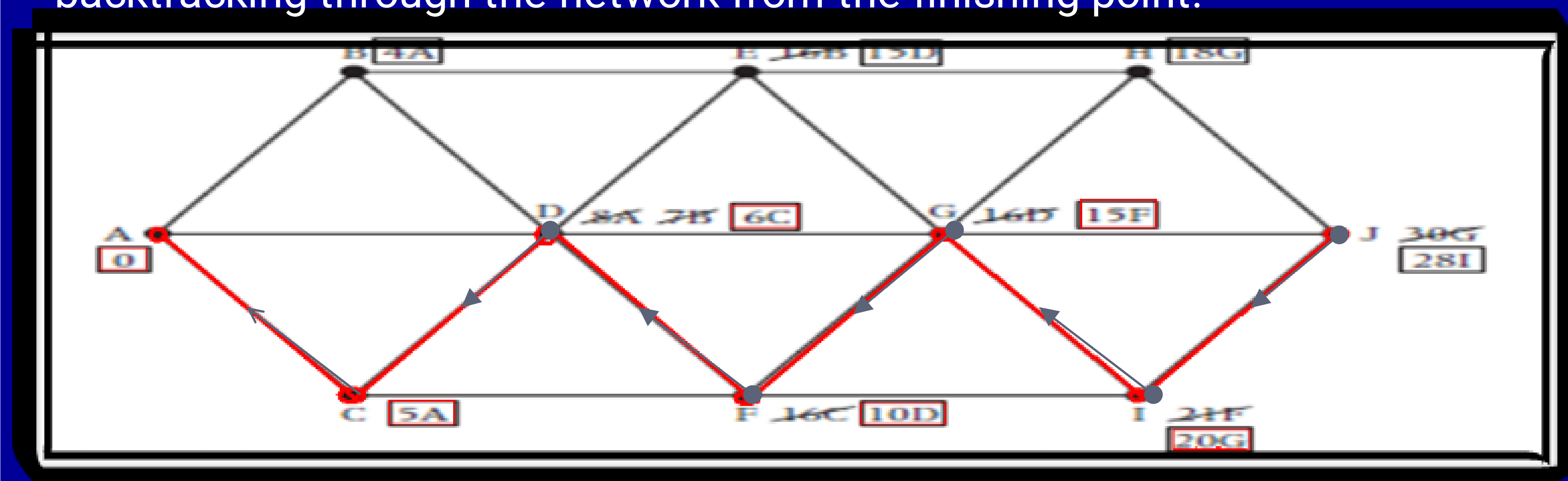
From B, the value at E becomes 16B. At D we get 7B. Box the smallest number, which is the value of 5A at C, so 5A is boxed at C.

From there, there is a value of 16C at F and a value of 6C at D. We box the smallest value, which is 6C at D. At D there is 8A crossed out, 7B crossed out and 6C, which has been boxed. This tells us that to get to D the smallest distance is 6 and we came from vertex C.

Working from the finishing point J we have a boxed value of 28I so we now look at vertex I. Here the boxed value is 20G so we now look at vertex G, and so on until we return to A. Hence the shortest path is ACDFGIJ, with length 28.



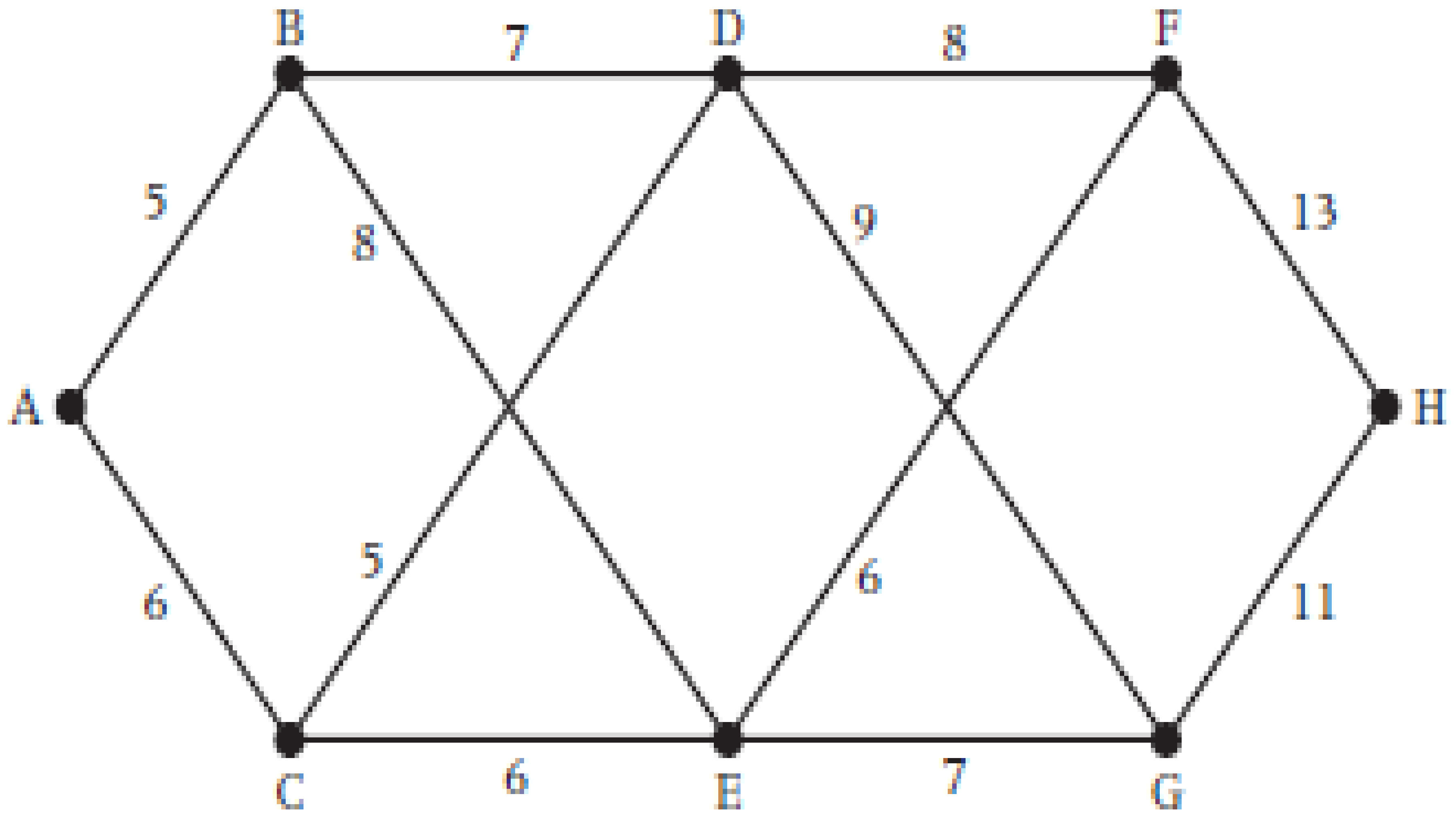
Instead of listing temporary values, we put a letter after each value, which indicates the preceding vertex on the route. We find the route by backtracking through the network from the finishing point.



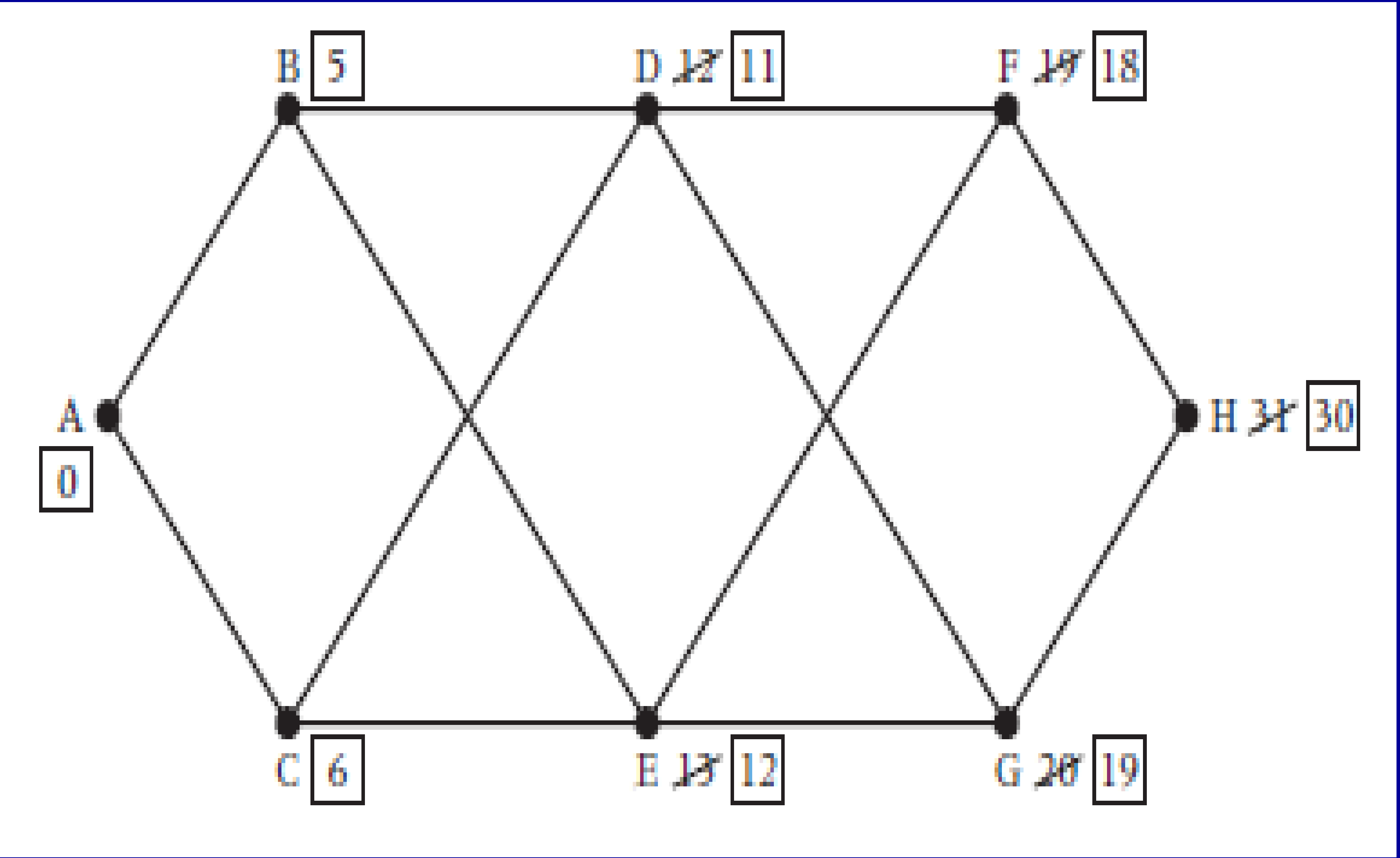
Step 7: The final vertex, in this case J. The boxed number at J is the shortest distance. The route corresponding to this distance of 28 is ACDFGIJ, but this is not immediately obvious from the network.

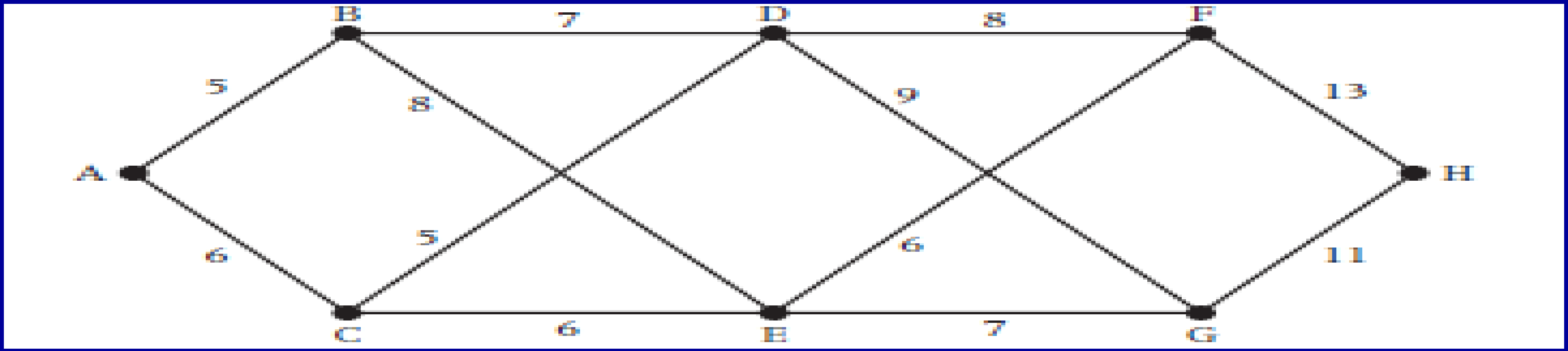
Class Work

Find the shortest distance from A to H on the network below.

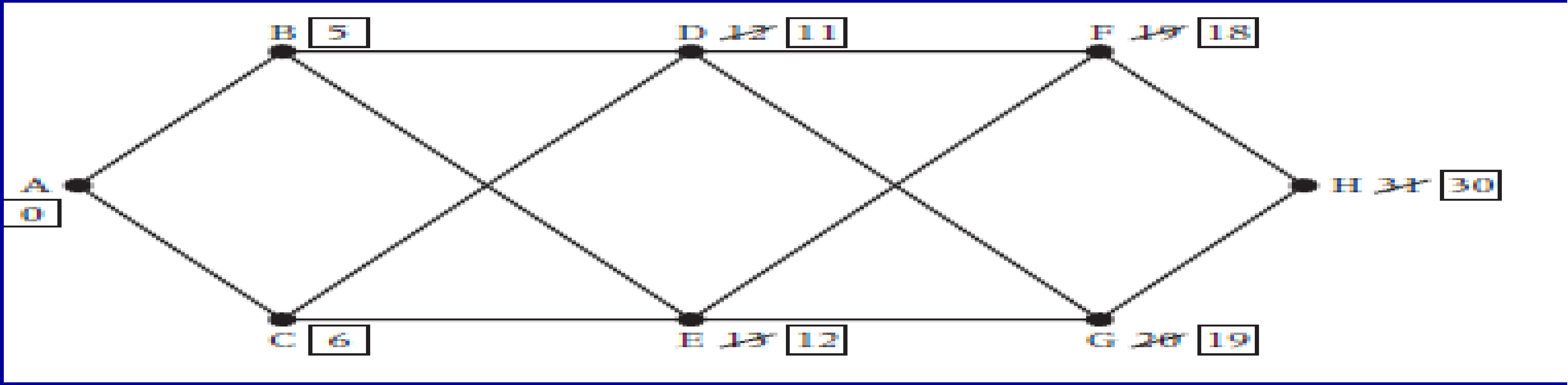


Solution





Instead of listing temporary values, we put a letter after each value, which indicates the preceding vertex on the route. We find the route by backtracking through the network from the finishing point.



The final vertex, in this case H. The boxed number at H is the shortest distance. The route corresponding to this distance of 30 is ACEGH, but this is not immediately obvious from the network.