Assignment 1

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1 Question 1

A dietitian is planning the menu for the noon meal at an elementary school. He plans to serve three main items, all having different nutritional content. The dietitian is interested in providing at least the minimum daily requirement of each of three vitamins in this one meal. Table 10.9 summarizes the vitamin content per ounce of each type of food, the cost per ounce, and the minimum daily requirement for each vitamin. Any combination of the three foods may be selected as long as the total serving size is at least 6.0 ounces.

Formulate the linear programming problem which, when solved, would determine the number of ounces of each food to serve. The objective is to minimize the cost of the meal while satisfying minimum daily requirement levels of the three vitamins as well as the restriction on the minimum serving size.

	Vitamins			
Food Item	**1**	**2**	**3**	**Cost per ounce**
1	20 mg	10 mg	20 mg	0.15
2	40 mg	25 mg	30 mg	0.18
3	30 mg	15 mg	25 mg	0.22
MDR	240 mg	120 ms	180 mg	

The decision variables are as follows:
x_1 = no of ounces of food item 1
x_2 = no of ounces of food item 2
x_3 = no of ounces of food item 3

Minimize

$$0.15x_1 + 0.18x_2 + 0.22x_3$$

The constraints are as follows:

$$x_1 + x_2 + x_3 \geq 6.0$$

$$20x_1 + 40x_2 + 30x_3 \geq 240$$

$$10x_1 + 25x_2 + 15x_3 \geq 120$$

$$20x_1 + 30x_2 + 25x_3 \geq 180$$

The complete linear programming formulation is:

Minimize

$$0.15x_1 + 0.18x_2 + 0.22x_3$$

Subject to :

$$x_1 + x_2 + x_3 \geq 6.0$$

$$20x_1 + 40x_2 + 30x_3 \geq 240$$

$$10x_1 + 25x_2 + 15x_3 \geq 120$$

$$20x_1 + 30x_2 + 25x_3 \geq 180$$

$$x_1, x_2, x_3 \geq 0$$

2 Question 2

question 2 A leading processor of sugar has two plants which supply four warehouses. Table 10.10 summarizes weekly capacities at each plant, weekly requirements at each warehouse, and shipping cost per ton (in dollars) between any plant and any warehouse. If x, equals the number of tons shipped from plant i to depot j, formulate the linear programming model which allows for determining the distribution schedule which results in minimum shipping cost.Weekly plant capacities are not to be violated, and warehouse requirements are to be satisfied.

	Warehouse				
Plant	1	2	3	4	Weekly supply,Tons
1	\$20	\$15	\$10	\$25	2800
2	\$30	\$25	\$20	\$15	3500
Weekly demand,Tons	1400	1600	1000	1500	

The decision variables are the number of tons shipped from each plant i to each warehouse j, denoted by x_{ij}.

Minimize

$$20x_{11} + 15x_{12} + 10x_{13} + 25x_{14} + 30x_{21} + 25x_{22} + 20x_{23} + 15x_{24}$$

The constraints are as follows:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 2800$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 3500$$

$$x_{11} + x_{21} = 1400$$

$$x_{12} + x_{22} = 1600$$

$$x_{13} + x_{23} = 1000$$

$$x_{14} + x_{24} = 1500$$

The complete linear programming formulation is:
Minimize

$$20x_{11} + 15x_{12} + 10x_{13} + 25x_{14} + 30x_{21} + 25x_{22} + 20x_{23} + 15x_{24}$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 2800$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 3500$$

$$x_{11} + x_{21} = 1400$$

$$x_{12} + x_{22} = 1600$$

$$x_{13} + x_{23} = 1000$$

$$x_{14} + x_{24} = 1500$$

$$x_{ij} \geq 0, \quad i = 1, 2; \; j = 1, 2, 3, 4$$

3 Question 3

A chemical company manufactures liquid oxygen at three different locations in the South. It must supply four storage depots in the same region. Table 10.11 summarizes shipping cost per 1,000 gallons between any plant and any depot as well as monthly capacity at each plant and monthly demand at each depot. If x, equals the number of gallons (in thousands) shipped from plant i to depot j, formulate the linear programming model which allows for determining the minimum cost allocation schedule.Plant capacities are not to be violated, and depot demands are to be satisfied by the schedule.

	Depot				
	1	**2**	**3**	**4**	**Supply, 1000 Gal**
plant1	50	40	35	20	1000
plant2	30	45	40	60	1400
plant1	600	25	50	30	1800
Demand, 1000 Gal	800	750	650	900	

Let x_{ij} = the number of gallons shipped from plant i to depot j.

Minimize:

$$50x_{11}+40x_{12}+35x_{13}+20x_{14}+30x_{21}+45x_{22}+40x_{23}+60x_{24}+60x_{31}+25x_{32}+50x_{33}+30x_{34}$$

Subject to :

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 1000$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 1400$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 1800$$
$$x_{11} + x_{21} + x_{31} = 800$$
$$x_{12} + x_{22} + x_{32} = 750$$
$$x_{13} + x_{23} + x_{33} = 650$$
$$x_{14} + x_{24} + x_{34} = 900$$

$$x_{ij} \geq 0, \forall i, j$$

4 Question 4

A firm manufactures three products which must be processed through some or all of four departments. Table 10.12 indicates the number of hours a unit of each product requires in the different departments and the number of pounds of raw material required. Also listed are labour and material costs per unit, selling price, and weekly capacities of both labour- hours and raw materials. If the objective is to maximize total weekly profit, formulate the linear programming model for this exercise.

	Product			
	A	**B**	**C**	**Weekly availability**
Department 1	2.5	4	2	120 h
Department 2	-	2	2	160 h
Department 3	3	1	-	100 h
Department 4	2	3	2.5	150 h
Pounds of raw material per unit	5.5	4.0	3.5	500lbs
Selling price	\$60	\$50	\$75	-
labor cost per unit	20	27	36	-
material cost per unit	21	8	7	-

Let x_i = the number of units of product i produced and sold, $i \in A, B, C$. Then, the objective is to maximize total weekly profit, which is given by:
Maximize

$$60x_A + 50x_B + 75x_C - (20 + 21)x_A - (27 + 8)x_B - (36 + 7)x_C$$

$$z = 19x_A + 15x_B + 32x_C$$

subject to the following constraints:

$$2.5x_A + 4x_B + 2x_C \leq 120$$
$$2x_B + 2x_C \leq 160$$
$$3x_A + x_B \leq 100$$

$$2x_A + 3x_B + 2.5x_C \leq 150$$
$$5.5x_A + 4x_B + 3.5x_C \leq 500$$
$$x_A, x_B, x_C \geq 0$$

Therefore, the linear programming model is:
Maximize

$$z = 19x_A + 15x_B + 32x_C$$

Subject to :

$$2.5x_A + 4x_B + 2x_C \leq 120$$
$$2x_B + 2x_C \leq 160$$
$$3x_A + x_B \leq 100$$
$$2x_A + 3x_B + 2.5x_C \leq 150$$
$$5.5x_A + 4x_B + 3.5x_C \leq 500$$
$$x_A, x_B, x_C \geq 0$$

5 Question 5

Referring to Exercise 4, write the constraints associated with each of the following conditions.
(a) Combined weekly production must be at least 50 units.

$$x_A + x_B + x_C \geq 50$$

(b) The number of units of product A must be no more than twice the quantity of product B.

$$x_A \leq 2x_B$$

(c) Since products B and C are usually sold together, production levels of both should be the same.

$$x_B = x_C$$

(d) The number of units of product B should be no more than half of the total weekly production.

$$x_B \leq 0.5(x_A + x_B + x_C)$$

6 Question 6

A regional truck rental agency is planning for a heavy demand during the summer months. The agency has taken truck counts at different cities and has compared these with projected needs for each city all trucks are the same size. Three metropolitan areas are expected to have more trucks than will be needed during the summer, although four cities are expected to have fewer trucks than will be demanded. To prepare for these months, trucks can be relocated from surplus areas to shortage areas by hiring drivers. Drivers are paid a flat fee which depends on the distance between the two cities. In addition, they receive per diem daily expenses. Table 10.13 summarizes costs of having a truck delivered between two cities. Also shown are the projected surpluses for each city which has an oversupply and projected shortages for each city needing additional trucks. Note that total surplus exceeds total shortage. If the objective is to minimize the cost of reallocating the trucks, formulate the linear programming model which would allow for solving the problem. Hint: Let xu number of trucks delivered from surplus area i to shortage area j.

	Shortage Areas				
	1	**2**	**3**	**4**	**Supply of trucks**
Surplus City 1	\$100	\$250	\$200	\$150	120
Surplus City 2	200	175	100	200	125
Surplus City 3	300	180	50	400	100
Shortage of trucks	60	80	75	40	-

Let x_{ij} = the number of trucks relocated from surplus area i to shortage area j

Minimize

$$Z = 100x_{11}+250x_{12}+200x_{13}+150x+14+200x_{21}+175x_{22}+100x_{23}+200x_{24}+300x_{31}+180x_{32}+50x_{33}+400x_{34}$$

constraints (Surplus):

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 120$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 125$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 100$$

constraints (Demand):

$$x_{11} + x_{21} + x_{31} = 60$$

$$x_{12} + x_{22} + x_{32} = 80$$

$$x_{13} + x_{23} + x_{33} = 75$$

$$x_{14} + x_{24} + x_{34} = 40$$

$$x_{ij} \geq 0$$

7 Question 7

A coffee manufacturer blends four component coffee beans into three final blends of coffee. The four component beans cost the manufacturer 0.65,0.80, $0.90 and 0.75$ pound, respectively. The weekly availabilities of the four components are 80,000, 40,000, 30,000, and 50,000 pounds, respectively. The manufacturer sells the three blends at wholesale prices of \$1.25, \$1.40, and \$1.80 per pound, respectively. Weekly output should include at least 50,000 pounds of final blend 3. The following are blending restrictions which must be followed by the brewmaster.
(a) Component 2 should constitute at least 30 percent of final blend 3 and no more than 20 percent of final blend 1.
(b) Component 3 should constitute exactly 25 percent of final blend 3.
(c) Component 4 should constitute at least 40 percent of final blend 1 and no more than 18 percent of final blend 2.
The objective is to determine the number of pounds of each component which should ach final blend so as to maximize weekly profit. Formulate this as an LP model, carefully defining your decision variables.

Let $x_{i,j}$ = the number of pounds of component i used in final blend j, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3$.
Maximize

$$1.25x_{11} + 1.4x_{12} + 1.8x_{13} - (0.65x_{11} + 0.8x_{21} + 0.9x_{31} + 0.75x_{41} + 0.65x_{12}$$

$$0.8x_{22} + 0.9x_{32} + 0.75x_{42} + 0.65x_{13} + 0.8x_{23} + 0.9x_{33} + 0.75x_{43}$$

subject to the following constraints:

$$x_{11} + x_{12} + x_{13} \leq 80000$$

$$x_{21} + x_{22} + x_{23} \leq 40000$$

$$x_{31} + x_{32} + x_{33} \leq 30000$$

$$x_{41} + x_{42} + x_{43} \leq 50000$$

$$x_{13} + x_{23} + x_{33} + x_{43} \geq 50000$$

$$x_{23} \geq 0.3(x_{13} + x_{23} + x_{33} + x_{43})$$

$$x_{21} \leq 0.2(x_{11} + x_{21} + x_{31} + x_{41})$$

$$x_{33} = 0.25(x_{13} + x_{23} + x_{33} + x_{43})$$

$$x_{41} \geq 0.4(x_{11} + x_{21} + x_{33} + x_{41})$$

$$x_{42} \leq 0.18(x_{12} + x_{22} + x_{32} + x_{42})$$

8 Question 8

A producer of machinery wishes to maximize the profit from producing two products, product A and product B. The three major inputs for each product are steel, electricity, and labor-hours. Table 10.14 summarizes the requirements per unit of each product, available resources, and profit margin per unit. The number of units of product A should be no more than 80 percent of the number of product B. Formulate the linear program- ming model for this situation.

	Product		
	A	**B**	**Monthly tool available**
Energy 1	100kWh	200kWh	20000kWh
Steal 1	60lb	80lb	10000lb
Labor 1	2.5h	2h	400h
profit per unit	\$30	\$40	-

Let x = the number of units of products manufactured.
maximize

$$z = 30x_1 + 40x_2$$

Subject to constraints :

$$100x_1 + 200x_2 \leq 20,000$$

$$60x_1 + 80x_2 \leq 10,000$$

$$2.5x_1 + 2x_2 \leq 400$$

$$x_1 \leq 0.80x_2$$

$$x_i \geq 0$$

9 Question 9

In a certain area there are two warehouses which supply food to five grocery stores. Table 10.15 summarizes the delivery cost per truckload from each warehouse to each store, the required number of truckloads per store per week, and the maximum number of truckloads available per week per warehouse. Formulate a linear programming model that would determine the number of deliveries from each warehouse to each store which would minimize total delivery cost.

	Store					
	1	**2**	**3**	**4**	**5**	**Maximum number of truckloads**
Warehouse A	\$40	\$30	\$45	\$25	\$50	100
Warehouse B	\$50	\$35	\$40	\$20	\$40	250
Required no. of truckloads	80	50	75	45	80	

Let $x_{i,j}$ = the number of food items supplied from warehouse i to store j.
maximize

$$z = 40x_{11}+30x_{12}+45x_{13}+25x_{14}+50x_{15}+50x_{21}+35x_{22}+40x_{23}+20x_{24}+40x_{25}$$

Constraints : (Supply)

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \leq 100$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \leq 250$$

Constraints : (Demand)

$$x_{11} + x_{21} = 80$$

$$x_{12} + x_{22} = 50$$

$$x_{13} + x_{23} = 75$$

$$x_{14} + x_{24} = 45$$

$$x_{15} + x_{25} = 80$$

$$x_{ij} \geq 0$$

10 Question 10

Capital Expansion A company is considering the purchase of some additional machinery as part of a capital expansion program. Four types of machines are being considered. Table 10.16 indicates relevant attributes of the four machines.The total budget for this program is $750,000. The maximum available floor space is 16,000 square feet. The company wants to maximize the output (total number of units produced) resulting from the purchase of the new machines. Define your decision variables carefully and formulate the LP model for this problem.

	Machine				
	A	**B**	**C**	**D**	
Cost	$50000	$35000	$60000	$80000	
Square footage required	200	150	250	280	
Daily output, units	10000	8000	25000	18000	

Let x_i = the number of units produced.
Let x_1 x_2 x_3 x_4 be products A,B,C,D.
maximize

$$z = 10,000x_1 + 8000x_2 + 25,000x_3 + 18000x_4$$

constraints :

$$200x_1 + 150x_2 + 250x_3 + 280x_4 \leq 16,000$$

$$50000x_1 + 35000x_2 + 60000x_3 + 80000x_4 \leq 750,000$$

11 Question 11

A national car rental firm is planning for the summer season. An analysis of current inventories of sub-compact cars in seven cities along with projections of demands during the summer in these same cities indicate that three of these areas will be short of their needs while four of the cities will have surplus numbers of sub-compact auto-mobiles. In order to prepare for the summer season, company officials have decided to relocate cars from those cities expected to have surpluses to those expected to have shortages. The cars can be relocated by contracting with an auto transport firm. Bids have been received from the trucking firm which indicate the cost of relocating a car from a given surplus city to a given shortage city. Table 10.17 summarizes these costs along with the surpluses and shortages for the mentioned cities. Let x equal the number of cars relocated from surplus area i to shortage area j. If the objective is to minimize the cost of relocating these cars such that each shortage area will have its needs satisfied, formulate the LP model for this problem.

	Shortage area				
	1	**2**	**3**	**4**	**suplus of cars**
Surplus city 1	\$50	\$40	\$25	\$30	300
Surplus city 2	\$40	\$25	\$35	\$45	150
Surplus city 3	\$35	\$50	\$40	\$25	250
Shortage of cars	250	150	125	175	-

Let $x_{i,j}$ = the number of cars relocated from surplus area i to shortage area j.

Minimize $50x_{11}+40x_{12}+25x_{13}+30x_{14}+40x_{21}+25x_{22}+35x_{23}+45x_{24}+35x_{31}+50x_{32}+40x_{33}+25x_{34}$

Constraints : (Surplus of cars)

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 300$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 150$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 250$$

Constraints : (demand)

$$x_{11} + x_{21} + x_{31} = 250$$

$$x_{12} + x_{22} + x_{32} = 150$$

$$x_{13} + x_{23} + x_{33} = 125$$

$$x_{14} + x_{24} + x_{34} = 175$$

$$x_{ij} \geq 0 \quad \text{for all } i, j$$

12 Question 12

Financial Portfolio A person is interested in investing \$500,000 in a mix of investments. Table 10.18 indicates the investment choices and estimated rates of return for each. The investor wants at least 35 percent of her investment to be in government bonds. Because of the higher perceived risk of the two stocks, she has specified that the combined investment in these not exceed \$80,000. The investor also has a hunch that interest rates are going to remain high and has specified that at least 20 percent of the investment should be in the money market fund. Her final investment condition is that the amount invested in mutual fund A should be no more than the amount invested in mutual fund B. The problem is to decide the amount of money to invest in each alternative so as to maximize total annual return (in dollars). Carefully define your variables and formulate the LP model for this problem.

investment	**Projeted rate of return**
Mutual fund A	0.12
Mutual fund B	0.09
Money market fund	0.08
Government bonds	0.085
Stocks A	0.16
Stocks B	0.18

Let x_i = the number of dollars to be invested.
i = 1,2,3,4,5,6
x_1,x_2,x_3,x_4,x_5,x_6

$$\text{maximize } z = 0.12x_1 + 0.09x_2 + 0.08x_3 + 0.08x_4 + 0.16x_5 + 0.18x_6$$

Subject to constraints :

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 500000$$

$$x_4 \geq 0.35(50000)$$

$$x_5 + x_6 \leq 80,000$$

$$x_3 \geq 0.20(50000)$$

$$x_1 \leq x_2$$

13 Question 13

Cargo Loading The owner of a cargo ship is considering the nature of the next shipment. Four different commodities are being offered for shipment. Table 10.20 sum- marizes their weight, volume, and revenue-generating characteristics.

The cargo ship has three cargo holds, each characterized by weight and volume capacities. The forward hold has a weight capacity of 100 tons and volume capacity of 6,000 cubic feet. The center hold has a weight capacity of 140 tons and volume capacity of 8,000 cubic feet. The aft hold has a weight capacity of 80 tons and volume capacity of 5,000 cubic feet. The problem is to decide how much of each commodity should be accepted for shipment if the objective is to maximize total revenue. Specifically, it must be decided how many tons of each commodity should be placed in each hold while not exceeding the weight and volume capacities. Let x, equal the number of tons of commodity i placed in hold j and formulate the LP model for this problem.

commodity	**Weight offered, Tons**	**Volume, Ft3/Ton**	**Revenue, $ per Ton**
1	200	70	$1250
2	100	50	$900
3	80	60	$1000
4	150	75	$1200

Let $x_{i,j}$ = the number of tons of commodity i placed in hold j.
maximize

$$Z = 1250(x_{11}+x_{12}+x_{13})+900(x_{21}+x_{22}+x_{23}+1000(x_{31}+x_{32}+x_{33})+1200(x_{41}+x_{42}+x_{43})$$

subject to :

$$200x_{11} + 100x_{21} + 80x_{31} + 150x_{41} \leq 100$$

$$forward - volume : 70x_{11} + 50x_{21} + 60x_{31} + 75x_{41} \leq 6000$$

$$center - weight : 200x_{12} + 100x_{22} + 80x_{32} + 150x_{42} \leq 140$$

$$volume : 70x_{12} + 50x_{22} + 60x_{32} + 75x_{42} \leq 8000$$

after hold :

$$weight : 200x_{13} + 100x_{23} + 80x_{33} + 150x_{43} \leq 80$$

$$volume : 700x_{13} + 50x_{23} + 60x_{33} + 75x_{43} \leq 5000$$

$$x_{ij} \geq 0 \quad \forall i, j$$

14 Question 14

A company manufactures and sells five products. Costs per unit, selling price, and hourly labor requirements per unit produced are given in Table 10.21. If the objective is to maximize total profit, formulate a linear programming model having the following constraints: at least 20 units of product A and at least 10 units of product B must be produced; sufficient raw materials are not available for total production in excess of 75 units; the number of units produced of products C'and E must be equal; combined production of A and B should be no more than 50 percent of combined production of C, D, and E; the amount produced of C must be at least that of A; and labor availability in departments 1 and 2 equal 120 and 150 hours, respectively.

	Product					
	A	**B**	**C**	**D**	**E**	
Cost per unit	\$50	\$80	\$300	\$25	\$10	
Selling price	\$70	\$90	\$350	\$50	\$12	
Dept.1 labor hours/unit	2	1	0.5	1.6	0.75	
Dept.2 labor hours/unit	1.5	0.8	1.5	1.2	2.25	

Let x_i = the number of dollars to be invested.
i = 1,2,3,4
x_1,x_2,x_3,x_4

maximize

$$z = 20x_1 + 10x_2 + 50x_3 + 25x_4 + 2x_5$$

Subject to :

$$2x_1 + x_2 + 0.5x_3 + 1.6x_4 + 0.75x_5 \leq 120$$

$$1.5x_1 + 0.8x_2 + 1.5x_3 + 1.2x_4 + 2.25x_5 \leq 120$$

$$x_1 \geq 20$$

$$x_2 \geq 10$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 75$$

$$x_3 = x_5$$

$$2x_1 + x_2 \leq 0.5(x_3 + x_4 + x_5)$$

$$x_3 \geq x_1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$