3. NOISE in ELECTRONIC CIRCUITS

ENEL434 Electronics 2

Summary

- Interference
- Noise basics
- Sources of noise
  - Thermal noise
  - Shot noise
  - Flicker noise
  - Burst noise
  - Avalanche noise
- Transistor noise models
- Circuit noise calculations
- Low noise amplifiers
- Minimum detectable signal

References

- P R Gray and R G Meyer, Analysis and design of analog integrated circuits, 3rd Edition, Wiley, 1993

Interference

Electronic circuits operate in an electromagnetic (EM) environment polluted with electrical noise – most of it man-made.

- **EM Interference** enters electronic circuits via:
  - **Power supply** – Immunity requires attention to decoupling
  - **Input signal cable** – Immunity requires shielding and filtering
  - **Antenna** – Immunity requires filtering
  - **Direct coupling** – Immunity requires shielding
- Circuits and Systems susceptible to EMI could also emit interfering signals
- Subsystems within a system may cause interference to other subsystems within a system:
  - particularly in mixed-signal environments
  - immunity requires attention to power supply decoupling and internal shielding

Interference

Electronic components also generate noise of their own by various fundamental physical processes. This is true noise.

This course deals predominantly with this internally generated noise rather than interference from external sources.

Noise Basics

Consider the deterministic sinusoidal waveform Acosωt

The time average over a period (T) is zero:

$$\langle A\cos\omega t\rangle = \frac{1}{T}\int_{-T/2}^{T/2} A\cos\omega t\, dt = 0$$

But the time average over a period of the square of y(t) gives the power into 1 Ω:

$$\left\langle (A\cos\omega t)^2 \right\rangle = \frac{1}{T}\int_{-T/2}^{T/2} A^2\cos^2\omega t\, dt = \frac{A^2}{2}$$

The time average over a period of the product of two sinusoids of different frequency is zero:

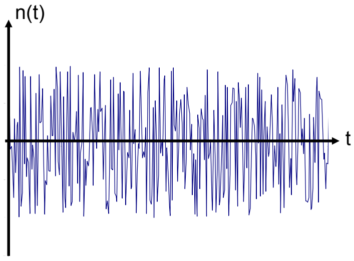
$$\langle \cos\omega_1 t\cos\omega_2 t\rangle = \frac{1}{T}\int_{-T/2}^{T/2} \cos\omega_1 t\cos\omega_2 t\, dt = 0$$

Noise Basics

A noise waveform cannot be described by a deterministic function but still has a number of important properties.

The time average is zero.

$$\langle n(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)dt = 0$$



The time average of the square of n(t) – called "mean square" - gives the power of the noise waveform into 1 Ω

$$\text{Power into } 1\Omega = \langle n(t)^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [n(t)]^2 dt$$

Square root is RMS value

Noise Basics

Consider two unrelated noise waveforms $n_1(t)$ and $n_2(t)$.

These waveforms will be uncorrelated and hence the time average of their product is zero:

$$\langle n_1(t) n_2(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n_1(t) n_2(t) dt = 0$$

A non-zero result for this time average means that they are correlated.

Thermal Noise

- In a conducting material, thermal energy manifests itself as random vibration of atoms and random motion of electrons.
- The electrons have thermal velocities significantly higher than drift velocity.
- The random motion of the electrons gives rise to a noise voltage across and noise current through the conductor.
- This noise is called thermal noise, or Johnson noise or Nyquist noise.

Thermal Noise

- **Mean-square** Thevenin voltage of a resistor with resistance R into bandwidth B is:

$$\left\langle v_n(t)^2 \right\rangle = 4kTBR$$

where k = 1.38 x 10^{-23} J/K (Boltzmann's constant)

T is absolute temperature (K)

B is the bandwidth (Hz)

R is the resistance in ohm

- Model is valid up to 1000 GHz
- A Norton equivalent is also possible

- eg. R = 1kΩ B = 1MHz T = 300 K

The RMS noise voltage across the resistor is 4μV

Shot Noise

- Shot noise is present in semiconductor devices when conducting current.
- The current of a pn junction for example, involves the passage minority carriers across the pn junction – which are random events.
- For a junction with DC current I_o, the mean-square noise current into bandwidth B is given by:

$$\langle i_n{}^2 \rangle = 2qI_oB$$

where q = 1.6 x 10^{-19} C

- eg. for I_o = 10 mA and B = 1 MHz

  The RMS noise current through the device is 57 nA

Flicker or 1/f Noise

- Flicker noise is present in all semiconductor devices and carbon resistors.
- Like Shot noise, Flicker noise is associated with current flow.
- Various origins but is essentially the result of contamination and defects present in manufacturing processes.
- Flicker noise varies with frequency.
- The means-square noise current power into an incremental bandwidth Δf is:

$$\langle i(t)^2 \rangle = K_1 \frac{I_o{}^a}{f^b} \Delta f$$

where K_1 a and b are process / technology dependent constants.

a is typically in the range 0.5 to 2

b is around unity – hence “1/f” noise

- Flicker noise is dominates other noise phenomena at very low frequencies.

Burst Noise

- Burst noise is present in all semiconductor devices.
- Is associated with current flow.
- Origins not well understood but possibly due to heavy metal impurities.
- Burst noise varies with frequency.
- The means-square noise current power into an incremental bandwidth Δf is:

$$\langle i(t)^2 \rangle = K_2 \frac{I_o^{\,c}}{1+\left(\frac{f}{f_c}\right)^2} \Delta f$$

where K_2, c and f_c are process / technology dependent constants.

c is typically in the range 0.5 to 2

Avalanche Noise

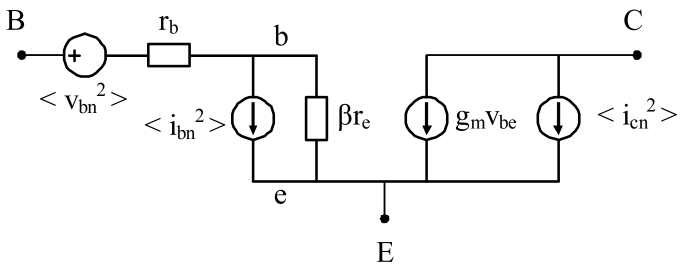
- Breakdown gives rise to large amounts of noise.
- Zener diodes and reverse biased BE junctions of BJTs are often used as noise sources.
- IMPATT diodes – a microwave negative resistance diode – can emit significant amount of noise at microwave frequencies.

Bipolar Junction Transistor Model

Simplified small-signal equivalent circuit including noise but ignoring parasitic effects.

Flicker and Burst noise sources are aggregated into the base noise current source.

Noise sources are uncorrelated



Bipolar Junction Transistor Model

Let I_B and I_C be the base and collector bias currents respectively:

$$\langle i_{bn}^2 \rangle = 2qI_B\Delta f + K_1 \frac{I_B^{\,a}}{f^{\,b}}\Delta f + K_2 \frac{I_B^{\,c}}{1+\left(\frac{f}{f_c}\right)^2}\Delta f$$

$$\langle i_{cn}^2 \rangle = 2qI_C\Delta f$$

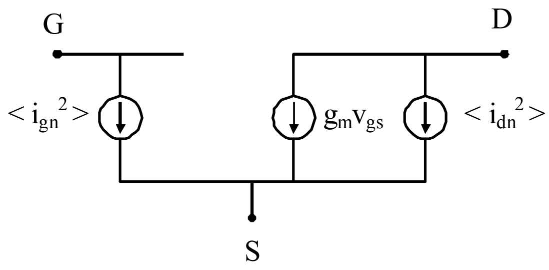
$$\langle v_{bn}^2 \rangle = 4kTr_b\Delta f$$

$$r_e = \frac{1}{g_m} = \frac{kT}{qI_C}$$

FET Model

Simplified small-signal equivalent circuit including noise but ignoring parasitic effects.

Noise sources are uncorrelated



FET Model

Let I_G and I_D be the gate and drain bias currents respectively:

$$\langle i_{gn}{}^2 \rangle = 2qI_G\Delta f \approx 0$$

$$\langle i_{dn}{}^2 \rangle = \underbrace{4K_{th}kTg_m\Delta f}_{} + K_1 \frac{I_D{}^a}{f^b}\Delta f$$

Thermal noise due to channel resistance

K_{th} is a constant between 0.67 and 5

Circuit Noise Calculations

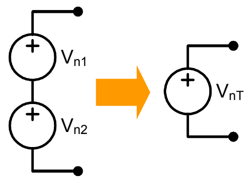
eg 1. Calculate the equivalent noise voltage of two noise voltages in in series.

$$\langle v_{nT}(t)^2 \rangle = \langle (v_{n1}(t) + v_{n2}(t))^2 \rangle$$

$$= \langle v_{n1}{}^2 + v_{n2}{}^2 + 2v_{n1}v_{n2} \rangle$$

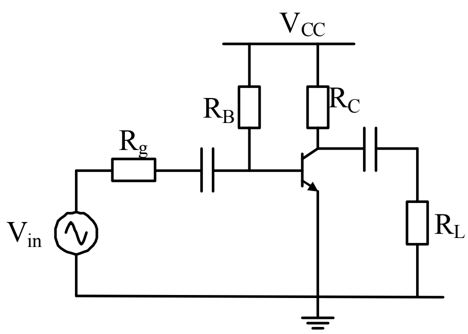
$$= \langle v_{n1}{}^2 \rangle + \langle v_{n2}{}^2 \rangle + 2\langle v_{n1}v_{n2} \rangle$$

Zero for uncorrelated noise sources



Circuit Noise Calculations

eg 2. Calculate the equivalent noise voltage across the load.



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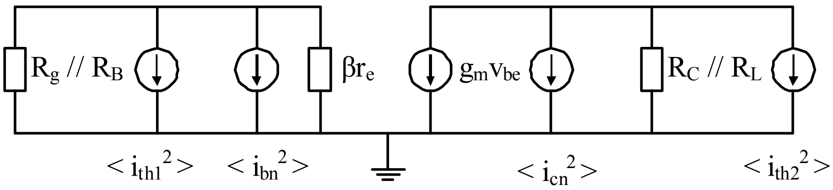


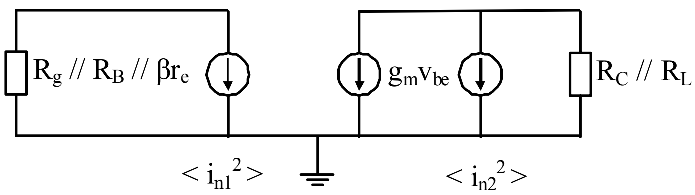
$$\langle i_{th1}{}^2 \rangle = \frac{4kT\Delta f}{R_g // R_B} \qquad \langle i_{cn}{}^2 \rangle = 2qI_C\Delta f \qquad \langle i_{th2}{}^2 \rangle = \frac{4kT\Delta f}{R_C // R_L}$$

$$\langle i_{bn}{}^2 \rangle = 2qI_B\Delta f + K_1 \frac{I_B{}^a}{f^b}\Delta f$$

For argument sake ignore burst noise and assume r_b is zero

Circuit Noise Calculations

eg 2. Calculate the equivalent noise voltage across the load.

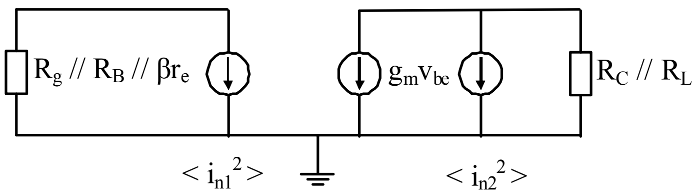


$$\langle i_{n1}{}^2 \rangle = \frac{4kT\Delta f}{R_g // R_B} + 2qI_B\Delta f + K_1 \frac{I_B{}^a}{f^b}\Delta f$$

$$\langle i_{th2}{}^2 \rangle = \frac{4kT\Delta f}{R_C // R_L} + 2qI_C\Delta f$$

Circuit Noise Calculations

eg 2. Calculate the equivalent noise voltage across the load.

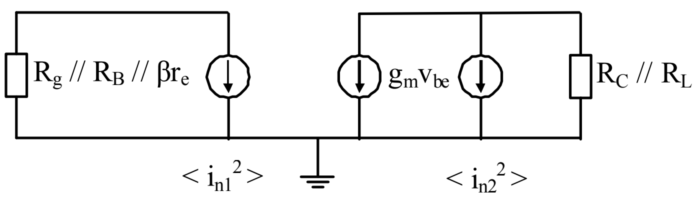


$$\langle v_{be}^2 \rangle = \langle i_{n1}^2 \rangle (R_g // R_R // \beta r_e)^2$$

$$= \left(\frac{4kT\Delta f}{R_g // R_B} + 2qI_B \Delta f + K_1 \frac{I_B^{\,a}}{f^b} \Delta f \right) (R_g // R_R // \beta r_e)^2$$

Circuit Noise Calculations

eg 2. Calculate the equivalent noise voltage across the load.



$$\langle v_{ce}^2 \rangle = \langle v_{be}^2 \rangle g_m^{\,2} (R_C // R_L)^2 + \langle i_{n2}^2 \rangle (R_C // R_L)^2$$

Circuit Noise Calculations

eg 2. Calculate the equivalent noise voltage across the load.

$$\langle v_{ce}{}^2 \rangle = g_m{}^2 \left(\frac{4kT\Delta f}{R_g // R_B} + 2qI_B\Delta f + K_1 \frac{I_B{}^a}{f^b}\Delta f \right) (R_g // R_R // \beta r_e)^2 (R_C // R_L)^2$$

$$+ \left(\frac{4kT\Delta f}{R_C // R_L} + 2qI_C\Delta f \right) (R_C // R_L)^2$$

Typically:

- R_g is much smaller than R_B and βr_e
- I_B is small

Recall that $g_m R_C // R_L$ is the amplifier gain

Circuit Noise Calculations

eg 2. So at midband frequencies:

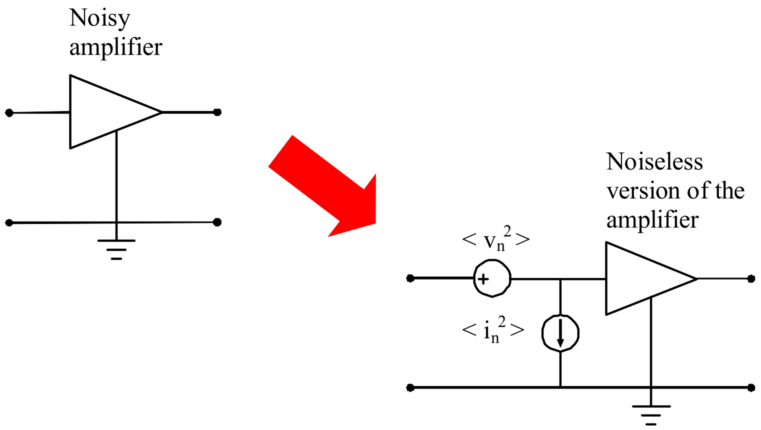
$$\langle v_{ce}{}^2 \rangle \approx g_m{}^2 (R_C // R_L)^2 4kTR_g\Delta f$$

$$+ 4kT(R_C // R_L)\Delta f + 2qI_C (R_C // R_L)^2 \Delta f$$

R_g has a significant impact on amplifier noise performance because its noise contribution is amplified by the transistor

Low Noise Amplifiers

An amplifier can be described by its gain, input and output impedances, and an equivalent noise current and noise voltage at its input.



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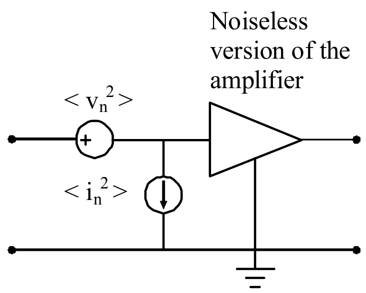


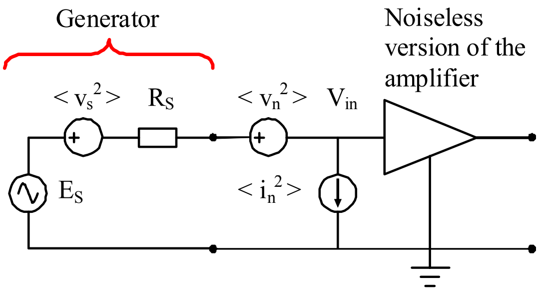
WARNING: v_n and i_n may be correlated: $v_n = v_{nu} + R_{cor} i_n$

where R_{cor} is a correlation coefficient whose dimensions is resistance

and v_{nu} is the uncorrelated component of noise voltage

Low Noise Amplifiers

Suppose we connect a generator to the input of the amplifier.

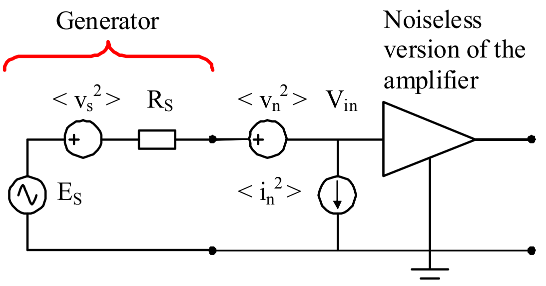


$$\langle v_S{}^2 \rangle = 4kT_{gen} R_S \Delta f$$

where T_{gen} is the effective temperature of the generator

Low Noise Amplifiers

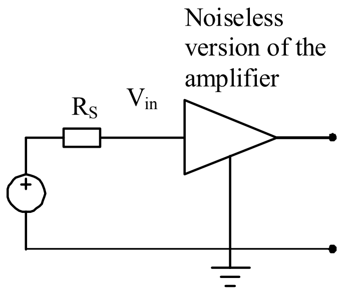
Suppose we connect a generator to the input of the amplifier.



Available Generator Noise Power: $N_{gen} = \dfrac{\langle V_S{}^2 \rangle}{4R_S} = kT_{gen} \Delta f$

Low Noise Amplifiers

Suppose we connect a generator to the input of the amplifier.



$$\langle v_T{}^2\rangle = \langle (v_s + v_n + i_n R_S)^2 \rangle$$

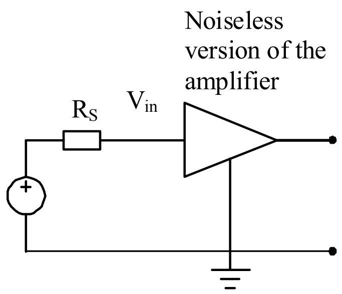
Taking the means square of v_T:

$$\langle v_T{}^2\rangle = \langle v_s{}^2\rangle + \langle v_n{}^2\rangle + \langle i_n{}^2\rangle R_S{}^2 + 2\langle v_n i_n\rangle R_S$$

$$= \langle v_s{}^2\rangle + \langle v_n{}^2\rangle + \langle i_n{}^2\rangle (R_S{}^2 + 2R_S R_{cor})$$

Low Noise Amplifiers

Suppose we connect a generator to the input of the amplifier.



$$\langle v_T{}^2\rangle = \langle (v_s + v_n + i_n R_S)^2 \rangle$$

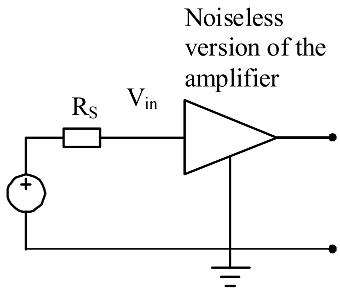
Available Total Noise Power:

$$N_{total} = \frac{\langle v_T{}^2\rangle}{4R_S} = \frac{\langle v_s{}^2\rangle + \langle v_n{}^2\rangle + \langle i_n{}^2\rangle (R_S{}^2 + 2R_S R_{cor})}{4R_S}$$

Low Noise Amplifiers

Suppose we connect a generator to the input of the amplifier.

$$\langle v_T^2 \rangle = \langle (v_s + v_n + i_n R_S)^2 \rangle$$



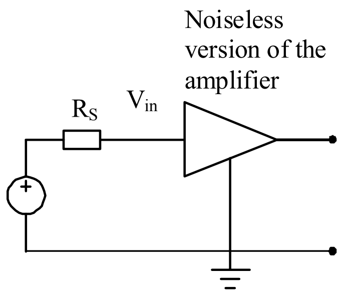
Available Total Noise Power:

$$N_{Total} = kT_{gen}\Delta f + \frac{\langle v_n^2 \rangle}{4R_S} + \frac{\langle i_n^2 \rangle}{4}(R_S + 2R_{cor})$$

Low Noise Amplifiers

Suppose we connect a generator to the input of the amplifier.

$$\langle v_T^2 \rangle = \langle (v_s + v_n + i_n R_S)^2 \rangle$$



Noise Factor:

$$F = \frac{N_{total}}{N_{gen}} = 1 + \frac{\langle V_n^2 \rangle}{4kT_{gen}R_S\Delta f} + \frac{\langle i_n^2 \rangle (R_S + 2R_{cor})}{4kT_{gen}\Delta f}$$

Low Noise Amplifiers

$$F = \frac{N_{total}}{N_{gen}} = 1 + \frac{\langle V_n^2 \rangle}{4kT_{gen} R_S \Delta f} + \frac{\langle i_n^2 \rangle (R_S + 2R_{cor})}{4kT_{gen} \Delta f}$$

Noise factor (F) is a measure of the deterioration of signal to noise ratio due to noise introduced by the amplifier.

F is a function of R_S and suggests an optimum value to minimise F.

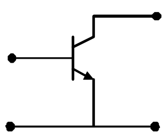
It can be shown that:

$$R_{S_{opt}} = \sqrt{\frac{\langle V_n^2 \rangle}{\langle i_n^2 \rangle}}$$

Low Noise Amplifiers

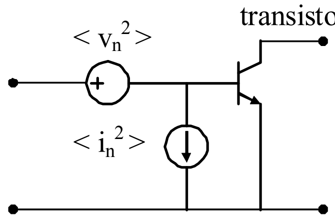
eg. Equivalent input noise generators of a transistor.

Noisy transistor



$< v_n^2 >$ and $< i_n^2 >$ are obtained by equating output noise currents for both circuits when the input is short-circuited and open-circuited

Noiseless version of the transistor

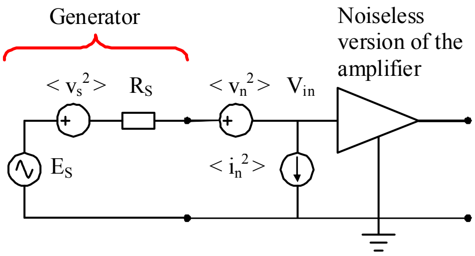


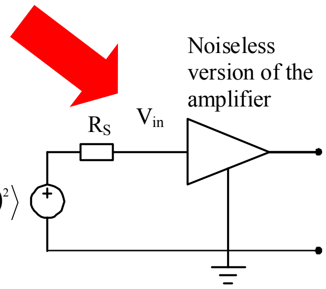
One outcome will be that v_n and i_n are correlated.

As a homework problem determine $< v_n^2 >$ and $< i_n^2 >$ for the BJT with non-zero r_b

Minimum Detectable Signal

Suppose we connect a generator to the input of the amplifier.





$$\langle v_T^2 \rangle = \langle (v_s + v_n + i_n R_S)^2 \rangle$$

Minimum Detectable Signal: $\sqrt{\langle V_T^2 \rangle}$