Question 1: **10 Marks**

Suppose that performing arithmetic with integers less than 100 on a certain processor is much quicker than doing arithmetic with larger integers. You can restrict almost all our computations to integers less than 100 if we represent integers using their remainders modulo pairwise relatively prime integers less than 100. For example, You can use the moduli of 99, 98, 97, and 95. (These integers are relatively prime pairwise, because no two have a common factor greater than 1.)

Write a new **addition** routine/function (in the programming language of your choice), that takes as input two large numbers, breaks those numbers in *4-tuples*, adds the 4-tuples *componentwise* and reduce each component with respect to the appropriate modulus and converts back the resultant 4-tuple into corresponding integer (using the chines remainder theorem).

You may assume that the result does not exceed the product of your chosen moduli.

Question 2: **10 Marks**

Consider this problem, which was originally posed by Leonardo Pisano, also known as Fibonacci, in the thirteenth century in his book *Liber abaci*. A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month, as shown in Figure 1. Find a recurrence relation for the number of pairs of rabbits on the island after *n* months, assuming that no rabbits ever die.

Question 3: **10 Marks**

A popular puzzle of the late nineteenth century invented by the French mathematician Édouard Lucas, called the Tower of Hanoi, consists of three pegs mounted on a board together with disks of different sizes. Initially these disks are placed on the first peg in order of size, with the largest on the bottom (as shown in Figure 2). The rules of the puzzle allow disks to be moved one at a time from one peg to another as long as a disk is never placed on top of a smaller disk. The goal of

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the puzzle is to have all the disks on the second peg in order of size, with the largest on the bottom.

Let H_n denote the number of moves needed to solve the Tower of Hanoi problem with n disks. Set up a recurrence relation for the sequence $\{H_n\}$.

Question 4: **10 Marks**

Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?

Question 5: **10 Marks**

A deposit of $100,000 is made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year.

a) Find a recurrence relation for $\{P_n\}$, where P_n is the amount in the account at the end of n years if no money is ever withdrawn.

b) How much is in the account after n years if no money has been withdrawn?

Question 6: **10 Marks**

A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

a) Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.

b) Find L_n if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2.

Question 7: **10 Marks**

Find the solution to the recurrence relation

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

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with the initial conditions $a_0 = 2$, $a_1 = 5$, and $a_2 = 15$.

Question 8: **10 Marks**

Find the solution to the recurrence relation

$$a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$$

with initial conditions $a_0 = 1$, $a_1 = -2$, and $a_2 = -1$.

Question 9: **10 Marks**

Find the solution to the recurrence relation

$$a_n = a_{n-1} + n$$

with initial conditions $a_1 = 1$.

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