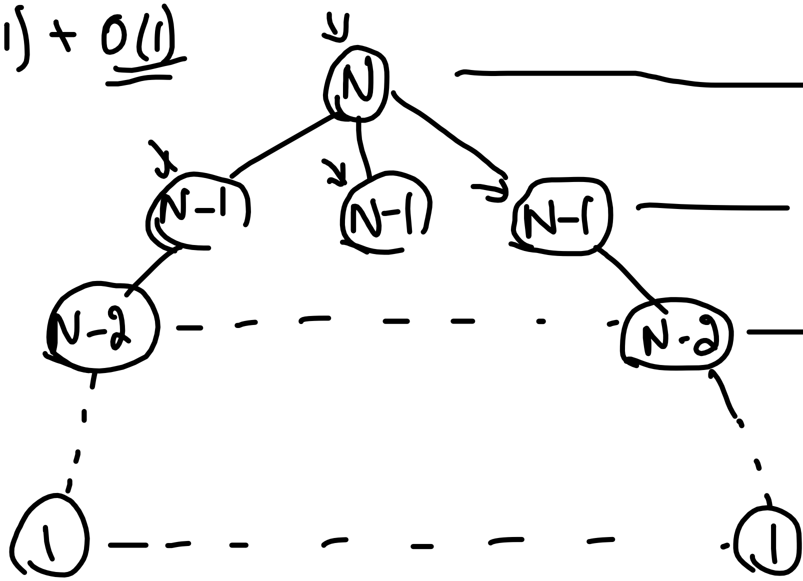
A#1(Solution)

Saturday, February 24, 2024 10:01 AM

$3T(N-1) + O(1)$



1

$1+1+1 = 3^1$

$9 = 3^2$

$27 = 3^3$

3^K

$N - K = 1$

$N-1 = K$

$K \approx N$

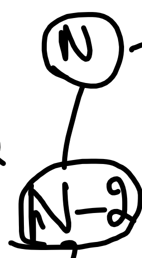
$3^0 + 3^1 + 3^2 + 3^3 + \dots + 3^{N-1} + 3^N$

$R > 1$ $\quad R = 3$

$a\left(\frac{R^N - 1}{R - 1}\right)$

$\frac{1 * (3^N - 1)}{3 - 1} = O(3^N)$

$T(N-2) + N^2$



N^2

$(N-2)^2$

$1^2 + 2^2 + 3^2 + \dots + N^2$

$\frac{N*(N+1)(2N+1)}{6}$

$1^2 + 3^2 + 5^2 + \dots + (N-2)^2 + N^2$

$\boxed{\sum^{N} i^2}$

Sum of sq each term

$\frac{N*(N+1)(2N+1)}{6}$

$\underline{\underline{O}}(N^3)$

$(N-4)$ —— $(N-4)^2$

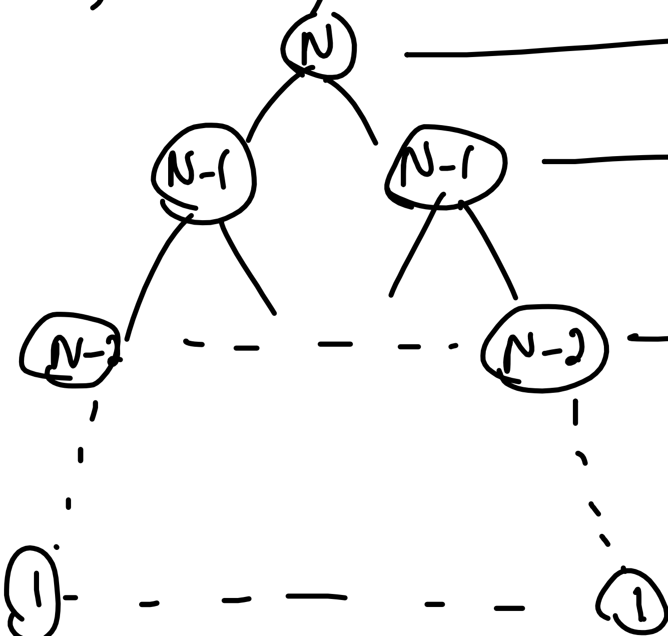
$(N-6)^2$

$\vdots$

(1) —— $(1)^2$

$\boxed{i=0}$

$2T(N-1) + \Theta(N)$



N

$2^1*(N-1)$

$4*(N-2) = 2^2*(N-2)$

$\vdots$

$2^K*(N-K)$

$$\sum_{i=0}^{N} 2^i(N-i) = \sum_{i=0}^{N} 2^i N - \cancel{i*2^i}$$

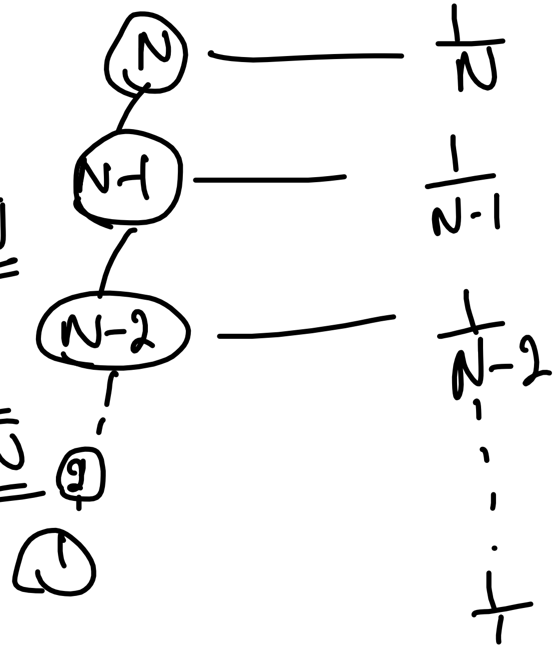
$r>1$

$$N\left[2^0+2^1+2^2+2^3+\dots+2^N\right]$$

$O(N*2^N)$

$O(N \times a)$

$T(N-1) + \frac{1}{N}$



$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N}$

$O(\log N)$

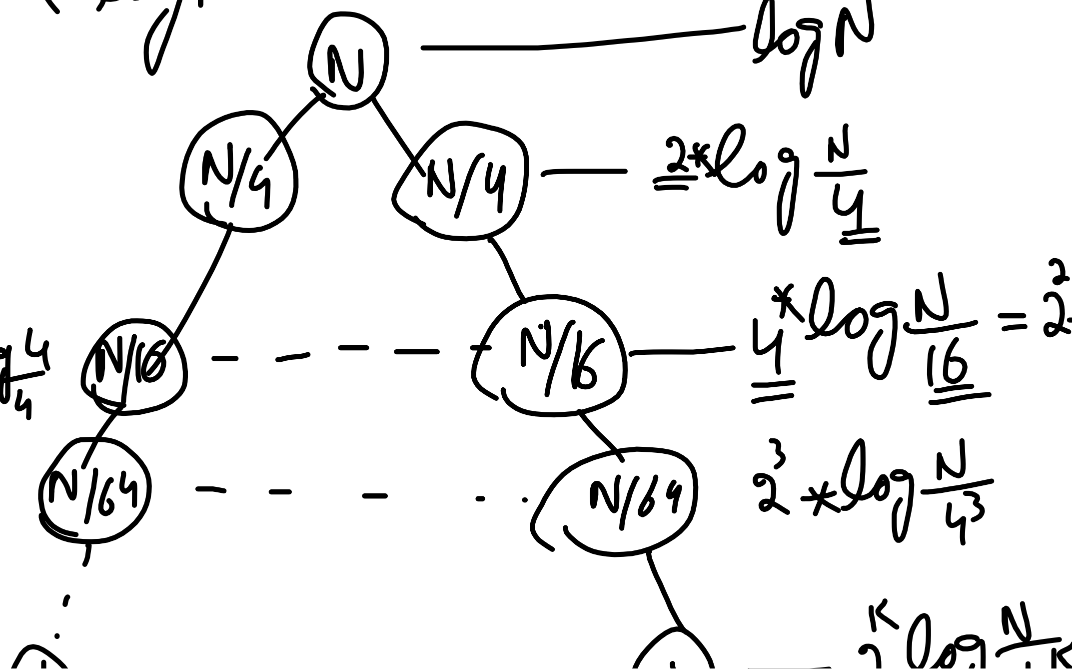
$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{\sqrt{N}}$

$\log \sqrt{N}$

$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{\log N}$

$\log \log N$

$2T(N/4) + \log N$



$\frac{N}{4^K} = 1$

$N = 4^K$

$\log_4 N = K * \frac{\log 4}{\log 4}$

$K = \log_4 N$

$$\sum_{i=0}^{\log_4 N} 2^i \log \frac{N}{4^i}$$

$$\log \frac{a}{b} = \log a - \log b$$

$$2^i * [\log N - \boxed{\log 4^i}]$$

$$i * \log_2 4 = \boxed{i2}$$

$$[2^i \log N] - [2^i * i * 2]$$

$$\log N \left[2^0 + 2^1 + 2^2 + \dots + 2^{\log_4 N}\right] \quad R > 1$$

$$\log N \left[2^{\log_4 N}\right] \rightarrow \log N \left[N^{\log_4 2}\right]$$

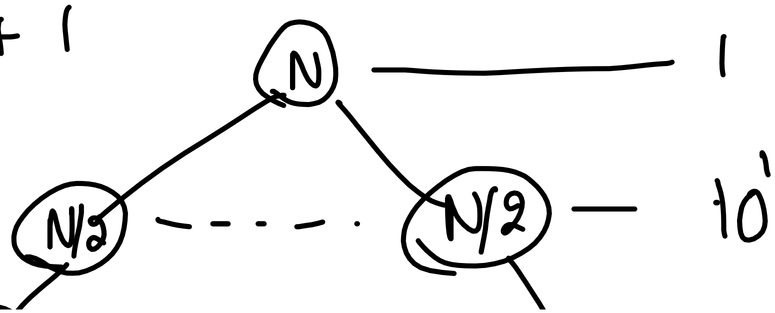
1/2

$$\log N \; N^{1/2}$$

$$O(\sqrt{N} * \log N)$$

$10T(N/2) + 1$

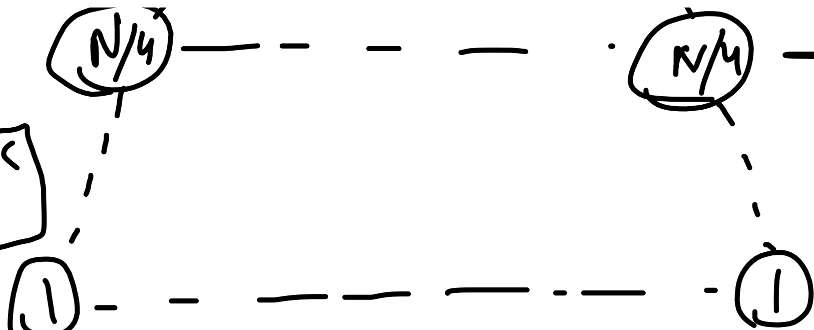
$K = \log_2 N$



$\frac{N}{2^K} = 1$

$\boxed{N = 2^K}$

$\log_2 N = K \times \log_2 2$



$100 = 10^2$

10^K

$\log_2 N$

$1 + 10 + 100 + \dots + 10^{\log_2 N}$

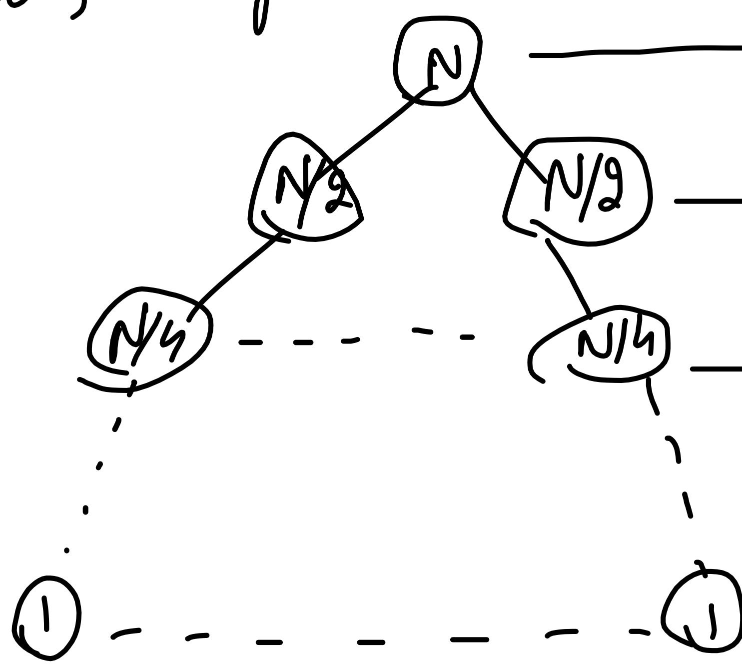
$\frac{1\left[10^{\log_2 N} - 1\right]}{9} = N^{\log_2 10}$

$O\left(N^{\log_2 10}\right)$

$O\left(N^{3.33}\right)$

3 :-

$2T\left(\frac{N}{2}\right) + \frac{N}{\log N}$



$N * \frac{1}{\log N}$

$\cancel{2} * \frac{N}{\cancel{2}} * \frac{1}{\log N/2}$

$\cancel{4} * \frac{N}{\cancel{4}} * \frac{1}{\log N/4}$

$N * \frac{1}{\log N/2^K}$

$$\frac{N}{\log N} + \frac{N}{\log N/2} + \cdots + \frac{N}{\log \frac{N}{2^k}}$$

$$= N\left[\frac{1}{\log N} + \frac{1}{\log N/2} + \frac{1}{\log N/4} + \cdots + \frac{1}{\log 8} + \frac{1}{\log 4} + \frac{1}{\log 2} + \cancel{\frac{1}{\log 1}}\right]$$

$N=32$

$$\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + \frac{1}{1}$$

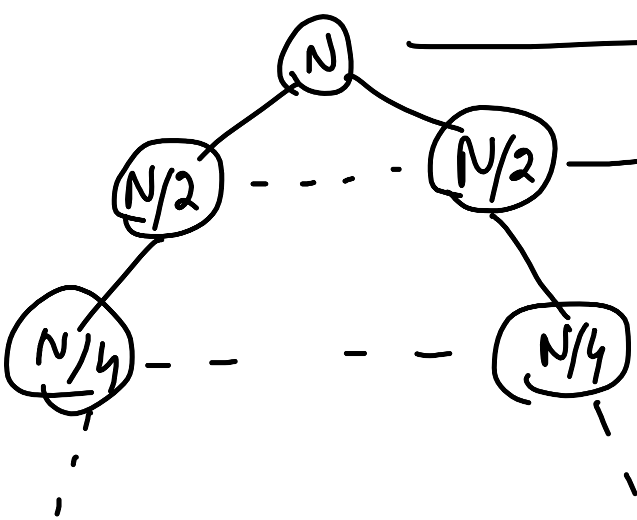
Harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{\log N}$$

$$\Theta(N \log \log N)$$

$$4T(N/2) + N^2\sqrt{N}$$

$$N^2\sqrt{\frac{N}{2^0}}$$



$$N^2\sqrt{N}$$

$$4*\left(\frac{N}{2}\right)^2\sqrt{\frac{N}{2}} = N^2\sqrt{\frac{N}{2}}$$

$$16*\left(\frac{N}{4}\right)^2\sqrt{\frac{N}{4}} = N^2\sqrt{\frac{N}{4}}$$

$$2\sqrt{\frac{N}{\;}}$$

1 - - - - - - 1 —— $N\sqrt{2^k}$

$$N^2\sqrt{N} + N^2\sqrt{\frac{N}{2}} + N^2\sqrt{\frac{N}{4}} + \dots + N^2\sqrt{\frac{N}{2^{\log N}}}$$

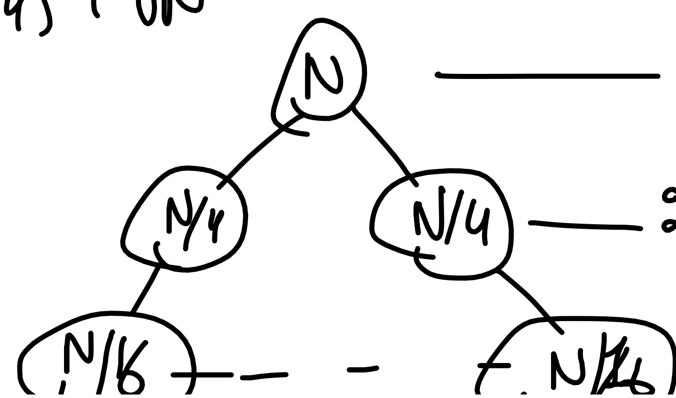
$$N^2\sqrt{N}\left(1+\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{2^2}}+\dots+\sqrt{\frac{1}{2^{\log N}}}\right)$$

$$\boxed{r<1}$$

$$N^2\sqrt{N}\left(\sum_{i=0}^{\log_2 N}\sqrt{\frac{1}{2^i}}\right) \qquad \sum_{i=0}^{\infty}\left(\frac{1}{\sqrt{2^i}}\right)$$

$$N^2\sqrt{N}\left(\frac{1}{1-r}\right) = O(N^2\sqrt{N})$$

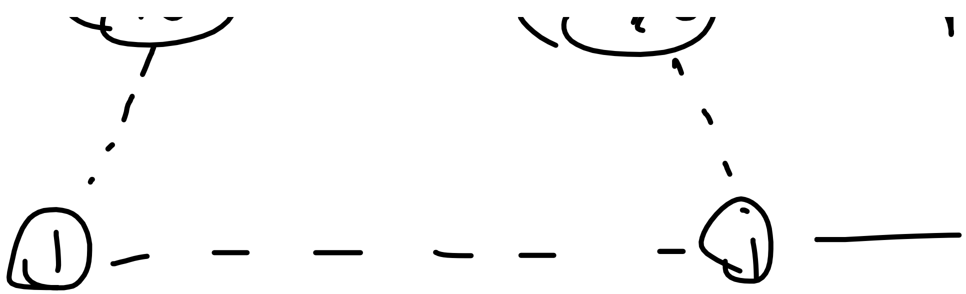
$2T(N/4)+\sqrt{N}$



—— $\sqrt{N}$

—— $2\times\sqrt{\frac{N}{4}} = \cancel{2}\times\frac{\sqrt{N}}{\cancel{2}}$

—— $4\times\sqrt{\frac{N}{16}} = \cancel{4}\times\frac{\sqrt{N}}{\cancel{4}}$

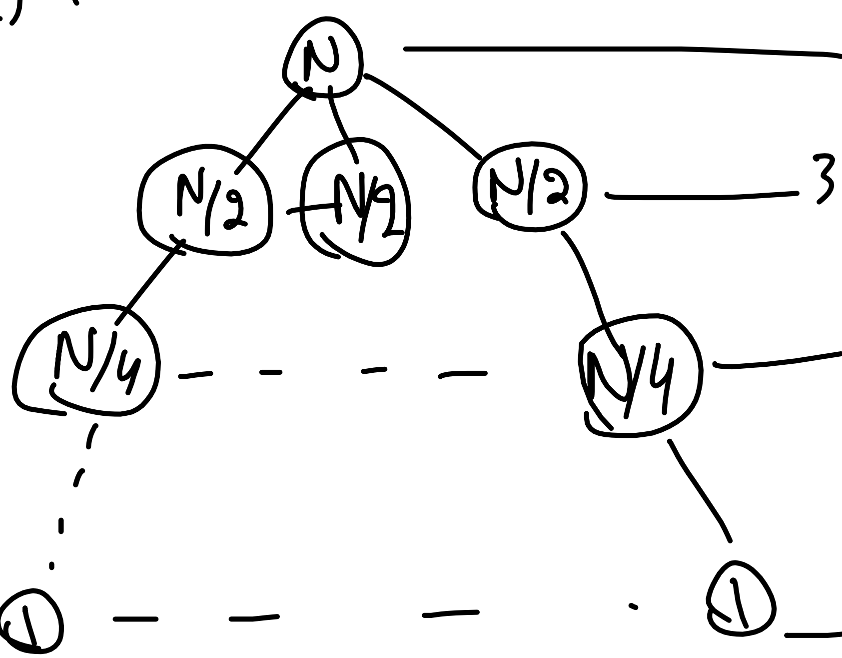


$\sqrt{N}$

$\sqrt{N} * \log_4 N$

$O(\sqrt{N} * \log N)$

$3T\left(\frac{N}{2}\right) + N^3$



$N^3 = \left(\frac{3}{8}\right)^0 N^3$

$3\left(\frac{N}{2}\right)^3 = 3\frac{N^3}{8} = \left(\frac{3}{8}\right)^1 N^3$

$9\left(\frac{N}{4}\right)^3 = \frac{9}{64}N^3 = \left(\frac{3}{8}\right)^2 N^3$

$\left(\frac{3}{8}\right)^3 N^3$

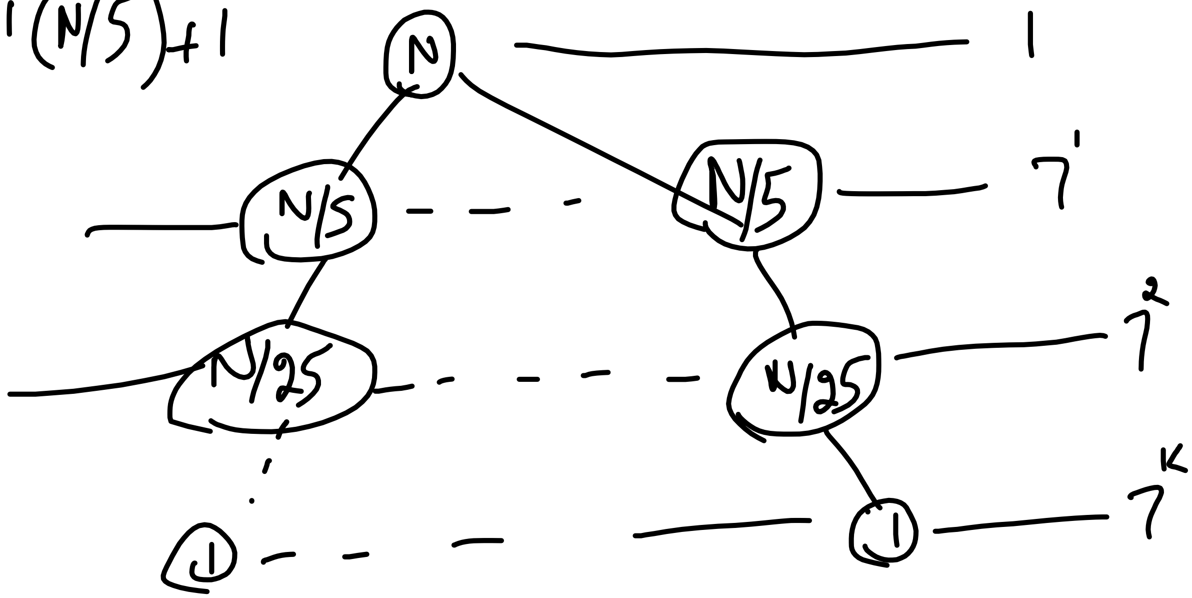
$\left(\frac{3}{8}\right)^k N^3$

$N^3\left(1 + \frac{3}{8} + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^3 + \dots + \left(\frac{3}{8}\right)^{\log_2 N}\right)$

$N^{3}\left(\sum_{i=0}\left(\frac{\;}{8}\right)\right)$

$N^{3}\left(\frac{1}{1-R}\right) = O(N^{3})$

$7T(N/5)+1$

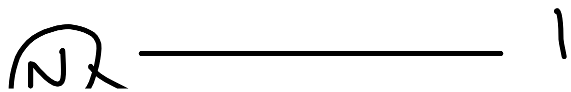


$1+7+7^{2}+7^{3}+\dots+7^{\log_5 N}$

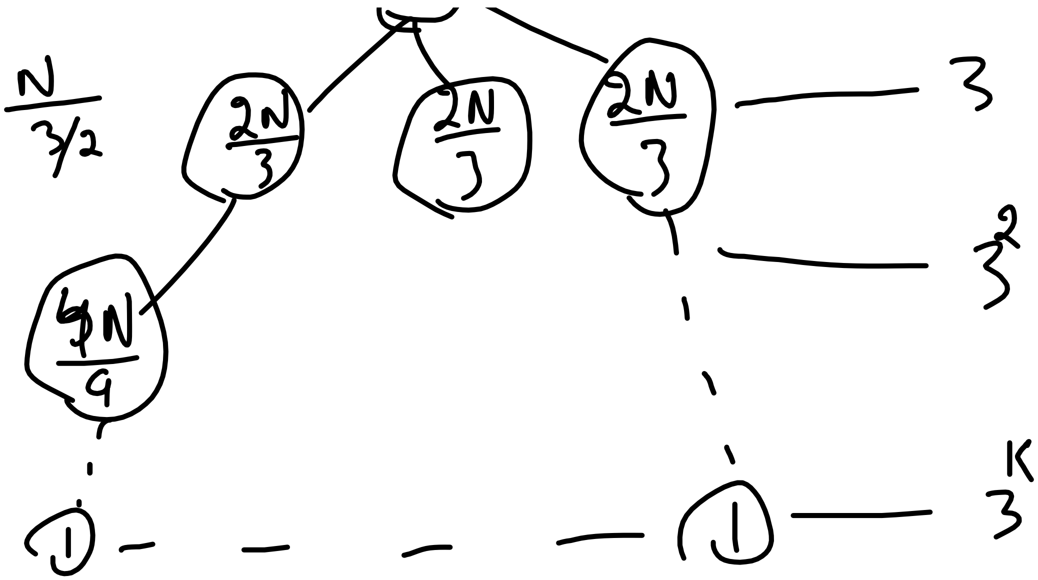
$R=7$

$\cancel{\frac{a(R^{N}-1)}{R-1}}$ $\qquad \frac{1\left(7^{\log_5 N}-1\right)}{7-1} = O\left(N^{\log_5 7}\right)$

$3T\left(\frac{2N}{3}\right)+1$



$\frac{2N}{3} = \frac{N}{3/2}$



$K = \log_{3/2} N$

$1\left(3^{\log_{3/2} N} - 1\right)$

$a = 1$
$r = 3$
$N = \log_{3/2} N$

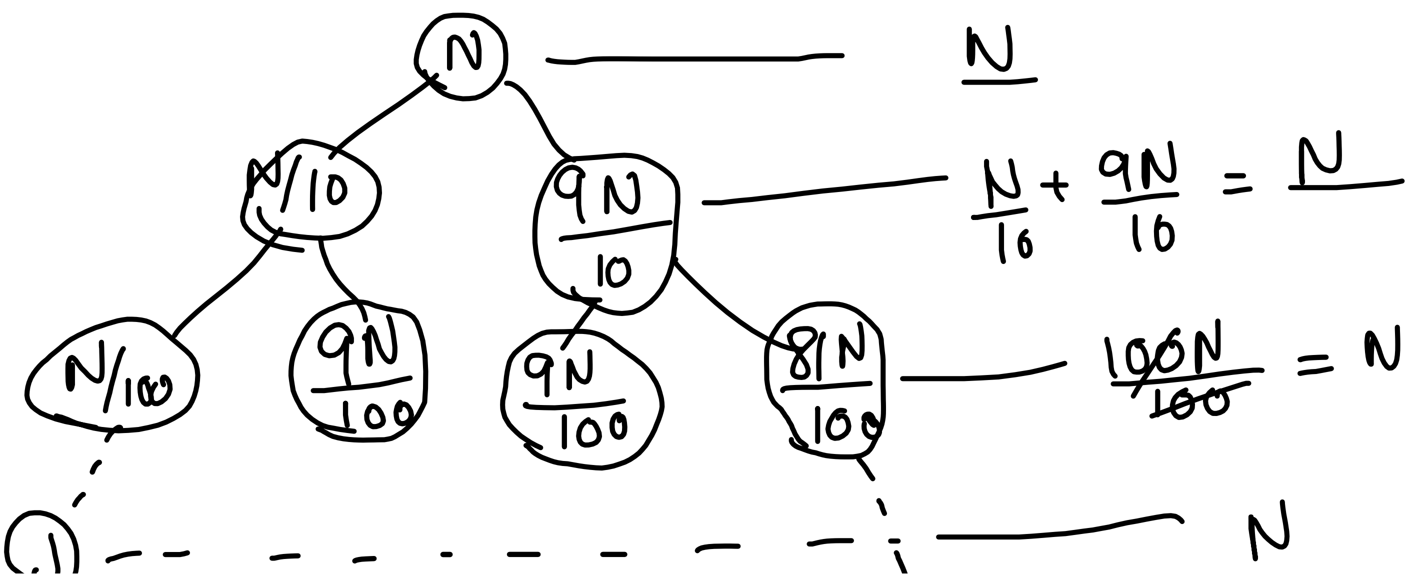
$O\left(N^{\log_{3/2} 3}\right)$

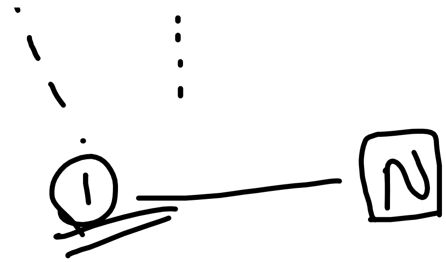
$3T\left(\frac{4N}{5}\right) + 1$

$\log N = K$

$$1\left(\frac{3^{\log_{5/4} N}-1}{3-1}\right) = O\left(N^{\log_{5/4} 3}\right)$$

$$T(N) = T\left(\frac{N}{10}\right) + T\left(\frac{9N}{10}\right) + \underline{N}$$



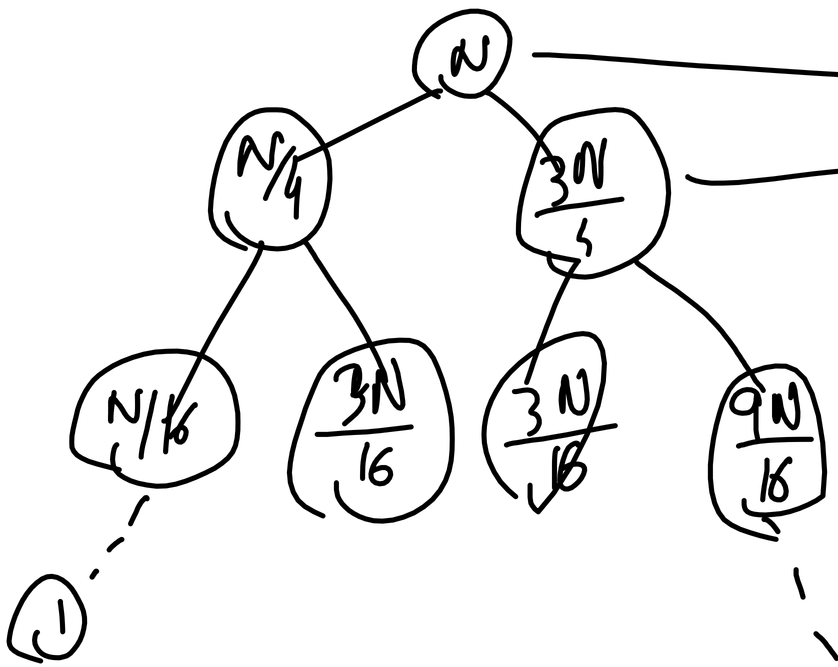


$N + N + N + N + \dots + N$

$N * \log_{10/9} N$

$O(N * \log N)$

$T\left(\frac{N}{4}\right) + T\left(\frac{3N}{4}\right) + N^2$



$N^2 = \left(\frac{10}{16}\right)^0 N^2$

$\frac{N^2}{16} + \frac{9N^2}{16} = \left(\frac{10}{16}\right)^1 N^2$

$\frac{N^2}{256} + \frac{81N^2}{256} = \frac{100}{256} N^2 = \left(\frac{10}{16}\right)^2 N^2$

$\left(\frac{10}{16}\right)^K N^2$

$$N^2\left[1+\left(\frac{10}{16}\right)+\left(\frac{10}{16}\right)^2+\cdots+\left(\frac{10}{16}\right)^{\log_{4/3} N}\right]$$

$$N^2\left[\sum_{i=0}^{\infty}\left(\frac{1}{1-R}\right)\right]$$

$$O(N^2)$$