Smith chart

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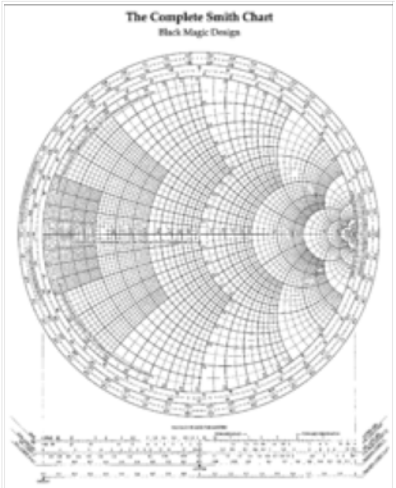
(Redirected from Smith Chart)



An impedance Smith chart (with no data plotted)

The **Smith Chart**, invented by Phillip H. Smith (1905-1987),[1][2] is a graphical aid or nomogram designed for electrical and electronics engineers specializing in radio frequency (RF) engineering to assist in solving problems with transmission lines and matching circuits.[3] Use of the Smith Chart utility has grown steadily over the years and it is still widely used today, not only as a problem solving aid, but as a graphical demonstrator of how many RF parameters behave at one or more frequencies, an alternative to using tabular information. The Smith Chart can be used to represent many parameters including impedances, admittances, reflection coefficients, S_{nn} scattering parameters, noise figure circles, constant gain contours and regions for unconditional stability.[4][5] The Smith Chart is most frequently used at or within the unity radius region. However, the remainder is still mathematically relevant, being used, for example, in oscillator design and stability analysis.[6]

Contents

Overview

The Smith Chart is plotted on the complex reflection coefficient plane in two dimensions and is scaled in normalised impedance (the most common), normalised admittance or both, using different colours to distinguish between them. These are often known as the Z, Y and YZ Smith Charts respectively.[7] Normalised scaling allows the Smith Chart to be used for problems involving any characteristic impedance or system impedance, although by far the most commonly used is 50 ohms. With relatively simple graphical construction it is straighforward to convert between normalised impedance (or normalised admittance) and the corresponding complex voltage reflection coefficient.

A network analyzer (HP 8720A) showing a Smith chart.

The Smith Chart has circumferential scaling in wavelengths and degrees. The wavelengths scale is used in distributed component
problems and represents the distance measured along the transmission line connected between the generator or source and the load to the point under consideration. The degrees scale represents the angle of the voltage reflection coefficient at that point. The Smith Chart may also be used for lumped element matching and analysis problems.

Use of the Smith Chart and the interpretation of the results obtained using it requires a good understanding of AC circuit theory and transmission line theory, both of which are pre-requisites for RF engineers.

As impedances and admittances change with frequency, problems using the Smith Chart can only be solved manually using one frequency at a time, the result being represented by a point. This is often adequate for narrow band applications (typically up to about 5% to 10% bandwidth) but for wider bandwidths it is usually necessary to apply Smith Chart techniques at more than one frequency across the operating frequency band. Provided the frequencies are sufficiently close, the resulting Smith Chart points may be joined by straight lines to create a locus.

A locus of points on a Smith Chart covering a range of frequencies can be used to visually represent:

- how capacitive or how inductive a load is across the frequency range
- how difficult matching is likely to be at various frequencies
- how well matched a particular component is.

The accuracy of the Smith Chart is reduced for problems involving a large spread of impedances or admittances, although the scaling can be magnified for individual areas to accommodate these.

Mathematical basis

Actual and normalised impedance and admittance

A transmission line with a characteristic impedance of Z_0 may be universally considered to have a characteristic admittance of Y_0 where

$$Y_0 = \frac{1}{Z_0}$$

Any impedance, Z_T
expressed in ohms, may be normalised by dividing it by the characteristic impedance, so the normalised impedance using the lower case z, suffix T is given by

$$z_T = \frac{Z_T}{Z_0}$$

Similarly, for normalised admittance

$$y_T = \frac{Y_T}{Y_0}$$

The SI unit of impedance is the ohm with the symbol of the upper case Greek letter Omega (Ω) and the SI unit for admittance is the siemens
with the symbol of an upper case letter S. Normalised impedance and normalised admittance are dimensionless. Actual impedances and admittances must be normalised before using them on a Smith Chart. Once the result is obtained it may be de-normalised to obtain the actual result.

The normalised impedance Smith Chart

Using transmission line theory, if a transmission line is terminated in an impedance (Z_T) which differs from its characteristic impedance (Z_0), a standing wave will be formed on the line comprising the resultant of both the forward (V_F) and the reflected (V_R) waves. Using complex exponential notation:

$V_F = A\exp(j\omega t)\exp(-\gamma l)$ and
$V_R = B\exp(j\omega t)\exp(\gamma l)$

where

$\exp(j\omega t)$ is the temporal part of the wave and
$\omega = 2\pi f$ where
ω is the angular frequency in radians per second (rad/s)
f is the frequency in hertz (Hz)
t is the time in seconds (s)
A and B are constants
l is the distance measured along the transmission line from the generator in metres (m)

Also

$\gamma = \alpha + j\beta$ is the propagation constant which has units 1/m

where

α is the attenuation constant in nepers per metre (Np/m)
β is the phase constant in radians per metre (rad/m)

The Smith Chart is used with one frequency at a time so the temporal part of the phase ($\exp(\omega t)$) is fixed. All terms are actually multiplied by this to obtain the instantaneous phase, but it is conventional and understood to omit it. Therefore

$V_F = A\exp(-\gamma l)$ and

$$V_R = B\exp(\gamma l)$$

The variation of complex reflection coefficient with position along the line

The complex voltage reflection coefficient ρ
is defined as the ratio of the reflected wave to the incident (or forward) wave. Therefore

$$\rho = \frac{V_R}{V_F} = \frac{B\exp(\gamma l)}{A\exp(-\gamma l)} = C\exp(2\gamma l)$$

where *C* is also a constant.

For a uniform transmission line (in which γ
is constant), the complex reflection coefficient of a standing wave varies according to the position on the line. If the line is lossy (α is finite) this is represented on the Smith Chart by a spiral path. In most Smith Chart problems however, losses can be assumed negligible ($\alpha = 0$) and the task of solving them is greatly simplified. For the loss free case therefore, the expression for complex reflection coefficient becomes

$$\rho = \rho_0\exp(2j\beta l)$$

The phase constant β may also be written as

$$\beta = \frac{2\pi}{\lambda}$$

where λ is the wavelength *within the transmission line* at the test frequency.

Therefore

$$\rho = \rho_0\exp(\frac{4j\pi}{\lambda}l)$$

This equation shows that, for a standing wave, the complex reflection coefficient and impedance repeats every half wavelength along the transmission line. The complex reflection coefficient is generally simply referred to as reflection coefficient. The outer circumferential scale of the Smith Chart represents the distance from the generator to the load scaled in wavelengths and is therefore scaled from zero to 0.50.

The variation of normalised impedance with position along the line

If V and I
are the voltage across and the current entering the termination at the end of the transmission line respectively, then

$$V_F + V_R = V \text{ and}$$
$$V_F - V_R = Z_0 I\,.$$

By dividing these equations and substituting for both the voltage reflection coefficient

$$\rho = \frac{V_R}{V_F}$$

and the normalised impedance of the termination represented by the lower case *z*, subscript T

$$z_T = \frac{V}{Z_0 I}$$

gives the result:

$$z_T = \frac{1+\rho}{1-\rho}.$$

Alternatively, in terms of the reflection coefficient

$$\rho = \frac{z_T - 1}{z_T + 1}$$

These are the equations which are used to construct the Z Smith Chart. Mathematically speaking ρ and z_T are related via a Möbius transformation.

Both ρ and z_T are expressed in complex numbers
without any units. They both change with frequency so for any particular measurement, the frequency at which it was performed must be stated together with the characteristic impedance.

ρ may be expressed in magnitude and angle on a polar diagram. Any actual reflection coefficient must have a magnitude of less than or equal to unity
so, at the test frequency, this may be expressed by a point inside a circle of unity radius. The Smith Chart is actually constructed on such a polar diagram. The Smith chart scaling is designed in such a way that reflection coefficient can be converted to normalised impedance or vice versa. Using the Smith Chart, the normalised impedance may be obtained with appreciable accuracy by plotting the point representing the reflection coefficient *treating the Smith Chart as a polar diagram* and then reading its value directly using the characteristic Smith Chart scaling. This technique is a graphical alternative to substituting the values in the equations.

By substituting the expression for how reflection coefficient changes along an unmatched loss free transmission line

$$\rho = \frac{B\exp(\gamma l)}{A\exp(-\gamma l)} = \frac{B\exp(j\beta l)}{A\exp(-j\beta l)}$$

for the loss free case, into the equation for normalised impedance in terms of reflection coefficient

$$z_T = \frac{1+\rho}{1-\rho}.$$

and using Euler's identity

$$\exp(j\theta) = \cos\theta + j\sin\theta$$

yields the impedance version transmission line equation for the loss free case:[8]

$$Z_{IN} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

where Z_{IN}
is the impedance 'seen' at the input of a loss free transmission line of length l, terminated with an impedance Z_L

Versions of the transmission line equation may be similarly derived for the admittance loss free case and for the impedance and admittance lossy cases.

The Smith Chart graphical equivalent of using the transmission line equation is to normalise Z_L, to plot the resulting point on a Z Smith Chart and to draw a circle through that point centred at the Smith Chart centre. The path along the arc of the circle represents how the impedance changes whilst moving along the transmission line. In this case the circumferential (wavelength) scaling must be used, remembering that this is the wavelength within the transmission line and may differ from the free space wavelength.

Regions of the Z Smith Chart

If a polar diagram is mapped on to a cartesian coordinate system it is conventional to measure angles relative to the positive x-axis using a counter-clockwise
direction for positive angles. The magnitude of a complex number is the length of a straight line drawn from the origin
to the point representing it. The Smith Chart uses the same convention, noting that, in the normalised impedance plane, the positive x-axis extends from the center of the Smith Chart at $z_T = 1 \pm j0$ to the point $z_T = \infty \pm j\infty$
. The region above the x-axis represents inductive impedances and the region below the x-axis represents capacitive impedances. Inductive impedances have positive imaginary parts and capacitive impedances have negative imaginary parts.

If the termination is perfectly matched, the reflection coefficient will be zero, represented effectively by a circle of zero radius or in fact a point at the centre of the Smith Chart. If the termination was a perfect open circuit or short circuit
the magnitude of the reflection coefficient would be unity, all power would be reflected and the point would lie at some point on the unity circumference circle.

Circles of Constant Normalised Resistance and Constant Normalised Reactance

The normalised impedance Smith Chart is composed of two families of circles: circles of constant normalised resistance and circles of constant normalised reactance. In the complex reflection coefficient plane the Smith Chart occupies a circle of unity radius centred at the origin. In cartesian coordinates therefore the circle would pass through the points (1,0) and (-1,0) on the x-axis and the points (0,1) and (0,-1) on the y-axis.

Since both ρ and z
are complex numbers, in general they may be expressed by the following generic rectangular complex numbers:

$$z = a + jb$$
$$\rho = c + jd$$

Substituting these into the equation relating normalised impedance and complex reflection coefficient:

$$\rho = \frac{z-1}{z+1}$$

gives the following result:

$$\rho = c + jd = \frac{a^2 + b^2 - 1}{(a+1)^2 + b^2} + j\left(\frac{2b}{(a+1)^2 + b^2}\right).$$

This is the equation which describes how the complex reflection coefficient changes with the normalised impedance and may be used to construct both families of circles.[9]

The Y Smith Chart

The Y Smith chart is constructed in a similar way to the Z Smith Chart case but by expressing values of voltage reflection coefficient in terms of normalised admittance instead of normalised impedance. The normalised admittance y_T is the reciprocal of the normalised impedance z_T, so

$$y_T = \frac{1}{z_T}$$

Therefore:

$$y_T = \frac{1-\rho}{1+\rho}$$

and

$$\rho = \frac{1-y_T}{1+y_T}$$

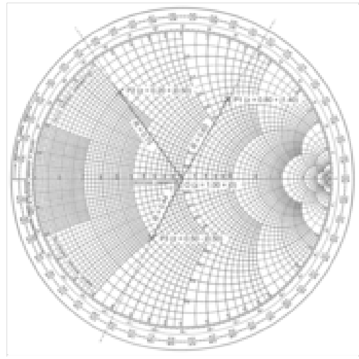
The Y Smith Chart appears like the normalised impedance type but with the graphic scaling rotated through $180°$, the numeric scaling remaining unchanged.

The region above the x-axis represents capacitive admittances and the region below the x-axis represents inductive admittances. Capacitive admittances have positive imaginary parts and inductive admittances have negative imaginary parts.

Again, if the termination is perfectly matched the reflection coefficient will be zero, represented by a 'circle' of zero radius or in fact a point at the centre of the Smith Chart. If the termination was a perfect open or short circuit the magnitude of the voltage reflection coefficient would be unity, all power would be reflected and the point would lie at some point on the unity circumference circle of the Smith Chart.

Practical examples

A point with a reflection coefficient magnitude 0.63 and angle $60°$, represented in polar form as $0.63\angle 60°$, is shown as point P_1 on the Smith Chart. To plot this, one may use the circumferential (reflection coefficient) angle scale to find the $\angle 60°$ graduation and a ruler to draw a line passing through this and the centre of the Smith Chart. The length of the line would then be scaled to P_1
assuming the Smith Chart radius to be unity. For example if the actual radius measured from the paper was 100 mm, the length OP_1 would be 63 mm.


Example points plotted on the normalised impedance Smith Chart

The following table gives some similar examples of points which are plotted on the Z Smith Chart. For each, the reflection coefficient is given in polar form together with the corresponding normalised impedance in rectangular form. The conversion may be read directly from the Smith Chart or by substitution into the equation.

Some examples of points plotted on the normalised impedance Smith Chart

Point Identity	Reflection Coefficient (Polar Form)	Normalised Impedance (Rectangular Form)
P_1 (Inductive)	$0.63\angle 60^\circ$	$0.80 + j1.40$
P_2 (Inductive)	$0.73\angle 125^\circ$	$0.20 + j0.50$
P_3 (Capacitive)	$0.44\angle -116^\circ$	$0.50 - j0.50$

Working with both the Z Smith Chart and the Y Smith Charts

In RF circuit and matching problems sometimes it is more convenient to work with admittances (representing conductances and susceptances) and sometimes it is more convenient to work with impedances (representing resistances and reactances). Solving a typical matching problem will often require several changes between both types of Smith Chart, using normalised impedance for series elements and normalised admittances for parallel
elements. For these a dual (normalised) impedance and admittance Smith Chart may be used. Alternatively, one type may be used and the scaling converted to the other when required. In order to change from normalised impedance to normalised admittance or vice versa, the point representing the value of reflection coefficient under consideration is moved through exactly 180 degrees at the same radius. For example the point P1 in the example representing a reflection coefficient of $0.63\angle 60^\circ$ has a normalised impedance of
$z_P = 0.80 + j1.40$
. To graphically change this to the equivalent normalised admittance point, say Q1, a line is drawn with a ruler from P1 through the Smith Chart centre to Q1, an equal radius in the opposite direction. This is equivalent to moving the point through a circular path of exactly 180 degrees. Reading the value from the Smith Chart for Q1, remembering that the scaling is now in normalised admittance, gives $y_P = 0.30 - j0.54$. Performing the calculation

$$y_T = \frac{1}{z_T}$$

manually will confirm this.

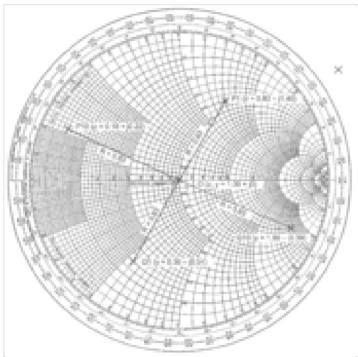
Once a transformation
from impedance to admittance has been performed the scaling changes to normalised admittance until such time that a later transformation back to normalised impedance is performed.

The table below shows examples of normalised impedances and their equivalent normalised admittances obtained by rotation of the point through 180°
. Again these may either be obtained by calculation or using a Smith Chart as shown, converting between the normalised impedance and normalised admittances planes.

Values of reflection coefficient as normalised impedances and the equivalent normalised admittances

Normalised Impedance Plane	Normalised Admittance Plane
P_1 ($z = 0.80 + j1.40$)	Q_1 ($y = 0.30 - j0.54$)
P_{10} ($z = 0.10 + j0.22$)	Q_{10} ($y = 1.80 - j3.90$)

Choice of Smith Chart type and component type



Values of complex reflection coefficient plotted on the normalised impedance Smith Chart and their equivalents on the normalised admittance Smith Chart

The choice of whether to use the Z Smith Chart or the Y Smith Chart for any particular calculation depends on which is more convenient. Impedances in series and admittances in parallel add whilst impedances in parallel and admittances in series are related by a reciprocal equation. If Z_{TS} is the equivalent impedance of series impedances and Z_{TP} is the equivalent impedance of parallel impedances, then

$$Z_{TS} = Z_1 + Z_2 + Z_3 + ...$$
$$\frac{1}{Z_{TP}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + ...$$

For admittances the reverse is true, that is

$$Y_{TP} = Y_1 + Y_2 + Y_3 + ...$$
$$\frac{1}{Y_{TS}} = \frac{1}{Y_1} + \frac{1}{Y_2} + \frac{1}{Y_3} + ...$$

Dealing with the reciprocals, especially in complex numbers, is more time consuming and error-prone than using linear addition. In general therefore, most RF engineers work in the plane where the circuit topography supports linear addition. The following table gives the complex expressions for impedance (real and normalised) and admittance (real and normalised) for each of the three basic passive circuit elements: resistance, inductance and capacitance. Knowing just the characteristic impedance (or characteristic admittance) and test frequency can be used to find the equivalent circuit from any impedance or admittance, or vice versa.

Expressions for Real and Normalised Impedance and Admittance with Characteristic Impedance Z_0 or Char

Element Type	Impedance (Z or z) or Reactance (X or x)		Admittance (Y or y) or S	
	Real (Ω)	Normalised (No Unit)	Real (S)	Normali
Resistance (R)	$Z = R$	$z = \frac{R}{Z_0} = RY_0$	$Y = G = \frac{1}{R}$	$y = g$
Inductance (L)	$Z = jX_L = j\omega L$	$z = jx_L = j\frac{\omega L}{Z_0} = j\omega LY_0$	$Y = -jB_L = \frac{-j}{\omega L}$	$y = -$
Capacitance (C)	$Z = -jX_C = \frac{-j}{\omega C}$	$z = -jx_C = \frac{-j}{\omega CZ_0} = \frac{-jY_0}{\omega C}$	$Y = jB_C = j\omega C$	$y = jl$

Using the Smith Chart to solve conjugate matching problems with

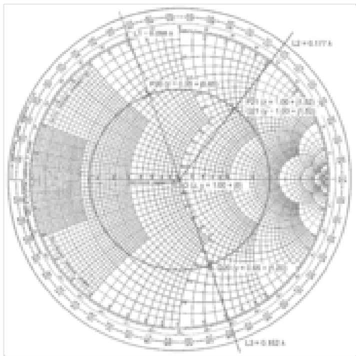
distributed components

Usually distributed matching is only feasible at microwave frequencies since, for most components operating at these frequencies, appreciable transmission line dimensions are available in terms of wavelengths. Also the electrical behavior of many lumped components becomes rather unpredictable at these frequencies.

For distributed components the effects on reflection coefficient and impedance of moving along the transmission line must be allowed for using the outer circumferential scale of the Smith Chart which is calibrated in wavelengths.

The following example shows how a transmission line, terminated with an arbitrary load, may be matched at one frequency either with a series or parallel reactive component in each case connected at precise positions.

Supposing a loss free air-spaced transmission line of characteristic impedance $Z_0 = 50\ \Omega$



Smith Chart construction for some distributed transmission line matching

, operating at a frequency of 800 MHz, is terminated with a circuit comprising a 17.5 Ω resistor in series with a 6.5 nanohenry (6.5 nH) inductor. How may the line be matched?

From the table above, the reactance of the inductor forming part of the termination at 800 MHz is

$$Z_L = j\omega L = j2\pi f L = j32.7\ \Omega$$

so the impedance of the combination (Z_T) is given by

$$Z_T = 17.5 + j32.7\ \Omega$$

and the normalised impedance (z_T)is

$$z_T = \frac{Z_T}{Z_0} = 0.35 + j0.65$$

This is plotted on the Z Smith Chart at point P_{20}. The line OP_{20} is extended through to the wavelength scale where it intersects at the point $L_1 = 0.098\lambda$
. As the transmission line is loss free, a circle centred at the centre of the Smith Chart is drawn through the point P_{20} to represent the path of the constant magnitude reflection coefficient due to the termination. At point P_{21} the circle intersects with the unity circle of constant normalised resistance at

$$z_{P21} = 1.00 + j1.52.$$

The extension of the line OP_{21} intersects the wavelength scale at $L_2 = 0.177\lambda$, therefore the distance from the termination to this point on the line is given by

$$L_2 - L_1 = 0.177\lambda - 0.098\lambda = 0.079\lambda$$

Since the transmission line is air-spaced, the wavelength at 800 MHz in the line is the same as that in free space and is given by

$$\lambda = \frac{c}{f}$$

where c is the velocity of electromagnetic radiation in free space and f is the frequency in hertz. The result gives $\lambda = 375\ \mathrm{mm}$, making the position of the matching component 29.6 mm from the load.

The conjugate match for the impedance at P_{21} (z_{match}) is

$z_{match} = -j(1.52)$,

As the Smith Chart is still in the normalised impedance plane, from the table above a series capacitor C_m is required where

$$z_{match} = -j1.52 = \frac{-j}{\omega C Z_0} = \frac{-j}{2\pi f C_m Z_0}$$

Rearranging, we obtain

$$C_m = \frac{-1}{(1.52)\omega Z_0} = \frac{-1}{(1.52)(2\pi f)Z_0}.$$

Substitution of known values gives

$$C_m = 2.6\ \mathrm{pF}$$

To match the termination at 800 MHz, a series capacitor of 2.6 pF must be placed in series with the transmission line at a distance of 29.6 mm from the termination.

An alternative shunt match could be calculated after performing a Smith Chart transformation from normalised impedance to normalised admittance. Point Q_{20} is the equivalent of P_{20} but expressed as a normalised admittance. Reading from the Smith Chart scaling, remembering that this is now a normalised admittance gives

$$y_{Q20} = 0.65 - j1.20$$

(In fact this value is not actually used). However, the extension of the line OQ_{20} through to the wavelength scale gives $L_3 = 0.152\lambda$
. The earliest point at which a shunt conjugate match could be introduced,moving towards the generator, would be at Q_{21}, the same position as the previous P_{21}, but this time representing a normalised admittance given by

$$y_{Q21} = 1.00 + j1.52.$$

The distance along the transmission line is in this case

$$L_2 + L_3 = 0.177\lambda + 0.152\lambda = 0.329\lambda$$

which converts to 123 mm.

The conjugate matching component is required to have a normalised admittance (y_{match}) of

$$y_{match} = -j1.52.$$

From the table it can be seen that a negative admittance would require to be an inductor, connected in parallel with the transmission line. If its value is L_m, then

$$-j1.52 = \frac{-j}{\omega L_m Y_0} = \frac{-jZ_0}{2\pi f L_m}$$

This gives the result

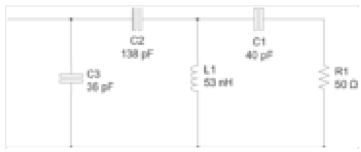
$$L_m = 6.5 \text{ nH}$$

A suitable inductive shunt matching would therefore be a 6.5 nH inductor in parallel with the line positioned at 123 mm from the load.

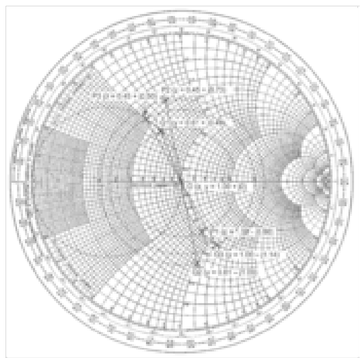
Using the Smith Chart to analyze lumped element circuits

The analysis of lumped element components assumes that the wavelength at the frequency of operation is much greater than the dimensions of the components themselves. The Smith Chart may be used to analyze such circuits in which case the movements around the chart are generated by the (normalized) impedances and admittances of the components at the frequency of operation. In this case the wavelength scaling on the Smith Chart circumference is not used. The following circuit will be analyzed using a Smith Chart at an operating frequency of 100 MHz. At this frequency the free space wavelength is 3 m. The component dimensions themselves will be in the order of millimetres so the assumption of lumped components will be valid. Despite there being no transmission line as such, a system impedance must still be defined to enable normalization and de-normalization calculations and $Z_0 = 50\ \Omega$ is a good choice here as $R_1 = 50\ \Omega$. If there were very different values of resistance present a value closer to these might be a better choice.

The analysis starts with a Z Smith Chart looking into R_1 only with no other components present. As $R_1 = 50\ \Omega$ is the same as the system impedance,



A lumped element circuit which may be analysed using a Smith Chart


Smith Chart with graphical construction for analysis of a lumped circuit

this is represented by a point at the centre of the Smith Chart. The first transformation is OP_1 along the line of constant normalized resistance in this case the addition of a normalized reactance of -*j*0.80, corresponding to a series capacitor of 40 pF. Points with suffix P are in the *Z* plane and points with suffix Q are in the *Y* plane. Therefore transformations P_1 to Q_1 and P_3 to Q_3
are from the Z Smith Chart to the Y Smith Chart and transformation Q_2 to P_2 is from the Y Smith Chart to the

Z Smith Chart. The following table shows the steps taken to work through the remaining components and transformations, returning eventually back to the centre of the Smith Chart and a perfect 50 ohm match.

Smith Chart steps for analysing a lumped element circuit

Transformation	**Plane**	***x* or *y* Normalized Value**	**Capacitance/Inductance**	**Formula to Solve**	**Result**
$O \rightarrow P_1$	Z	$-j0.80$	Capacitance (Series)	$-j0.80 = \dfrac{-j}{\omega C_1 Z_0}$	$C_1 = 40$ pF
$Q_1 \rightarrow Q_2$	Y	$-j1.49$	Inductance (Shunt)	$-j1.49 = \dfrac{-j}{\omega L_1 Y_0}$	$L_1 = 53$ nH
$P_2 \rightarrow P_3$	Z	$-j0.23$	Capacitance (Series)	$-j0.23 = \dfrac{-j}{\omega C_2 Z_0}$	$C_2 = 138$ pF
$Q_3 \rightarrow O$	Y	$+j1.14$	Capacitance (Shunt)	$+j1.14 = \dfrac{j\omega C_3}{Y_0}$	$C_3 = 36$ pF

References

1. ^ Smith, P. H.; Transmission Line Calculator; Electronics, Vol. 12, No. 1, pp 29-31, January 1939
2. ^
   Smith, P. H.; An Improved Transmission Line Calculator; Electronics, Vol. 17, No. 1, p 130, January 1944
3. ^
   Ramo, Whinnery and Van Duzer (1965); "Fields and Waves in Communications Electronics"; John Wiley & Sons; pp 35-39. ISBN
4. ^ Pozar, David M. (2005); *Microwave Engineering, Third Edition* (Intl. Ed.); John Wiley & Sons, Inc.; pp 64-71. ISBN 0-471-44878-8.
5. ^ Gonzalez, Guillermo (1997); *Microwave Transistor Amplifiers Analysis and Design, Second Edition*; Prentice Hall NJ; pp 93-103. ISBN 0-13-254335-4.
6. ^ Gonzalez, Guillermo (1997) (op. cit);pp 98-101
7. ^ Gonzalez, Guillermo (1997) (op. cit);p 97
8. ^
   Hayt, William H Jr.; "Engineering Electromagnetics" Fourth Ed;McGraw-Hill International Book Company; pp 428 433. ISBN 0-07-027395-2.
9. ^
   Davidson, C. W.;"Transmission Lines for Communications with CAD Programs";Macmillan; pp 80-85. ISBN 0-333-47398-1

- P.H.Smith 1969 *Electronic Applications of the Smith Chart.* Kay Electric Company

External links

- A Collection of Smith Chart Resources (http://www.sss-mag.com/smith.html) Tutorials, graphics and other info on Smith Chart
- linSmith (http://www.jcoppens.com/soft/linsmith/index.en.php) Smith charting program for Linux.
- Smith Chart (http://www.printfreegraphpaper.com/) Print free Smith Charts from your computer.
- Black Magic (http://www.sss-mag.com/pdf/smithchart.pdf) Smith Chart - Vector-graphic (infinitely scalable) Smith Chart for practical use.
- The Java Smith-Chart-Tool (http://www.ate.uni-duisburg.de:8080/hp-renny/download/jsct/) - A free Java-Tool to paint s-parameters in a Smith-Chart.
- Smith Chart Applet (http://www.ocf.berkeley.edu/%7Ejoydip/smithchart/) - a Java applet that simulates

operations on a Smith chart
- Stub Matching Applet (http://www.ee.iitb.ac.in/uma/%7Espalit/SmithChart.html) - Java applet that simulates stub matching on a Smith chart
- Smith Excel Graph (http://www.sss-mag.com/txt/SmithChartRevD.xls) plots reflection coefficient data in real and imaginary formats on a customizable Smith Chart (Microsoft Excel Spreadsheet 53K)
- PostScript functions (http://www.nikeserver.net/gian.leonardo.solazzi/smith.html) Functions to plot dots, lines, gamma circle, constant real and imaginary path in PostScript format to make vectorial images.
- An online educational interactive Smith chart (http://cgi.www.telestrian.co.uk/cgi-bin/www.telestrian.co.uk/smiths.pl) . A choice of impedance and admittance charts with a chart marker.
- A Matlab m file to generate a standard color smith chart (http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=324&objectType=FILE) .
- QuickSmith (http://www.nathaniyer.com/qsdw.htm) - a rapid and efficient Smith Chart matching tool
- Printfreegraphpaper.com (http://www.printfreegraphpaper.com/) Print free Smith Chart Graph Paper.
- **(Italian)**Smith Chart demo 1.0 (http://bart99.altervista.org/smith/) Interactive Smith Chart using Canvas