5. NONLINEARITY in RF CIRCUITS and SYSTEMS

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Summary

- Background
- Linear phenomena
- Nonlinear phenomena
- Single-tone excitation
- Two-tone excitation
- Multitone excitation
- Large-signal small-signal excitation
- Power amplifier behaviour
- Mixers

References

- D M Pozar, Microwave Engineering, 3rd Edition, John Wiley 2005 [Sections 10.2, 11.5, 12.6]
- S A Maas, Nonlinear Microwave and RF Circuits, Artech House 2003.
- H L Krauss, C W Bostian & F H Raab, Solid State Radio Engineering, John Wiley 1980.

Background

- All electronic circuits employing transistors and diodes are inherently nonlinear.
- The small-signal approximation, assumes that for a given dc bias, the transistor or diode appears linear to the AC signal of very small amplitude.
- Nonlinear means that the **output signal level** is **NOT directly proportional** to the **input signal level**. This means that the **signal is distorted**.
- Distortion by a nonlinearity is not to be confused with linear distortion phenomena such as linear channel dispersion.
- Nonlinearity can either be **undesirable or essential** for circuit operation.

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Background

Nonlinearity is **undesirable** in what are **supposedly linear circuits**:

- Low-noise and small to medium signal amplifiers
- Class-A power amplifiers

Nonlinearity devices are **essential elements** in:

- Oscillators (ensures stable oscillation)
- Mixers (multiplies LO with an input signal)
- Class-B, class-C, class-D, class-E and class-F power amplifiers

  (shapes collector / drain voltage and current waveforms so as to maximise efficiency)

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Linear Phenomena

A linear system is defined by the following relationships:

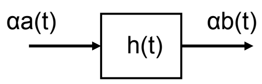
$$v_{out}(t) = h(t) * v_{in}(t) \quad \leftrightarrow \quad V_{out}(\omega) = H(\omega)\, V_{in}(\omega)$$

where h(t) is the impulse response and H(ω) is the transfer function being the Fourier transform of h(t).

Important properties:

If:

1.



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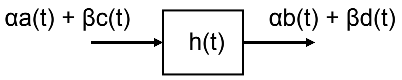


Nonlinear Phenomena

- The (output) response of nonlinear system is related by a nonlinear function to the (input) excitation.
- Concepts such as scaling and superposition do not hold.
- The response contains spectral components that are NOT present in the excitation.
- Fortunately, the frequencies of the response spectral components are easily predicted – their amplitudes are not.
- It is useful to know that the response can be represented by a Fourier series.
- It is useful to know that a nonlinear function can be represented by its Taylor series.

Single-Tone Excitation – Example 1

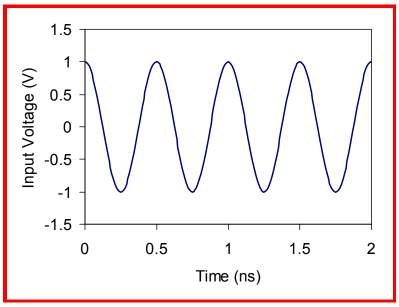
The output voltage of an amplifier is related to the input voltage in the following manner:

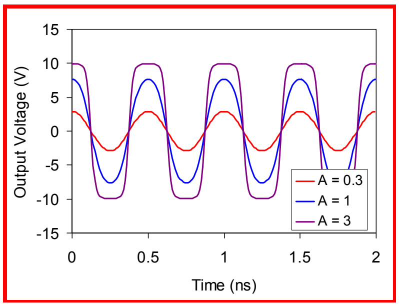
$$v_{out}(t) = 10\tanh\left(v_{in}(t)\right)$$

Suppose the input signal is a **single-tone excitation**: $v_{in}(t) = A\cos\omega t$

Hence the output voltage will be:

$$v_{out}(t) = 10\tanh\left(A\cos\omega t\right)$$





Single-Tone Excitation

- When the input signal amplitude a is small, $v_{out}(t)$ looks like the input signal – ie the small-signal concept.
- For increasing input signal amplitude, the output voltage remains periodic with same period as the input waveform.
- This means that $v_{out}(t)$ can be represented by a Fourier series:

$$v_{out}(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega t) + b_m \sin(m\omega t)$$

  - Fundamental: ω
  - Harmonics: 2ω, 3ω, ….
- That is the output has frequency components that are not present at the input.
- Knowledge of the Fourier coefficients is important as they tell us the level of each harmonic.
- Harmonics are undesired output signals in an amplifier but may be desired in other circuits such as a frequency multiplier.

Single-Tone Excitation

We could **directly calculate the Fourier coefficients** either:

- **Analytically**
  - Results in expressions for fundamental and harmonic levels in terms of the input level a.
  - May be impossible depending on the transfer function.
- **Numerically** using the FFT
  - Input data is the sampled output voltage waveform.

We could **indirectly calculate the Fourier coefficients** using a polynomial approximation of the transfer function and the application of trigonometric identities.

The polynomial approximations are obtained from Taylor series.

Single-Tone Excitation – Example 2

The output voltage of an amplifier is related to the input voltage in the following manner:

$$v_{out}(t) = a_1 v_{in}(t) + a_2 (v_{in}(t))^2 + a_3 (v_{in}(t))^3$$

Suppose the input signal is of the form: $v_{in}(t) = A \cos\omega t$

In this case we have a polynomial.

Recall:

$$2\cos x \cos y = \cos(x + y) + \cos(x - y)$$

$$4\cos x \cos^2 y = 2\cos x + \cos(x + 2y) + \cos(x - 2y)$$

So: $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x \qquad \cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos 3x$

Hence:

$$v_{out}(t) = \frac{a_2 A^2}{2} + \left(a_1 A + \frac{3a_3 A^3}{4}\right)\cos\omega t + \frac{a_2 A^2}{2}\cos 2\omega t + \frac{a_3 A^3}{4}\cos 3\omega t$$

Single-Tone Excitation

From the previous example a nonlinearity defined by:

$$v_{out}(t) = a_1 v_{in}(t) + a_2 (v_{in}(t))^2 + a_3 (v_{in}(t))^3$$

with a **single-tone input signal** of $A\cos\omega t$ results in the following waveform:

$$v_{out}(t) = \underbrace{\frac{a_2 A^2}{2}}_{\text{dc component}} + \underbrace{\left(a_1 A + \frac{3a_3 A^3}{4}\right)}_{\omega\text{ component}}\cos\omega t + \underbrace{\frac{a_2 A^2}{2}}_{2\omega\text{ component}}\cos 2\omega t + \underbrace{\frac{a_3 A^3}{4}}_{3\omega\text{ component}}\cos 3\omega t$$

- The above expression is a Fourier series derived from the polynomial transfer function.
- The amplitudes of each term are the Fourier coefficients.

Single-Tone Excitation

In general for **single-tone excitation** of a system described by a n^{th} order polynomial:

- The spectral components will be at:

  $0, \omega, 2\omega, 3\omega \ldots n\omega$

- The **odd spectral components** (ω, 3ω, 5ω …) are related to the **odd degree terms** (a_1, a_3, a_5 …) of the polynomial.
- The **even spectral components** (0, 2ω, 4ω …) are related to the **even degree terms** (a_2, a_4, …) of the polynomial.
- The dc term is often called “self-biasing” as it may constitute a change in bias with input signal.

Single-Tone Excitation – Example 1

The output voltage of an amplifier is related to the input voltage in the following manner:

$$v_{out}(t) = 10\tanh(v_{in}(t))$$

Recall:

$$\tanh(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \ldots$$

If we assume that input signal is sufficiently small so that the 5^{th} order and higher terms can be neglected:

$$\tanh(x) \approx x - \frac{x^3}{3}$$

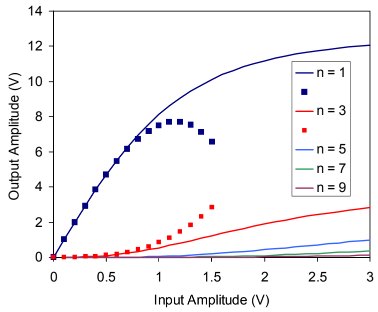
Hence:

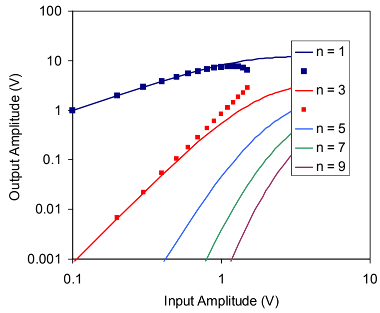
$$v_{out}(t) \approx 10v_{in}(t) - 3.33(v_{in}(t))^3$$

Comparing with example 2: a_1 = 10 and a_3 = -3.33, hence:

$$v_{out}(t) = (10A - 2.5A^3)\cos\omega t - 0.833A^3\cos 3\omega t$$

Nonlinear Phenomena – Example 1





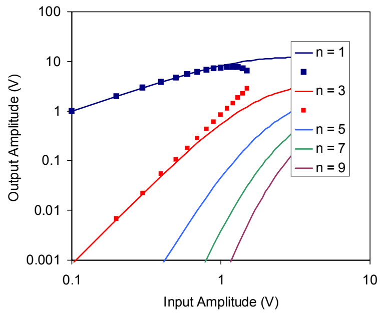
- Solid lines represent the coefficient obtained from FFT (sample size = 256) of waveforms generated using the exact formula:

$$v_{out}(t) = 10\tanh\left(v_{in}(t)\right) = 10\tanh(A\cos\omega t)$$

- The dotted lines represent analytic approach using the 3rd order model:

$$v_{out}(t) \approx 10v_{in}(t) - 3.33\left(v_{in}(t)\right)^3 = \left(10A - 2.5A^3\right)\cos\omega t - 0.833A^3\cos 3\omega t$$

Single-Tone Excitation



At **very low input levels**:

- The fundamental increases by 1 dB per 1 dB increase of input level.
- The third harmonic increases by 3 dB per 1 dB increase of input level.
- Moreover, the **n^{th} order output product increases by n dB per 1 dB increase of input level**.

Two-Tone Excitation – Example 2

The output voltage of an amplifier is related to the input voltage in the following manner:

$$v_{out}(t) = a_1 v_{in}(t) + a_2 (v_{in}(t))^2 + a_3 (v_{in}(t))^3$$

Suppose a **two-tone input signal** is applied:

$v_{in}(t) = A(\cos\omega_1 t + \cos\omega_2 t)$; $\omega_2 > \omega_1$

In this case we have a polynomial.

Recall:

$2\cos x \cos y = \cos(x + y) + \cos(x - y)$

$4\cos x \cos^2 y = 2\cos x + \cos(x + 2y) + \cos(x - 2y)$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$$

$$\cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos 3x$$

Two-Tone Excitation – Example 2

$$(\cos\omega_1 t + \cos\omega_2 t)^2 = \cos^2\omega_1 t + 2\cos\omega_1 t\cos\omega_2 t + \cos^2\omega_2 t$$

$$= \frac{1}{2} + \frac{1}{2}\cos 2\omega_1 t + \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t + \frac{1}{2} + \frac{1}{2}\cos 2\omega_2 t$$

$$= 1 + \cos(\omega_1 - \omega_2)t + \frac{1}{2}\cos 2\omega_1 t + \cos(\omega_1 + \omega_2)t + \frac{1}{2}\cos 2\omega_2 t$$

Two-Tone Excitation – Example 2

$$(\cos\omega_1 t + \cos\omega_2 t)^3$$

$$= \cos^3\omega_1 t + 3\cos\omega_1 t\cos^2\omega_2 t + 3\cos\omega_2 t\cos^2\omega_1 t + \cos^3\omega_2 t$$

$$= \frac{9}{4}\cos\omega_1 t + \frac{1}{4}\cos 3\omega_1 t + \frac{3}{4}\cos(2\omega_2 + \omega_1)t + \frac{3}{4}\cos(2\omega_2 - \omega_1)t$$

$$+ \frac{9}{4}\cos\omega_2 t + \frac{1}{4}\cos 3\omega_2 t + \frac{3}{4}\cos(2\omega_1 + \omega_2)t + \frac{3}{4}\cos(2\omega_1 - \omega_2)t$$

Two-Tone Excitation – Example 2

DC:	a_2A^2	"Self-biasing"
$\omega_2 - \omega_1$:	a_2A^2	2nd order intermodulation product
$2\omega_1 - \omega_2$:	$\frac{3a_3A^3}{4}$	3nd order intermodulation product
ω_1:	$a_1A + \frac{9a_3A^3}{4}$	fundamental 1
ω_2:	$a_1A + \frac{9a_3A^3}{4}$	fundamental 2
$2\omega_2 - \omega_1$:	$\frac{3a_3A^3}{4}$	3nd order intermodulation product

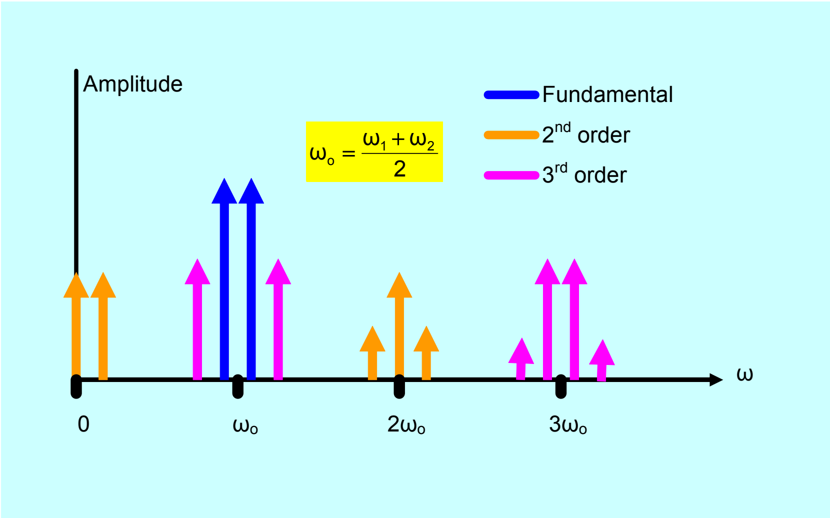
Two-Tone Excitation – Example 2

$2\omega_1$:	$\frac{a_2A^2}{2}$	2^{nd} harmonic of ω_1
$\omega_1+\omega_2$:	a_2A^2	2^{nd} order intermodulation product
$2\omega_2$:	$\frac{a_2A^2}{2}$	2^{nd} harmonic of ω_2

Two-Tone Excitation – Example 2

$3\omega_1$:	$\frac{a_3A^3}{4}$	3^{rd} harmonic of ω_1
$2\omega_1 + \omega_2$:	$\frac{3a_3A^3}{4}$	3^{rd} order mixing product
$2\omega_2 + \omega_1$:	$\frac{3a_3A^3}{4}$	3^{rd} order mixing product
$3\omega_2$:	$\frac{a_3A^3}{4}$	3^{rd} harmonic of ω_2

Two-Tone Excitation – Example 2



Two-Tone Excitation

In general for two-tone excitation of a system described by a n^{th} order polynomial and with excitation frequencies ω_1 and ω_2:

- The spectral components will be at:

  $p\omega_1 + q\omega_2$:

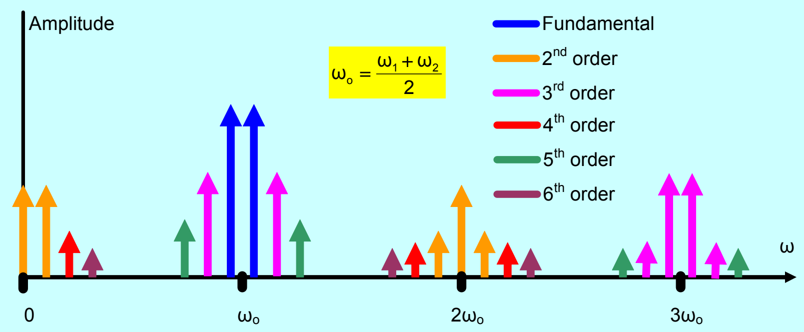
  where $p = \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

  $q = \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

  subject to **$|p| + |q| \leq n$**

- **Odd order** spectral components: $|p| + |q|$ is odd (eg. $2\omega_1 - \omega_2$)
- **Even order** spectral components: $|p| + |q|$ is even (eg. $\omega_2 - \omega_1$)
- The **odd order spectral components** are related to the **odd degree terms** (a_1, a_3, a_5 …) of the polynomial.
- The **even order spectral components** are related to the **even degree terms** (a_2, a_4, …) of the polynomial.

Two-Tone Excitation



$$v_{out}(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos(m\omega_\Delta t) + b_m \sin(m\omega_\Delta t)$$

$$\omega_\Delta = \omega_2 - \omega_1$$

Multi-Tone Excitation

- In this case there are multiple tones in the input excitation.
- It should be apparent from the single and two tone case that this would result in far greater numbers of spectral components in the response.
- The principles that have been developed for single and two tone excitation apply it is just that the mathematics becomes more tedious.

Large-Signal Small-Signal Excitation

This is a special case of two-tone and multi-tone excitation in which the amplitude of one excitation is far greater than the rest.

eg. $v_{in}(t) = 5\cos\omega_{LO}t + 0.001\cos\omega_{RF}t$

- This type of problem is often encountered in mixers.
- We will show that the analytic complexity can be reduced compared to pure two-tone and multi-tone excitation analysis.

LS / SS Excitation – Example 2

The output voltage of an amplifier is related to the input voltage in the following manner:

$$v_{out}(t) = a_1 v_{in}(t) + a_2\left(v_{in}(t)\right)^2 + a_3\left(v_{in}(t)\right)^3$$

Suppose the input excitation is of the form:

$$v_{in}(t) = \underbrace{A\cos\omega_1 t}_{\text{Large-signal (LS) component}} + \underbrace{\alpha\cos\omega_2 t}_{\text{Small-signal (SS) component}} \quad ; \quad \omega_2 > \omega_1 \text{ and } |\alpha| << |A|$$

$$\begin{aligned}
\left(A\cos\omega_1 t + \alpha\cos\omega_2 t\right)^2 &= A^2\cos^2\omega_1 t + 2A\alpha\cos\omega_1 t\cos\omega_2 t + \alpha^2\cos^2\omega_2 t \\
&\approx A^2\cos^2\omega_1 t + 2A\alpha\cos\omega_1 t\cos\omega_2 t \\
&= \frac{A^2}{2} + \frac{A^2}{2}\cos 2\omega_1 t + A\alpha\cos\left(\omega_2 - \omega_1\right)t + A\alpha\cos\left(\omega_2 + \omega_1\right)t
\end{aligned}$$

LS / SS Excitation – Example 2

$$(A\cos\omega_1 t + \alpha\cos\omega_2 t)^3$$
$$= A^3\cos^3\omega_1 t + 3A\alpha^2\cos\omega_1 t\cos^2\omega_2 t + 3A^2\alpha\cos\omega_2 t\cos^2\omega_1 t + \alpha^3\cos^3\omega_2 t$$
$$\approx A^3\cos^3\omega_1 t + 3A^2\alpha\cos\omega_2 t\cos^2\omega_1 t$$
$$= \frac{3A^3}{4}\cos\omega_1 t + \frac{A^3}{4}\cos 3\omega_1 t + \frac{3A^2\alpha}{2}\cos\omega_2 t$$
$$+ \frac{3A^2\alpha}{4}\cos(2\omega_1 + \omega_2)t + \frac{3A^2\alpha}{4}\cos(2\omega_1 - \omega_2)t$$

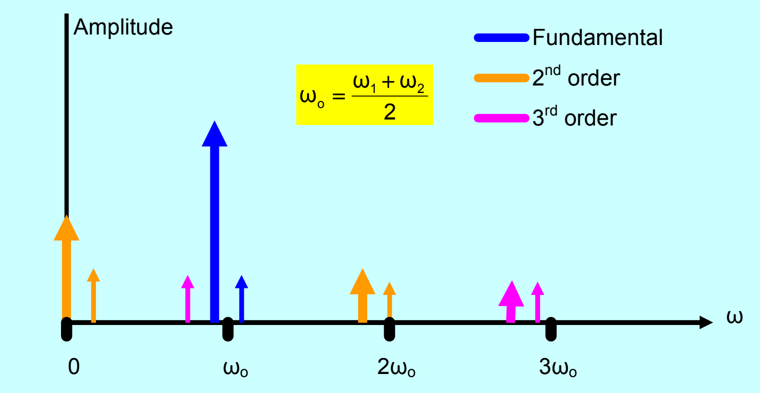
LS / SS Excitation – Example 2

DC:	$\frac{a_2 A^2}{2}$	"Self-biasing"
$\omega_2 - \omega_1$:	$a_2 A\alpha$	2nd order intermodulation product
$2\omega_1 - \omega_2$:	$\frac{3a_3 A^2\alpha}{4}$	3nd order intermodulation product
ω_1:	$a_1 A + \frac{3a_3 A^3}{4}$	fundamental 1
ω_2:	$a_1\alpha + \frac{3a_3 A^2\alpha}{2}$	fundamental 2

LS / SS Excitation – Example 2

$2\omega_1$:	$\frac{a_2A^2}{2}$	2^{nd} harmonic of ω_1
$\omega_1+\omega_2$:	$a_2A\alpha$	2^{nd} order intermodulation product
$3\omega_1$:	$\frac{a_3A^3}{4}$	3^{rd} harmonic of ω_1
$2\omega_1+\omega_2$:	$\frac{3a_3A^2\alpha}{4}$	3^{nd} order mixing product

LS / SS Excitation – Example 2



If $\alpha = 0$ then we are left with the single-tone case (with $\omega = \omega_1$)

LS / SS Excitation – Example 2

We see that $v_{out}(t)$ can be resolved into two waveforms:

$$v_{out}(t) = v_{outLS}(t) + v_{outSS}(t)$$

$$v_{outLS}(t) = \frac{a_2A^2}{2} + \left(a_1A + \frac{3a_3A^3}{4}\right)\cos\omega_1 t + \frac{a_2A^2}{2}\cos 2\omega_1 t + \frac{a_3A^3}{4}\cos 3\omega_1 t$$

$$\begin{aligned} v_{outSS}(t) = \alpha\Bigg(& a_2A\cos(\omega_2 - \omega_1)t + \frac{3a_3A^2}{4}\cos(2\omega_1 - \omega_2)t \\ & + \left(a_1 + \frac{3a_3A^2}{2}\right)\cos\omega_2 t + a_2A\cos(\omega_1 + \omega_2)t \\ & + \frac{3a_3A^2}{4}\cos(2\omega_1 + \omega_2)t\Bigg) \end{aligned}$$

LS / SS Excitation

Clearly the LS component is the single-tone response and we saw that this was relatively easy to obtain.

Our attention is now focussed on the SS component.

Can we have:

$$v_{outSS}(t) = v_{inSS}(t)\, A_v(t)$$

where $A_v(t)$ is a time-dependent small-signal voltage gain:

$$A_v(t) = \left.\frac{df(v_{in})}{dv_{in}}\right|_{v_{in}(t)=A\cos\omega_1 t}$$

LS / SS Excitation – Example 2

In our case:

$$f(v_{in}) = a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3$$

So:

$$A_v(v_{in}) = a_1 + 2a_2 v_{in} + 3a_3 v_{in}^2$$

Hence:

$$A_v(t) = a_1 + \frac{3a_3A^2}{2} + 2a_2A\cos\omega_1 t + \frac{3a_3A^2}{2}\cos 2\omega_1 t$$

Multiplying by $\alpha\cos\omega_2 t$:

$$A_v(t)\alpha\cos\omega_2 t = \alpha\left(\left(a_1 + \frac{3a_3A^2}{2}\right)\cos\omega_2 t + 2a_2A\cos\omega_1 t\cos\omega_2 t + \frac{3a_3A^2}{2}\cos 2\omega_1 t\cos\omega_2 t\right)$$

and we see this indeed gives $v_{outSS}(t)$. Clearly this approach is easier than the brute force way especially if the SS excitation has a complicated spectrum.

Power Amplifier Behaviour

Gain-Compression:

The output voltage of an amplifier cannot increase indefinitely with increasing input voltage.

The output voltage is limited by the power supply and also inherent nonlinear properties such as:

1. Diode junction clamping effect

   eg. gate-channel junction of a GaAs FET and the base-emitter junction of a BJT behaves as diodes.

2. Breakdown in BJTs and FETs.
3. The saturation region of a BJT I_C vs V_{CE} characteristic.
4. The triode region of a FET I_D vs V_{DS} characteristics.

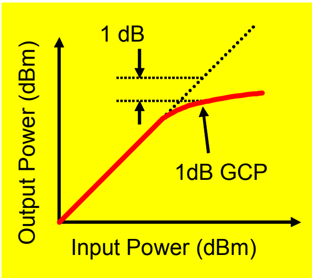
For example, for an amplifier described by: $v_{out}(t) = 10\tanh(v_{in}(t))$

v_{out} is restricted to takes values in the range: -10 to 10 V

Power Amplifier Behaviour

Gain-Compression:

- The limitation of the output voltage means that there is a limitation of output power – known as output power saturation.
- At the point where the amplifier saturates, gain will decrease. This is called gain compression.
- The 1 dB gain compression point (GCP) is defined as the point where the gain falls by 1 dB from its small-signal value.



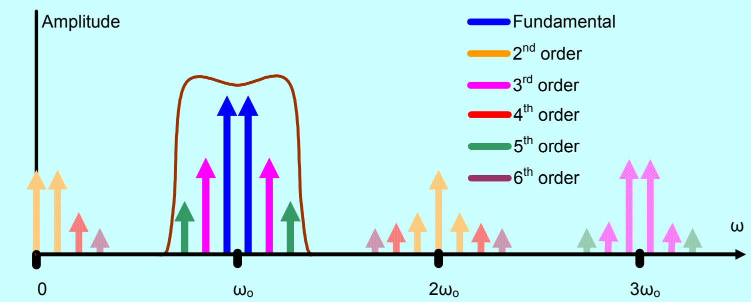
- Sometimes the gain increases with increasing input power. This is called gain expansion.
- Taylor series coefficient a_3 strongly effects gain compression.

$$P_{dBm} = 10\log\left(\frac{P}{0.001}\right)$$

Power Amplifier Behaviour

Odd-order intermodulation distortion:

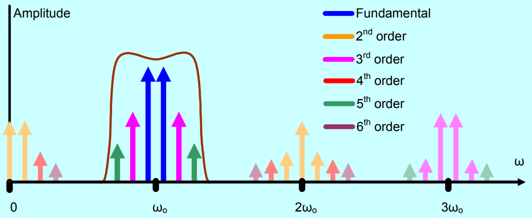
- Due to modulation (eg. AM or FM), or the presence of several carriers, there will be several spectral tones present at the input signal.
- With several tones present at the input, intermodulation will occur.
- Usually RF and microwave amplifiers for communications purposes have narrow bandwidths of about 5 – 10 %.



Power Amplifier Behaviour

Odd-order intermodulation distortion:

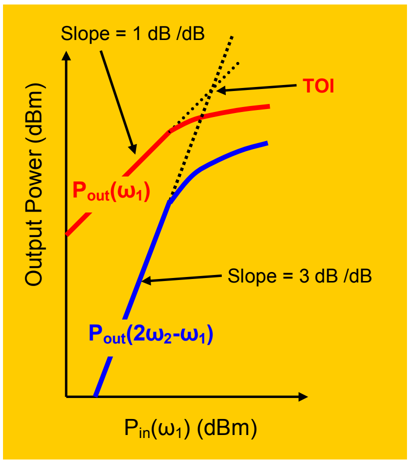
- The bandpass response of the amplifier will suppresses harmonics BUT NOT odd-order intermodulation products that fall within the pass-band.
- Odd-order intermodulation products appear within the passband will interfere with the desired signal or signals.
- Odd-order intermodulation distortion is the root of inter-channel interferance and determines ACPR (adjacent channel power ratio).
- Filtering cannot be used since filtering would also suppress the desired signals.



Power Amplifier Behaviour

Odd-order intermodulation distortion:

- 3rd order intermodulation products (IM3) are the most intense of the intermodulation products since:
  - They are related to a_3 of the polynomial which is normally greater in magnitude than a_5 etc.
  - 3rd order intermodulation products are closest to the fundamentals.
- Intermodulation products, like output fundamental, will show saturation behaviour.
- The polynomial coefficient a_5 is primarily responsible for IM3 saturation.



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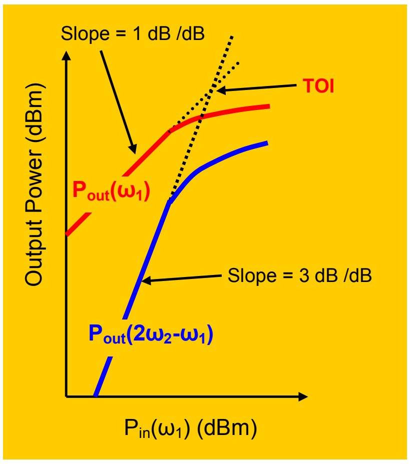
Power Amplifier Behaviour

Odd-order intermodulation distortion:

- At very low input levels, output power in the fundamental increases by 1 dB per 1 dB increase in input power.
- At very low input levels, output power in the IM3 increases by 3 dB per 1 dB increase in input power.
- The intersection of the extrapolations of the small-signal transfer function gives the 3rd order intercept (TOI).
- The TOI is a figure of merit.
- The fundamental transfer (and gain) characteristic is different than that under single-tone excitation.



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Power Amplifier Behaviour

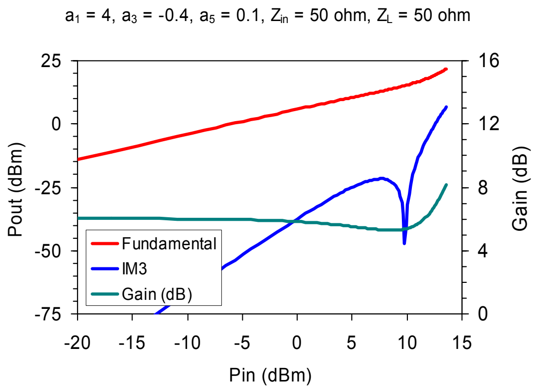
Odd-order intermodulation distortion:

- Over some ranges of input power, the IM3 may actually dip into a null.
- This is an optimum operating range for systems that demand low intermodulation distortion, or moreover demand low ACPR.
- This is caused by certain values of combinations of polynomial coefficients.
- Another cause is nonlinear capacitances found in all transistors.

Power Amplifier Behaviour – Example 3

Say we have an amplifier described by a polynomial transfer function with: a_1 = 4, a_3 = -0.4 and a_5 = 0.1.

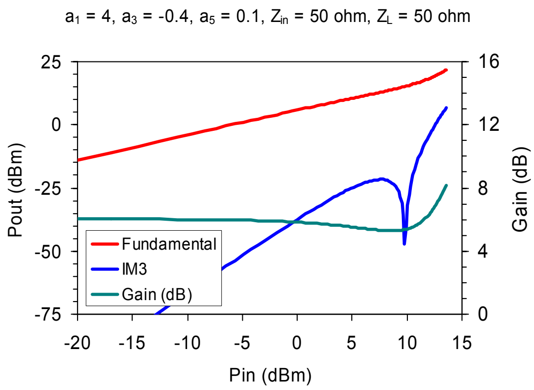
The amplifier has a 50 Ω input impedance and load.



Power Amplifier Behaviour – Example 3

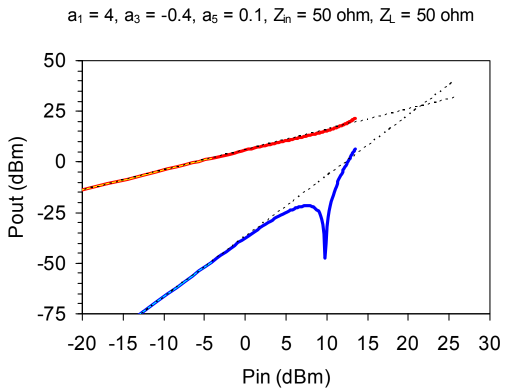
This amplifier displays gain expansion and gain compression.

There is a null in the IM3 response.



Power Amplifier Behaviour – Example 3

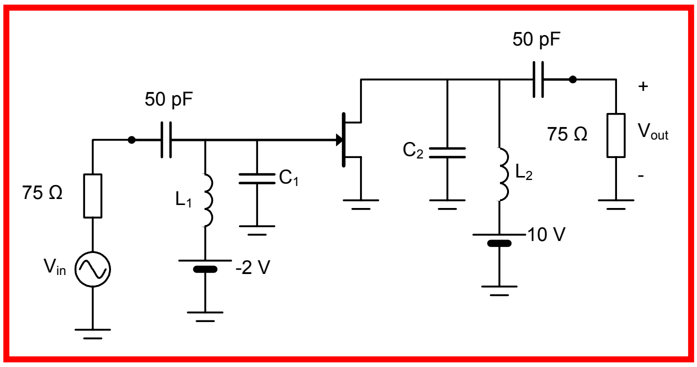
The third-order intercept (TOI) is at an output power of 21.4 dBm.



Example 4

The following circuit is an amplifier that employs a power FET whose input and output capacitances are 1.5 pF and 0.7 pF respectively, and the drain conduction current is related to gate-voltage by:

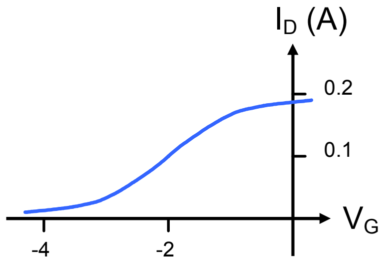
$$I_D = 0.1 + 0.1\tanh(V_G + 2) \quad ; V_D > 1V$$



Example 4

The transfer function between drain current and gate voltage is:

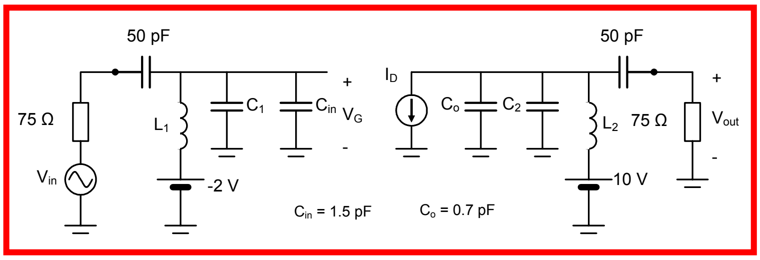
$$I_D = 0.1 + 0.1\tanh(V_G + 2) \quad ; V_D > 1\text{V}$$



Determine the values of L_1, C_1, L_2 and C_2 so that the following amplifier has a centre frequency of 2.45 GHz and a bandwidth of 300 MHz.

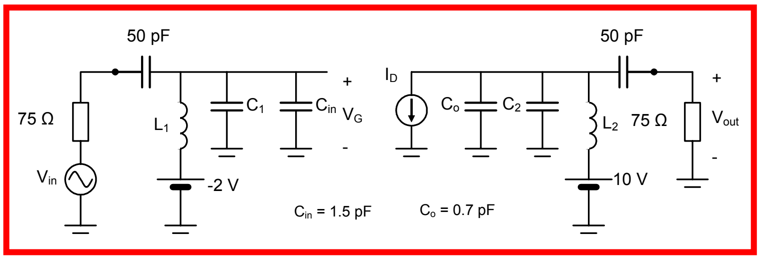
Determine the transfer function within the passband.

Example 4



- L_1 and C_1 resonate the input circuit. We want C_1 to be about twice as much as C_{in} so as to reduce the effect of the C_{in} nonlinearity.
- L_2 and C_2 not only resonate the output circuit but are used to suppress harmonics – so we want this to have a bandwidth of B.

Example 4

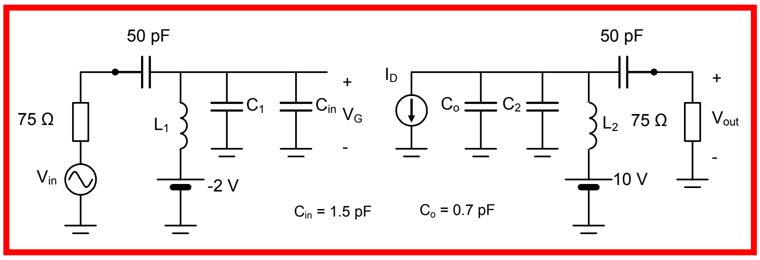


For the output circuit (set B = 300 MHz):

$$Q = \frac{f_o}{B} = 75\omega_o(C_o + C_2) \quad \rightarrow C_2 = \ 6.4 \text{ pF}$$

$$\omega_o = \frac{1}{\sqrt{L_2(C_o + C_2)}} \quad \rightarrow L_2 = 0.59 \text{ nH}$$

Example 4

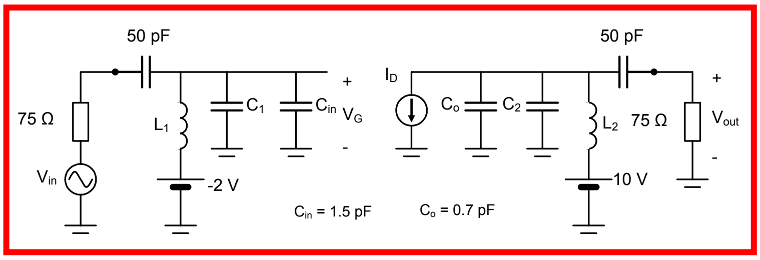


For the input circuit (set C_1 = 1.5 pF):

$$Q = \frac{f_o}{B} = 75\omega_o(C_{in} + C_1) \quad \rightarrow B = \ 707 \text{ MHz}$$

$$\omega_o = \frac{1}{\sqrt{L_1(C_{in} + C_1)}} \quad \rightarrow L_1 = 1.4 \text{ nH}$$

Example 4



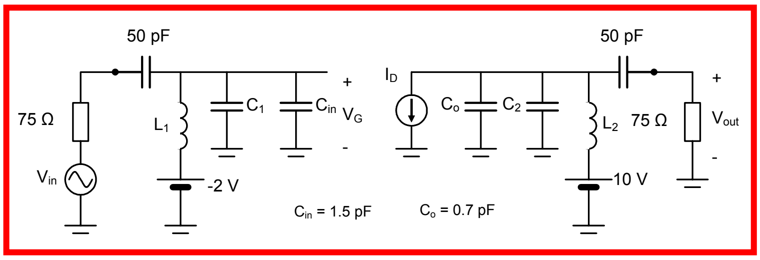
At 2.45 GHz, the input circuit is at parallel resonance:

$$v_G(t) = -2 + v_{in}(t)$$

where the input signal $v_{in}(t)$ only contains spectral components within the amplifier passband. Hence:

$$i_D(t) = 0.1 + 0.1\tanh(-2 + v_{in}(t) + 2) \quad ; V_D > 1V$$

Example 4

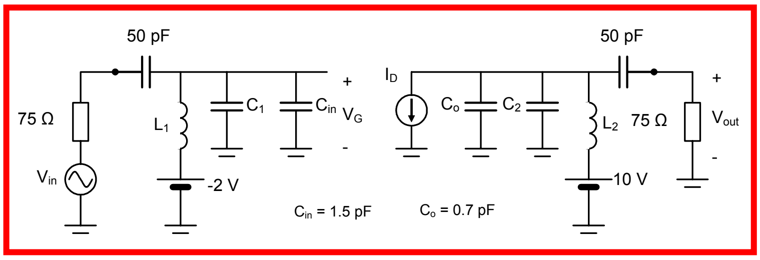


Hence:

$$i_D(t) = 0.1 + 0.1\tanh(v_{in}(t)) \quad ; V_D > 1V$$

Does this relationship look familiar ?

Example 4

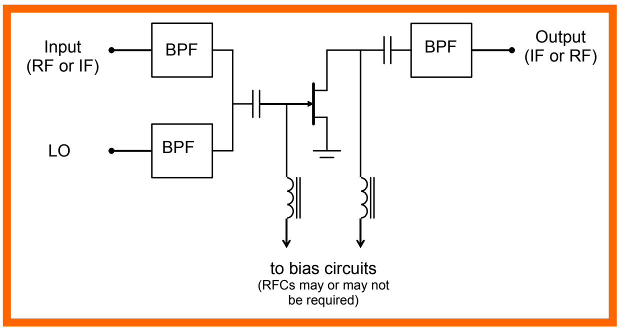


At 2.45 GHz, the output circuit is at parallel resonance:

- dc component of drain current flows only through L_2.
- passband components of drain current flow into 75 Ω load
- harmonic components of drain current bypass load due to parallel resonant circuit.

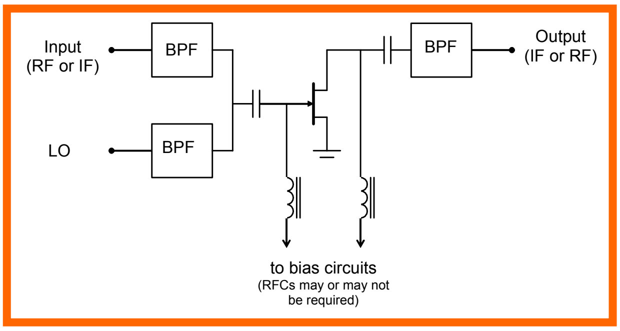
The output voltage is nearly sinusoidal with maximum amplitude of 7.5 V.

Mixers



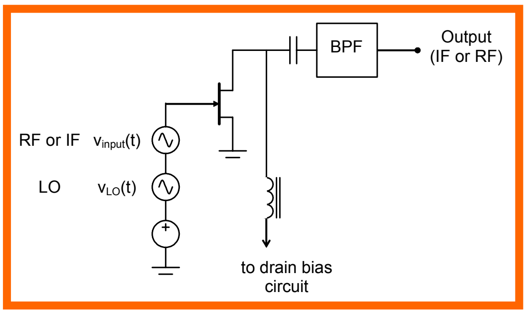
- The circuit above is a generic FET mixer topology. Other topologies are also possible.
- The BPFs select only the band of interest and resonate either the input or output circuit.
- The input BPF and LO BPF also have the task of isolating the input source and LO.

Mixers



- The FET provides the nonlinearity (a diode could be used instead) which essentially achieves multiplication of the LO and the input waveform.
- The “a_2” term of the Taylor series is the essential term of the nonlinearity.
- The FET and BJT mixers are non-reciprocal unlike a diode mixer.

Mixers

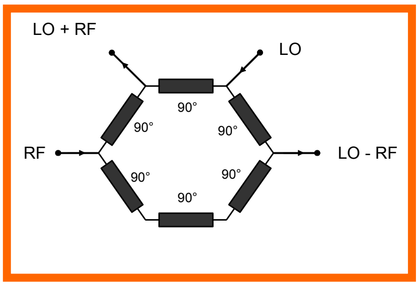


As far as the FET is concerned, the LO, input signal (RF or IF) and gate dc bias are all applied to the gate of the FET.

Mixers

What if the input RF and LO frequencies are nearly identical?

A rat-race hybrid can be used to feed the RF and LO to the FET gate …

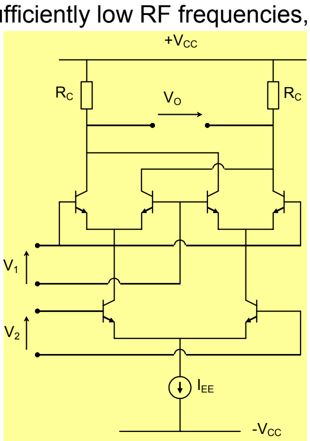


- Isolation between the LO and RF ports can be achieved without using filters.
- The rat-race hybrid is the microwave analogue of a transformer with centre-tapped secondary and is used in balanced mixers.

Mixers

What if the input RF and LO frequencies are nearly identical?

At sufficiently low RF frequencies, a four-quadrant multiplier can be used …

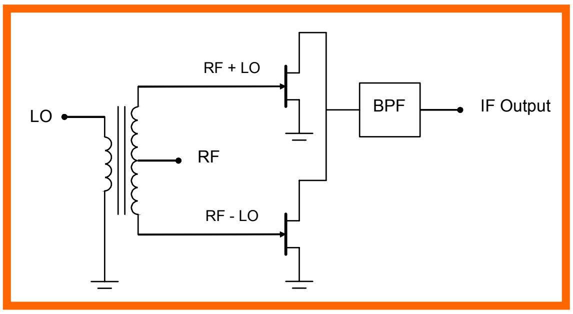


$$V_o = I_{EE} R_C \left[\tanh\left(\frac{V_1}{2V_T} \right) \right] \left[\tanh\left(\frac{V_2}{2V_T} \right) \right]$$

where $V_T = \frac{kT}{q} = 26\,\text{mV}$ at 300K

Mixers

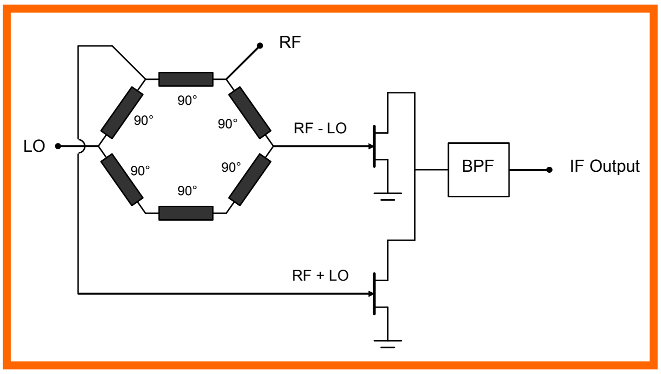
Single-Balanced Mixer (bias circuitry and resonators not shown):



The LO is inherently suppressed at the output of the FETs.

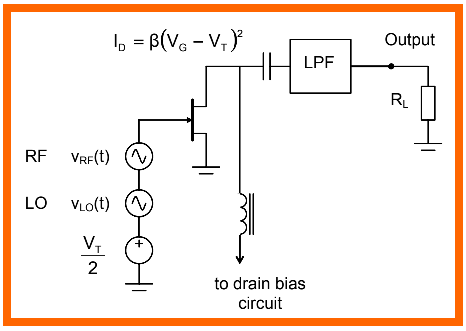
Mixers

Single-Balanced Mixer (bias circuitry and resonators not shown):



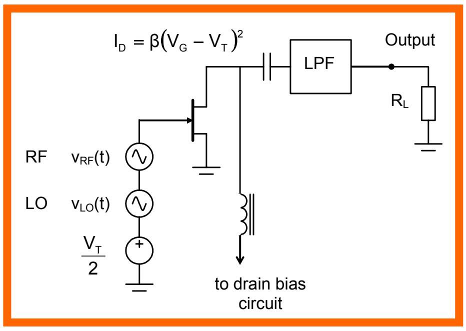
The LO is inherently suppressed at the output of the FETs.

Mixers – Example 5



Calculate the conversion voltage gain if a LC LPF filter is used, the LO amplitude is $V_T/2$, and assuming a small-signal RF.

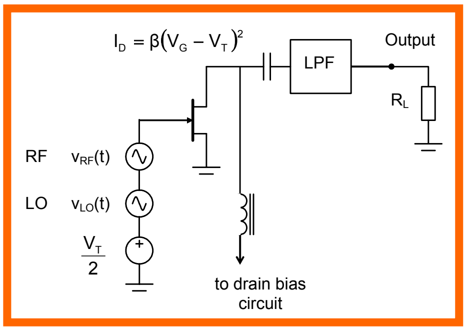
Mixers – Example 5



$$v_G(t) = v_{LO}(t) + v_{RF}(t) + \frac{V_T}{2}$$

$$= V_{LO}\cos\omega_{LO}t + \frac{V_T}{2} + V_{RF}\cos\omega_{RF}t$$ where $V_T < 0$ and $|V_{RF}| << |V_{LO}|$

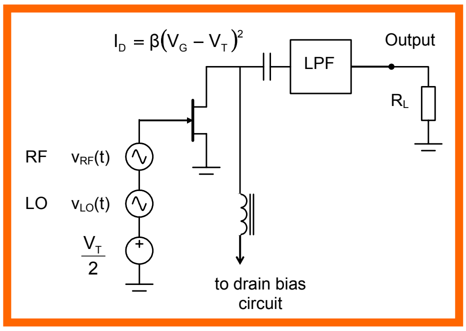
Mixers – Example 5



We can split $v_G(t)$ into small-signal and large-signal components, and noting that V_{LO} is equal to $V_T/2$:

$$v_{G_{LS}}(t) = \frac{V_T}{2}\cos\omega_{LO}t + \frac{V_T}{2} \qquad v_{G_{SS}}(t) = V_{RF}\cos\omega_{RF}t$$

Mixers – Example 5



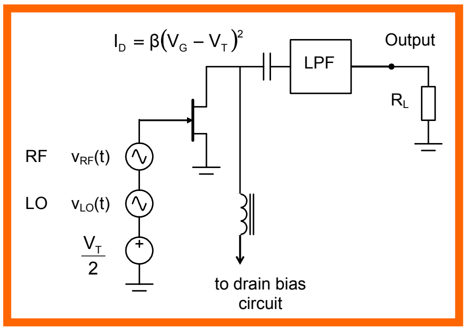
The FET transconductance is:

$$g_m = 2\beta(V_G - V_T)$$

and hence:

$$g_m(t) = 2\beta\left(\frac{V_T}{2}\cos\omega_{LO}t + \frac{V_T}{2} - V_T\right) = \beta V_T(\cos\omega_{LO}t - 1)$$

Mixers – Example 5

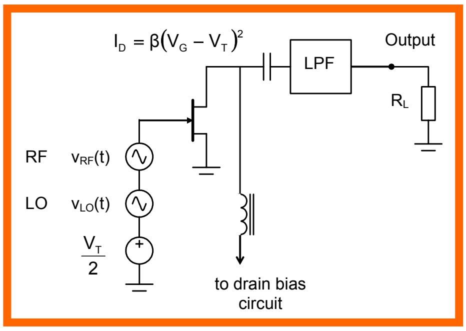


The small-signal voltage gain is $-g_m(t)R_L$ so:

$$v_{D_{ss}}(t) = -g_m(t)R_L v_{G_{ss}}(t) = -\beta V_T R_L(\cos\omega_{LO}t - 1)V_{RF}\cos\omega_{RF}t$$

which will have components at ω_{RF}, $\omega_{IF} = \omega_{RF} - \omega_{LO}$ and at $\omega_{RF}+\omega_{LO}$

Mixers – Example 5



Due to the LP response of the output filter (and the dc block):

$$v_o(t) = -\frac{\beta V_T R_L}{2} V_{RF}\cos\omega_{IF}t$$

Finally, the conversion gain: $$A_{V_{conv}} = \left|\frac{V_{IF}}{V_{RF}}\right| = \frac{\beta V_T R_L}{2}$$