

X Answers.

## TEST

<b>Prescription Number:</b>	<b>ENEL434</b>
<b>Paper Title:</b>	<b>Electronics II</b>

Time Allowed: 50 Minutes

Number of Pages: 5

Formulae Sheet Supplied

Smith Chart Supplied

Department Calculators are supplied

Answer both questions

Each question is worth 20 marks

Q1

Given the transistor amplifier circuit shown in Figure 1 and using the following S parameters taken at  $Z_0 = 50\Omega$  and a frequency of 2GHz

$$S_{11} = 0.8 \angle -120^\circ, \quad S_{12} = 0.0, \quad S_{21} = 2.77 \angle +90^\circ, \quad S_{22} = 0.44 \angle -63^\circ$$

answer the following questions.

- a) Using one of the Smith chart supplied, find
  - (i) The shortest length of a short circuit input stub D1 and
  - (ii) the length of transmission line L1 that conjugate matches the input of the transistor (both answers in wavelengths)
- b) Using the second Smith chart, find
  - (i) the shortest length of a short circuit output stub D2 and
  - (ii) the length of the transmission line L2 that conjugate matches the output of the transistor to the complex load
- c) Calculate the transducer gain in dB
  - (i) with the two conjugate matching networks and
  - (ii) without the two matching networks. (NB I have chosen values to make this calculation easy.)

You must name, label and hand in both Smith charts with your answer book.

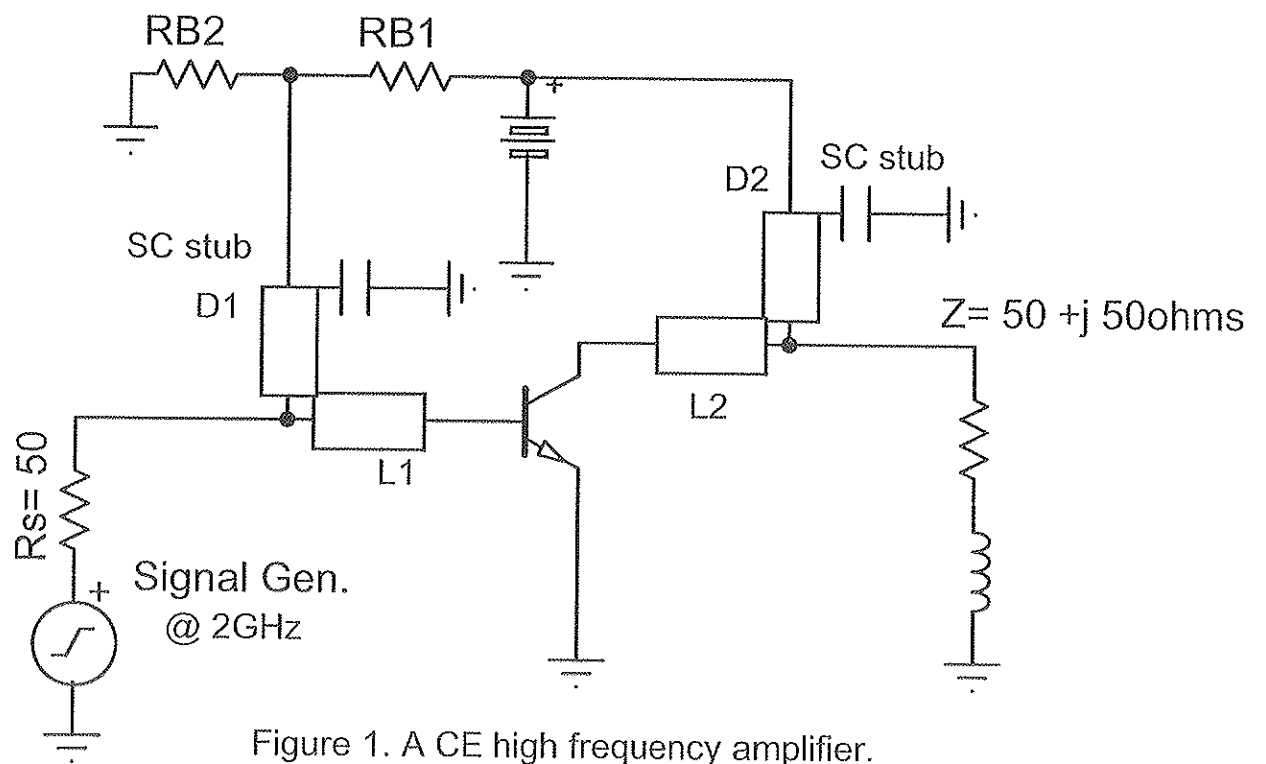


Figure 1. A CE high frequency amplifier.

Q2

Please refer to Figure 2. Assuming the gain is sufficient for oscillation, calculate:

- a) the DC biasing conditions specifically
  - i)  $I_c$
  - ii)  $V_{CE}$
  - iii)  $r_e$
- b) the resonant frequency  $\omega_0$  in rads /s stating any assumptions you make,
- c)  $R_L'$  the load as seen through the inductive transformer,
- d)  $R_{coil}$ ,
- e)  $R_{in}'$  the input as seen through the capacitive transformer,
- f)  $R_C'$  the effective ac load on the collector,
- g) the ac gain from the emitter to the collector,
- h) the feedback ratio  $\beta$ , and
- i) if the gain is sufficient for the oscillator to start.

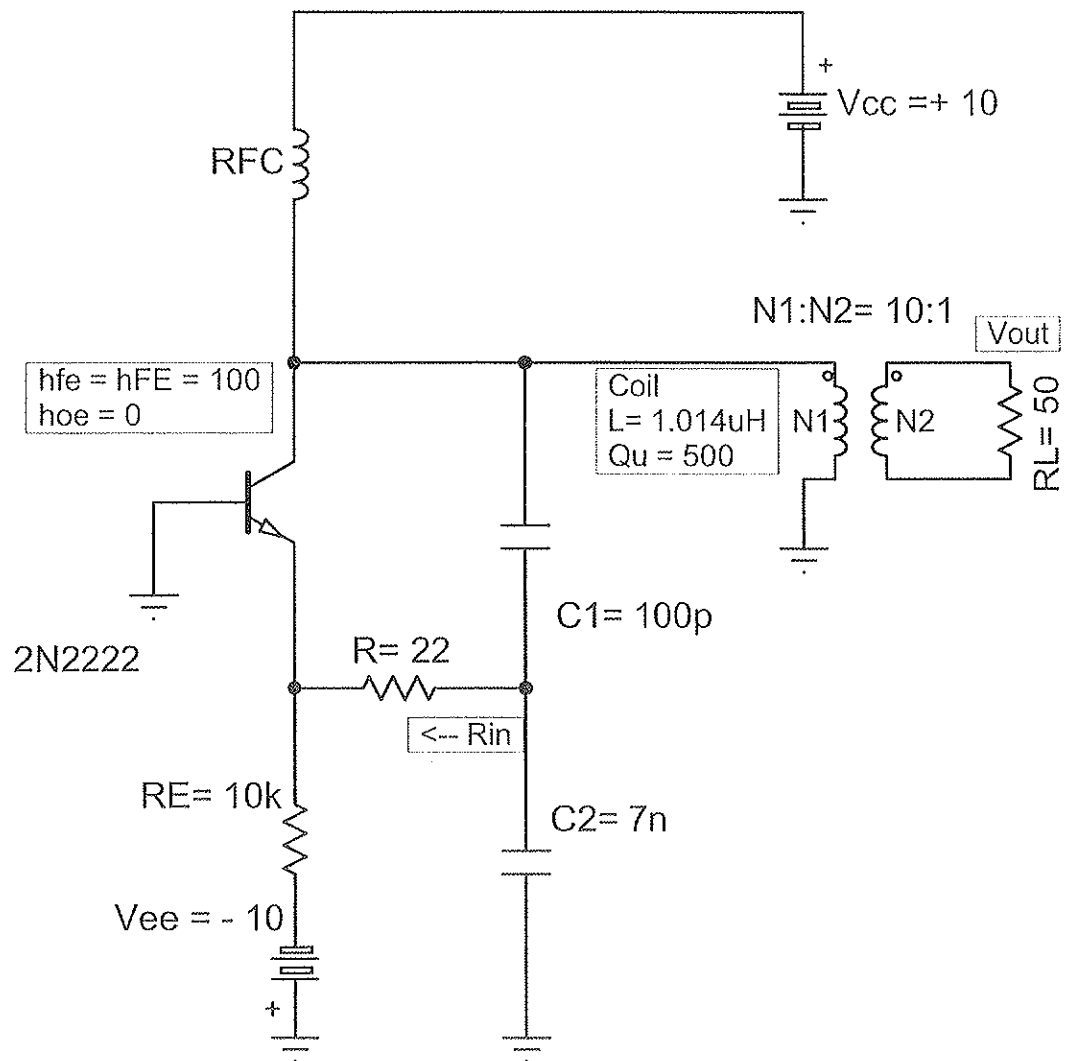


Figure 2 A Colpitts Oscillator

### Some useful expansions and maths identities

$$2 \sin x \sin y = -\cos(x+y) + \cos(x-y)$$

$$2 \cos x \cos y = \cos(x+y) + \cos(x-y)$$

$$2 \sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$4 \cos x \cos^2 y = 2 \cos x + \cos(x+2y) + \cos(x-2y)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \text{and} \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

### CE, CC and CB transistor amplifiers

Very small signal Voltage Gain

$A_v = -g_m \cdot (\text{Resistive load on collector}) = -g_m \cdot (R_c \parallel R_L \parallel r_o)$  where  $g_m = I_C / 0.026$  for CE amplifier with emitter resistor bypass capacitor.

$A_v$  for a CB amplifier is the same as that for the CE amplifier but positive.

Without the emitter resistor bypass capacitor,  $g_m$  is replaced by  $G_m = 1/(r_e + R_E)$

Miller Capacitance for a CE BJT amplifier

$$C_{eq} = C_{BC}(1 + |A_v|) \text{ thus } C_{in} = C_{eq} + C_{BE} + C_{stray}$$

Small signal Voltage Gain for a differential amplifier;  $A_v \equiv v_{out1}/v_{in} = -\frac{g_m}{2} \cdot (R_c \parallel R_L \parallel r_o)$

Small signal Voltage Gain for a differential amplifier;  $A_v \equiv v_{out2}/v_{in} = \frac{g_m}{2} \cdot (R_c \parallel R_L \parallel r_o)$

Small signal Voltage Gain for CC BJT amplifiers;  $A_v = \frac{R_E \parallel R_L}{r_e + R_E \parallel R_L}$ .

Power gain  $A_p = A_v \cdot A_i = A_v^2 R_{in} / R_L$

Power Gain in dB =  $20 \log(A_v) + 10 \log(R_{in}/R_L)$

Large signal Voltage Gain

For large input voltages ( $v_{in} \geq 0.026V$ ), the gain formulas for small signals are valid but with  $g_m$  replaced by  $G_m(x)$  where the input signal ratio  $x \equiv v_{in}/V_T = v_{in}/0.026$ .  $G_m(x)/g_m$  is usually read from a graph or tables.

Useful rules of thumb for BJT amplifiers with voltage divider bias.

$V_E \approx 0.1 \cdot V_{cc}$ ,  $I_{RB2} \approx 10 \cdot I_B$  both modifications to increase the stability factor.

For a tuned-load, Class A amplifier,  $P_{BJT+load} \approx 0.90 V_{cc} \cdot I_c$  watts

and so  $P_{load} \approx 0.5 P_{BJT+load}$  at max efficiency (i.e., 50% of 90% of the power supplied).

### Resonance and tuned load amplifiers

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{and} \quad Q = \frac{f_0}{BW} = \frac{R_p}{X_L(@\omega_0)} \text{ (for parallel)} = \frac{X_L(@\omega_0)}{R_s} \text{ (for series)}.$$

Parallel to series (and the inverse)

$$L_{SE} = \frac{Q^2}{(1+Q^2)} L_P, \quad C_{SE} = \frac{(1+Q^2)}{Q^2} C_P \text{ and } R_{SE} = R_P / (1 + Q^2)$$

so when  $Q > 10$ , the narrowband case,  $L_{SE} \approx L_P, C_{SE} \approx C_P$  and  $R_{SE} \approx R_P / Q^2$ .

## Matching Networks for Two Ports

$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$  looking towards the source and  $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$  looking towards the load.  
It is easier to use the Smith Chart; i.e., plot  $Z_s/Z_0$  or  $Z_L/Z_0$  and read off  $\Gamma_s$  or  $\Gamma_L$ .

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L}$$

so  $\Gamma_{in} = s_{11}$  when  $Z_L = Z_0$  or for the unilateral amplifier when  $s_{12} = 0$ .

$$\Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} = s_{22} + \frac{s_{12}s_{21}\Gamma_s}{1 - s_{11}\Gamma_s}$$

so  $\Gamma_{out} = s_{22}$  when  $Z_s = Z_0$  or for the unilateral amplifier when  $s_{12} = 0$ .

**Power Gain** (i.e., power in the load  $P_L$  / power supplied to the input  $P_{in}$ )

$$G = \frac{P_L}{P_{in}} = \frac{1}{(1 - |\Gamma_{in}|^2)} \cdot |s_{21}|^2 \cdot \frac{(1 - |\Gamma_L|^2)}{|1 - s_{22}\Gamma_L|^2} \quad \text{see Pozar (11.8), page 538.}$$

$$= G_S \cdot G_0 \cdot G_L$$

where  $G_S$  = source network gain,  $G_0$  = device gain and  $G_L$  = load network gain;

**Transducer Power Gain** (relative to the max that could be taken from the input)

$$G_T = \frac{P_L}{P_{avs}} = \frac{(1 - |\Gamma_s|^2)}{|1 - \Gamma_{in}\Gamma_s|^2} \cdot |s_{21}|^2 \cdot \frac{(1 - |\Gamma_L|^2)}{|1 - s_{22}\Gamma_L|^2} \quad \text{given that } P_{avs} \equiv P_{in}|_{\Gamma_s=\Gamma_{in}^*} \quad \text{see Pozar (11.9)}$$

**Available Power Gain** (when source and load are both conjugate matched)

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{(1 - |\Gamma_s|^2)}{|1 - s_{11}\Gamma_s|^2} \cdot |s_{21}|^2 \cdot \frac{1}{(1 - |\Gamma_{out}|^2)} \quad \text{given } P_{avs} \equiv P_{in}|_{\Gamma_s=\Gamma_{in}^*} \text{ and } P_{avn} = P_L|_{\Gamma_L=\Gamma_{out}^*}$$

## Special Cases of Transducer Power Gain

### 1. Source and load at Characteristic Impedance

(where  $\Gamma_s = \Gamma_L = 0$ , i.e.,  $Z_L = Z_0$  and  $Z_s = Z_0$ )

$$G_T = |s_{21}|^2$$

### 2. Unilateral Transducer Power Gain

(where  $s_{12} = 0$ , so  $\Gamma_{in} = s_{11}$  and  $\Gamma_{out} = s_{22}$  but  $\Gamma_L$  and  $\Gamma_s$  could be anything)

$$G_{TU} = \frac{(1 - |\Gamma_s|^2)}{|1 - s_{11}\Gamma_s|^2} \cdot |s_{21}|^2 \cdot \frac{(1 - |\Gamma_L|^2)}{|1 - s_{22}\Gamma_L|^2} = G_{S(TU)} \cdot G_0 \cdot G_L$$

### 3. Maximum Unilateral Transducer Power Gain

(for when  $s_{12} = 0$  as well as the input and output conjugate matched)

$$G_{TUmax} = \frac{1}{1 - |s_{11}|^2} \cdot |s_{21}|^2 \cdot \frac{1}{1 - |s_{22}|^2} = G_{S(TUmax)} \cdot G_0 \cdot G_{L(TUmax)}$$





⑤ 1

$$(a) \quad \Gamma_{in} = S_{11} = 0.8 \angle -120^\circ$$

$$\therefore \Gamma_s = \Gamma_{in}^* = 0.8 \angle +120^\circ$$

$$\therefore Y_s = 0.42 - j1.65. \quad \text{This is the aim (or end) point.}$$

$$Y_{A.} = Y_0 = 1.0. \quad \text{This is the start pt.}$$

$$\text{Add a s.c. stub} = (0.308 - 0.250)\lambda \text{ long} \\ = 0.058\lambda.$$

$$Y_B = 1.0 - j2.6.$$

Now travel down transmission line from B. to  $Y_0$ .

$$= (0.333 - 0.302)\lambda = 0.031\lambda$$

$$(i) \therefore D1 = 0.058\lambda$$

$$(ii) \text{ and } L1 = 0.031\lambda$$

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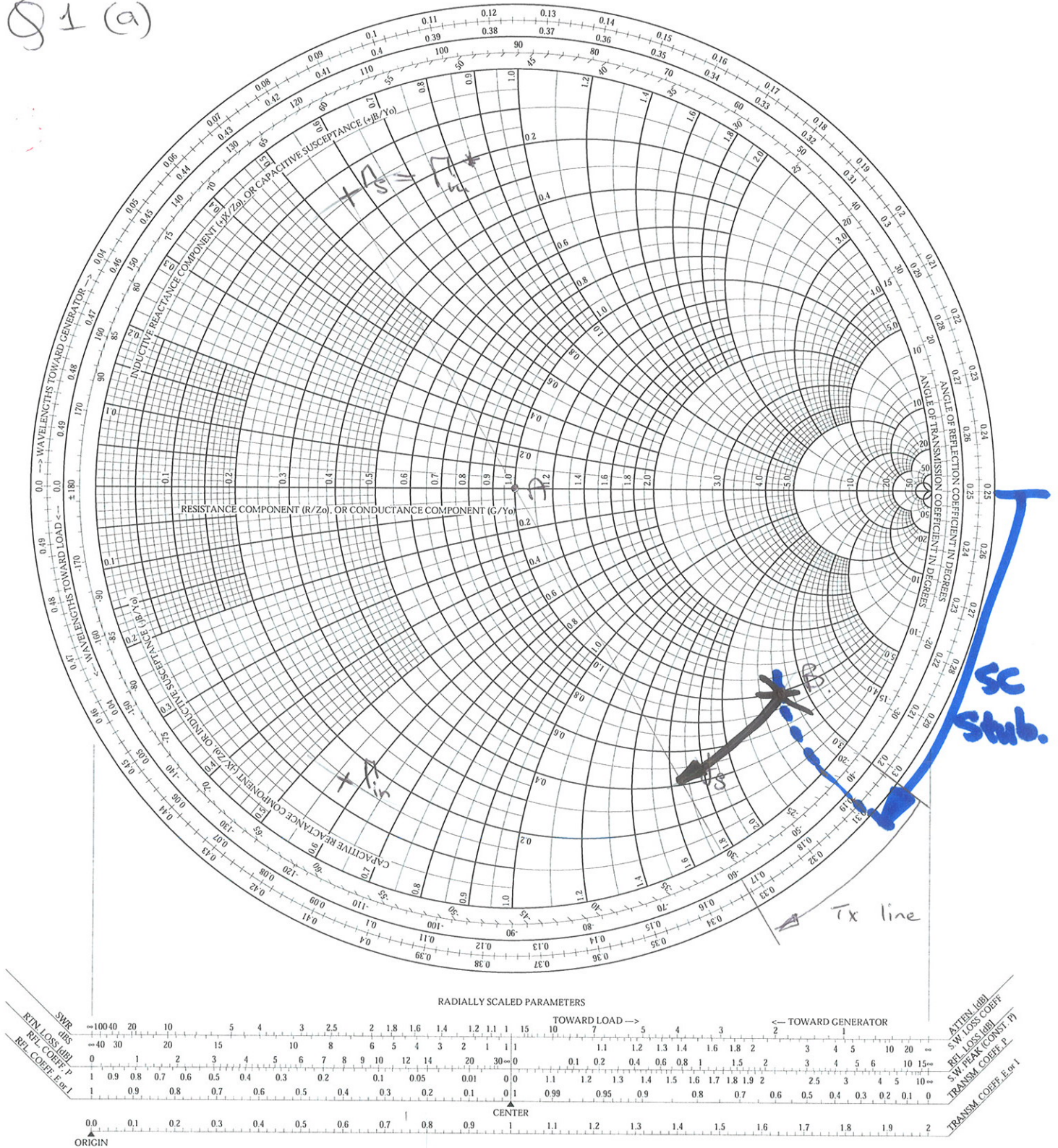
Student Name.....P.T. Gough.....

ENEL434

Student ID.....

## Smith Chart

Q1 (a)



Q1 (b)  $Z = 50 + j50$

$\therefore Z = 1 + j$  plot on Smith chart.

Now turn into an admittance

$Y = 0.5 - j0.5$ . This is our start pt.

now  $\Gamma_{out} = S_{22} = 0.44 \angle -63^\circ$ .

$\therefore \Gamma_L = \Gamma_{out}^* = 0.44 \angle +63^\circ$ .

$\therefore Y_e = 0.5 - j0.5$  This is our end pt!

Thus  $D2$  and  $L2 = 0\lambda$ , and we don't need them at all. The load as specified is the conjugate match.

(c) (i)  $G_{in} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} \quad (S_{12} = 0)$

$= \frac{1}{1 - 0.64} \cdot |2.77|^2 \cdot \frac{1}{(1 - 0.49)}$

$= 2.78 \times 7.67 \times 1.23 = 26.2x$

$= 4.4 \text{ dB} + 8.84 \text{ dB} + 0.89 \text{ dB}$

$= 14.13 \text{ dB}$

(ii)  $G = \frac{|S_{21}|^2}{1 - |S_{22}|^2} = 9.43$   
 $= 7.67 \times 1.23 = 9.43 = 9.75 \text{ dB}$

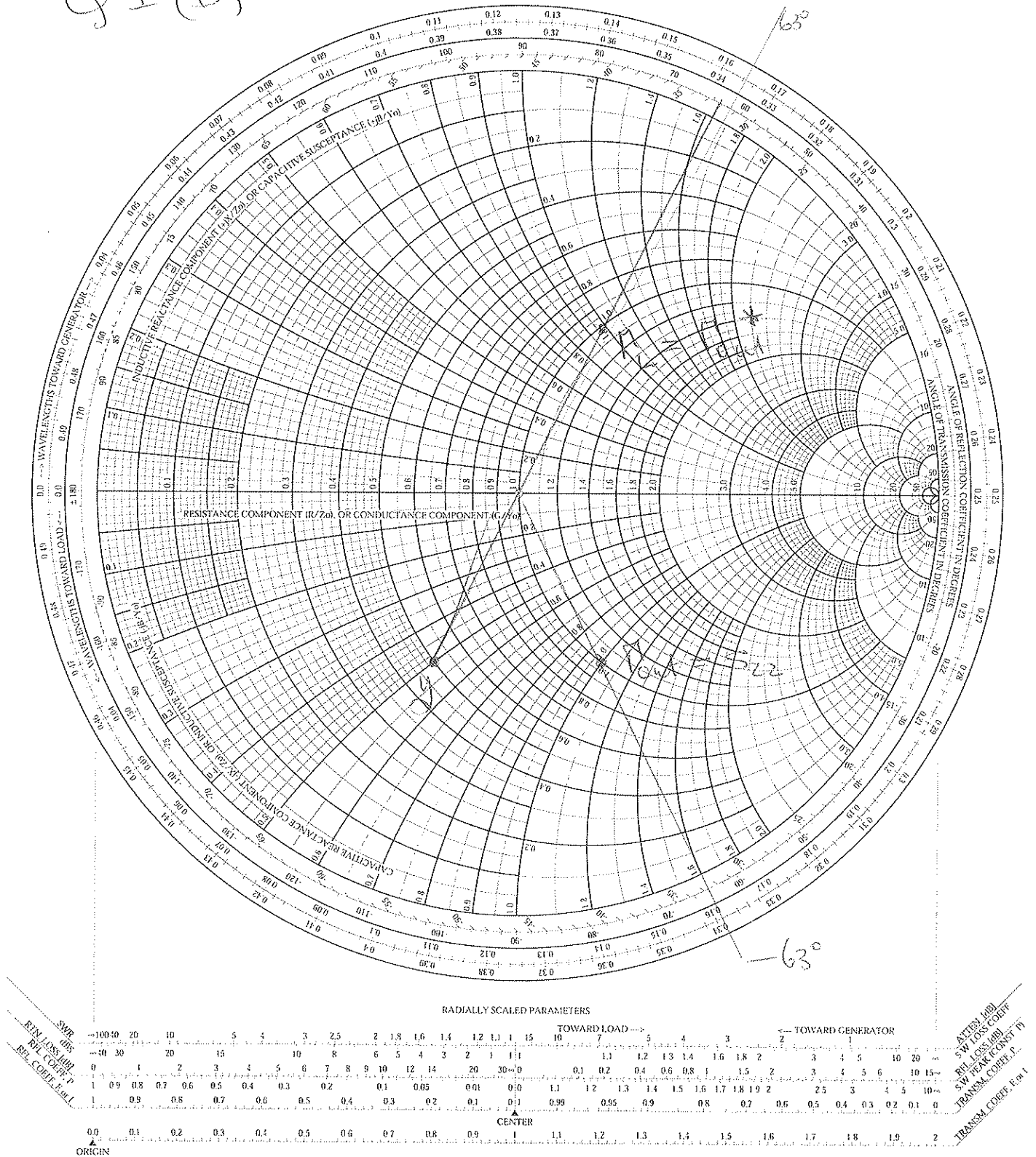
Student Name..... P.T. Gray

Student ID.....

ENEL434

# Smith Chart

§ 1 (b)



## Answers

2.

(a) (i)  $V_{CE} \cdot I_c = 0.93 \text{ mA}$

(ii)  $V_{CE} = 10 + 0.7 = 10.7 \text{ V}$

(iii)  $r_e = 0.026 / 0.93 \text{ mA} = 28 \Omega$

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(b)  $C_{eq} = 98.59 \text{ pF}$

$L = 1.014 \text{ } \mu\text{H}$

$\omega_0 = 100 \text{ M rad/sec}$

Assumes  $X_{C2} \ll 50 \Omega$

checking  $X_{C2} @ \omega_0 = 10^8 = 1.43 \Omega$

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(c)

$R_L' = 50 \times 100 = 5 \text{ k}\Omega$

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$R_{in}' = \frac{(71)^2}{22+28} \times 50 = 252 \text{ k}\Omega$

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(d)

$R_{coil} = Q \times X_L = 500 \times 101.4$   
 $= 50.7 \text{ k}\Omega$

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(f)

$R_{e'} = R_L' \parallel R_{in}' \parallel R_{coil}$   
 $= 5 \times 10^3 \parallel 42.2 \times 10^3$   
 $= 4.47 \text{ k}\Omega$

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(g)  $a_{v_{mid}} \frac{V_{d(t)}}{V_{e(t)}} = -159 \times$

$$\beta = \frac{1}{71} \times \frac{28}{50} = \cancel{(0.788)} 7.88 \times 10^{-3}$$

$$\text{or } 1/\beta = 129$$

$$\text{c) } A\beta = 159 \times 129 = 1.23$$

So there is just enough gain for the circuit to oscillate.

Note: There is a DC short from the power supply through the RFC and the transformer. To work the transformer would need to be connected to the AC power supply or a coupling capacitor used.