4. NOISE in RF CIRCUITS and SYSTEMS

ENEL434 Electronics 2

Summary

- Random variables (review)
- Noise basics
- Thermal noise
- Shot noise
- Flicker noise
- Circuit noise calculations
- Equivalent noise sources and noise temperature
- Representation of a noisy 2-port
- Noise figure and noise temperature
- Noise figure of an attenuator
- Cascaded communication system
- Examples

References

- D M Pozar, Microwave Engineering, 3rd Edition, John Wiley 2005 [Section 10.1]
- R E Collin, Foundations for Microwave Engineering, 2nd Edition, McGraw-Hill 1992 [Section 10.8]
- P Z Peebles, Jr, Probability, Random Variables and Random Signal Principles, 3rd Edition, McGraw-Hill 1993

Random Variables

Consider a **continuous random variable** X:

- It is **possible** for X to take any value on the range: (-∞, ∞)
- The statistical behaviour is fully described by its **probability density function** f(x).
- The **probability** that X is within the range [a, b] is given by:

$$P(a \le X \le b) = \int_a^b f(x)dx$$

- Hence: $$\int_{-\infty}^{\infty} f(x)dx = 1$$
- The **mean** (or loosely average value) μ_X, and **variance** (spread) σ_X^2 are given by:

$$\mu_X = \int_{-\infty}^{\infty} xf(x)dx \qquad \sigma_X^{\ 2} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

Random Variables

Consider two **continuous random variables** X and Y:

- It is **possible** for X and Y to take any value on the range: $(-\infty, \infty)$
- The statistical behaviour of X and Y are fully described by their **joint probability density function** f(x,y).
- The **probability** that X is within the range [a, b] AND Y is within the range [c, d] is given by:

$$P(a \le X \le b \quad \cap \quad c \le Y \le d) = \int_a^b \int_c^d f(x,y)dydx$$

- Hence:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)dydx = 1$$

- X and Y are **independent** (outcome for X has no bearing on the outcome of Y) if **f(x,y) = f(x)f(y)**

Random Variables

Consider two **continuous random variables** X and Y:

- An important statistical property is the mean of the product XY called the **correlation**:

$$R_{XY} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x,y)dydx$$

- If $\mathbf{R_{XY} = \mu_X \mu_Y}$ then X and Y are **uncorrelated**.
- Independence of X and Y is sufficient to imply X and Y are uncorrelated. But the reverse statement is in general not true.

Random Variables

Consider the sum of multiple **independent continuous random variables**

$Y = X_1 + X_2 + \ldots + X_n$

- The mean and variance of Y are:

$$\mu_Y = \mu_1 + \mu_2 + \ldots + \mu_n$$

$$\sigma_Y^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2$$

where μ_k and σ_k^2 are the mean and variance respectively of X_k

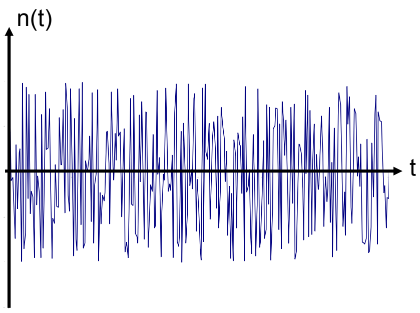
- **Central Limit Theorem:** Y has a normal distribution as n tends to infinity regardless of the distributions of the components of Y (X_1, X_2 … X_n)
- If X_1, X_2 … X_n all have normal distributions, then Y will have a normal distribution for any value of n

Noise Basics

- Noise is a random low level disturbance present in all real electrical circuits.

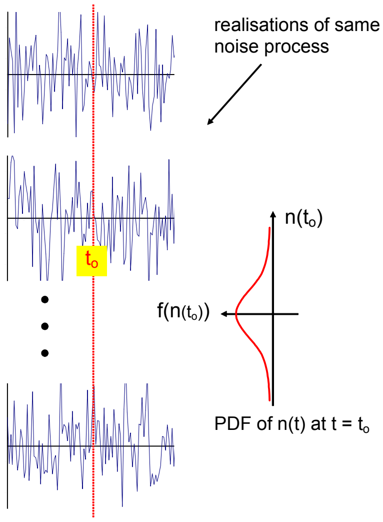
$$v(t) = v_{Signal}(t) + n(t)$$

- Noise becomes an issue when the level of the signal is comparable to n(t).
- n(t) is a **random process**. That is:
  - n(t) is a random variable at time t
  - $n(t_1)$ and $n(t_2)$ are a pair of random variables



Noise Basics

- n(t) can be characterised by a probability density function at time t.
- That is we could consider an infinite number of realisations of the same noise process.
- The **mean μ_n of n(t) is zero** for noise.
- The **variance σ_n^2 is non-zero** and is the noise power into a 1 Ω resistor (since mean is zero).
- The correlation of $n(t_1)$ and $n(t_2)$, called **autocorrelation $R_{nn}(t_1,t_2)$**,is of interest.
- We assume that noise is the sum of many random events. Hence, n(t) will have a **normal distribution**.
- Noise is called **white noise** if its power spectral density is constant.



Noise Basics

- We may assume that the statistics are independent of time – ie **stationary**. In the case of autocorrelation dependent only on time difference.
- A useful assumption is that the noise source is **ergodic**.
- This means that the **statistics of n(t) can be obtained from time averages of one realisation of n(t).**

$$\mu_n = \langle n(t) \rangle = 0 \qquad \langle n(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)dt$$

$$\sigma_n^2 = \langle (n(t)-\mu_n)^2 \rangle = \langle n(t)^2 \rangle \qquad \langle n(t)^2 \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [n(t)]^2 dt$$

$$R_{nn}(t_1,t_2) = R_{nn}(\tau) = \langle n(t)n(t+\tau) \rangle$$

$$\langle n(t)n(t+\tau) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} n(t)n(t+\tau)dt$$

Noise Basics

- Another important aspect of n(t) is its power spectral density $S_{nn}(\omega)$ and may be defined so that the power into a 1Ω resistor is given by:

$$P_{nn} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{nn}(\omega)d\omega$$

- The power into 1Ω over a small bandwidth Δf centred at f = ω/2π:

$$dP_{nn} = 2S_{nn}(\omega)\Delta f$$

- It can be shown that

$$S_{nn}(\omega) = \int_{-\infty}^{\infty} R_{nn}(\tau)e^{-j\omega\tau}d\tau$$

- It can be shown that:
  1. R_{nn} is maximum at τ = 0
  2. R_{nn} is even, so therefore, **S_{nn} is pure real and an even function**.

Noise Basics

Say that we have two noise sources $n_1(t)$ and $n_2(t)$.

- The **cross-correlation** is given by:

$$R_{n_1n_2}(\tau) = \langle n_1(t)n_2(t+\tau)\rangle$$

- The **cross-power spectral density** is:

$$S_{n_1n_2}(\omega) = \int_{-\infty}^{\infty} R_{n_1n_2}(\tau)e^{-j\omega\tau}d\tau$$

- Unlike autocorrelation, cross-correlation is in general not even, and hence the cross-power spectral density is in general complex.
- $\mathrm{Re}S_{n1n2}$ is and even function, $\mathrm{Im}S_{n1n2}$ is an odd function.
- The cross-power into 1Ω over a small bandwidth Δf centred at f = ω/2π:

$$dP_{n_1n_2} = 2\mathrm{Re}\left(S_{n_1n_2}(\omega)\right)\Delta f$$

Thermal Noise

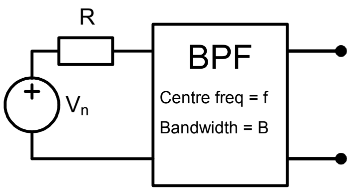
- In a conducting material, thermal energy manifests itself as random vibration of atoms and random motion of electrons.
- The electrons have thermal velocities significantly higher than drift velocity.
- The random motion of the electrons gives rise to a noise voltage across and noise current through the conductor.
- This noise is called thermal noise, or Johnson noise or Nyquist noise.
- **Mean-square** Thevenin voltage of a resistor with resistance R is:

$$\langle v_n(t)^2 \rangle = \frac{4hfBR}{e^{hf/kT} - 1}$$

where h = 6.626 x 10^{-34} Js (Planck's constant)
k = 1.38 x 10^{-23} J/K (Boltzmann's constant)
T is absolute temperature (K)
f is the centre frequency (Hz) of the system whose bandwidth is B (Hz)

Thermal Noise

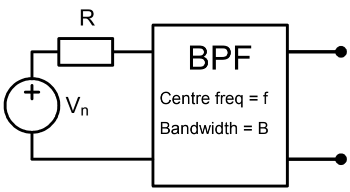
$$\langle v_n(t)^2 \rangle = \frac{4hfBR}{e^{hf/kT} - 1}$$



- Up to about 1000 GHz thermal noise is white: $\langle v_n(t)^2 \rangle \approx 4kTBR$
- The power spectral density of V_n is: $S_{V_n V_n}(\omega) = 2kTR$
- The noise power into bandwidth Δf is: $dP_{v_n v_n} = d\langle v_n(t)^2 \rangle = 4kTR\,\Delta f$
- A Norton equivalent is also possible.

Thermal Noise

$$\langle v_n(t)^2 \rangle = \frac{4hfBR}{e^{hf/kT} - 1}$$

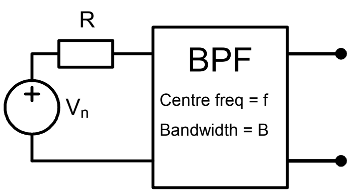


Because the resistor can be represented by a generator:

- Available power: $P = kTB$
- Available power spectral density: $S(\omega) = \frac{kT}{2}$

Thermal Noise

$$\langle v_n(t)^2 \rangle = \frac{4hfBR}{e^{hf/kT} - 1} \approx 4kTBR$$



- eg. T = 300 K, B = 100 MHz, N = 0.414 pW ≡ -93.8 dBm
- Often engineers immerse circuitry in liquid nitrogen or even liquid helium to lower T and hence noise voltage.

Shot Noise

- Shot noise is present in semiconductor devices when conducting current.
- The current of a pn junction for example, involves the passage minority carriers across the pn junction – which are random events.
- For a junction with DC current I_o, the mean-square noise current into bandwidth B is given by:

$$\langle i_n^2 \rangle = 2qI_oB$$

where q = 1.6 x 10^{-19} C

and the power spectral density is given by: $S_{I_nI_n}(\omega) = qI_o$

- Shot noise is white.
- eg. for I_o = 10 mA and B = 100 MHz, $\langle i_n^2 \rangle = 0.32 \text{ x} 10^{-12} A^2$

Flicker or 1/f Noise

- Flicker noise is present in all semiconductor devices and resistive elements.
- Like Shot noise, Flicker noise is associated with current flow.
- Various origins but is essentially the result of contamination and defects present in manufacturing processes.
- Flicker noise is NOT white and nor is it Gaussian.
- The power spectral density of the noise current is given by:

$$S_{I_nI_n}(\omega) = K\frac{I_o^a}{\omega^b}$$

where K, a and b are process / technology dependent constants.

a is typically in the range 0.5 to 2

b is around unity – hence “1/f” noise

- Flicker noise is negligible in microwave amplifiers but Flicker noise can be up-converted in nonlinear microwave circuits such as mixers and oscillators.

Equivalent Noise Sources

- More often in microwave and RF engineering we are interested in the level of noise and not its physical origin.
- In this case, we would represent the noise source by an equivalent thermal noise source.
- **eg.** Suppose that the available noise power from an antenna is 0.1pW into a bandwidth of 100 MHz.

  The available noise power N from a resistor at temperature T_e is:

  $N = kT_eB$

  Hence to achieve N = 0.1 pW into B = 100 MHz, T_e needs to be 72 K.

  ie. taking a resistor and cooling it to -201°C would achieve the same noise power.

Circuit Noise Calculations

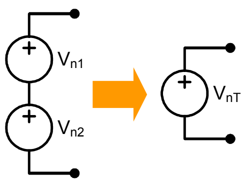
eg. Calculate the equivalent noise voltage of two noise voltages in in series.

From a **time-domain** point of view …

$$\langle v_{nT}(t)^2\rangle = \langle (v_{n1}(t) + v_{n2}(t))^2\rangle$$

$$= \langle v_{n1}{}^2 + v_{n2}{}^2 + 2v_{n1}v_{n2}\rangle$$

$$= \langle v_{n1}{}^2\rangle + \langle v_{n2}{}^2\rangle + 2\langle v_{n1}v_{n2}\rangle$$



where $\langle v_{n1}v_{n2}\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} v_{n1}(t)v_{n2}(t)dt$

$\langle v_{n1}v_{n2}\rangle = 0$ for independent noise sources. Why?

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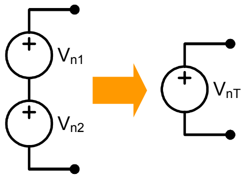
Circuit Noise Calculations

eg. Calculate the equivalent noise voltage of two noise voltages in in series.

From a **spectral domain** point of view …

$V_{nT} = V_{n1} + V_{n2}$

$$|V_{nT}|^2 = |V_{n1} + V_{n2}|^2 = |V_{n1}|^2 + |V_{n2}|^2 + 2\mathrm{Re}\left[V_{n1}V_{n2}^*\right]$$



For **deterministic signals into 1 Ω** :

- $|V_{nT}|^2$ is the power of source V_{nT}
- $|V_{n1}|^2$ is the power of source V_{n1}
- $|V_{n2}|^2$ is the power of source V_{n2}
- $\mathrm{Re}(V_{n1}V_{n2}^*)$ is a “cross-power”

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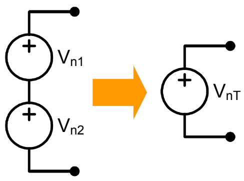
Circuit Noise Calculations

eg. Calculate the equivalent noise voltage of two noise voltages in in series.

From a **spectral domain** point of view …

$V_{nT} = V_{n1} + V_{n2}$

$$|V_{nT}|^2 = |V_{n1} + V_{n2}|^2 = |V_{n1}|^2 + |V_{n2}|^2 + 2\mathrm{Re}\left[V_{n1}V_{n2}^*\right]$$



For **noise signals into 1 Ω over bandwidth Δf**:

$2S_{V_{nT}V_{nT}}\Delta f$ is the power from source V_{nT}

$2S_{V_{n1}V_{n1}}\Delta f$ is the power from source V_{n1}

$2S_{V_{n2}V_{n2}}\Delta f$ is the power from source V_{n2}

$2\mathrm{Re}\left(S_{V_{n1}V_{n2}}\right)\Delta f$ is the cross-power from sources V_{n1} and V_{n2} noting that

$$\mathrm{Re}\left(S_{V_{n1}V_{n2}}(-\omega)\right) = \mathrm{Re}\left(S_{V_{n1}V_{n2}}(\omega)\right)$$

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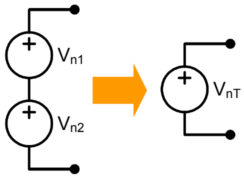
Circuit Noise Calculations

eg. Calculate the equivalent noise voltage of two noise voltages in in series.

From a **spectral domain** point of view …

$$|V_{nT}|^2 = |V_{n1} + V_{n2}|^2 = |V_{n1}|^2 + |V_{n2}|^2 + 2\mathrm{Re}\left[V_{n1} V_{n2}^{*}\right]$$



$$2S_{V_{nT}V_{nT}}\Delta f = 2S_{V_{n1}V_{n1}}\Delta f + 2S_{V_{n2}V_{n2}}\Delta f + 4\mathrm{Re}\left(S_{V_{n1}V_{n2}}\right)\Delta f$$

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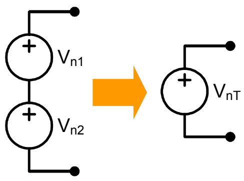
Circuit Noise Calculations

eg. Calculate the equivalent noise voltage of two noise voltages in in series.

From a **spectral domain** point of view …



V_{n1} and V_{n2} can be described by their:

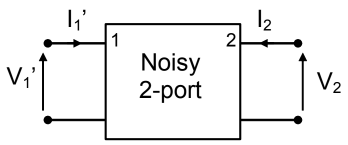
- auto-correlations ↔ power spectral densities
- cross-correlation ↔ cross power spectral density

$$2S_{V_{nT}V_{nT}}\Delta f = 2S_{V_{n1}V_{n1}}\Delta f + 2S_{V_{n2}V_{n2}}\Delta f + 4\mathrm{Re}\left(S_{V_{n1}V_{n2}}\right)\Delta f$$

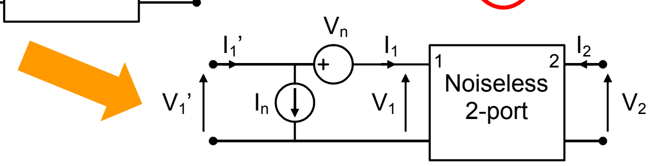
$$R_{V_{n1}V_{n2}} = \langle v_{n1}(t) v_{n2}(t+\tau) \rangle \qquad S_{V_{n1}V_{n2}}(\omega) = \int_{-\infty}^{\infty} R_{V_{n1}V_{n2}}(\tau) e^{-j\omega\tau} d\tau$$

Representation of a Noisy 2-Port

- The 2-port could be a transistor, an amplifier, a coaxial cable etc.
- Due to the physical processes inherent in the 2-port, the 2-port will be noisy.
- We know that from circuit analysis that the terminal behaviour of a 2-port can be described by ABCD parameters in the frequency (spectral) domain.
- We include two noise sources at the input. These sources are correlated. Why?

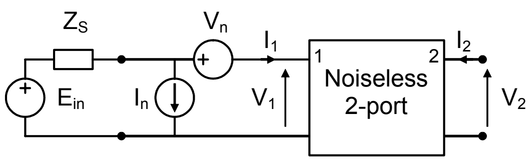


$$V_1' = A\,V_2 - B\,I_2 + V_n$$
$$I_1' = C\,V_2 - D\,I_2 + I_n$$

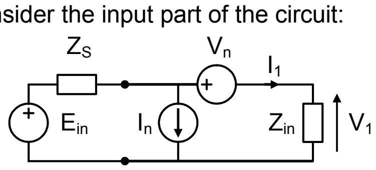


Representation of a Noisy 2-Port

- We would normally connect a generator (Thevenin voltage E_{in} and impedance Z_S) to the input of the noisy-two port.
- The generator supplies the input signal and input noise (effectively due to $\mathrm{Re}Z_S$).



- Let us consider the input part of the circuit:

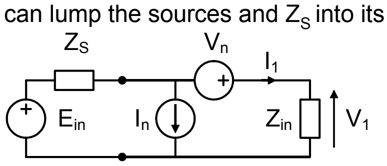


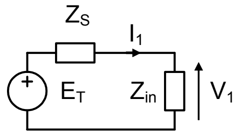
where Z_{in} is the input impedance to the 2-port and will be a function of load connected to port 2.

Representation of a Noisy 2-Port

Let us analyse the input part of the circuit.

- We can arbitrarily change the polarity of V_n.
- We can lump the sources and Z_S into its Thevenin equivalent:



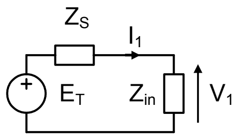


where $E_T = E_{in} + V_n + I_n Z_S$

Representation of a Noisy 2-Port

- The power available from the equivalent generator is:

$E_T = E_{in} + V_n + I_n Z_S$



$$P_A = \frac{|E_T|^2}{4 \mathrm{Re}\, Z_S}$$

- The square of $|E_T|$ is:

$$|E_T|^2 = |E_{in} + V_n + I_n Z_S|^2 = |E_{in}|^2 + |V_n|^2 + |I_n|^2 |Z_S|^2 + 2\mathrm{Re}\left[E_{in}(V_n + I_n Z_S)^*\right] + 2\mathrm{Re}\left[V_n (I_n Z_S)^*\right]$$

$$2S_{E_T E_T}\Delta f = 2S_{E_{in}E_{in}}\Delta f + 2S_{V_n V_n}\Delta f + 2|Z_S|^2 S_{I_n I_n}\Delta f + 4\mathrm{Re}\left(S_{E_{in}V_n}\right)\Delta f + 4\mathrm{Re}\left(S_{E_{in}I_n} Z_S^*\right)\Delta f + 4\mathrm{Re}\left(S_{V_n I_n} Z_S^*\right)\Delta f$$

Representation of a Noisy 2-Port

- Note that above we use the result that:

$$\mathrm{Re}\left(S_{V_nI_n}(-\omega)Z_S^*(-\omega)\right)=\mathrm{Re}\left(S_{V_nI_n}^*(\omega)Z_S(\omega)\right)=\mathrm{Re}\left(S_{V_nI_n}(\omega)Z_S^*(\omega)\right)$$

as well as the other properties of power spectral density functions.

- E_{in} is uncorrelated with V_n and I_n

$$2S_{E_TE_T}\Delta f = 2S_{E_{in}E_{in}}\Delta f + 2S_{V_nV_n}\Delta f + 2|Z_S|^2 S_{I_nI_n}\Delta f + 4\mathrm{Re}\left(S_{E_{in}V_n}\right)\Delta f + 4\mathrm{Re}\left(S_{E_{in}I_n}Z_S^*\right)\Delta f + 4\mathrm{Re}\left(S_{V_nI_n}Z_S^*\right)\Delta f$$

$$\Downarrow$$

$$2S_{E_TE_T}\Delta f = 2S_{E_{in}E_{in}}\Delta f + 2S_{V_nV_n}\Delta f + 2|Z_S|^2 S_{I_nI_n}\Delta f + 4\mathrm{Re}\left(S_{V_nI_n}\right)\mathrm{Re}\left(Z_S\right)\Delta f + 4\mathrm{Im}\left(S_{V_nI_n}\right)\mathrm{Im}\left(Z_S\right)\Delta f$$

Representation of a Noisy 2-Port

- Finally:

$$2S_{E_TE_T}\Delta f = 2S_{E_{in}E_{in}}\Delta f + 2S_{V_nV_n}\Delta f + 2|Z_S|^2 S_{I_nI_n}\Delta f + 4\mathrm{Re}\left(S_{V_nI_n}\right)\mathrm{Re}\left(Z_S\right)\Delta f + 4\mathrm{Im}\left(S_{V_nI_n}\right)\mathrm{Im}\left(Z_S\right)\Delta f$$

$$\Downarrow$$

$$2S_{E_TE_T}\Delta f = 2\Delta f\left[S_{E_{in}E_{in}} + a + b|Z_S|^2 + c\,\mathrm{Re}\,Z_S + d\,\mathrm{Im}\,Z_S\right]$$

where a, b, c and c are real constants related to the noise properties of the 2-port network

Noise Figure and Noise Temperature

- Noise figure (F) is a figure of merit of a 2-port element (such as an amplifier or down-converter).
- The signal to noise ratio always deteriorates through its passage of linear processes.
- **F measures this deterioration of SNR**:

$$\boxed{F = \frac{SNR_{input}}{SNR_{output}}}$$

- **A low F is better than a high F. The best F we could ever obtain is 1.**
- We normally talk about available powers:
  - S_{input} is the signal power available from the input generator
  - S_{output} is the signal power available at the output of the 2-port
  - N_{input} is the noise power available from the generator
  - N_{output} is the noise power available at the output of the 2-port

Noise Figure and Noise Temperature

- The 2-port will amplify (or attenuate) the input signal and noise.
- The 2-port will contribute noise.
- We can refer this contributed noise to the input of the 2-port so that it is amplified (or attenuated) by the 2-port.
- $S_{output} = G_A \, S_{input}$

  where G_A is the available power gain of the 2-port
- $N_{output} = G_A \, N_{input} + G_A N_{added}$

  where N_{added} is the noise added by the 2-port REFERED to its input
- Hence:

$$F = \frac{N_{input} + N_{added}}{N_{input}} = 1 + \frac{N_{added}}{N_{input}}$$

- nb. F is a power ratio so $F_{dB} = 10\log_{10}F$

Noise Figure and Noise Temperature

- Earlier we talked about equivalent noise source as though the origin of all noise is thermal.
- ie N = kTB where T is the effective noise temperature.
- $N_{input} = kT_S B$ where T_S is the effective noise temperature of the generator.
- $N_{added} = kT_e B$ where T_e is the effective noise temperature of the 2-port referred to its input port.
- Hence:

$$F = 1 + \frac{N_{added}}{N_{input}} = 1 + \frac{T_e}{T_S}$$

Noise Figure and Noise Temperature

$$F = 1 + \frac{N_{added}}{N_{input}} = 1 + \frac{T_e}{T_S}$$

- T_e is a parameter of the 2-port. T_S is a parameter of the generator.
- Suppose we have an amplifier whose T_e is 290 K.
  - Salesman A sets T_S = 290 K and hence advertises amplifier F = 2
  - Salesman B sets T_S = 2900 K and hence advertises amplifier F = 1.1
  - Clearly Salesman B gets the most sales
  - Do you see the dilemma the customer faces when F is specified?
- **For this reason whenever you calculate, write or read F it is understood that T_S = 290 K (≡ 17°C).**

$$\boxed{F = 1 + \frac{T_e}{290}}$$

Noise Figure and Noise Temperature

- Recall:

$$\underbrace{\frac{2S_{E_TE_T}\Delta f}{4\mathrm{Re}Z_S}}_{N_{total}} = \underbrace{\frac{2S_{E_{in}E_{in}}\Delta f}{4\mathrm{Re}Z_S}}_{N_{input}} + \underbrace{\frac{2\Delta f}{4\mathrm{Re}Z_S}\left(a + b|Z_S|^2 + c\,\mathrm{Re}Z_S + d\,\mathrm{Im}Z_S\right)}_{N_{added}}$$

- Assuming that $2S_{EinEin}\Delta f$ is effectively thermal due to $\mathrm{Re}Z_S$ with effective temperature T_S (= 290):

$$\underbrace{\frac{2S_{E_TE_T}\Delta f}{4\mathrm{Re}Z_S}}_{N_{total}} = \underbrace{\frac{2\Delta f(2k \times 290 \times \mathrm{Re}Z_s)}{4\mathrm{Re}Z_S}}_{N_{input}} + \underbrace{\frac{2\Delta f}{4\mathrm{Re}Z_S}\left(a + b|Z_S|^2 + c\,\mathrm{Re}Z_S + d\,\mathrm{Im}Z_S\right)}_{N_{added}}$$

Noise Figure and Noise Temperature

- Hence:

$$F = \frac{N_{input} + N_{added}}{N_{input}} = 1 + \frac{\alpha|Z_S|^2 + \beta\mathrm{Re}Z_S + \gamma\mathrm{Im}Z_S + \varepsilon}{\mathrm{Re}Z_S}$$

where α, β, γ and ε are real constants related to the noise properties of the 2-port network

- With some algebraic manipulation:

$$\boxed{F = F_{min} + G_m\frac{|Z_S - Z_{opt}|^2}{\mathrm{Re}Z_S}}$$

where F_{min} is the minimum noise figure
Z_{opt} is the generator impedance that minimises F, and
G_m is a proportionality constant with dimensions conductance.

- nb. G_m is not a physical conductance

Noise Figure and Noise Temperature

But:

$$\Gamma_S = \frac{Z_S - Z_o}{Z_S + Z_o} \qquad \Gamma_{opt} = \frac{Z_{opt} - Z_o}{Z_{opt} + Z_o}$$

where Z_o is the reference impedance for reflection coefficient and S-parameters. Substitution into:

$$F = F_{min} + G_m \frac{|Z_S - Z_{opt}|^2}{\operatorname{Re} Z_S}$$

yields:

$$\boxed{F = F_{min} + \frac{4R_n}{Z_o} \frac{|\Gamma_S - \Gamma_{opt}|^2}{\left(1 - |\Gamma_S|^2\right)|1 + \Gamma_{opt}|^2}}$$

where F_{min} is the minimum noise figure

Γ_{opt} is the value of Γ_S that minimises F, and

R_n is a proportionality constant called the "noise resistance" and is NOT a physical resistance.

Noise Figure and Noise Temperature

$$F = F_{min} + \frac{4R_n}{Z_o} \frac{|\Gamma_S - \Gamma_{opt}|^2}{\left(1 - |\Gamma_S|^2\right)|1 + \Gamma_{opt}|^2}$$

- The above expressions tell us how F varies with choice of Γ_S
- Recall that gain is also dependent on Γ_S
- This means that amplifier design requires trade-off of gain and F
- Fortunately ultra-low noise transistors are designed so that:
  - Γ_S in the vicinity of Γ_{opt} is associated with high gain
  - R_n is low so that F increases at a low rate as Γ_S moves away from Γ_{opt}

Noise Figure and Noise Temperature

$$F = F_{min} + \frac{4R_n}{Z_o}\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)\left|1+\Gamma_{opt}\right|^2}$$

$F = F_{min}$

- With respect to Γ_S we find that F increases monotonically with distance between Γ_S and Γ_{opt}.

eg. FHX04X 10 GHz …

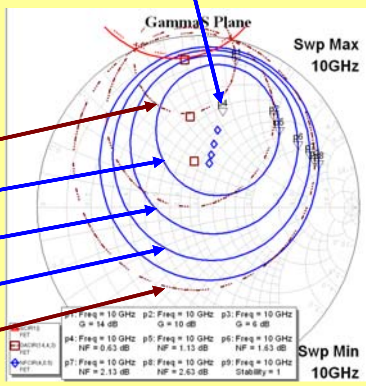
G_A = 14 dB

F = 1.13 dB

F = 1.63 dB

F = 2.13 dB

G_A = 6 dB



Example

Fujitsu *High Electron Mobility Transistor* (HEMT): FHX14X

Source: www.eudyna.com

At V_{DS} = 2 V, I_D = 10 mA, 20 GHz and ambient temperature of 25°C:

S_{11} = 0.523 / -140.0° S_{12} = 0.133 / 58.4°

S_{21} = 2.314 / 70.8° S_{22} = 0.335 / -59.4°

Γ_{opt} = 0.52 / 136° $\frac{R_N}{Z_o} = 0.07$ F_{min} = 1.03 dB = 1.268

What is the noise figure if simultaneous conjugate matching is used?

Based upon the S-parameters at 16 GHz, the generator and load reflection coefficients that accomplish simultaneous conjugate matching are:

Γ_S = 0.835 / 146° Γ_L = 0.775 / 72°

Example

Fujitsu *High Electron Mobility Transistor* (HEMT): FHX14X

Source: www.eudyna.com

At V_{DS} = 2 V, I_D = 10 mA, 20 GHz and ambient temperature of 25°C:

$S_{11} = 0.523 \angle -140.0°$ $\quad S_{12} = 0.133 \angle 58.4°$

$S_{21} = 2.314 \angle 70.8°$ $\quad S_{22} = 0.335 \angle -59.4°$

$\Gamma_{opt} = 0.52 \angle 136°$ $\quad \frac{R_N}{Z_o} = 0.07$ $\quad F_{min} = 1.03\text{ dB} = 1.268$

What is the noise figure if simultaneous conjugate matching is used?

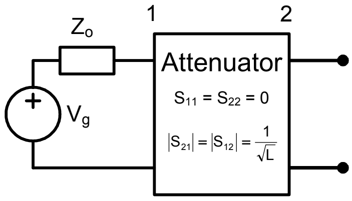
With Γ_S = **0.835 / 146°** the noise figure is:

$$F = F_{min} + \frac{4R_n}{Z_o}\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1-\left|\Gamma_S\right|^2\right)\left|1+\Gamma_{opt}\right|^2} = 1.268 + 4\times 0.07\times\frac{\left|0.835\angle 146° - 0.52\angle 136°\right|^2}{\left(1-0.835^2\right)\left|1+0.52\angle 136°\right|^2}$$

$$= 1.466 \equiv 1.66\text{ dB}$$

Noise Figure of an Attenuator

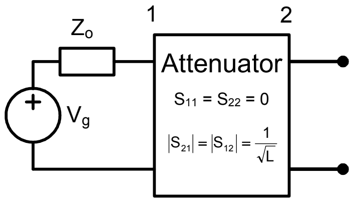
Consider an attenuator with matched ports and driven by a matched generator (reference impedance Z_o is also pure real):



- We assume that the generator and attenuator are at temperature T.
- Hence, N_{in} = kTB
- Looking into port 2 – the equivalent is a generator with Thevenin impedance Z_o and we would expect it to have a noise temperature of T.
- Hence, N_o = kTB

Noise Figure of an Attenuator

Consider an attenuator with matched ports and driven by a matched generator (reference impedance Z_o is also pure real):

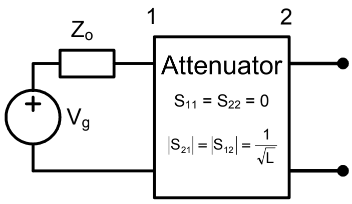


- But: $N_o = \frac{1}{L}\left(N_{in} + N_{added}\right) = \frac{1}{L}\left(kTB + N_{added}\right)$
- Hence: $N_{added} = (L - 1)\,kTB = kT_eB$

  where $T_e = (L - 1)T$

Noise Figure of an Attenuator

Consider an attenuator with matched ports and driven by a matched generator (reference impedance Z_o is also pure real):



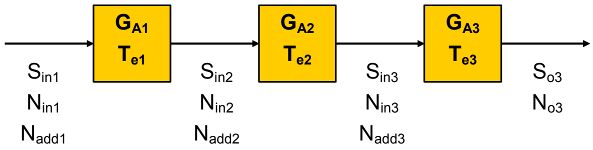
- Using the relationship for F (= 1 + T_e/290):

$$\boxed{F = 1 + (L - 1)\frac{T}{290}}$$

- This means that **F is equal to its insertion loss L when it has a physical temperature of 290 K**.

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:

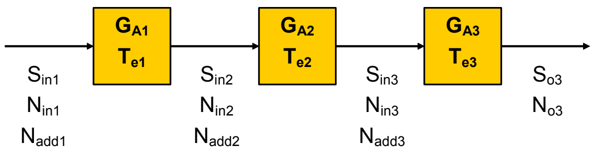


We make the assumption that over the bandwidth B:

- Thermal noise models can be used. eg:
  - $N_{in1} = kT_SB$ where T_S is the noise temperature of the input source
  - $N_{add1} = kT_{e1}B$
- T_S, T_{e1}, G_{A1} etc are flat

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:



Considering the signal powers:

$S_{in2} = G_{A1}S_{in1}$

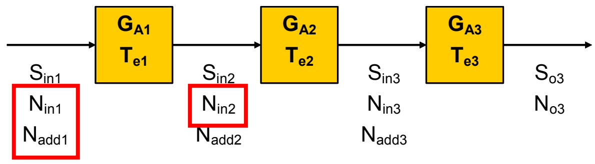
$S_{in3} = G_{A2}S_{in2} = G_{A1}G_{A2}S_{in1}$

$S_{o3} = G_{A3}S_{in3} = G_{A1}G_{A2}G_{A3}S_{in1}$

So the **overall available gain is $G_A = G_{A1}G_{A2}G_{A3}$**

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:



Considering the noise powers:

$N_{in1} = kT_SB$ where T_S is the noise temperature of the input source

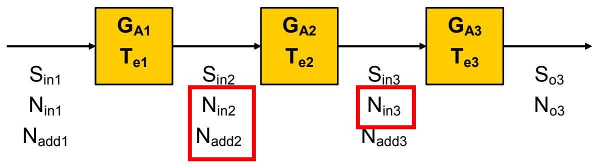
$N_{add1} = kT_{e1}B$

$N_{in2} = G_{A1}(N_{in1} + N_{add1})$

$= kG_{A1}(T_S + T_{e1})B$

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:



Considering the noise powers:

$N_{in2} = kG_{A1}(T_S + T_{e1})B$

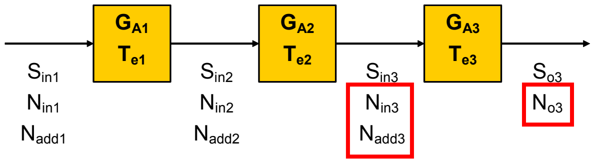
$N_{add2} = kT_{e2}B$

$N_{in3} = G_{A2}(N_{in2} + N_{add2})$

$= kG_{A1}G_{A2}(T_S + T_{e1})B + kG_{A2}T_{e2}B$

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:



Considering the noise powers:

$N_{in3} = kG_{A1}G_{A2}(T_S + T_{e1})B + kG_{A2}T_{e2}B$

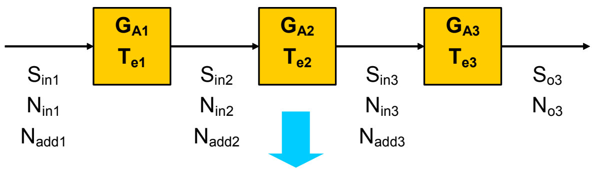
$N_{add3} = kT_{e3}B$

$N_{o3} = G_{A3}(N_{in3} + N_{add3})$

$$= k\left[G_{A1}G_{A2}G_{A3}\left(T_S + T_{e1}\right) + G_{A2}G_{A3}T_{e2} + G_{A3}T_{e3}\right]B$$

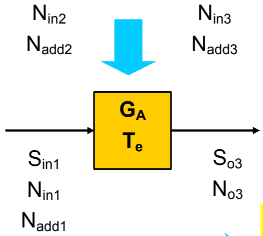
Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:



$S_{o3} = G_A S_{in1}$

$G_A = G_{A1}G_{A2}G_{A3}$



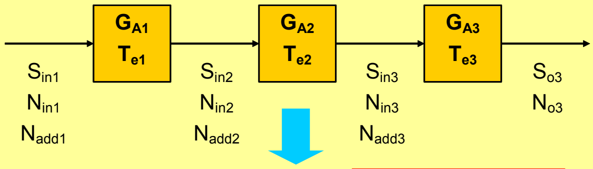
$$T_e = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}}$$

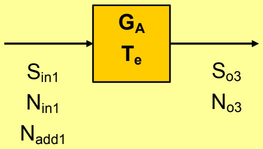
$N_{o3} = kG_A(T_S + T_e)\,B$

$$= k\left[G_{A1}G_{A2}G_{A3}\left(T_S + T_{e1}\right) + G_{A2}G_{A3}T_{e2} + G_{A3}T_{e3}\right]B$$

Cascaded Communication System

Consider a 3-stage communications system with signal (S) and noise (N) powers as indicated:





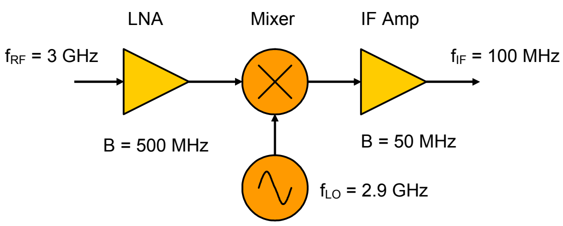
$$\boxed{G_A = G_{A1}G_{A2}G_{A3}}$$

$$\boxed{T_e = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}}}$$

- The result for T_e is significant as it says that the **first stage has the most impact on system noise performance**.
- Modification of formulae for different number of stages should be apparent

Example 1

Calculate the gain and noise figure of the following system:



LNA:
G_A = 20 dB
F = 3 dB

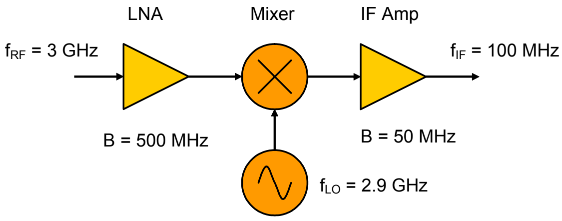
Mixer:
Conversion Loss = 5 dB
F = 8 dB

IF Amp:
G_A = 30 dB
F = 10 dB

- **The block with the narrowest bandwidth determines B for noise calculations.** Hence B = 50 MHz.
- Although the mixer employs a nonlinear element, the down-conversion process is linear provided input signal is small-signal.

Example 1

Calculate the gain and noise figure of the following system:



LNA:
G_A = 20 dB
F = 3 dB

Mixer:
Conversion Loss = 5 dB
F = 8 dB

IF Amp:
G_A = 30 dB
F = 10 dB

G_{A1} = 20 dB = **100**
F_1 = 3 dB = **2**

G_{A2} = - 5 dB = **0.316**
F_2 = 8 dB = **6.31**

G_{A3} = 30 dB = **1000**
F_3 = 10 dB = **10**

Example 1

G_{A1} = 20 dB = **100**
F_1 = 3 dB = **2**

G_{A2} = - 5 dB = **0.316**
F_2 = 8 dB = **6.31**

G_{A3} = 30 dB = **1000**
F_3 = 10 dB = **10**

$T_{e1} = 290(2 - 1) = 290$ K

$T_{e2} = 290(6.31 - 1) = 1540$ K

$T_{e3} = 290(10 - 1) = 2610$ K

$$
\begin{aligned}
G_A &= G_{A1}G_{A2}G_{A3} \\
&= 100 \times 0.316 \times 1000 \\
&= 31600 \\
&\equiv 45 \text{ dB}
\end{aligned}
$$

Example 1

G_{A1} = 20 dB = **100**
F_1 = 3 dB = **2**

G_{A2} = - 5 dB = **0.316**
F_2 = 8 dB = **6.31**

G_{A3} = 30 dB = **1000**
F_3 = 10 dB = **10**

$T_{e1} = 290(2 - 1)$
$= 290$ K

$T_{e2} = 290(6.31 - 1)$
$= 1540$ K

$T_{e3} = 290(10 - 1)$
$= 2610$ K

$$T_e = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}}$$

$$= 290 + \frac{1540}{100} + \frac{2610}{100 \times 0.316}$$

$$= 290 + 15.4 + 83\ \text{K}$$

$$= 388\ \text{K}$$

$$F = 1 + \frac{T_e}{290}$$

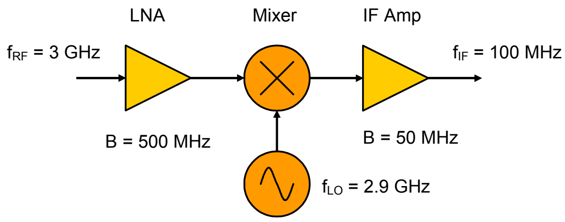
$$= 1 + \frac{388}{290}$$

$$= 2.34$$

$$\equiv 3.7\ \text{dB}$$

Example 2

Calculate the minimum input signal power so that the output SNR is greater than 15 dB when the input source has a noise temperature of 100 K.



LNA:
G_A = 20 dB
F = 3 dB

Mixer:
Conversion Loss = 5 dB
F = 8 dB

IF Amp:
G_A = 30 dB
F = 10 dB

We know from the previous example:

T_e = 388 K G_A = 31600 and B = 50 MHz

Example 2

Output signal power:

$S_o = G_A \, S_{in}$

Output noise power:

$N_o = k(T_e + T_S)BG_A$

Output SNR:

$$SNR_o = \frac{G_A S_{in}}{k(T_e + T_S)BG_A}$$

Hence:

$$\begin{aligned} S_{in} &= k(T_e + T_S)B\, SNR_o \\ &= 1.38 \times 10^{-23}(388 + 100)50 \times 10^6 \times 31.6 \\ &= 1.064 \times 10^{-11} \text{ W} \\ &\equiv -80 \text{ dBm} \end{aligned}$$

Example 2

Note the input SNR:

$$\begin{aligned} SNR_{in} &= \frac{S_{in}}{kT_S B} \\ &= \frac{1.064 \times 10^{-11}}{1.38 \times 10^{-23} \times 100 \times 50 \times 10^6} \\ &= 154 \\ &\equiv 22 \text{ dB} \end{aligned}$$

Example 3

You are to design a satellite TV receiver front-end low-noise block (LNB) and integrate it with the antenna which is to be located outdoors and 5 m away from the indoor unit (containing mixer, IF amp, decoder etc).

Specifications and considerations include:

- Centre frequency = 11.4 GHz
- Bandwidth = 300 MHz
- Maximum noise figure of 4 dB
- Minimum gain of 55 dB
- Antenna is to be located 5 m away from the indoor unit
- Indoor unit contains down-converter, IF amplifier, decoder etc.
- The box to contain outdoor electronics needs to be weather proof and its cost increases with size.

Example 3

You are restricted to use the following:

1 x Antenna: T_e = 150 K

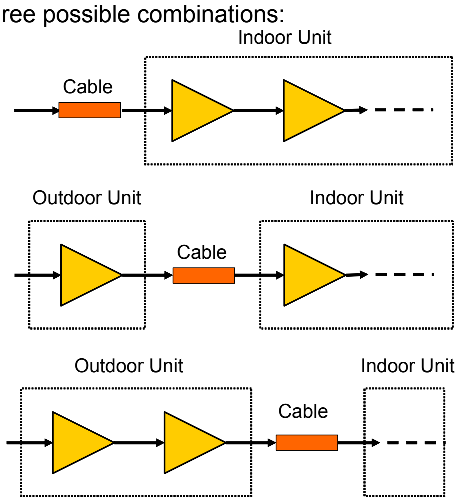
2 x Amplifier: G_A = 40 dB F = 2 dB

1 x Coaxial cable: Length = 5 m Loss = 20 dB

- The low noise bock can therefore use two amplifiers and there will be a coaxial cable connecting the antenna / outdoor unit to the indoor unit.
- The issues are whether we have electronics (amplifier(s)) outside in an outdoor unit or place them in the indoor unit.
- The overall gain of the two amplifiers and the coaxial cable is 60 dB thereby satisfying the gain specification.
- The overall gain does not depend on what order the these blocks are arranged.

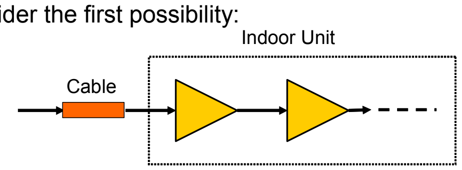
Example 3

There are three possible combinations:



Example 3

Let us consider the first possibility:



G_{A1} = -20 dB ≡ 0.01	F_1 = 20 dB ≡ 100	T_{e1} = 28710 K
G_{A2} = 40 dB ≡ 10^4	F_2 = 2 dB ≡ 1.58	T_{e2} = 170 K
G_{A3} = 40 dB ≡ 10^4	F_3 = 2 dB ≡ 1.58	T_{e3} = 170 K

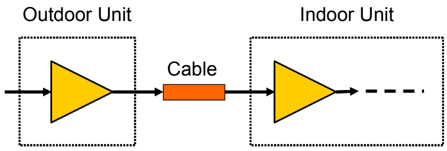
$$T_e = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}} = 28710 + \frac{170}{0.01} + \frac{170}{0.01 \times 10^4} = 45710\ \text{K}$$

$$F = 1 + \frac{T_e}{290} = 1 + \frac{45710}{290} = 159 \equiv 22\ \text{dB}$$

Cheap but bad!!

Example 3

Let us consider the second possibility:



G_{A1} = 40 dB ≡ 10^4 F_1 = 2 dB ≡ 1.58 T_{e1} = 170 K

G_{A2} = -20 dB ≡ 0.01 F_2 = 20 dB ≡ 100 T_{e2} = 28710 K

G_{A3} = 40 dB ≡ 10^4 F_3 = 2 dB ≡ 1.58 T_{e3} = 170 K

$$T_e = T_{e1} + \frac{T_{e2}}{G_{A1}} + \frac{T_{e3}}{G_{A1}G_{A2}} = 170 + \frac{28710}{10^4} + \frac{170}{10^4 \times 0.01} = 175\ \text{K}$$

$$F = 1 + \frac{T_e}{290} = 1 + \frac{175}{290} = 1.6 \equiv 2.04\ \text{dB}$$

Achieves spec at lowest cost