Q2)

a) $$\lim_{(x,y)\to(0,0)} \frac{7(5x^2-y^2)}{\sqrt[4]{x^2+y^2}}$$

$$= \lim_{r\to a} \frac{7(5r^2\cos^2\theta - r^2\sin^2\theta)}{\sqrt[4]{r^2}}$$

$$= \lim_{r\to a} 7r^{1.5}(5\cos^2\theta - \sin^2\theta). \quad (*)$$

IF $a \in \mathbb{R}$. but $a \neq 0$, then

$\lim_{r\to a} (*)$ approaches

$$7a^{1.5}(5\cos^2\theta - \sin^2\theta)$$

But this value depends on any value of θ.

$\therefore$ We cannot make $n(x,y)$ continuous.

So when $a=1$, $r=1$, so $x = \frac{1}{\sqrt{2}}$ and

$$\lim_{r\to 1} (*) = 7(5\cos^2\theta - \sin^2\theta).$$

b) ~~No~~, If we can redefine $r(x,y)$ so that the $\lim_{r \to a} (*)$ does not depend on θ.

We can make

$$r(x,y) = \frac{2(5x^2+5y^2)}{4\sqrt{x^2+y^2}}$$

so that $\cos^2\theta + \sin^2\theta = 1$, and exp does not depend on θ

Q3) $\lim_{(x,y)\to(-1,3)} \frac{y-3}{x+1}$ $= \frac{2y^4}{2y^4} = 1$

A) Let $y=3$:

$\lim_{x\to -1} \left(\frac{3-3}{x+1}\right) = \lim_{x\to -1}\left(\frac{0}{x+1}\right) = \lim_{x\to -1} 0 = 0$
$= L_1$

Let $y = x+4$

$\lim_{x\to -1} \frac{(x+4)-3}{x+1} = \lim_{x\to -1} \frac{\cancel{x+1}}{\cancel{x+1}} = \lim_{x\to -1} (1) = 1$
$= L_2$

Since $L_1 \neq L_2$, limit DNE.

B) $\lim_{(x,y)\to(0,0)} \frac{2x\cdot y^2}{x^2+y^4}$

Let $y=0$ (along x axis)

$\lim_{(x,0)\to(0,0)} \frac{2x(0)}{x^2+0} = \lim_{x\to 0} \frac{0}{x^2} = 0 = L_1$

Let $x = y^2$

$$\lim_{(y^2,y)\to(0,0)} \left(\frac{2xy^2}{x^2+y^4}\right) = \lim_{(y^2,y)\to(0,0)} \frac{2y^2y^2}{y^4+y^4} =$$

$$= \lim_{y\to 0} \frac{\cancel{2y^4}}{\cancel{2y^4}} = \lim_{y\to 0} 1 = 1 = L_2.$$

Since $L_1 \neq L_2$, the limit DNE.

(Q4)

Q4) $z + \sin(z) = xy$

Compute $\frac{\partial^2 z}{\partial x \partial y}$ in terms of z, by first finding $\frac{dz}{dx}$ and $\frac{\partial z}{\partial y}$

$$\frac{\partial z}{\partial x} + \cos(z)\frac{dz}{dx} = y$$

$$\frac{dz}{dx}\left(1 + \cos(z)\right) = y$$

$$\frac{dz}{dx} = \frac{y}{1+\cos(z)}$$

$$\frac{dz}{dy} + \cos(z)\frac{dz}{dy} = x$$

$$\frac{dz}{dy}\left(1+\cos(z)\right) = x$$

$$\frac{dz}{dy} = \frac{x}{1+\cos(z)}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{x}{1+\cos(z)}\right)$$

$$= \frac{1\left(1+\cos(z)\right) - x\left(-\sin(z)\frac{\partial z}{\partial x}\right)}{\left(1+\cos(z)\right)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1+\cos(z) + x\sin(z)\frac{\partial z}{\partial x}}{\left(1+\cos(z)\right)^2}$$

$$= \frac{1+\cos(z) + x\sin(z)\left(\frac{y}{1+\cos(z)}\right)}{\left(1+\cos(z)\right)^2}$$

$$= \frac{1+\cos(z) + xy\sin(z)}{\left(1+\cos(z)\right)^3}$$

Q5) $4x + 2y - z = 3$ plane

$z = x^2 + xy + y$ surface.

$z = f(x, y) = x^2 + xy + y$.

~~We can~~

We can take $f_x(x_0, y_0) = 2x_0 + y_0$

and $f_y(x_0, y_0) = x_0 + 1$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

So Match this with

$z + 3 = 4x + 2y$

So $f_x(x_0, y_0) = 4 = 2x + y$

$f_y(x_0, y_0) = 2 = x + 1$

$2x_0 + y_0 = 4$ $\quad x_0 = 1$

$x_0 + 1 = 2 \Rightarrow x_0 = 1$ $\quad y_0 = 2$

so $2(1) + y_0 = 4$

$y_0 = 4 - 2 = 2$.

So

$$\Rightarrow z - z_0 \overset{+3}{=} 4(x-1) + 2(y-2) -$$

$= 4x - 4 + 2y - 4$

$= 4x + 2y - 8$

~~$z + 8$~~ ~~4~~

$z + 8 - z_0 = 4x + 2y - 8$

$z + 8 - z_0 = 3$

so $z_0 = 5$

$\therefore$ Plane is tangent to surface at point $(1, 2, 5)$.