C h a p t e r   E l e v e n

Microwave Amplifier Design

Amplification is one of the most basic and prevalent microwave circuit functions in modern RF and microwave systems. Early microwave amplifiers relied on tubes, such as klystrons and traveling-wave tubes, or solid-state reflection amplifiers based on the negative resistance characteristics of tunnel or varactor diodes. But due to the dramatic improvements and innovations in solid-state technology that have occurred since the 1970s, most RF and microwave amplifiers today use transistor devices such as Si or SiGe BJTs, GaAs HBTs, GaAs or InP FETs, or GaAs HEMTs [1]–[4]. Microwave transistor amplifiers are rugged, low-cost, reliable, and can be easily integrated in both hybrid and monolithic integrated circuitry. As discussed in more detail in Chapter 10, transistor amplifiers can be used at frequencies in excess of 100 GHz in a wide range of applications requiring small size, low-noise figure, broad bandwidth, and low to medium power capacity. Although microwave tubes are still required for very high power and/or very high frequency applications, continuing improvement in the performance of microwave transistors is steadily reducing the need for microwave tubes.

Our discussion of transistor amplifier design will rely on the terminal characteristics of transistors, as represented by either S parameters or one of the equivalent circuit models introduced in the previous chapter. We will begin with some general definitions of two-port power gains that are useful for amplifier design, and then discuss the subject of stability. These results will then be applied to single-stage transistor amplifiers, including designs for maximum gain, specified gain, and low noise figure. Broadband balanced and distributed amplifiers are discussed in Section 11.4. We conclude with a brief treatment of transistor power amplifiers.

11.1 TWO-PORT POWER GAINS

In this section we develop several expressions for the gain and stability of a general two-port amplifier circuit in terms of the S parameters of the transistor. These results will be used in the following sections for amplifier and oscillator design.

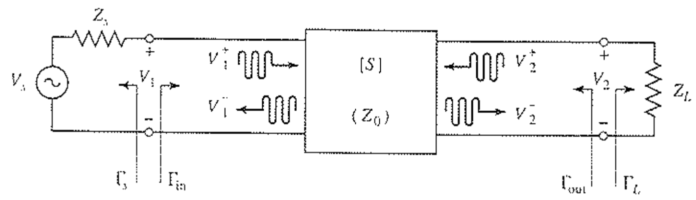

FIGURE 11.1 A two-port network with general source and load impedances.

Definitions of Two-Port Power Gains

Consider an arbitrary two-port network $[S]$ connected to source and load impedances Z_S and Z_L, respectively, as shown in Figure 11.1. We will derive expressions for three types of power gain in terms of the S parameters of the two-port network and the reflection coefficients, Γ_S and Γ_L, of the source and load.

- *Power Gain* $= G = P_L/P_{in}$ is the ratio of power dissipated in the load Z_L to the power delivered to the input of the two-port network. This gain is independent of Z_S, although some active circuits are strongly dependent on Z_S.
- *Available Gain* $= G_A = P_{avn}/P_{avs}$ is the ratio of the power available from the two-port network to the power available from the source. This assumes conjugate matching of both the source and the load, and depends on Z_S but not Z_L.
- *Transducer Power Gain* $= G_T = P_L/P_{avs}$ is the ratio of the power delivered to the load to the power available from the source. This depends on both Z_S and Z_L.

These definitions differ primarily in the way the source and load are matched to the two-port device; if the input and output are both conjugately matched to the two-port, then the gain is maximized and $G = G_A = G_T$.

With reference to Figure 11.1, the reflection coefficient seen looking toward the load is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{11.1a}$$

while the reflection coefficient seen looking toward the source is

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}, \tag{11.1b}$$

where Z_0 is the characteristic impedance reference for the S parameters of the two-port network.

In general, the input impedance of the terminated two-port network will be mismatched with a reflection coefficient given by Γ_{in}, which can be determined using a signal flow graph (see Example 4.7), or by the following analysis. From the definition of the S parameters that $V_2^+ = \Gamma_L V_2^-$, we have

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ + S_{12}\Gamma_L V_2^-, \tag{11.2a}$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-. \tag{11.2b}$$

Eliminating V_2^- from (11.2a) and solving for V_1^-/V_1^+ gives

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}, \tag{11.3a}$$

where Z_{in} is the impedance seen looking into port 1 of the terminated network. Similarly, the reflection coefficient seen looking into port 2 of the network when port 1 is terminated by Z_S is

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}. \tag{11.3b}$$

By voltage division,

$$V_1 = V_S \frac{Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in}).$$

Using

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}},$$

from (11.3a) and solving for V_1^+ in terms of V_S gives

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \Gamma_{in})}. \tag{11.4}$$

If peak values are assumed for all voltages, the average power delivered to the network is

$$P_{in} = \frac{1}{2Z_0} \left|V_1^+\right|^2 \left(1 - |\Gamma_{in}|^2\right) = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{in}|^2} \left(1 - |\Gamma_{in}|^2\right) \tag{11.5}$$

where (11.4) was used. The power delivered to the load is

$$P_L = \frac{\left|V_2^-\right|^2}{2Z_0} \left(1 - |\Gamma_L|^2\right). \tag{11.6}$$

Solving for V_2^- from (11.2b), substituting into (11.6), and using (11.4) gives

$$P_L = \frac{\left|V_1^+\right|^2 |S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{2Z_0 \quad |1 - S_{22}\Gamma_L|^2} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right) |1 - \Gamma_S|^2}{|1 - S_{22}\Gamma_L|^2 \, |1 - \Gamma_S\Gamma_{in}|^2}. \tag{11.7}$$

The power gain can then be expressed as

1

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{in}|^2\right) |1 - S_{22}\Gamma_L|^2}. \tag{11.8}$$

The power available from the source, P_{avs}, is the maximum power that can be delivered to the network. This occurs when the input impedance of the terminated network is conjugately matched to the source impedance, as discussed in Section 2.6. Thus, from (11.5),

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{\left(1 - |\Gamma_S|^2\right)}. \tag{11.9}$$

Similarly, the power available from the network, P_{avn}, is the maximum power that can be delivered to the load. Thus, from (11.7),

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_{out}|^2\right) |1 - \Gamma_S|^2}{|1 - S_{22}\Gamma_{out}^*|^2 \, |1 - \Gamma_S\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*}. \tag{11.10}$$

In (11.10), Γ_{in} must be evaluated for $\Gamma_L = \Gamma_{out}^*$. From (11.3a), it can be shown that

$$|1 - \Gamma_S\Gamma_{in}|^2\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{out}|^2\right)^2}{\left|1 - S_{22}\Gamma_{out}^*\right|^2},$$

which reduces (11.10) to

$$P_{avn} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 |1 - \Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{out}|^2\right)}. \tag{11.11}$$

Observe that P_{avs} and P_{avn} have been expressed in terms of the source voltage, V_S, which is independent of the input or load impedances. There would be confusion if these quantities were expressed in terms of V_1^+, since V_1^+ is different for each of the calculations of P_L, P_{avs}, and P_{avn}.

Using (11.11) and (11.9), the available power gain is then

2

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2 \left(1 - |\Gamma_{out}|^2\right)}. \tag{11.12}$$

3

From (11.7) and (11.9), the transducer power gain is

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right) \left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{in}|^2 \, |1 - S_{22}\Gamma_L|^2}. \tag{11.13}$$

A special case of the transducer power gain occurs when both the input and output are matched for zero reflection (in contrast to conjugate matching). Then $\Gamma_L = \Gamma_S = 0$, and (11.13) reduces to

SC(1)

$$G_T = |S_{21}|^2. \tag{11.14}$$

Another special case is the unilateral transducer power gain, G_{TU}, where $S_{12} = 0$ (or is negligibly small). This nonreciprocal characteristic is common to many practical amplifier circuits. From (11.3a), $\Gamma_{in} = S_{11}$ when $S_{12} = 0$, so (11.13) gives the unilateral transducer gain as

input matched

SC(2)

$$G_{TU} = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right) \left(1 - |\Gamma_L|^2\right)}{|1 - S_{11}\Gamma_S|^2 \, |1 - S_{22}\Gamma_L|^2}. \tag{11.15}$$

EXAMPLE 11.1 COMPARISON OF POWER GAIN DEFINITIONS

A microwave transistor has the following S parameters at 10 GHz, with a 50 Ω reference impedance:

$$S_{11} = 0.45\angle 150°$$
$$S_{12} = 0.01\angle -10°$$
$$S_{21} = 2.05\angle 10°$$
$$S_{22} = 0.40\angle -150°$$

The source impedance is $Z_S = 20\ \Omega$ and the load impedance is $Z_L = 30\ \Omega$. Compute the power gain, the available gain, and the transducer power gain.

Solution
From (11.1a,b) the reflection coefficients at the source and load are

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{20-50}{20+50} = -0.429,$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30-50}{30+50} = -0.250.$$

From (11.3a,b) the reflection coefficients seen looking at the input and output of the terminated network are

$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.45\angle 150^\circ + \frac{(0.01\angle{-10^\circ})(2.05\angle 10^\circ)(-0.250)}{1-(0.40\angle{-150^\circ})(-0.250)}$$

$$= 0.455\angle 150^\circ,$$

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.40\angle 150^\circ + \frac{(0.01\angle{-10^\circ})(2.05\angle 10^\circ)(-0.429)}{1-(0.45\angle 150^\circ)(-0.429)}$$

$$= 0.408\angle{-151^\circ}.$$

Then from (11.8) the power gain is

$$G = \frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left(1-|\Gamma_{\text{in}}|^2\right)|1-S_{22}\Gamma_L|^2} = \frac{(2.05)^2\left[1-(0.250)^2\right]}{\left|1-(0.40\angle{-150^\circ})(-0.250)\right|^2\left[1-(0.455)^2\right]}$$

$$= 5.94.$$

From (11.12) the available power gain is

$$G_A = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{\text{out}}|^2\right)} = \frac{(2.05)^2\left[1-(0.429)^2\right]}{\left|1-(0.45\angle{-150^\circ})(-0.429)\right|^2\left[1-(0.408)^2\right]}$$

$$= 5.85.$$

From (11.13) the transducer power gain is

$$G_T = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_{\text{in}}|^2\,|1-S_{22}\Gamma_L|^2}$$

$$= \frac{(2.05)^2\left[1-(0.429)^2\right]\left[1-(0.250)^2\right]}{\left|1-(0.40\angle{-150^\circ})(-0.250)\right|^2\left|1-(-0.429)(0.455\angle{-150^\circ})\right|^2} = 5.49.$$

Further Discussion of Two-Port Power Gains

A single-stage microwave transistor amplifier can be modeled by the circuit of Figure 11.2, where a matching network is used on both sides of the transistor to transform the input and

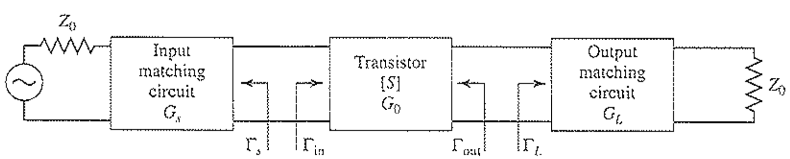

FIGURE 11.2 The general transistor amplifier circuit.

output impedance Z_0 to the source and load impedances Z_S and Z_L. The most useful gain definition for amplifier design is the transducer power gain of (11.13), which accounts for both source and load mismatch. Thus, from (11.13), we can define separate effective gain factors for the input (source) matching network, the transistor itself, and the output (load) matching network as follows:

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{\text{in}}\Gamma_S|^2}, \tag{11.16a}$$

$$G_0 = |S_{21}|^2, \tag{11.16b}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}. \tag{11.16c}$$

Then the overall transducer gain is $G_T = G_S G_0 G_L$. The effective gains from G_S and G_L are due to the impedance matching of the transistor to the impedance Z_0.

If the transistor is unilateral, so that $S_{12} = 0$ or is small enough to be ignored, then (11.3) reduces to $\Gamma_{\text{in}} = S_{11}$, $\Gamma_{\text{out}} = S_{22}$, and the unilateral transducer gain reduces to $G_{TU} = G_S G_0 G_L$, where

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}, \tag{11.17a}$$

$$G_0 = |S_{21}|^2, \tag{11.17b}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}. \tag{11.17c}$$

The above results have been derived using the S parameters of the transistor, but it is possible to obtain alternative expressions for gain in terms of the equivalent circuit parameters of the transistor. As an example, consider the evaluation of the unilateral transducer gain for a conjugately matched GaAs FET using the equivalent circuit of Figure 10.34 (with $C_{gd} = 0$). To conjugately match the transistor we choose source and load impedances as shown in Figure 11.3. Setting the series source inductive reactance $X = 1/\omega C_{gs}$ will make $Z_{\text{in}} = Z_S^*$, and setting the shunt load inductive susceptance $B = -\omega C_{ds}$ will make $Z_{\text{out}} = Z_L^*$; this effectively eliminates the reactive elements from the FET equivalent circuit. Then by voltage division $V_c = V_S/2j\omega R_i C_{gs}$, and the gain can be easily evaluated as

$$G_{TU} = \frac{P_L}{P_{\text{avs}}} = \frac{\frac{1}{8}|g_m V_c|^2 R_{ds}}{\frac{1}{8}|V_S|^2/R_i} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i}\left(\frac{f_T}{f}\right)^2, \tag{11.18}$$

where the last step has been written in terms of the cutoff frequency, f_T, from (10.78). This

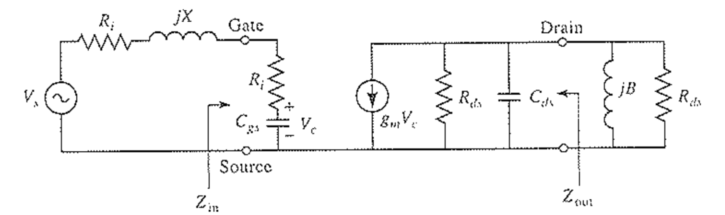

FIGURE 11.3 Unilateral FET equivalent circuit and source and load terminations for the calculation of unilateral transducer power gain.

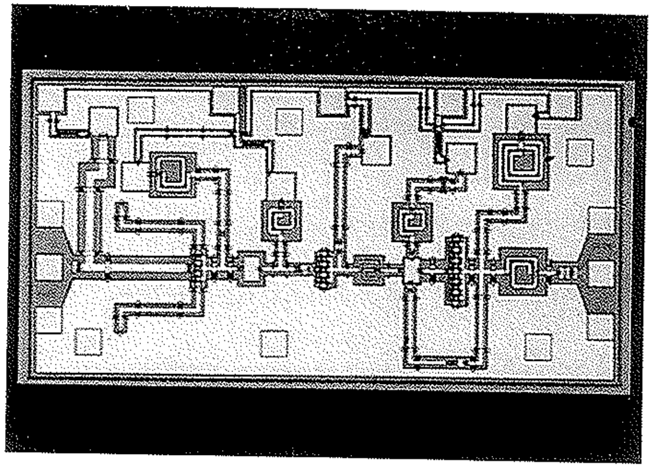

FIGURE 11.4 Photograph of a low noise MMIC amplifier using three HEMTs with coplanar waveguide circuitry. The amplifier has a gain of 20 dB from 20 to 24 GHz. The contact pads on the left and right of the chip are for RF input and output, with DC bias connections at the top. Chip dimensions are 1.1 × 2.0 mm.

Courtesy of R. W. Jackson and B. Hou of the University of Massachusetts–Amherst, and J. Wendler of M/A-COM, Lowell, Mass.

shows the interesting result that the gain of a conjugately matched FET amplifier drops off as $1/f^2$, or 6 dB per octave. A photograph of a MMIC low-noise amplifier is shown in Figure 11.4.

11.2 STABILITY

We now discuss the necessary conditions for a transistor amplifier to be stable. In the circuit of Figure 11.2, oscillation is possible if either the input or output port impedance has a negative real part; this would then imply that $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$. Because Γ_{in} and Γ_{out} depend on the source and load matching networks, the stability of the amplifier depends on Γ_S and Γ_L as presented by the matching networks. Thus, we define two types of stability:

- *Unconditional stability*: The network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances (i.e., $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$).
- *Conditional stability*: The network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for a certain range of passive source and load impedances. This case is also referred to as *potentially unstable*.

Note that the stability condition of an amplifier circuit is usually frequency dependent, since the input and output matching networks generally depend on frequency. Thus it is possible for an amplifier to be stable at its design frequency, but unstable at other frequencies. Careful amplifier design should consider this possibility. We must also point out that the following discussion of stability is limited to two-port amplifier circuits of the type shown in Figure 11.2, and where the S parameters of the active device can be measured without oscillations over the frequency band of interest. The rigorous general treatment of stability requires that the network S parameters (or other network parameters) have no poles in the right-half complex frequency plane, in addition to the conditions that $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ [6]. This can be a difficult assessment in practice, but for the special case considered here, where the S parameters are known to be pole-free (as confirmed by measurability), the following stability conditions are adequate.

Stability Circles

Applying the above requirements for unconditional stability to (11.3) gives the following conditions that must be satisfied by Γ_S and Γ_L if the amplifier is to be unconditionally stable:

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1, \tag{11.19a}$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1. \tag{11.19b}$$

If the device is unilateral ($S_{12} = 0$), these conditions reduce to the simple results that $|S_{11}| < 1$ and $|S_{22}| < 1$ are sufficient for unconditional stability. Otherwise, the inequalities of (11.19) define a range of values for Γ_S and Γ_L where the amplifier will be stable. Finding this range for Γ_S and Γ_L can be facilitated by using the Smith chart, and plotting the input and output *stability circles*. The stability circles are defined as the loci in the Γ_L (or Γ_S) plane for which $|\Gamma_{in}| = 1$ (or $|\Gamma_{out}| = 1$). The stability circles then define the boundaries between stable and potentially unstable regions of Γ_S and Γ_L. Γ_S and Γ_L must lie on the Smith chart ($|\Gamma_S| < 1$, $|\Gamma_L| < 1$ for passive matching networks).

We can derive the equation for the output stability circle as follows. First use (11.19a) to express the condition that $|\Gamma_{in}| = 1$ as

$$\left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1, \tag{11.20}$$

or

$$|S_{11}(1 - S_{22}\Gamma_L) + S_{12}S_{21}\Gamma_L| = |1 - S_{22}\Gamma_L|.$$

Now define Δ as the determinant of the scattering matrix:

$$\Delta = S_{11}S_{22} - S_{12}S_{21}. \tag{11.21}$$

Then we can write the above result as

$$|S_{11} - \Delta\Gamma_L| = |1 - S_{22}\Gamma_L|. \tag{11.22}$$

Now square both sides and simplify to obtain

$$|S_{11}|^2 + |\Delta|^2|\Gamma_L|^2 - (\Delta\Gamma_L S_{11}^* + \Delta^*\Gamma_L^* S_{11}) = 1 + |S_{22}|^2|\Gamma_L|^2 - (S_{22}^*\Gamma_L^* + S_{22}\Gamma_L)$$

$$(|S_{22}|^2 - |\Delta|^2)\Gamma_L\Gamma_L^* - (S_{22} - \Delta S_{11}^*)\Gamma_L - (S_{22}^* - \Delta^* S_{11})\Gamma_L^* = |S_{11}|^2 - 1$$

$$\Gamma_L\Gamma_L^* - \frac{(S_{22} - \Delta S_{11}^*)\Gamma_L + (S_{22}^* - \Delta^* S_{11})\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}. \tag{11.23}$$

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11.3 SINGLE-STAGE TRANSISTOR AMPLIFIER DESIGN

Design for Maximum Gain (Conjugate Matching)

After the stability of the transistor has been determined, and the stable regions for Γ_S and Γ_L have been located on the Smith chart, the input and output matching sections can be designed. Since G_0 of (11.16b) is fixed for a given transistor, the overall gain of the amplifier will be controlled by the gains, G_S and G_L, of the matching sections. Maximum gain will be realized when these sections provide a conjugate match between the amplifier source or load impedance and the transistor. Because most transistors appear as a significant impedance mismatch (large $|S_{11}|$ and $|S_{22}|$), the resulting frequency response will be narrowband. In the next section we will discuss how to design for less than maximum gain, with a corresponding improvement in bandwidth. Broadband amplifier design will be discussed in Section 11.4.

With reference to Figure 11.2 and our discussion in Section 2.6 on conjugate impedance matching, we know that maximum power transfer from the input matching network to the transistor will occur when

$$\Gamma_{in} = \Gamma_S^*, \tag{11.36a}$$

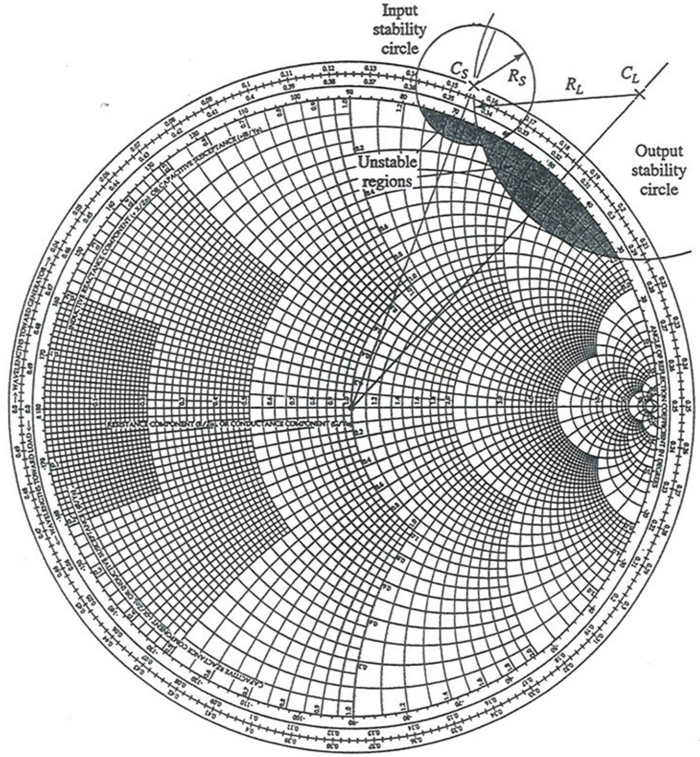


FIGURE 11.6 Stability circles for Example 11.2.

and the maximum power transfer from the transistor to the output matching network will occur when

$$\Gamma_{out} = \Gamma_L^*. \tag{11.36b}$$

Then, assuming lossless matching sections, these conditions will maximize the overall transducer gain. From (11.13), this maximum gain will be given by

$$G_{T_{max}} = \frac{1}{1 - |\Gamma_S|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}. \tag{11.37}$$

In the general case with a bilateral transistor, Γ_{in} is affected by Γ_{out}, and vice versa, so that the input and output sections must be matched simultaneously. Using (11.36) in (11.3) gives the necessary equations:

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}, \tag{11.38a}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}. \tag{11.38b}$$

We can solve for Γ_S by first rewriting these equations as follows:

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{1/\Gamma_L^* - S_{22}^*},$$

$$\Gamma_L^* = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S},$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$. Substituting this expression for Γ_L^* into the expression for Γ_S and expanding gives

$$\Gamma_S\left(1 - |S_{22}|^2\right) + \Gamma_S^2(\Delta S_{22}^* - S_{11}) = \Gamma_S\left(\Delta S_{11}^* S_{22}^* - |S_{11}|^2 - \Delta S_{12}^* S_{21}^*\right) + S_{11}^*\left(1 - |S_{22}|^2\right) + S_{12}^* S_{21}^* S_{22}.$$

Using the result that $\Delta(S_{11}^* S_{22}^* - S_{12}^* S_{21}^*) = |\Delta|^2$ allows this to be rewritten as a quadratic equation for Γ_S:

$$(S_{11} - \Delta S_{22}^*)\Gamma_S^2 + \left(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1\right)\Gamma_S + (S_{11}^* - \Delta^* S_{22}) = 0. \quad (11.39)$$

The solution is

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}. \quad (11.40a)$$

Similarly, the solution for Γ_L can be written as

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}. \quad (11.40b)$$

The variables $B_1,\ C_1,\ B_2,\ C_2$ are defined as

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2, \quad (11.41a)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2, \quad (11.41b)$$

$$C_1 = S_{11} - \Delta S_{22}^*, \quad (11.41c)$$

$$C_2 = S_{22} - \Delta S_{11}^*. \quad (11.41d)$$

Solutions to (11.40) are only possible if the quantity within the square root is positive, and it can be shown that this is equivalent to requiring $K > 1$. Thus unconditionally stable devices can always be conjugately matched for maximum gain, and potentially unstable devices can be conjugately matched if $K > 1$ and $|\Delta| < 1$. The results are much simpler for the unilateral case. When $S_{12} = 0$, (11.38) shows that $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, and then maximum transducer gain of (11.37) reduces to

$$G_{TU_{\max}} = \frac{1}{1 - |S_{11}|^2}|S_{21}|^2\frac{1}{1 - |S_{22}|^2}. \quad (11.42)$$

The maximum transducer power gain given by (11.37) occurs when the source and load are conjugately matched to the transistor, as given by the conditions of (11.36). If the transistor is unconditionally stable, so that $K > 1$, the maximum transducer power gain of (11.37) can be simply rewritten as follows:

$$G_{T_{\max}} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right). \quad (11.43)$$

This result can be obtained by substituting (11.40) and (11.41) for Γ_S and Γ_L into (11.37) and simplifying. The maximum transducer power gain is also sometimes referred to as the *matched gain*. The maximum gain does not provide a meaningful result if the device is only conditionally stable, since simultaneous conjugate matching of the source and load is not possible if $K < 1$ (see Problem 11.7). In this case a useful figure of merit is the *maximum stable gain*, defined as the maximum transducer power gain of (11.43) with $K = 1$. Thus,

$$G_{msg} = \frac{|S_{21}|}{|S_{12}|}. \quad (11.44)$$

The maximum stable gain is easy to compute, and offers a convenient way to compare the gain of various devices under stable operating conditions.

EXAMPLE 11.3 CONJUGATELY MATCHED AMPLIFIER DESIGN

Design an amplifier for maximum gain at 4.0 GHz using single-stub matching sections. Calculate and plot the input return loss and the gain from 3 to 5 GHz. The GaAs FET has the following S parameters ($Z_0 = 50\ \Omega$):

f(GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3.0	$0.80\angle{-89^\circ}$	$2.86\angle{99^\circ}$	$0.03\angle{56^\circ}$	$0.76\angle{-41^\circ}$
4.0	$0.72\angle{-116^\circ}$	$2.60\angle{76^\circ}$	$0.03\angle{57^\circ}$	$0.73\angle{-54^\circ}$
5.0	$0.66\angle{-142^\circ}$	$2.39\angle{54^\circ}$	$0.03\angle{62^\circ}$	$0.72\angle{-68^\circ}$

Solution

We first check the stability of the transistor by calculating Δ and K at 4.0 GHz:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.488\angle{-162^\circ},$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.195.$$

Since $|\Delta| < 1$ and $K > 1$, the transistor is unconditionally stable at 4.0 GHz. There is no need to plot the stability circles.

For maximum gain, we should design the matching sections for a conjugate match to the transistor. Thus, $\Gamma_S = \Gamma_{\text{in}}^*$ and $\Gamma_L = \Gamma_{\text{out}}^*$, and $\Gamma_S,\ \Gamma_L$ can be determined from (11.40):

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872\angle{123^\circ}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876\angle{61^\circ}.$$

Then the effective gain factors of (11.16) can be calculated as

$$G_S = \frac{1}{1 - |\Gamma_S|^2} = 4.17 = 6.20 \text{ dB},$$

$$G_0 = |S_{21}|^2 = 6.76 = 8.30 \text{ dB},$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = 1.67 = 2.22 \text{ dB}.$$

So the overall transducer gain will be

$$G_{T_{\max}} = 6.20 + 8.30 + 2.22 = 16.7 \text{ dB}.$$

The matching networks can easily be determined using the Smith chart. For the input matching section, we first plot Γ_S, as shown in Figure 11.7a. The impedance, Z_S, represented by this reflection coefficient is the impedance seen looking into the matching section toward the source impedance, Z_0. Thus, the matching section must transform Z_0 to the impedance Z_S. There are several ways of doing this, but

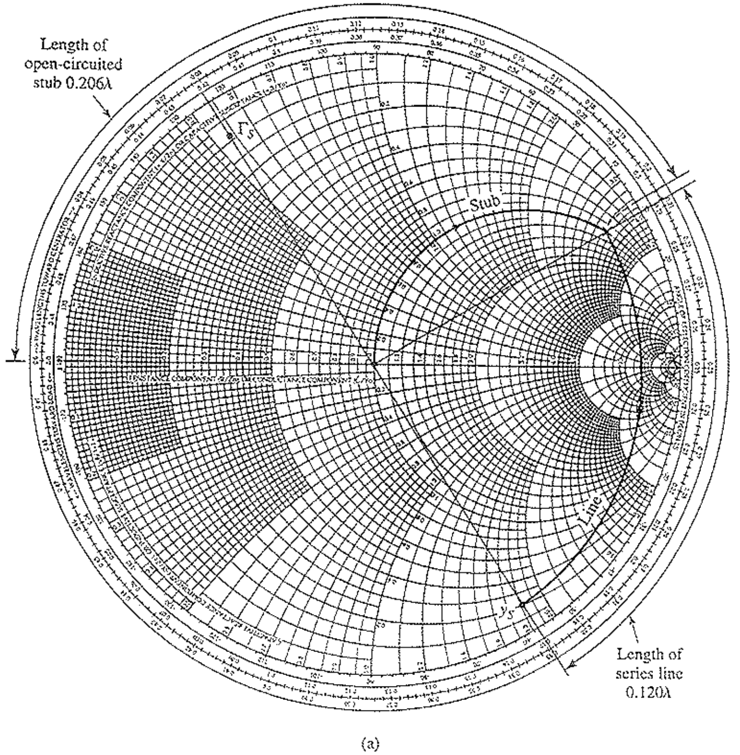


FIGURE 11.7 Circuit design and frequency response for the transistor amplifier of Example 11.3. (a) Smith chart for the design of the input matching network.

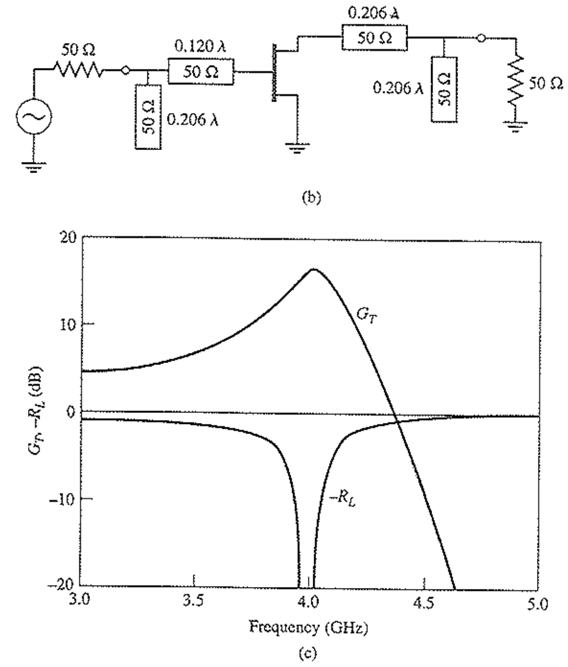


FIGURE 11.7 Continued. (b) RF circuit. (c) Frequency response.

we will use an open-circuited shunt stub followed by a length of line. Thus we convert to the normalized admittance y_s, and work backward (toward the load on the Smith chart) to find that a line of length 0.120λ will bring us to the $1 + jb$ circle. Then we see that the required stub admittance is $+j3.5$, for an open-circuited stub length of 0.206λ. A similar procedure gives a line length of 0.206λ and a stub length of 0.206λ for the output matching circuit.

The final amplifier circuit is shown in Figure 11.7b. This circuit only shows the RF components; the amplifier will also require some bias circuitry. The return loss and gain were calculated using a CAD package, interpolating the necessary S parameters from the table on page 551. The results are plotted in Figure 11.7c, and show the expected gain of 16.7 dB at 4.0 GHz, with a very good return loss. The bandwidth where the gain drops by 1 dB is about 2.5%. ■

Constant Gain Circles and Design for Specified Gain

In many cases it is preferable to design for less than the maximum obtainable gain, to improve bandwidth or to obtain a specific value of amplifier gain. This can be done by designing the input and output matching sections to have less than maximum gains; in other words, mismatches are purposely introduced to reduce the overall gain. The design procedure is facilitated by plotting *constant gain circles* on the Smith chart, to represent loci of Γ_S and Γ_L that give fixed values of gain (G_S and G_L). To simplify our discussion, we will only treat the case of a unilateral device; the more general case of a bilateral device must sometimes be considered in practice and is discussed in detail in references [1], [2], and [3].

In many practical cases $|S_{12}|$ is small enough to be ignored, and the device can then be assumed to be unilateral. This greatly simplifies the design procedure. The error in the transducer gain caused by approximating $|S_{12}|$ as zero is given by the ratio G_T/G_{TU}. It can be shown that this ratio is bounded by

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}, \tag{11.45}$$

where U is defined as the *unilateral figure of merit*,

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}. \tag{11.46}$$

Usually an error of a few tenths of a dB or less justifies the unilateral assumption.

The expression for G_S and G_L for the unilateral case are given by (11.17a) and (11.17c):

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2},$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$

These gains are maximized when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, resulting in the maximum values given by

$$G_{S_{max}} = \frac{1}{1-|S_{11}|^2}, \tag{11.47a}$$

$$G_{L_{max}} = \frac{1}{1-|S_{22}|^2}. \tag{11.47b}$$

Now define normalized gain factors g_S and g_L as

$$g_S = \frac{G_S}{G_{S_{max}}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}(1-|S_{11}|^2), \tag{11.48a}$$

$$g_L = \frac{G_L}{G_{L_{max}}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}(1-|S_{22}|^2). \tag{11.48b}$$

Then we have that $0 \le g_S \le 1$, and $0 \le g_L \le 1$.

For fixed values of g_S and g_L, (11.48) represents circles in the Γ_S or Γ_L plane. To show this, consider (11.48a), which can be expanded to give

$$g_S|1-S_{11}\Gamma_S|^2 = (1-|\Gamma_S|^2)(1-|S_{11}|^2),$$

$$(g_S|S_{11}|^2 + 1 - |S_{11}|^2)|\Gamma_S|^2 - g_S(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*) = 1 - |S_{11}|^2 - g_S,$$

$$\Gamma_S\Gamma_S^* - \frac{g_S(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*)}{1-(1-g_S)|S_{11}|^2} = \frac{1-|S_{11}|^2-g_S}{1-(1-g_S)|S_{11}|^2}. \tag{11.49}$$

Now add $(g_S^2|S_{11}|^2)/[1-(1-g_S)|S_{11}|^2]^2$ to both sides to complete the square:

$$\left|\Gamma_S - \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}\right|^2 = \frac{(1-|S_{11}|^2-g_S)[1-(1-g_S)|S_{11}|^2] + g_S^2|S_{11}|^2}{[1-(1-g_S)|S_{11}|^2]^2}.$$

Simplifying gives

$$\left|\Gamma_S - \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}, \tag{11.50}$$

which is the equation of a circle with its center and radius given by

$$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}, \tag{11.51a}$$

$$R_S = \frac{\sqrt{1-g_S}(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}. \tag{11.51b}$$

The results for the constant gain circles of the output section can be shown to be,

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}, \tag{11.52a}$$

$$R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}. \tag{11.52b}$$

The centers of each family of circles lie along straight lines given by the angle of S_{11}^* or S_{22}^*. Note that when g_S (or g_L) $= 1$ (maximum gain), the radius R_S (or R_L) $= 0$, and the center reduces to S_{11}^* (or S_{22}^*), as expected. Also, it can be shown that the 0 dB gain circles ($G_S = 1$ or $G_L = 1$) will always pass through the center of the Smith chart. These results can be used to plot a family of circles of constant gain for the input and output sections. Then Γ_S and Γ_L can be chosen along these circles to provide the desired gains. The choices for Γ_S and Γ_L are not unique, but it makes sense to choose points close to the center of the Smith chart to minimize the mismatch and thus maximize the bandwidth. Alternatively, as we will see in the next section, the input network mismatch can be chosen to provide a low-noise design.

EXAMPLE 11.4 AMPLIFIER DESIGN FOR SPECIFIED GAIN

Design an amplifier to have a gain of 11 dB at 4.0 GHz. Plot constant gain circles for $G_S = 2$ dB and 3 dB, and $G_L = 0$ dB and 1 dB. Calculate and plot the input return loss and overall amplifier gain from 3 to 5 GHz. The FET has the following S parameters ($Z_0 = 50\ \Omega$):

f (GHz)	S_{11}	S_{21}	S_{12}	S_{22}
3	$0.80\angle{-90^\circ}$	$2.8\angle{100^\circ}$	0	$0.66\angle{-50^\circ}$
4	$0.75\angle{-120^\circ}$	$2.5\angle{80^\circ}$	0	$0.60\angle{-70^\circ}$
5	$0.71\angle{-140^\circ}$	$2.3\angle{60^\circ}$	0	$0.58\angle{-85^\circ}$

Solution

Since $S_{12} = 0$ and $|S_{11}| < 1$ and $|S_{22}| < 1$, the transistor is unilateral and unconditionally stable. From (11.47) we calculate the maximum matching section gains as

$$G_{S_{max}} = \frac{1}{1-|S_{11}|^2} = 2.29 = 3.6 \text{ dB},$$

$$G_{L_{max}} = \frac{1}{1-|S_{22}|^2} = 1.56 = 1.9 \text{ dB}.$$

The gain of the mismatched transistor is

$$G_o = |S_{21}|^2 = 6.25 = 8.0 \text{ dB},$$

so the maximum unilateral transducer gain is

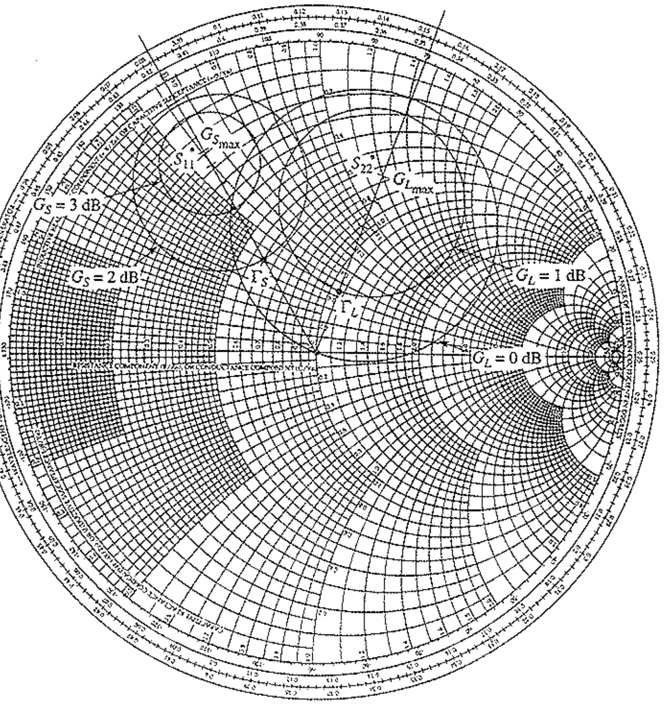
$$G_{TU_{max}} = 3.6 + 1.9 + 8.0 = 13.5 \text{ dB}.$$

Thus we have 2.5 dB more available gain than is required by the specifications.

We use (11.48), (11.51), and (11.52) to calculate the following data for the constant gain circles:

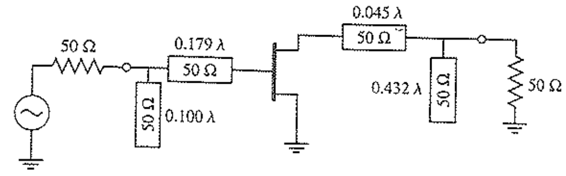
$G_S = 3$ dB	$g_S = 0.875$	$C_S = 0.706\angle 120^\circ$	$R_S = 0.166$
$G_S = 2$ dB	$g_S = 0.691$	$C_S = 0.627\angle 120^\circ$	$R_S = 0.294$
$G_L = 1$ dB	$g_L = 0.806$	$C_L = 0.520\angle 70^\circ$	$R_L = 0.303$
$G_L = 0$ dB	$g_L = 0.640$	$C_L = 0.440\angle 70^\circ$	$R_L = 0.440$

The constant gain circles are shown in Figure 11.8a. We choose $G_S = 2$ dB and $G_L = 1$ dB, for an overall amplifier gain of 11 dB. Then we select Γ_S and Γ_L

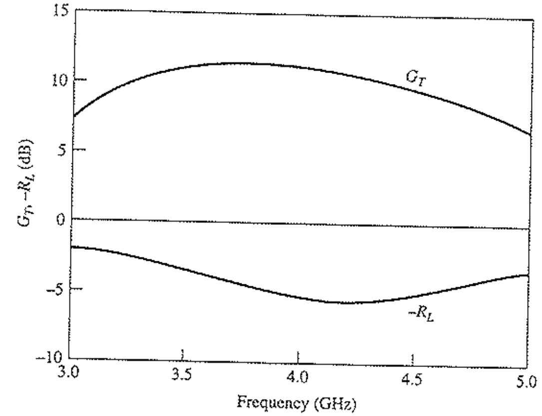


(a)

FIGURE 11.8 Circuit design and frequency response for the transistor amplifier of Example 11.4. (a) Constant gain circles.



(b)



(c)

FIGURE 11.8 Continued. (b) RF circuit. (c) Transducer gain and return loss.

along these circles as shown, to minimize the distance from the center of the chart (this places Γ_S and Γ_L along the radial lines at 120° and 70°, respectively). Thus, $\Gamma_S = 0.33\angle 120^\circ$ and $\Gamma_L = 0.22\angle 70^\circ$, and the matching networks can be designed using shunt stubs as in Example 11.3.

The final amplifier circuit is shown in Figure 11.8b. The response was calculated using CAD software, with interpolation of the given S parameter data. The results are shown in Figure 11.8c, where it is seen the desired gain of 11 dB is achieved at 4.0 GHz. The bandwidth over which the gain varies by ±1 dB or less is about 25%, which is considerably better than the bandwidth of the maximum gain design in Example 11.3. The return loss, however, is not very good, being only about 5 dB at the design frequency. This is due to the deliberate mismatch introduced into the matching sections to achieve the specified gain. ■

Low-Noise Amplifier Design

Besides stability and gain, another important design consideration for a microwave amplifier is its noise figure. In receiver applications especially, it is often required to have a preamplifier with as low a noise figure as possible since, as we saw in Section 10.1, the first stage of a receiver front end has the dominant effect on the noise performance of the overall system. Generally it is not possible to obtain both minimum noise figure and maximum gain for an amplifier, so some sort of compromise must be made. This can be done by using constant gain circles and *circles of constant noise figure* to select a usable trade-off between noise figure and gain. Here we will derive the equations for constant noise figure circles, and show how they are used in transistor amplifier design.

As derived in references [4] and [5], the noise figure of a two-port amplifier can be expressed as

$$F = F_{min} + \frac{R_N}{G_S}|Y_S - Y_{opt}|^2, \tag{11.53}$$

where the following definitions apply:

$Y_S = G_S + jB_S$ = source admittance presented to transistor.
Y_{opt} = optimum source admittance that results in minimum noise figure.
F_{min} = minimum noise figure of transistor, attained when $Y_S = Y_{opt}$.
R_N = equivalent noise resistance of transistor.
G_S = real part of source admittance.

Instead of the admittance Y_S and Y_{opt}, we can use the reflection coefficients Γ_S and Γ_{opt}, where

$$Y_S = \frac{1}{Z_0}\frac{1 - \Gamma_S}{1 + \Gamma_S}, \tag{11.54a}$$

$$Y_{opt} = \frac{1}{Z_0}\frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}. \tag{11.54b}$$

Γ_S is the source reflection coefficient defined in Figure 11.1. The quantities F_{min}, Γ_{opt}, and R_N are characteristics of the particular transistor being used, and are called the *noise parameters* of the device; they may be given by the manufacturer, or measured.

Using (11.54), the quantity $|Y_S - Y_{opt}|^2$ can be expressed in terms of Γ_S and Γ_{opt}:

$$|Y_S - Y_{opt}|^2 = \frac{4}{Z_0^2}\frac{|\Gamma_S - \Gamma_{opt}|^2}{|1 + \Gamma_S|^2|1 + \Gamma_{opt}|^2}. \tag{11.55}$$

Also,

$$G_S = \text{Re}\{Y_S\} = \frac{1}{2Z_0}\left(\frac{1 - \Gamma_S}{1 + \Gamma_S} + \frac{1 - \Gamma_S^*}{1 + \Gamma_S^*}\right) = \frac{1}{Z_0}\frac{1 - |\Gamma_S|^2}{|1 + \Gamma_S|^2}. \tag{11.56}$$

Using these results in (11.53) gives the noise figure as

$$F = F_{min} + \frac{4R_N}{Z_0}\frac{|\Gamma_S - \Gamma_{opt}|^2}{(1 - |\Gamma_S|^2)|1 + \Gamma_{opt}|^2}. \tag{11.57}$$

For a fixed noise figure, F, we can show that this result defines a circle in the Γ_S plane. First define the *noise figure parameter*, N, as

$$N = \frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2} = \frac{F - F_{min}}{4R_N/Z_0}|1 + \Gamma_{opt}|^2, \tag{11.58}$$

which is a constant, for a given noise figure and set of noise parameters. Then rewrite (11.58) as

$$(\Gamma_S - \Gamma_{opt})(\Gamma_S^* - \Gamma_{opt}^*) = N(1 - |\Gamma_S|^2),$$

$$\Gamma_S\Gamma_S^* - (\Gamma_S\Gamma_{opt}^* + \Gamma_S^*\Gamma_{opt}) + \Gamma_{opt}\Gamma_{opt}^* = N - N|\Gamma_S|^2,$$

$$\Gamma_S\Gamma_S^* - \frac{(\Gamma_S\Gamma_{opt}^* + \Gamma_S^*\Gamma_{opt})}{N + 1} = \frac{N - |\Gamma_{opt}|^2}{N + 1}.$$

Now add $|\Gamma_{opt}|^2/(N+1)^2$ to both sides to complete the square to obtain

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N + 1}\right| = \frac{\sqrt{N(N + 1 - |\Gamma_{opt}|^2)}}{(N + 1)}. \tag{11.59}$$

This result defines circles of constant noise figure with centers at

$$C_F = \frac{\Gamma_{opt}}{N + 1}, \tag{11.60a}$$

and radii of

$$R_F = \frac{\sqrt{N(N + 1 - |\Gamma_{opt}|^2)}}{N + 1}. \tag{11.60b}$$

EXAMPLE 11.5 LOW-NOISE AMPLIFIER DESIGN

A GaAs FET is biased for minimum noise figure, and has the following S parameters and noise parameters at 4 GHz ($Z_0 = 50\,\Omega$): $S_{11} = 0.6\angle{-60^\circ}$, $S_{21} = 1.9\angle{81^\circ}$, $S_{12} = 0.05\angle{26^\circ}$, $S_{22} = 0.5\angle{-60^\circ}$; $F_{min} = 1.6$ dB, $\Gamma_{opt} = 0.62\angle{100^\circ}$, $R_N = 20\,\Omega$. For design purposes, assume the device is unilateral, and calculate the maximum error in G_T resulting from this assumption. Then design an amplifier having a 2.0 dB noise figure with the maximum gain that is compatible with this noise figure.

Solution

We first compute the unilateral figure of merit from (11.46):

$$U = \frac{|S_{12}S_{21}S_{11}S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = 0.059.$$

Then from (11.45) the ratio G_T/G_{TU} is bounded as

$$\frac{1}{(1 + U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1 - U)^2},$$

or

$$0.891 < \frac{G_T}{G_{TU}} < 1.130.$$

In dB, this is

$$-0.50 < G_T - G_{TU} < 0.53 \text{ dB},$$

where G_T and G_{TU} are now in dB. Thus, we should expect less than about ± 0.5 dB error in gain.

Next, we use (11.58) and (11.60) to compute the center and radius of the 2 dB noise figure circle:

$$N = \frac{F - F_{min}}{4R_N/Z_0}|1 + \Gamma_{opt}|^2 = \frac{1.58 - 1.445}{4(20/50)}|1 + 0.62\angle 100^\circ|^2$$
$$= 0.0986,$$

$$C_F = \frac{\Gamma_{opt}}{N+1} = 0.56\angle 100^\circ$$

$$R_F = \frac{\sqrt{N(N + 1 - |\Gamma_{opt}|^2)}}{N+1} = 0.24.$$

This noise figure circle is plotted in Figure 11.9a. Minimum noise figure ($F_{min} =$ 1.6 dB) occurs for $\Gamma_S = \Gamma_{opt} = 0.62\angle 100^\circ$.

Next we calculate data for several input section constant gain circles. From (11.51),

G_S(dB)	g_S	C_S	R_S
1.0	0.805	$0.52\angle 60^\circ$	0.300
1.5	0.904	$0.56\angle 60^\circ$	0.205
1.7	0.946	$0.58\angle 60^\circ$	0.150

These circles are also plotted in Figure 11.9a. We see that the $G_S = 1.7$ dB gain circle just intersects the $F = 2$ dB noise figure circle, and that any higher gain will result in a worse noise figure. From the Smith chart the optimum solution is then $\Gamma_S = 0.53\angle 75^\circ$, yielding $G_S = 1.7$ dB and $F = 2.0$ dB.

For the output section we choose $\Gamma_L = S_{22}^* = 0.5\angle 60^\circ$ for a maximum G_L of

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}.$$

The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB},$$

so the overall transducer gain will be

$$G_{TU} = G_S + G_0 + G_L = 8.53 \text{ dB}.$$

A complete AC circuit for the amplifier, using open-circuited shunt stubs in the matching sections, is shown in Figure 11.9b. A computer analysis of the circuit gave a gain of 8.36 dB. ∎

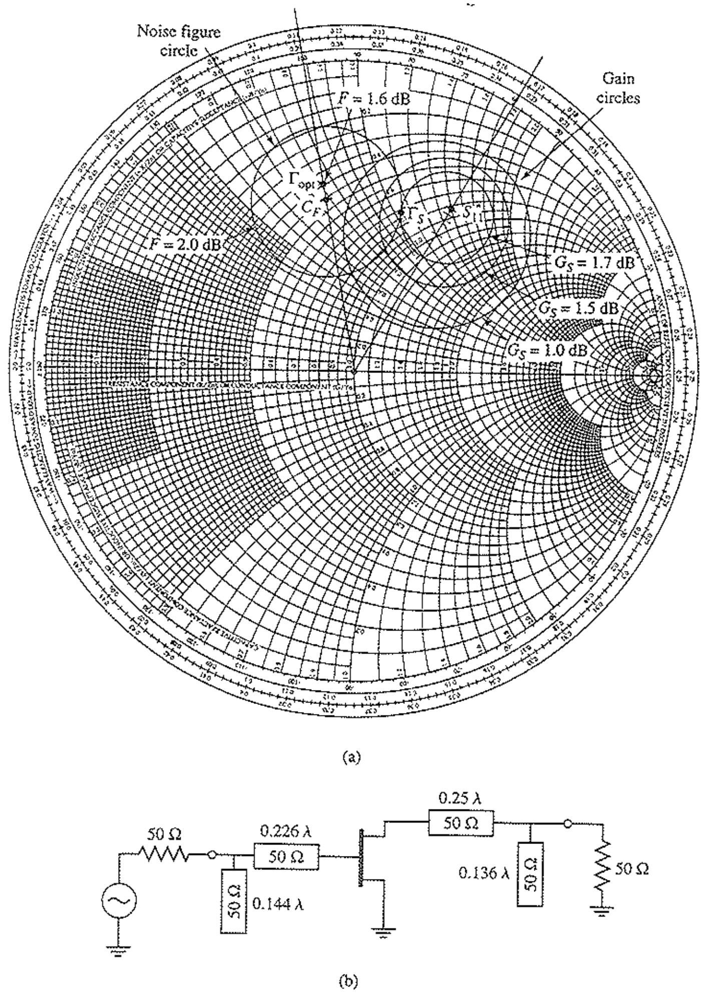


FIGURE 11.9 Circuit design for the transistor amplifier of Example 11.5. (a) Constant gain and noise figure circles. (b) RF circuit.

11.4 BROADBAND TRANSISTOR AMPLIFIER DESIGN

The ideal microwave amplifier would have constant gain and good input matching over the desired frequency bandwidth. As the examples of the last section have shown, conjugate matching will give maximum gain only over a relatively narrow bandwidth, while designing for less than maximum gain will improve the gain bandwidth, but the input and output ports of the amplifier will be poorly matched. These problems are primarily a result of the fact