

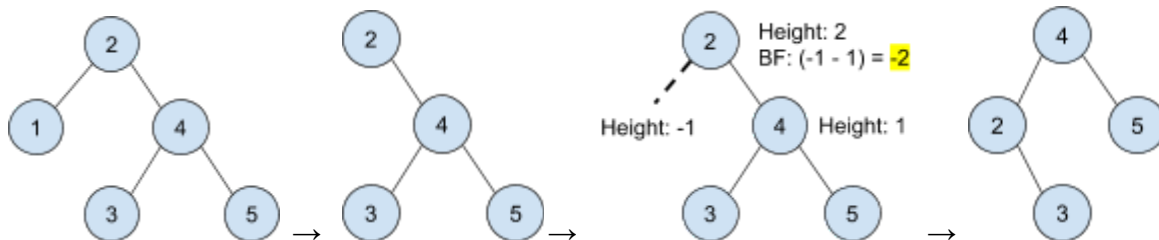
Topics

I. AVL Remove

A. The algorithm:

1. Remove the node like you are removing from a BST (considering all the same 0, 1, and 2 child cases).
2. When tracing back up from the node you removed, update heights and balance factors, and check for and perform rotations.
 - a) If this was a two child remove case, make sure to update and rotate when coming back from removing the predecessor/successor.

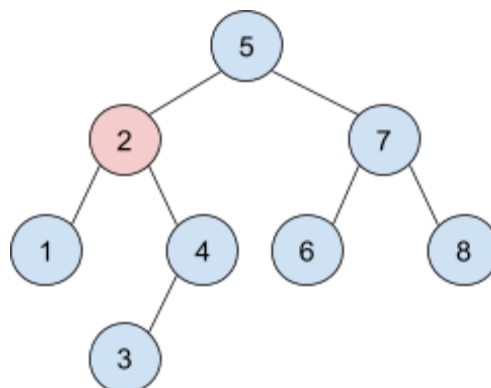
B. Example: remove(1)



1. Remove like you're removing from a BST
2. Update the height and balance factor of the node containing 2 (the only node we traversed to find the node containing 1)
 - a) Height = $\max(-1, 1) + 1$
 - b) Balance factor = $-1 - 1 = -2$
3. Since the balance factor is -2 and its right child's balance factor is not > 0 , we just need to do a single left rotation on the node containing 2

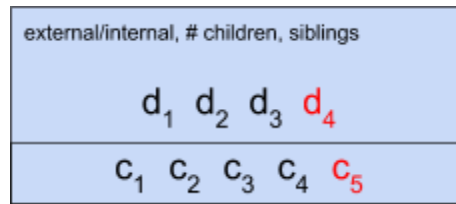
C. Example: remove(2) using predecessor

1. Hint: we need to do a double rotation



II. 2-4 Trees / B-Trees

A. 2-4 nodes:



- Can have 2 to 4 children (c_i) and can store 1 to 3 data values (d_i)
 - A node with m data values must have $m + 1$ children
 - Data 4 and Child 5 are in red because they are allowed to exist momentarily, but will be immediately fixed
- Data is stored within a node in ascending order ($d_1 < d_2 < d_3$)
- Nodes can also store whether they're internal or external, the number of children they have, and references to their siblings or parent

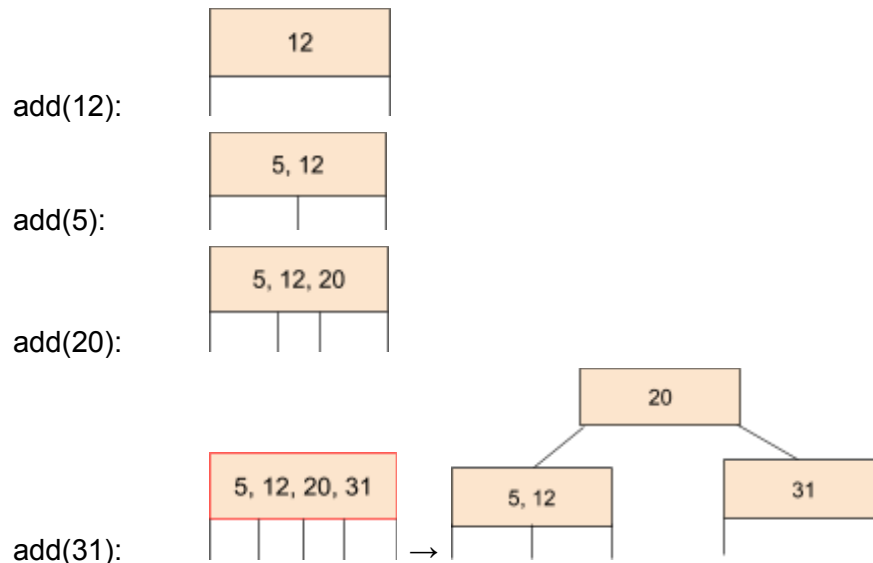
B. Order Property:

- $c_1 \text{ data} < d_1 < c_2 \text{ data} < d_2 < c_3 \text{ data} < d_3 < c_4 \text{ data} < d_4 < c_5 \text{ data}$

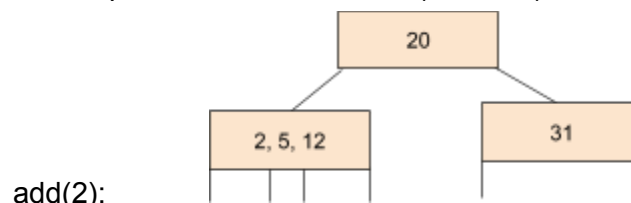
C. Shape Property:

- Every leaf MUST reside on the same level
- Run times are guaranteed to be $O(\log n)$

D. Example: add 12, 5, 20, 31, 2, 18, 42, 45, 49, 33, 36

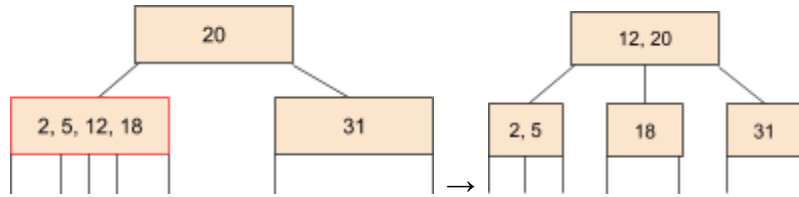


Promotion: occurs when we have overflow in a node; we will always promote a middle node (12 or 20), the choice is arbitrary

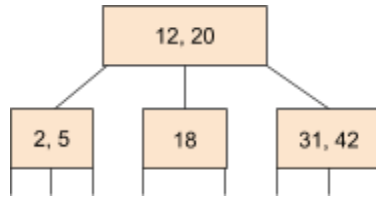


We always add new data to a leaf node.

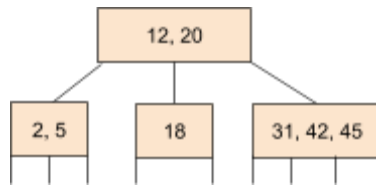
add(18):



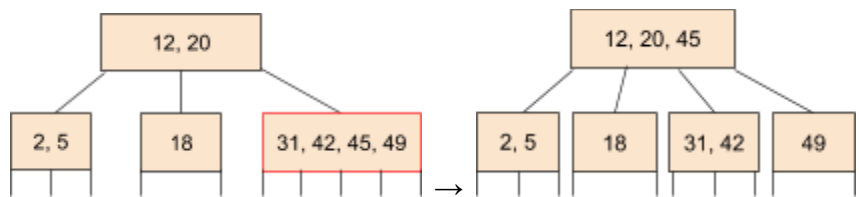
add(42):



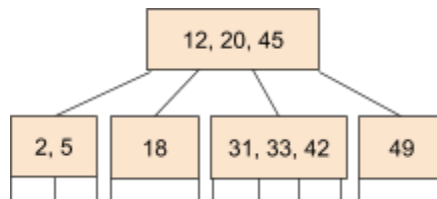
add(45):



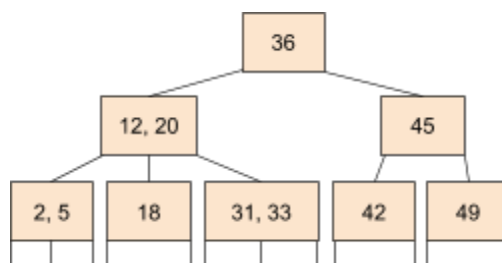
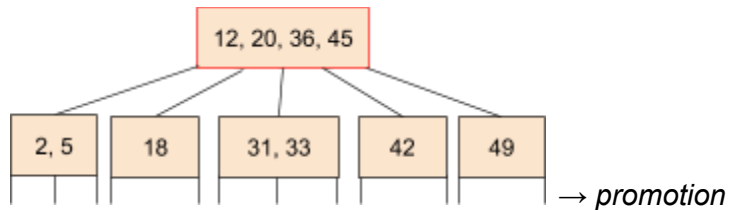
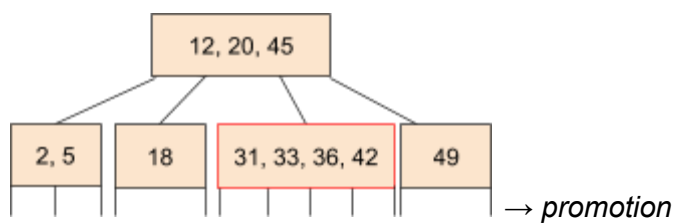
add(49):



add(33):



add(36):



Topics

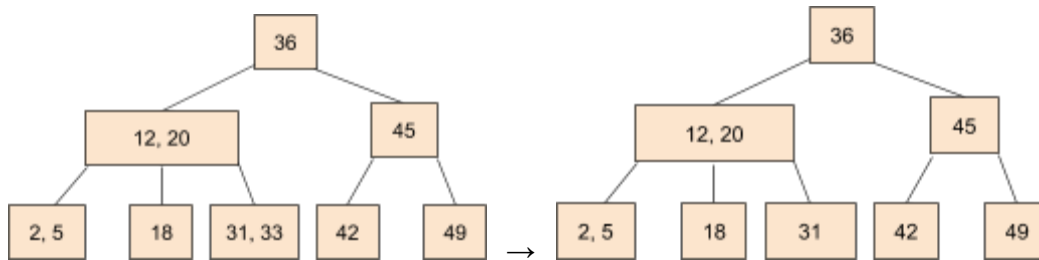
I. 2-4 Trees

A. Add - can cause *overflow*, we fix overflow with *promotion* (move a middle value to its parent or create a new node if the overflow is in the root)

B. Remove - can cause *underflow*, we fix underflow with *transfers* and *fusion*

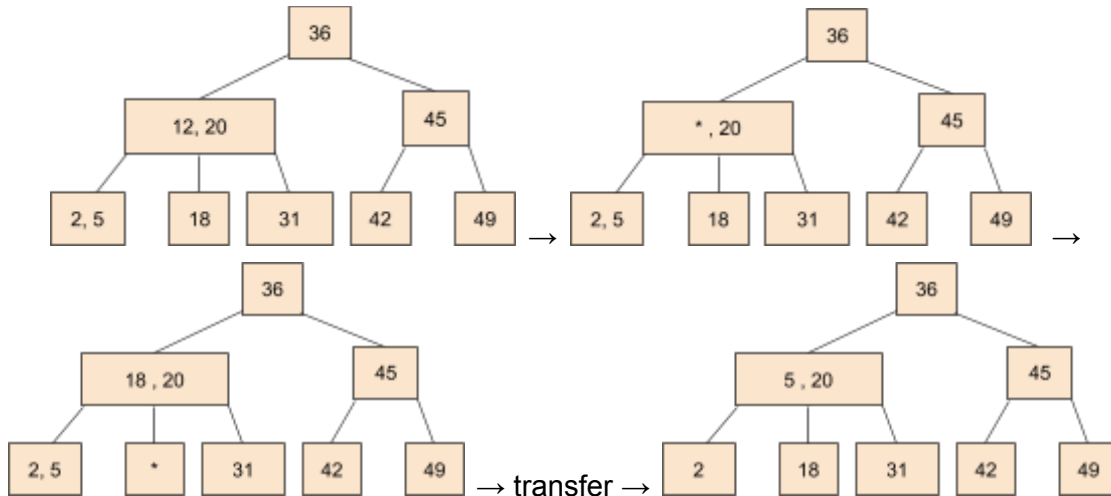
1. Removing from a leaf with > 1 values (easiest case)

a) remove (33)



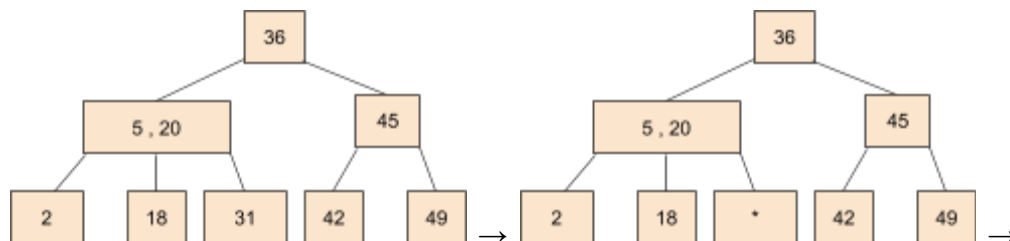
2. Removing from an internal node with a child with > 1 value

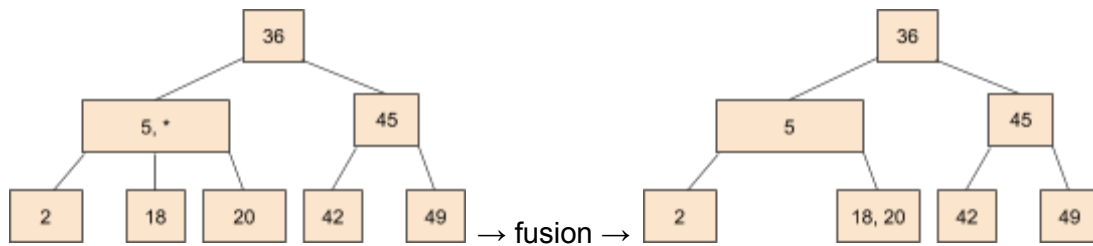
a) remove(12) replacing with successor



3. Removing from a leaf with a parent with multiple data

a) remove(31)





Removing from a 2-4 Tree Flow Diagram

Produced by: Kevin Chen

