APMA 1170: Introduction to Computational Linear Algebra

Instructor: Peter Sentz, Postdoctoral Associate in Applied Mathematics

Fall 2023

Time/Location: M/W/F 9-9:50am (Sept 6. - Dec. 11), Barus & Holley 163
Instructor email: `peter_sentz@brown.edu`

Course Description

This course focuses on fundamental algorithms in computational linear algebra with relevance to all science concentrators. Basic linear algebra and matrix decompositions (Cholesky, LU, QR, etc.), round-off errors and numerical analysis of errors and convergence. Iterative methods and conjugate gradient techniques. Computation of eigenvalues and eigenvectors, and an introduction to least squares methods.

Learning Goals

By the end of the semester, students will be able to

- Implement the fundamental algorithms covered in the course using MATLAB and similar programming languages
- Understand how conditioning and stability affect the reliability of numerical linear algebra techniques, particularly within the context of floating-point arithmetic
- Compare and contrast different approaches for linear algebra problems, and identify which methods are appropriate by considering the structure of a given matrix
- Apply the concepts of this course to linear algebra problems arising in advanced STEM courses (e.g., efficiently solve a linear system which arises from the discretization of a partial differential equation).

Prerequisites

- **MATH 0100**: Single Variable Calculus, Part II (or equivalent).
- **MATH 0520**: Linear Algebra (or equivalent)
- Experience with a programming language is **strongly recommended**, but not required.

Materials

- The required text for the course is "Numerical Linear Algebra" by Lloyd N. Trefethen and David Bau, III published by SIAM (Society for Industrial and Applied Mathematics). The content of the course will broadly follow the presentation of the book.
  - Note: Either the "25th anniversary edition" (2022, ISBN: 978-1-611977-15-8) or the "1st edition" (1997, ISBN: 978-0-898713-61-9) are fine; the content is essentially the same.
- Another book I recommend is "Applied Numerical Linear Algebra" by James W. Demmel, also published by SIAM (1997, ISBN: 978-0-898713-89-3).
- Lecture slides will be uploaded to Canvas.
- MATLAB will be used throughout the course, including demonstrations in lecture. You must use Matlab for programming problems in homework assignments. I will upload example code discussed in class to Canvas.

Cost of Materials

- "Numerical Linear Algebra" is available from the Brown bookstore for between $55.30 and $79.00
- Used copies of the first edition of the book can found for cheaper. For example, see https://isbnsearch.org/isbn/9780898713619
- MATLAB is freely available through Brown's IT software catalog.

Course Topics

The topics covered in the course will broadly follow the presentation in the text. The following is a tentative outline, with corresponding textbook chapters. You are encouraged to read the relevant portions of the text before lectures.

- Brief review of linear algebra basics. Interpreting matrix operations in terms of column vectors (Chapter 1)
- Orthonormal vectors and bases (Chapter 2)
- Vector and matrix norms (Chapter 3)
- Singular value decomposition and applications (Chapters 4 and 5)
- Least-squares problems and projectors (Chapters 11 and 6)
- QR factorization using Gram-Schmidt and Householder algorithms. Application to least-squares problems (Chapters 7, 8, 10, 11)
- Errors. Conditioning of problems and stability of algorithms (Chapters 12-15)
- Stability and conditioning of least-squares problems (Chapters 16-19)

- LU factorization (Chapter 20)
- LU with pivoting. Stability of LU (Chapters 21 and 22)
- Cholesky Factorization (Chapter 23)
- Overview of eigenvalue problems. Reduction to Hessenberg/Tridiagonal form (Chapters 24-26)
- Power iteration and related methods for solving eigenvalue problems (Chapter 27)
- QR Algorithm for finding eigenvalues (Chapters 28 and 29)
- Overview and motivation of iterative methods (Chapter 32)
- Iterative methods for non-hermitian matrices - Arnoldi iteration and GMRES (Chapters 33-35)
- Iterative methods for real, symmetric matrices - Lanczos iteration and conjugate gradient method (Chapters 36-38)

Meeting Times

Lectures are the primary mode of instruction, and will be held on Mondays, Wednesdays, and Fridays, 9am-9:50am in Barus & Holley 163. Questions are always encouraged and appreciated during lecture.

Lecture Recordings: I will attempt to record lectures using Panopto. However, I have experienced many issues with this system, and it has routinely failed to record lectures in the past. I may also try lower quality Zoom recordings if Panopto continues to have issues. **There are no guarantees that any given lecture will be recorded**. If successful, recorded lectures will be uploaded to the course Canvas page.

Course Staff and Office Hours

Instructor: Peter Sentz, Postdoctoral Associate in Applied Mathematics
email: `peter_sentz@brown.edu`
Office Hours: Wednesdays and Fridays, 4pm-5pm or by appointment
Office Hour Location: Room 118, 170 Hope Street

As of right now, there are no TAs assigned to the course. This may change after shopping period if the number of registered students reaches 25.

Course Webpage

We will use Canvas for the class, `canvas.brown.edu`. Here you will find Homework (and posted solutions), announcements, any successful recordings of lectures, lecture slides, and additional information.

Assessments

Grades will be based on the following assessments:

- Homework (50% of grade)
- Mid-term (20% of grade)
- Final project (30% of grade)

Homework (50% of grade)

There will be homework assignments given out **roughly every week**. The problem sets will include both theoretical and computational (i.e. programming) problems.

The assignments will be posted on Canvas and are **due by 8 p.m. on Fridays**, and must be submitted online through Gradescope. Solutions will be posted promptly after the due date.

Show your work; for a calculation, this means you should display the process used, not just a final result. For questions starting with "Show that..." you should write a complete mathematical argument.

The assignments are meant to be part of the learning process and are not tests as such. Allow adequate time and start them **well in advance** of the due date. Some answers may not come to you immediately but may become clearer a day or two later. Please do not hesitate to seek help in office hours, or to discuss homework with other students. If you are unable to complete a question, write a short note describing what you tried and which concepts you think may be important for the solution. Demonstrating a serious effort on problems will be a factor in awarding credit for assignments. **Your lowest homework grade will be dropped.**

Late assignment policy: Late assignments will not be accepted, with a caveat. If you submit a late assignment, you will receive no credit. However, at the end of the semester, if your final grade would be impacted by your late assignments, I may take them into account when calculating your final grade. This is up to my discretion; if you routinely submit late assignments, I will not recalculate your grade. Again, your lowest homework grade will be dropped.

You are encouraged to discuss HW assignments with other students, but you must write up your own answer independently. **When you have collaborated with other students, please acknowledge this by adding a note such as "I discussed question X with A and B".**

Midterm (20% of grade)

There will be a take-home midterm exam which will be released approximately mid-semester. The exact date will be announced later. It will include both programming and theoretical questions. **You may collaborate with 1 or 2 other students. Please acknowledge this upon submission of the exam.**

Final Project (30% of grade)

This is described as a final "project", but is essentially a take-home final or a longer homework assignment. The content is not finalized, but I will stress utilizing computational linear algebra techniques within scientific/engineering applications.

The final project will be due **Noon on December 21st, 2023**, and will be released roughly 10 days prior to the due date. This should be plenty of time to complete the final, **but please start as early as possible**.

Unlike other assessments, you should not collaborate with other students on the final.

Grading Scale

The percentage breakdown of grades is described above under "Assessments". For final grades, the following tentative scale will be used:

- **A**: 90% or higher
- **B**: Less than 90%, but greater than or equal to 80%
- **C**: Less than 80%, but greater than or equal to 65%
- **NC**: Less than 65%

Note: The scales above are a **guarantee**, in that your grade cannot be lower than what is dictated by the list above. However, the final grade calculation is up to my discretion. (So the cut-off for an "A" may end up being 88% for example, but will not be higher than 90% under any circumstances).

For students taking the course S/NC, a minimum grade of 65% is required to guarantee a grade of "S".

Required Course Activities and Expected Times

- Scheduled class meetings: 3 hours/week; 40 hours total
- Reading and reviewing class material: 3.5 hours/week; 45 hours total
- Weekly assignments: 5 hours/week: 65 hours total
- Mid-term and Preparation: 12 hours
- Final Project and preparation: 20 hours
- Total: 182 hours/semester

Accessibility and Accommodations Statement

Brown University is committed to full inclusion of all students. Please inform me early in the term if you may require accommodations or modification of any of course procedures. You may speak with me after class, during office hours, or by appointment. If you need accommodations around online learning or in classroom accommodations, please be sure to reach out to Student Accessibility Services (SAS) for their assistance (seas@brown.edu, 401-863-9588). Undergraduates in need of short-term academic advice or support can contact an academic dean in the College by emailing college@brown.edu. Graduate students may contact one of the deans in the Graduate School by emailing graduate_school@brown.edu.

Books, Supplies, and Materials

If your Brown undergraduate financial aid package includes the Book/Course Material Support Pilot Program (BCMS), concerns or questions about the cost of books and course materials for this or any other Brown course (including RISD courses via cross-registration) can be addressed to bcms@brown.edu. For all other concerns related to non-tuition course-related expenses, whether or not your Brown undergraduate financial aid package includes BCMS, please visit the Academic Emergency Fund in E-GAP (within the umbrella of "E-Gap Funds" in UFunds) to determine options for financing these costs, while ensuring your privacy.

Academic Integrity

It is requested of all students that they review the **Brown Academic Code**, available at `https://www.brown.edu/academics/college/degree/policies/academic-code`

Collaboration on homework and discussion of class topics is encouraged, as noted already. It often helps students to work together on portions of homework assignments. However, each student must turn in their own solutions or write-ups, which should not be identical copies of their classmates. The in-class midterm is an individual assessment of a student's ability.

Class Recording and Distribution of Course Materials

I would like to record our discussion because some students may be in different time zones, have poor internet connections, or have health issues. This means that we will record all classes to make them available to all students that are enrolled but cannot be present. If you have questions or concerns about this protocol, please contact me so that we can talk through those to also ensure your full participation in this course.

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