FluiDB: Adaptive storage layout using reversible relational operators

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If the scientists, brought to heel by self-interested rulers, limit themselves to piling up knowledge for knowledge's sake, then science can be crippled and your new machines will lead to nothing but new impositions. You may in due course discover all that there is to discover, and your progress will nonetheless be nothing but a progress away from mankind. The gap between you and it may one day become so wide that your cry of triumph at some new achievement will be echoed by a universal cry of horror.

(Bertolt Brecht – Life of Galileo)

Abstract

It is a popular practice to use materialized intermediate results to improve the performance of RDBMSes. Work in this area has focused either optimisers matching existing materialized results in the cache and selecting intermediate results from a plan to survive the plan execution. To our knowledge, few attempts have been made to create plans with cached intermediate results in mind, and none that make any attempt to deduplicate the stored data to alleviate the storage cost of maintaining possibly large queries.

We built *FluiDB* to explore a novel approach to integrating the selection of materialized results with the planner to optimize the logical representation of data in memory. FluiDB materializes common intermediate results and deduplucates data to alleviate the cost of maintaining them. This is achieved by introducing *reversible operations*: versions of normal relational operators that may produce complementary tables alongside the normal output, that allow the reconstruction of the input relations. A planner aware of such operations can build query plans that dynamically adapt the data layout to the plan under a constrained memory budget. This thesis revolves around four main chapters, each of which describes in detail a different part of FluiDB and a final one that goes into evaluation of the system.

The first chapter focuses on query processing and the relational algebra semantics that FluiDB operates under. FluiDB parses queries into DAGs of sub-queries connected by reversible operators. Each such graph of the workload is merged into a global query graph that is used to infer properties of each relation like cardinality and extent.

The next chapter is dedicated to the planner and a novel monad for weighted backtracking that the planer is based on. The planner attempts to generate a plan by traversing the query graph so that, besides solving the query at hand, it leaves in memory a curated set of queries aiming to maximize the amortized performance of the workload being run.

In this chapter, the garbage collector is also discussed, which is the part of the planner responsible for inserting plan fragments that delete nodes when required such that the available storage budget is respected while no information is lost from the database.

After that, we go into *Antisthenis*, a framework we built for defining incremental computation systems. Antisthenis is used to build modules of the planner that efficiently determine whether a relation materializable, and estimate the cost of materializing a relation, given a set of materialized relations. Besides computation reuse, Antisthenis is able to prune the computation taking advantage of properties of the operators involved like absorbing group elements and bounded partial results. These techniques are also used to allow evaluation of some classes of self-referential computations.

The final chapter about the FluiDB architecture describes the transpilation of plans generated by the planner to C++, as well as the supporting libraries that enable the transpilation of queries to highly specialized C++ code, and the low level data organization of the database.

The thesis closes with a chapter that describes our methods for benchmarking and some experimental results.

Lay Summary

A database is a computer program that can answer questions(*queries*) based on a data stored in a medium accessible by the computer running the database, as well as provide an interface to manipulate said data. A *relational database* is a database that organizes the underlying data into tables or *relations*. Database systems rephrase the queries they receive in such a way that they can be answered efficiently, breaking them down to sub-queries, each subquery defined by an operator and one or more input operands, the sequence of subqueries generated being called a *query plan*.

A well-studied method of increasing efficiency is to store the answers to commonly encountered subqueries (*materialized views*) so they are readily available. This work describes the development of FluiDB, an experimental relational database system that explores the idea that the design decisions pertaining to planning and to maintaining a set of materialized views are interdependent and therefore should be addressed simultaneously from the start.

The design of FluiDB aspires to take one more step in that direction. Instead of asking which materialized views should be allowed to survive the execution of a query plan, we frame the problem as one of adapting the layout of the data to maximize efficiency of answering the the sequence of queries as a whole.

Acknowledgments

This thesis is dedicated to all the people that directly or indirectly supported me though my PhD studies, but also to all the people without whose support and hard work I would never have started this work to begin with.

It is customary to thank the supervisors first regardless of the circumstances but I would like to underline my gratitude to Stratis who really understood my idiosyncratic way of working and provided precisely the quality and quantity of guidance that I required. This is a rare quality and I cherish the opportunity I had to work with him. I would also like to express my gratitude to Boris Grot, not just in the general sense but because, especially during the first years of my thesis he went out of his way to make me feel included and gave value to my voice. For the similar reasons I would like to thank Christophe Dubach whose door was always open and his whiteboard always available, I deeply appreciate all the times he was willing to patiently listen to my crazy ideas.

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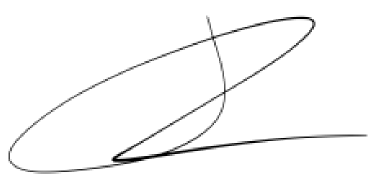
Special thank you to my flatmate for two years Christos Velanis who got me safe and sound through some fairly turbulent times, and for the ideological steeling brought about by this cohabitation. Thanks to him and to Christos Maniatis for the heated philosophical debates that would typically end after 6am.

Last but not least I would like to thank my friends that were there though all of it, the thick and the thin Antreas, Giannis, Aisha, Maria, Irini, Iordanis, Nelly, Christos, Pigi, Niki, The President, and Nikolas.

Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.



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CHAPTER 1

Introduction

In [dialectical] movement, consciousness experiences the emergence of individuality in the unchangeable, and of the unchangeable in individuality.

(G. W. F. Hegel – Phenomenology of Spirit)

Chapter summary

- FluiDB is an in-memory RDBMSes optimizes data layout for space efficiency w.r.t. the workload
- The main novelty relates to the introduction of reversible relational operations which affords a new perspective on query planning and view selection.
- FluiDB materializes all intermediate results and deletes garbage collects when she runs out of space.

With the advent of technologies that make access to information scalable and affordable, the mental and temporal gap between collection of data and their analysis grows rapidly. At least two of the biggest players in the tech industry, Facebook and Google, base their competitive advantage almost exclusively on vast amounts of information that they have collected and their capacity for such collection and processing. In cases like these, the layout of the stored data is independent of the growing number of applications taking advantage of it.

A mantra in the database community used to be that "storage is cheap" and while that is true a more complete version of the mantra might be that "slow storage is cheap".

Databases have traditionally been dealing with the trade-off between memory and time efficiency within monetary constraints, especially with the use of intermediate result recycling technologies which employ sophisticated ways of choosing parts of computation to be stored for reuse. This model has had some impressive results in OLAP workloads. We adopt a slightly different view of the problem: executing query plans does not only leave some specific intermediate results as residue, but rather *transitions the entire storage state from one where the result of the query is not materialized to one where it is*. With such a notion of query planning what new dimensions open up in the design space of a query planner? The creation of FluiDB is an attempt to study some aspects of this question. In particular FluiDB is based on the following pillars:

- Introduce reversible query operations that allow for more sophisticated plans based on available materialized views.
- The planner itself involves a garbage collector that will delete materialized views or primary tables that can be materialized from the remaining relations.
- An incremental numeric evaluation system allows the planner to efficiently and repeatedly infer the cost of materializing queries under a rapidly changing inventory of materialized views.
- Query execution is based on code generation.

These concepts allow FluiDB to dynamically adapt the data layout to the workload in ways that would not be possible in traditional intermediate view recycling systems. To make this more concrete let's look at an example.

Consider the following query over the TPC-H dataset that computes the average discount per country:

```
select      n_name, avg(l_discount)
from        lineitem, customer, nation, order
where       l_orderkey = o_orderkey
and         c_custkey = o_custkey
and         c_nationkey = n_nationkey
and         l_shipdate > 10-11-2015
group by    n_name
```

An optimizer considering this query in isolation would come up with some plan resembling the following following plan following:

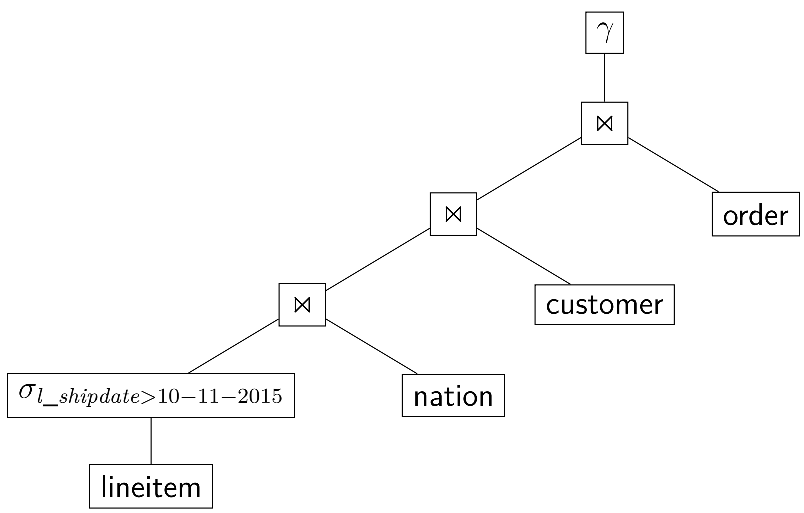


Figure 1.1.: An efficient logical plan for a single query.

Notice how the selection ($\sigma_{(l\_shipdate)>10-11-2015}$) is pushed all the way to the bottom of the tree because it is a cheap operation (worst case a full scan over the input) and can potentially shrink the input by a lot rendering the joins higher in the tree much cheaper.

Consider a workload that repeatedly runs a query generated from the following template:

```sql
select      n_name, avg(l_discount)
from        lineitem, customer, nation, order
where       l_orderkey = o_orderkey
and         c_custkey = o_custkey
and         c_nationkey = n_nationkey
and         l_shipdate > ${min_date}
group by    n_name
```

It is clear in this case that it would be beneficial if the workload is large enough for the cost of $lineitem \bowtie order \bowtie customer \bowtie nation$ to be amortized we would like to materialize the large join and only run $\gamma\sigma$ for each query attached as show in figure 1.2.

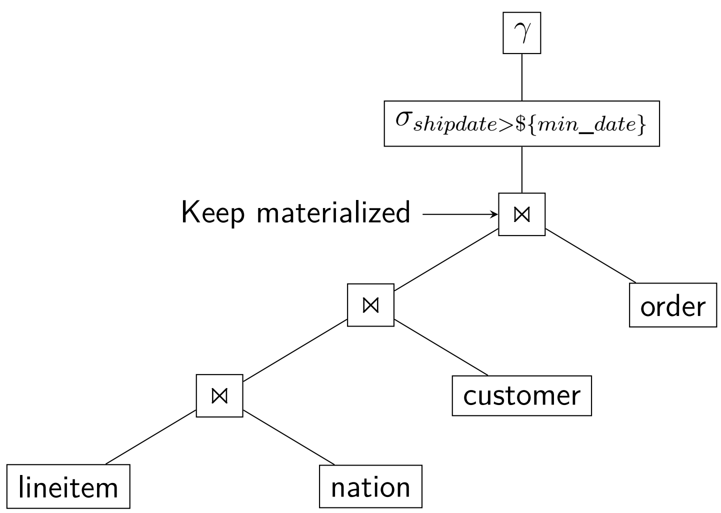


Figure 1.2.: An efficient logical plan for a template based workload. The relational algebra operators used are: γ for aggregations. σ for selection, and $\bowtie$ for joins.

This shift of focus from per-query optimization to considering the amortized cost of materialization of expensive relations is already a tall order for most RDBMSes that feature materialized views. A simple but effective approach to integrating incremental query materialization and the optimization processes, was presented in [112]. In their approach they maintain a *history pool* (a list of all the past queries) that is used to decide the benefit of materializing a sub-expression, and a *view pool* that keeps track of the materialized tables at every moment. Both these sets are taken into account during planning to produce a plan that will likely minimize the amortized cost of the workload. After each query is planned the sets are updated. A limitation of such an approach is that when dealing with relations like $lineitem \bowtie order$ in budget restricted settings, materialized view storage can quickly become a scarce resource.

There is an opportunity to reduce the effect of this problem by exploiting another common workload attribute: certain tables are frequently subsumed by the same intermediate result. In our example workload $lineitem$ is fully subsumed by $lineitem \bowtie order$, $lineitem \bowtie order \bowtie customer$, and $lineitem \bowtie order \bowtie customer \bowtie nation$. Similarly $order$, $customer$, and $nation$ are fully or partially subsumed by similar relations. It seems wasteful to always keep the rows of all primary tables separately *and* concatenated in the rows of $lineitem \bowtie order \bowtie customer \bowtie nation$.

In particular we can retrieve the rows of $lineitem$ by projecting on the latter relation and deduplicating the resulting rows we can obtain any of the former. In this – contrived yet demonstrative – example a plan taking into account amortized costs could be:

```
Query $\gamma\sigma_{p_0}(lineitem \bowtie order \bowtie customer \bowtie nation)$ {
  $Q_0$ := Materialize[$lineitem \bowtie order$]
  $Q_1$ := Materialize[$customer \bowtie Q_0$]
  $Q_2$ := Materialize[$nation \bowtie Q_1$]
  # Not enough space to continue. Delete relations that we can rebuild.
  GC {
    Delete[$lineitem$]
    Delete[$Q_0$]
  }
  $Q_3$ := Materialize[$\sigma_{p_0} Q_2$]
  $Q_4$ := Materialize[$\gamma Q_3$]
}
Query $\gamma\sigma_{p_1}(lineitem \bowtie order \bowtie customer \bowtie nation)$ {
  GC {
    Delete[$Q_1$]
  }
  $Q_5$ := Materialize[$\sigma_{p_1} Q_2$]
  $Q_6$ := Materialize[$\gamma Q_5$]
}
Query $\gamma\sigma_{p_2}(lineitem \bowtie order \bowtie customer \bowtie nation)$ {
   GC {
    Delete[$customer$]
   }
  $Q_7$ := Materialize[$\sigma_{p_2} Q_2$]
  $Q_8$ := Materialize[$\gamma Q_7$]
}
...
# Since the large join has all the columns of \li we should be able
# to create it by simply getting slicing and deduplicating
Query $lineitem$ {
  $lineitem$ := Materialize[$uniq\{\pi_{cols(lineitem)} Q_2\}$]
}
```

Figure 1.3.: A sequence of plans optimizing the workload amortized cost and involving reverse operations. These queries are the same save for the predicate p_i selecting $lineitem$

By incorporating reverse relational operations where possible FluiDB can indeed generate workload plans similar to the one described.

The solution we experiment with by implementing FluiDB resembles the solution provided in [29] by Gou et.al for multi-query optimization (MQO). In their work they embed aggregations `group by x1, ..., xk` into the $\subseteq$-lattice that arises from the power set $P(\{x_1, ..., x_k\})$. Thereby they encode the fact that `group by A, B, C` is subsumed, or can be computed from of:

- `group by A, B`,
- `group by A, C`
- `group by B, C`.

...

Once the lattice is constructed a variant of the $A^\star$ path finding is used algorithm to search for the optimal aggregation plan. However they make no attempt to recycle tables, i.e.. garbage collect tables, whose data can be found in other relations, and narrows it's attention to aggregations.

From the aforementioned work we keep the basic notion of using a graph to represent relationships between queries and to derive the benefit of materializing a relation. We also use path finding techniques in that graph to create plans. However we introduce a more complex and ad-hoc hierarchy of relations to account for subsumptions in the entire relational algebra, rather than just aggregations that is very similar to AND-OR dags as found in [20]. The relations we express in that graph are bidirectional, so rather than only finding paths towards the goal and deleting relations when they are no longer needed via GC plan fragments, we simultaneously plan for moving towards the goal query and performing "backwards" operations to enrich the space of possible plans. This process will become clear with an example which demonstrates a slightly simplified version of our system's functionality:

In figure 1.4 we provide representation of a subsumption graph that our RDBMS might create after witnessing selections on $lineitem$:

- `select * from lineitem where l_quantity < 24`
- `select * from lineitem where l_shipdate > 10-11-2015`
- `select * from lineitem where l_discount < 0.06`
- `select * from lineitem where l_discount < 0.06 and l_shipdate > 10-11-2015`

For brevity let $p := shipdate > 10-11-2015$, $q := quantity < 24$ and $r := discount < .06$. In relational algebraic representation the queries the RDBMS has encountered are $\sigma_p(lineitem)$, $\sigma_q(lineitem)$ and $\sigma_q\sigma_r(lineitem)$. For the purposes of this example we assume that according to the workload, the planner has determined that the priority in terms of usefulness of each query involved is in descending order of "usefulness":

View	Size
$\sigma_q lineitem$	5
$\sigma_p lineitem$	6
$\sigma_{\neg p} lineitem$	4
$\sigma_{p\wedge r} lineitem$	1
$\sigma_q lineitem$	6
$\sigma_{\neg q} lineitem$	5
$lineitem$	10

Table 1.1.: The of figure 1.4 annotated with their sizes and ordered in descending order of "usefullness" as determined by the planner.

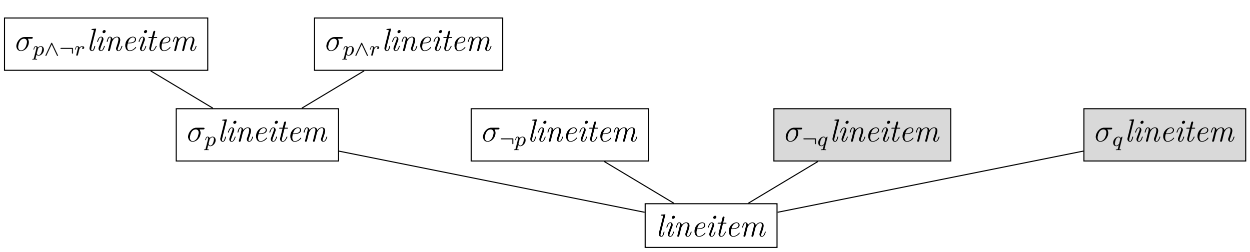


Figure 1.4.: The currently materialized relations are marked with gray. Assuming the absence of null values and assuming set semantics $lineitem = \sigma_p lineitem \cup \sigma_{\neg p} lineitem$, FluiDB can find a plan to generate any relation involving $lineitem$..

The query that we are planning is $\sigma_{quantity<24}\sigma_{discount<.06}lineitem$, which is denoted in the figure as $\sigma_{p\wedge r}lineitem$. Our total size budget is 25. The plan described is codified in 1.5.

Following the edges in the graph, to plan $\sigma_{p\wedge r}lineitem$ we need $\sigma_p lineitem$ and for that we need $lineitem$. So first we plan the union

$$\sigma_{\neg q}lineitem \cup \sigma_q lineitem \rightarrow lineitem$$

Then we need $\sigma_p lineitem$ but we are now using 20 units of space and adding 6 more would exceed our space budget of 25 units. $lineitem$ is the least beneficial of our materialized views but it is required for our next step, i.e., creating $\sigma_p lineitem$, $\sigma_{\neg q}lineitem$ is deleted instead since it is derivable from $lineitem$. We can now create not only $\sigma_p(lineitem)$, but also $\sigma_{\neg p}(lineitem)$ so we are now using 25 units of space.

$$lineitem \rightarrow \{\sigma_p lineitem, \sigma_{\neg p} lineitem\}$$

Finally, we need to create the final relation $\sigma_{p\wedge r}lineitem$. However the available storage is all used up. We can now delete $lineitem$ because its data is duplicated in the combination if $\sigma_p lineitem$ and $\sigma_{\neg p}lineitem$. Once that deletion is carried out we can create $\sigma_{p\wedge r}lineitem$ which was the requested query.

```
Inventory {
  Q_0 := σ_{¬q} lineitem
  Q_1 := σ_q lineitem
}

# Budget: 25, used: 10
Query σ_{q∧r} lineitem {
  lineitem := Materialize[Q_0 ∪ Q_1]
  # used: 20; we need 26 to proceed...
  GC { Delete[Q_0] }
  # used: 15 we can now materialize both
  Q_2, Q_3 := Materialize[{σ_p lineitem, σ_{¬p} lineitem}]
  # used: 25; we can delete lineitem now
  GC { Delete[lineitem] }
  # used: 15; we can now materialize the requested
  Q_4 := Materialize[σ_r Q_2]
}
```

Figure 1.5.: A sequence of plans optimizing the workload amortized cost and involving reverse operations.

This radical approach to adaptation of data to the workload is particularly useful in contexts where (fast) memory is scarce compared to the data, particularly in-memory databases. In that vain we lean heavily on executing plans by transpiling the plan to C++ code.

The overall system is comprised of three internal their interlocation being described in diagram 1.6. An SQL query is initially parsed and passed to the *Graph Builder* which correlates it and its possible subqueries to historical queries using the *QNF processor* which evaluates the equivalence between subplans. This way the *Relational Algebra (RA) Graph* is populated with nodes corresponding to relations connected by often bidirectional operators.

The next level in the pipeline is the planner, which traverses the graph using an $A^{\star}$

inspired branch and bound algorithm in order to come up with a plan that a) efficiently computes the requested query and b) optimizes the data layout to provide opportunity for efficient future plans. Pruning of the search space is assisted by *Antisthenis*, an incremental framework optimized a) for calculating the expected cost of materializing a relation without actually planning for it as well as b) calculating whether a relation is materializable at all given the currently materialized relations. The plan generated has the form of a sequence of operators and relation deletions which are finally translated to C++ and compiled with an off-the-shelf compiler.

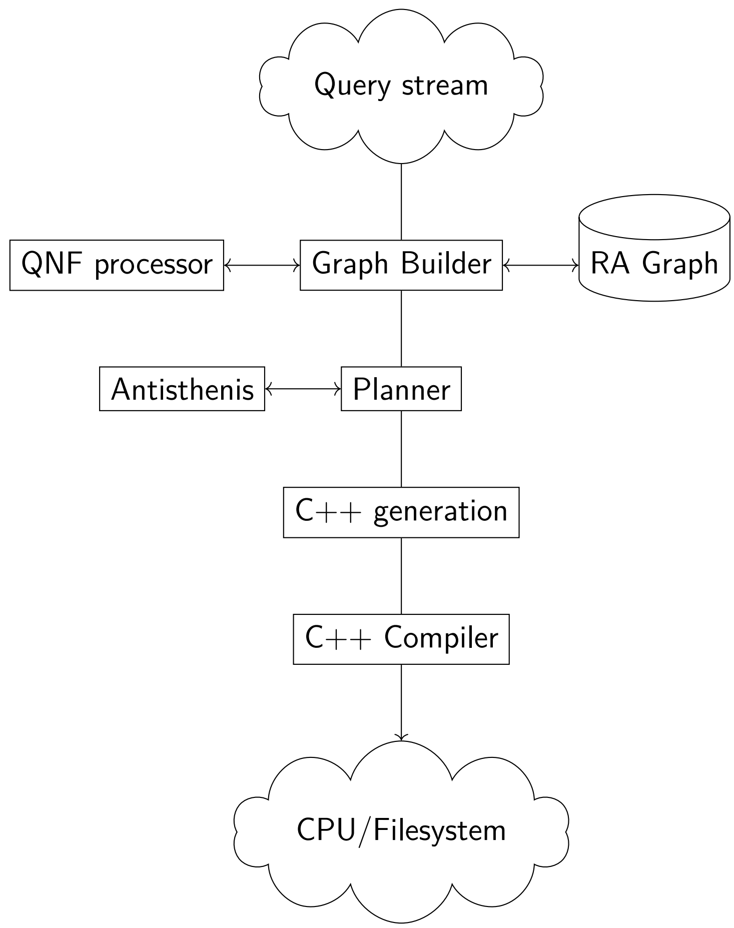


Figure 1.6.: The system architecture

In this thesis we highlight the following contributions:

- We introduce the concept of reversible relational operators. FluiDB is built around leveraging this idea to adapt the layout of primary data to the workload. This allows FluiDB to efficiently answer complex queries and share computation between different queries with minimal memory footprint. It is discussed in detail in chapter 3.

- We present a monadic framework for branch and bound search. While branch and bound is an algorithm common in rewrite systems it has not been implemented based on a monad. This approach affords flexibility and composability to easily implement and tweak complex heuristics and search space pruning techniques. The `HCntT` monad transformer that captures this is discussed in chapter 4.

- We introduce an incremental computation system that focuses on incrementally computing the cost estimation of logical plans for relational queries. There is little work on incremental cost estimation functions and none that applies to either recursive (non-DAG) operator graphs or that takes into account materialized intermediate results. To make FluiDB's optimization tractable we introduce a novel, highly specialized, incremental computation framework we call Antisthenis. Antisthenis is discussed in detail in chapter 5.

This thesis describes in detail how we implemented FluiDB, a system targeted at performing this kind of reasoning for planning and executing queries. In chapter 2 we provide a brief overview of the state of the art and common practices that pertain to query processing; in chapter 3 we describe how queries are processed and stored at a logical level and various methods we employ for overcoming the challenges of assembling queries in a graph; in chapter 4 we describe how FluiDB performs planning and garbage collection to come up with a concrete physical plan for each query; in chapter 5 we go over an incremental evaluation system we built to facilitate the requirements of the physical planner; in chapter 6 we go over the algorithms and particular techniques that FluiDB employs to transpile the physical plan into C++ code; finally at chapter 7 we describe our experimental evaluation of FluiDB on the benchmark SSB-TPC-H. We present a conclusion and some future directions in the final chapter 8

CHAPTER 2

Background

I am not the sow of chance, the forger of new life
I am a child of necessity, a mature offspring of rage

(K. Varnalis – The Guide (from "The Light That Burns")*)*

Chapter summary

- Relational databases are question answering systems that deal with information organized in tables or relations.
- Query optimization and planning revolves around finding an efficient algorithm for answering a query.
- In-memory databases keep all data in main memory so they are closer to the processor
- Many in-memory databases employ some variant of code generation to execute the query plans.
- Intermediate result recycling is the practice of reusing computation between queries.

FluiDB is a system that generally focuses on relational query optimization and planning. This chapter aims to give the reader some idea about where FluiDB fits in the design space and the historical context in which it was developed. First, we outline a very high level overview of the *query language and operators* involved in relational database management systems (RDBMSes) as well as the the overall architecture of such systems. Following that, we will focus on the *query planning* subsystems of relational query databases and some traditional approaches to query evaluation. Afterwards, we will focus particularly on *in-memory relational databases* and the trade-offs that govern their design. Finally, we look into the systems that utilize *intermediate result recycling* and how they solve the problem of automatically selecting and maintaining intermediate results for sharing computation within workloads.

2.1. Relational databases

Databases are more than just a method of accessing data. In their essence they are question answering machines. The typical database works in a perpetual loop of reading queries and coming up with answers based on a set of data points. There are two important aspects that every database needs to define in order to delineate its operational semantics:

- The language in which the queries are expressed in
- The representation of data in terms of which the queries are expressed and the results are presented.

The oldest, most studied, and most common model is the *relational* model which defines queries in terms of *relational algebra* and organizes data in *tables* or relations.

A relation is typically defined as an unordered set of tuples $\{d_1, d_2, ..., d_k\}$ where each element d_i represents an attribute. The core relational algebra is an extension of the algebra of sets (that defines operators of set union $\cup$, intersection $\cap$, product $\times$, and difference $-$) that includes the operators of joins $\bowtie$, projection π and selection σ. Upon this foundation relational databases typically define extra operators to increase the expressive power of the language like aggregations γ, semijoins $\ltimes$, sorting, limiting, etc.

A typical relational database processes operates in a cascading fashion. It initially receives a query in a textual form. While relational algebra is the language that underpins the operation of a relational database, it is rarely the language in which users interact with it. Instead, most relational databases expect queries written in a variant of SQL, a query language that is parsed into a tree of relational algebra operators.

This tree is processed by the query optimizer which rewrites it into a representation that is efficient to be executed by taking advantage of the mathematical properties of RA and often gathering statistics about the underlying data itself. This process leads to the formation of a relational algebra expression called the *logical plan*. It is a high level description of a sequence of operations required to produce a result. A logical plan only assumes the denotational semantics of these operations, making no assumptions about their implementation.

Each RA operator is typically implemented by several different algorithms, each being efficient and even possible in some situations but not in others. For example, a join can be implemented by nested loops or by the merge join algorithm, among others. While the former algorithm is general and can implement any join, it is very inefficient. On the other hand a merge join is much more efficient but can only implement joins of the form $\bowtie_{a=b}$ (equijoins or natural joins). Furthermore, a *logical plan* typically contains only implied information about the scheduling of the operators, for example, the logical plan $(\pi A) \bowtie (\sigma B)$ implies that the join can not begin being evaluated before the projection and selection but the latter can be evaluated in any order, or even simultaneously. All these details about the execution of the query are resolved by the *physical planner*. The physical planner accepts a logical plan and emits an unambiguous algorithm that will produce the result of the query, called the *physical plan*.

The final step is actually executing the physical plan which is handled by the *query execution engine*. The execution engine simply evaluates the physical plan on top of the data, making use of the correct algorithms, auxiliary structures like indexes managing memory, handling page-level caching, etc.

2.2. Query optimization and planning

Query optimization and planning generally is a set of processes that take place between reading a query and executing a physical plan. The concerns of query optimization are the correctness of the final result and the efficiency of the plan generated, usually in terms of time, but also in terms of space.

Finding an optimal plan is in the general case NP-complete [15], but query optimisers can do a good job of finding good plans using heuristics. The selection and organization of these heuristics as well as query cost estimation are the main problems that make the design of a query optimizer nontrivial. In particular one could separate the the tasks of a query optimizer into three broad categories:

- Query rewriting
- Query plan enumeration
- Size and cost estimation (cost model)

At a very high level, the architecture of a query optimizer is demonstrated in figure 2.1 and is broadly comprised of the following components:

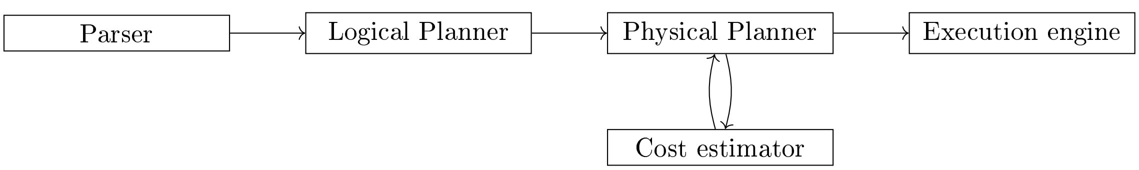


Figure 2.1.: Common architecture of a query optimizer.

- The *parser* which receives the query in textual form and produces a logical plan in the form of an abstract syntax tree.
- The *deterministic optimizer* or *logical planner* that rewrites the logical plan applying optimizations that are based solely on the general properties of the relational algebra.
- The *physical planner* that transforms a logical plan into a physical plan that can be unambiguously executed by the execution engine to produce a result. The physical

planner typically has at least some information about the underlying data that the plan will operate on like estimations about the statistics of the values or the cardinality of the relations.

- The information about the data being manipulated by the plan is inferred by the *cost estimator*. It uses a cost model to predict the cost of plans and the cardinality of relations by taking into account the provenance of relations as well as physical properties of the data like the presence of indexes, the data layout, etc.

These subsystems are presented here as separate for simplicity and because in many database systems they are clearly delineated, but it is also common that they blur into each other. For example, in some RDBMSes the physical and logical planners are merged into one [8, 60, 70]. In fact, it is increasingly common for systems to intersperse query planning with query execution, adapting the optimization strategy [5] to concrete information about the intermediate results evaluated rather than purely relying on estimations and predictions [33, 47, 78, 85]. The degree to which these subsystems are separate is a major concern in the design space of query optimisers.

Another important aspect to be considered, and which is indeed important in the design of FluiDB, is the number of queries considered at a time during optimization and the way in which they are considered. The optimizer usually considers one query at a time and maintains little or no state between executions. Although it has found little adoption in main stream databases, multi-query optimization has been researched expensively [19, 65, 82, 100]. What is more common in recent years is recycling intermediate results. RDBMSes that incorporate this technique materialize and cache intermediate relations, reusing them when they appear as sub-queries in later queries [48, 64, 68].

The final query processing concern that is relevant to the design of FluiDB regards the traversal and pruning of the query plan search space. As mentioned, query optimization is generally NP-complete, so the viable options DBMS designers are left with are randomized algorithms, ML approaches, and heuristics. Virtually all systems implement heuristics entirely or to some degree, while recently more and more also incorporate randomized algorithms [53] and machine learning [89, 98].

2.2.1. Logical and physical query optimization

The *logical planner* transforms a syntax tree in the form of relational algebraic expressions, where the operators only contain information about the semantic meaning of the operations and none relating to, the algorithms that will eventually be executed on the input data. It is for all intents and purposes a rewrite engine for relational algebra. Typical transformations performed by the logical query optimizer are

- Predicate normalization to conjunctive normal form, e.g.

$$(a_0 = 1 \wedge a_1 = b_1) \vee b_2 = 3 \hookrightarrow (a_0 = 1 \vee b_2 = 3) \wedge (a_1 = b_1 \vee b_2 = 3)$$

- Predicate push-down e.g. $A \bowtie_{a_0=1 \wedge p} B \hookrightarrow (\sigma_{a_0=1} A) \bowtie_p B$.
- Cartesian product to joins $\sigma_p(A \times B) \hookrightarrow A \bowtie_p B$
- Searching the join ordering space.

The *physical query planner*, on the other hand, will specialize the logical operators deciding on the particular algorithm that should be used. A physical query planner therefore requires low level information about the query that relates to the indexes available, possible ordering of the data, materialized views, etc.

Each of the planners needs to enumerate the plans under consideration while traversing the search space. There are two major approaches to this:

- The *top-down* approach, where the planner establishes the top level operator and branches searching the children, backtracking as necessary. This was the approach of the descendants of the *volcano optimizer* [7] where they implement a "search engine" that uses a branch and bound approach to optimization with caching.
- The *bottom-up* approach where the planner builds and connects fragments of a plan accumulating those fragments into a larger plan in a traditional dynamic programming approach. [54, 90]

An important optimizer, on which most modern optimisers are still being based, is the *cascades optimizer* [8]. In short, cascades keeps track of *groups* of equivalent query expressions and uses those as the fundamental atom that it manipulates. For example, instead of keeping track of $A \bowtie (B \bowtie C)$ and $(A \bowtie B) \bowtie C$ separately they would be part of the same query group. Cascades then uses a global hash table (the "memo" structure) to match the best plan that corresponds to each group. These plans are stitched together to form larger plans until the plan for the required query is created in the memo structure.

Many database systems use optimisers similar to cascades (like the Microsoft SQL server, Postgres, MemSQL[75], and Greenplum – now Orca [71] to name a few). Notable among the descendants of cascades is Apache Calcite [84], a framework for implementing query planning used by a number of commercial and research databases [91].

Probably the hardest aspect of the planner design is the *cost estimation algorithm*, which is required by the optimisers, especially the physical planner. As cost estimation, we generally refer to a number of different related procedures that are broadly the *estimation of the cost* of an arbitrary plan and include the prediction of the *cardinality* of not yet materialized relations. The role of cost estimation is detrimental to plan selection and to the navigation of the plan search space. A good cost model can help basic planners make decent plans, and a bad one can cause sophisticated planners make horrible plans [74].

It seems that the most important challenges involved in the design of a cost model relate to the fact that during planning we are fundamentally operating with sparse and highly uncertain information. Especially relating to cardinality estimation, uncertainty and bad predictions propagate and make make it extremely hard for the planners to make correct decisions. Consider, for example, a join. Some joins are similar to Cartesian products, producing large output tables, and some joins are more similar to lookups producing only a few rows. A cost model that confuses the kind of join will make very bad predictions w.r.t. the cost of any relational algebra expression that uses the said join, causing an optimizer to select an expensive plan or avoid a cheap one. What is worse, misestimating the input of an operator caps our ability to estimate the output.

2.3. In-memory relational databases

In-memory databases are databases where all data lives in main memory. The design of in-memory databases is different from the design of a disk-backed database in a number of respects. To name a few:

- Page buffers have little use and caching of data in general has very different goals. While in disk-backed databases caches are mainly used to avoid disk IO, in in-memory databases they focus on reuse of computation.
- Concurrency control is much simpler as storage synchronization concerns are almost entirely eliminated.
- In disk-backed databases, only a small percentage of time is spend on actual computation [86]. Much of the non-compute latency is directly linked to the persistent storage.

On the other hand, there are concerns that are specific to the *lack of backing database storage*. To name a few:

- Should the system rely on record IDs like in a persistent database or can it use direct pointers to records?
- Error prone software can bring the data to an irrecoverable state.
- Query execution algorithms have fundamentally different characteristics. While in persistent databases IO dominates the runtime the bottlenecks for an in-memory database are much more complex and they can include things like locking, cache misses, predicate evaluation, and data movements.
- As main memory is a much less abundant resource than persistent storage, in-memory databases are often distributed, making network performance a major concern.

One increasingly common technique to address many of these issues, which is also used by FluiDB, is *code generation*. Since workloads for in-memory databases are typically CPU bound, there are major gains in performance to be had by specializing the code being executed. A typical interpreter-based plan execution engine makes heavy use of virtual function calls and conditionals inside tight loops which kills performance on virtually all modern architectures. The value proposition of code generation is to inline or hard-code the virtual functions and erase the conditionals at runtime to reduce the number of operations and make better use of hardware optimization.

We identify 4 different approaches in the literature to solving this problem:

2.3.1. Transpilation

Transpilation of a physical plan to a systems language like C or C++ and then fed to an off-the-shelf compiler [49] is expensive but it generates highly efficient code and more easily debuggable execution plans. The most notable complete database system that used this technique were Microsoft Hekaton that generates C code from SQL queries and older versions of MemSQL. For FluiDB, we reuse many techniques introduced in HIQUE [49] to translate physical plans to template-heavy C++ code, making the assumption that query compilation will be much faster than the query runtime.

2.3.2. Third party JIT compilers

JIT compilation has received a lot of attention in the compiler community, especially in the context of the JVM. A number of database systems have taken advantage of this trend to speed up query execution. Most notable of these are SPARK [72], which generates Scala AST which is then converted to JVM byte code, and Neo4j that directly generates bytecode out of the queries.

Systems like Peloton [81] and recently Postgres [83] compile query plans to LLVM IR code that is then passed to the LLVM compiler to generate high performance machine code. Notable among the systems that use this approach is HyPer [111] which addresses the problem of query compilation overhead with an adaptive execution approach: they built an IR interpreter that starts running the query while the compiler does proper optimization and compilation of the LLVM IR program being interpreted. Cheap queries are thus completed reasonably fast, while in the case of complex queries the interpreted program is seamlessly replaced by the compiled one once the LLVM compiler finishes generating machine code.

2.3.3. Direct machine code generation

Some databases do not reuse any compiler or JITing VM, but rather directly generate highly specific machine code out of the physical plan. The first system to attempt that, like most techniques used today, was System-R, which originally compiled SQL statements directly to machine code by stitching together code fragments from a "fragment library" [2]. The technique was quickly deprecated, however, due to the large engineering effort required. Oracle also includes similar to some of their databases and MemSQL express their plans in a custom language called MPL (MemSQL Plan Language) for which they have a custom compiler that translates it directly to machine code.

2.3.4. Custom execution engines

The final category of databases that use code generation is is per database code generation rather than per-query code generation. Volcano/EXODUS [7] and more recently SageDB [109] generate an optimizer and execution engine that is specific to the database schema but not to the queries. Essentially, these database systems compile the database schema into a custom RDBMS.

2.4. Intermediate result recycling

A materialized view is a relation defined by a query that is persistently stored while a view that is not stored is said to be *virtual*. *View selection* is the process of selecting an appropriate set of materialized views to improve the performance of a workload [59]. Automated materialized view selection or intermediate view recycling has occupied the database research community for a while now. A few approaches to this problem have to do with AND/OR directed acyclic graphs [12], modeling the problem as a state optimization [14], and lattices to represent data cube operations (i.e. multiple aggregations over the same relation) [106].

A related problem is *multi-query optimization (MQO)* [14] that attempts to plan multiple queries simultaneously. An efficient solution using AND/OR DAGs was proposed by Roy in [19] where they insert queries and their sub-queries in a graph and attempt to evaluate a plan by traversing that graph from multiple sources in a fashion similar to volcano optimization. Building on that Kathuria et. al. [79] present an approximation algorithm that runs in time quadratic to the number of common subexpressions and provides theoretical guarantees on the quality of the solution obtained.

Researchers have further looked at opportunistically reusing intermediate results that would be materialized. Both in traditional databases [48, 64] and on non-database contexts like MapReduce [56], and there have been attempts to unify the planner with the materialized view selection engine [69]. Notable in this field is Nectar [46] which is also not an RDBMS which automatically compresses rarely used data into programs that generate that data. Nectar focuses on sharing computation and data as much as possible.

Work such as MRShare [51] tries to bridge the gap between intermediate result recycling and MQO by automatically grouping queries in a workload in such a way that computation can be maximally shared.

2.5. Related work

Our work on FluiDB focuses on intermediate result recycling and query optimization but on our way to a solution to this particular problem we had to also work in fields that are fall outside the scope of database research. In this section we present the prior art in the three main fields in which the main contributions have been made:

- Intermediate result recycling (corresponding to chapter 3)
- Monadic backtracking (corresponding to chapter 4)
- Incremental computations (corresponding to antisthenis described in chapter 5)

2.5.1. Intermediate result recycling

The fundamental problem being solved by FluiDB falls under the very general umbrella of *optimizing multiple queries simultaneously*. This problem is central to database research and there is a very wide diversity of approaches to solving it. It is useful, however, to understand this less like a single problem more like a family of problems. To help the reader understand the philosophy of FluiDB a high level view we propose an organization of the proposed algorithms that solve subsets of that problem exist in a space organized around two poles:

- The algorithm solves a *set* of queries simultaneously
- The algorithm solves one query at a time from a *stream* of queries.

While many design decisions are informed by the space a piece of work occupies in relation to this dipole, virtually all approaches involve identification of common subexpressions or sub-plans that can be shared between different queries and avoiding duplicate evaluation. This is indeed the unifying element of the work in the field.

The former problem has heavily researched for decades and is a major subfield in database research [4, 19, 104]. It's focus is slightly narrower as it focuses on identifying common subqueries in an unordered set of queries.

The latter (amortized optimization of a *stream* of queries) has also been dubbed the View Selection Problem (VSP) but research has mostly focused on static query workloads, i.e. information about the query workload is known in advance and views are pre-materialized [35]. Recently there have been attempts to mitigate this based on reinforcement learning [94] and genetic algorithms[17, 31, 61] due to high volumes, diversity and unpredictability of queries in modern workloads [99]. These approaches have important advantages and shape a field that is rapidly advances. So far, however, they improve on the heuristics based solutions. FluiDB's novelty, or at least certain aspects of it, is orthogonal in that it aspires to reshape the search space of plans, by introducing the GC and the reversible operations.

On the MQO side, Harizopoulos' two projects StagedDB [23] and QPipe [24] share between queries that arrive within a time window. This work is seminal and often cited in the field of MQO.

CJoin [38] uses an "always-on" plan of join operations to execute the joins of all concurrent queries. Similarly Crescando [45] attempts to share individual table scans, rather than joins, between queries to reduce IO. Crescando and other database systems that take similar approaches, like the aforementioned ones, aim at sharing computation between temporally overlapping query executions.

DataPath [44] creates what they call a "data-centric approach" to query processing. It transposes the order in which a per-query system operates (what they call "computation-centric approach") where the system effectively iterates over the queries in an outer loop and each query iterates over the relevant data. Instead DataPath iterates over the data in an outer loop and an inner loop iterates over the queries looking for a query that has use for that data. This is a very different approach to the other but it boils down to sharing data fetches between the queries.

SharedDB [57], BatchDB [80] and MQJoin [77] use a global query plan for grouped queries. Sloth [76] CrocodileDB [93] exploit lazy or deferred execution to increase the opportunities for batching and therefore sharing. This way of viewing query planning is conceptually much closer to FluiDB's approach. In fact the global query plan has many parallels to the QDAG that we will see in the next chapter. FluiDB, however, fundamentally differs from these systems as it's aim is processing streaming queries, rather than requiring ahead of time knowledge of the entire workload. This leads to radically different design decisions like reversible queries and the introduction of the garbage collector.

A slightly more proactive approach is taken by database systems like MonetDB [58] and CoScan [55] that try schedule queries so similar or overlapping queries are planned and executed simultaneously, allowing the system to more easily share computation.

A step closer to the aims of FluiDB is a parallel tradition roughly started in the 90s by DynaMat [16] and watchman [11] which do dynamic view selection, choosing which views maintain and which to evict aiming to reuse them in future queries. These and even more recent work of view selection focuses on opportunistically reusing intermediate results that would be materialized [48, 64], taking advantage of tried-and-true optimization techniques that are however geared towards query at a time processing.

There have been attempts to unify the planner with the materialized view selection engine like HAWC [69], which philosophically is the closest to FluiDB but, in contrast to FluiDB, neither incorporates a fully specialized optimizer towards nor takes into account selected views and intermediate results memory requirements during planning.

Notable in this field is also Nectar [46] which is not RDBMs. It automatically compresses rarely used data into programs that generate that data. Nectar focuses on sharing computation and data as much as possible.

2.5.2. Backtracking and branch-and-bound

The fundamental framework on which we build our optimizer is a based on a backtracking monad. Backtracking monads have been researched quite a bit in the past decades [18, 39, 105, 107]. The simplest backtracking monad is essentially the list monad which combined with haskell's lazy semantics amounts simply to nested loops traversing the search space. More complex search patterns are introduced by using monad transformers [67] and particularly the continuation monad transformer [10]. This allows us to jump between branches (continuations) of the search space. The main problem being addressed by the literature is that some branches of the search can be large or infinite. The most popular approach for mitigating this, other than continuations, comes in the form of interleaving the execution of different branches via special operators [26].

Only recently [97] has prioritized search been introduced to this area. Since we are able to quantify the priority of partially constructed query execution plans, this is fundamentally the approach we require, but we needed from our monad to support cut-like features. The combination of the latter and weighted search has, to our knowledge, not been addressed by the literature. In chapter 4 we present a monad that fits this particular bill.

2.5.3. Antisthenis

An important part of FluiDB is Antisthenis, our highly specialized incremental computation engine. Incremental computation is in it's essence an advanced conception of caching. Incremental improvements on simple memoization techniques [6] have led the modern notion of self adjusting computation [22]. Self-adjusting computation usually models the computation as a graph of variables and parameters and focuses on propagating changes of the parameters to the variables in the graph.

While there has been work on defining general frameworks for incrementalizing computation like [66] and [95], most systems focus on incrementalizing specific algorithms like trees[21], kinetic data structures[28], convex hulls [37], Huffman coding[43], relational databases [87] and Bayesian inference [30].

None of the previous works have unified evaluation strategy with incremental computation in the context of recursive functions, particularly ones composed boolean of AND/OR and min/sum operators.

2.6. Conclusion

We presented an overview of the various techniques and challenges of problems that are directly related to the design of FluiDB and also some adjacent ones. FluiDB draws inspiration from all these systems to explore a new approach to managing in-memory data being queried. It subscribes to the relational model of data and queries and parallels many techniques introduced for intermediate result recycling and mutli-query optimization. As an in-memory database, it employs code generation and takes it to the furthest extreme, generating highly generic C++ like the HIQUE system, fully compromising on the performance of the C++ compiler.

CHAPTER 3

Logical planning

At first glance [Odradek] looks like a flat star-shaped spool for thread, and indeed it does seem to have thread wound upon it; to be sure, they are only old, broken-off bits of thread, knotted and tangled together, of the most varied sorts and colors. [...] He does no harm to anyone that one can see; but the idea that he is likely to survive me I find almost painful»

(F. Kafka – The Cares of a Family Man)

Chapter summary

- FluiDB expresses queries in relational algebra and defines reverse operations for most operations.
- Query plan trees are accumulated in a bipartite DAG (QDAG) where the types of nodes are tables and bidirectional operations.
- Queries are normalized to hashable objects (QNF) that abstract a wide range of rewrites. This allows efficient merging query plan trees into the QDAG.
- Sub-graphs of the QDAG (clusters) represent higher level operations and are used as *propagators* to infer information of the n-nodes like cardinality, column type, and uniqueness of subtuples.

FluiDB is focused on query optimization and planning when the queries in a workload being served can have common subqueries. To exploit these similarities between queries, FluiDB transforms them to bipartite graphs of subqueries and bidirectional operators, which she incrementally merges into a global query DAG and she uses to enumerate the possible plans. In this chapter, we will dive into how FluiDB processes and merges these graphs. and into a detailed description of the graphs themselves.

We will start by looking into what kinds of queries FluiDB can understand and how she processes them. Then we will look at how she relates the queries to each other to construct a tightly interrelated story of the workflow by looking at the basic query graph, the heart of FluiDB.

For manipulating a graph of queries, it is important that FluiDB can distinguish between semantically different queries and and to be able to tell when they are the equivalent. We will look at how FluiDB normalizes queries to efficiently determine this equivalence relation.

Then we will see how FluiDB organizes the nodes in the query graphs into clusters representing single operations, how these clusters are formed within the query graph, and how those are used to maintain consistency among information about intermediate results.

Finally, we conclude by describing how the query processing infrastructure interfaces with the rest of the components: the planner, Antisthenis,and the code generator, described in later chapters.

In this chapter, we introduce the concept of reversible relational operations. We go into detail about how a traditional AND/OR query DAG augmented with reversible relational operators can facilitate a new approach workload optimization focusing on memory efficiency and throughput.

3.1. Relational Algebra

Much like most other RDBMSs, FluiDB deals with queries expressed in terms of a *relational algebra*. Here we define the FluiDB relational algebra semantics, that is, the traditional relational algebra extended by unary operators for ordering $s_e(X)$, limiting $l_i(X)$, anti-semijoin-ing $\not\ltimes$ and aggregating $\gamma_e(X)$, as listed in table 3.1.

Description	RA	SQL
Aggregation	$\gamma_e(X)$	`select * from X group by e`
Ordering	$s_e(X)$	`select * from X order by e`
Limit	$l_i(X)$	`select * from X limit i`
Join	$X \bowtie_p Y$	`select * from X, Y where p`
Anti-semijoin	$X \not\ltimes_p Y$	
Union	$X \cup Y$	
Selection	$\sigma_p(X)$	`select * from X where p`
Projection	$\pi_r(X)$	`select r from X`

Table 3.1.: Correspondence between FluiDB's relational algebra expressions and SQL.

Before going any further, it is important to clarify some assumptions made to accommodate the design of FluiDB. Particularly that

$$\sigma_p A \cup \sigma_{\neg p} A \equiv A$$

This means that there no `NULL` values are allowed since a predicate over a `NULL` value always evaluates to `False`, breaking the excluded middle axiom. This allows the definition of a bidirectional variant of the select (σ) operator.

Furthermore, we assume set semantics for all relations, meaning that a valid relation needs to have at least one unique sub-tuple. The user is not burdened with the responsibility of enforcing this, it is rather enforced by a preprocessing stage on projections and aggregations, exposing extra columns than the ones specified by the user such that the resulting relation has at least one sub-tuple that is unique for each row (see section 3.3 on query pre-processing).

The semantics of the sorting operation are preserved by making a small concession regarding the set unordered semantics: The *only* time when records of a relation are assumed by the planner to be ordered is for expressions of the form $s_e(X)$ and only for the duration between it running and the next operation. In practice, this means that the FluiDB planner generally treats s like an identity operation.

Besides these caveats, all operators known to FluiDB behave as expected. The first of the extensions adopted is the antijoin operator $\not\ltimes_p$. It is equivalent to $A \not\ltimes_p B \equiv A - \pi_A(A \bowtie_p B)$ and it is primarily useful for creating a bidirectional join by capturing the rows left over from a join (see subsection 3.2 on the QDAG). The important property we will be interested in is:

$$A \not\ltimes_p B \cup A \ltimes_p B \equiv A \not\ltimes_p B \cup \bar{\pi}_{cols(A)}(A \bowtie_p B) \equiv A$$

$\bar{\pi}_{cols(A)}$ meaning "group by the unique columns of A" We also define:

$$A \not\ltimes_p B := A \not\rtimes_p B$$

3.1.1. Expressions

Relational algebra expressions contain numeric, Boolean, string, etc expressions that operate at the field level. These appear in projection column definitions, selection predicates, aggregation column definitions, etc. These expressions are organized in up to 4 nested layers based on the kinds of operators:

- The top layer is purely Boolean operations `Prop` that can be rewritten in Cartesian normal form.
- The intermediate layer `Rel` describes relations between values ($\equiv$, `like`, $<$, $\leq$, etc). This layer is not nested, i.e. we do not allow relations between relations.
- The lowest layer `Expr` is expressions of any kind that may return any value. The above layers can be expressed in terms of this layer to support the ternary operator

`if <expr> then <expr> else <expr>`.

- The symbol layer describes the literal and symbolic atoms involved in the query. Its type is parametric in most parts of FluiDB to allow us to tag the symbols in the expression without breaking compatibility.

Predicates of selections and joins (joins are always θ joins at the type level) have type `Prop (Rel (Expr e))` for some type `e` and projections have type `QProj [(e,Expr e)]`. A special layer is provided for aggregations `Aggr` that makes it possible to disallow nested aggregation functions at the type level making the aggregation operator `QGroup [(e,Expr (Aggr (Expr e)))] [Expr e]`, where the first argument of the constructor is the aggregation functions and their names, and the second argument is the expressions on which the grouping should happen.

The detailed definitions of the algebraic expressions are presented in listing 3.1.

```
-- | Algebraic type for a relational algebra query.
-- `e` denotes the type of the column symbol and `s` denotes
-- the type of the primary table symbol.
data Query e s = Q2 (BQOp e) (Query e s) !(Query e s)
 | Q1 (UQOp e) !(Query e s)
 | Q0 s

 -- | Algebraic type of a proposition
 data Prop e = P2 BPOp (Prop e) (Prop e)
   | P1 UPOp (Prop e)
   | P0 e
 data UPOp = PNot
 data BPOp = PAnd | POr

 -- | Algebraic type of an expression
 data Expr s = E2 BEOp (Expr s) (Expr s)
   | E1 UEOp (Expr s)
   | E0 s
 data UEOp =
   EFun ElemFunction
```

```
  | ENot
  | EAbs
  | ESig
  | ENeg
data ElemFunction
  = ExtractDay -- erased after parsing
  | ExtractMonth -- erased after parsing
  | ExtractYear -- erased after parsing
  | Prefix Int
  | Suffix Int
  | SubSeq Int Int
  | AssertLength Int
data BEOp =
  -- Boolean (truthy)
  EEq | ELike | EAnd | EOr | ENEq
  -- Numeric
  | EAdd | ESub | EMul | EDiv

-- | Algebraic type of a relation.
data Rel e = R2 BROp e e
data BROp
  = REq
  | RGt
  | RLt
  | RGe
  | RLe
  | RLike
  | RSubstring

-- | Algebraic expression of an aggregation function
data AggrFunction = AggrSum
  | AggrCount
  | AggrAvg
  | AggrMin
```

```
    | AggrMax
    | AggrFirst
```

Listing 3.1.: Types of algebraic expressions that compose the non-RA parts of the queries.

3.2. QDAG: The unified query graph

The operation of FluiDB revolves around an AND-OR graph of relations and algebraic operations, not unlike the ones commonly used in multiquery optimization literature. This graph encodes all plans that the query optimizer and planner will enumerate by means of traversal. Said graph is a bipartite graph where *t-nodes* (similar to AND nodes in other literature) represent relational operations and *n-nodes* (similar to OR nodes elsewhere) represent relations that will act as intermediate results. A set of extensions is introduced to the traditional AND-OR DAG to facilitate the operation of FluiDB. In the interest of motivating those extensions, we will take a very high level view of the operations on the QDAG before describing them in detail.

There are multiple ways of composing relational operations to materialize a certain relation. Each one is a relational algebraic representation, we commonly refer to them as logical plans and they are represented by trees similar to ASTs. These trees are made of two kinds of nodes: operation nodes (t-nodes), and relation nodes (n-nodes). For example, the query $A \bowtie B \bowtie C$ may be represented as $(A \bowtie B) \bowtie C$ or $A \bowtie (B \bowtie C)$ due to associativity of $\bowtie$. Those two expressions represent two tree forms where the join operators $\bowtie$ are represented as t-nodes and each of A, B, C, $A \bowtie B$, $B \bowtie C$ and $A \bowtie B \bowtie C$ are represented as n-nodes.

While t-nodes can be triggered bidirectionally, they are not symmetric: a subset of n-nodes are on the output side and another on the input side. Each n-node can be materialized by *triggering* one of the input t-nodes, by running a connected operation to which said n-node is on the output. It is clear then where the name OR-node found in other literature originates: *any* of the input nodes may participate to a plan involving the OR-node (n-node for us). Conversely, each t-node can be triggered iff all of it's input n-nodes are materialized. Because *all* input nodes must be materialized, t-nodes are elsewhere referred to as AND-nodes. We depart from the standard nomenclature to avoid confusion in the presence of the extensions introduced by FluiDB. The planning of a query amounts to searching for a sequence of t-nodes to be triggered that would result in the corresponding node being materialized.

The QDAG of a query is created by unifying all logical plans that would derive it by means of merging n-nodes that are equivalent. For example the mentioned trees $(A \bowtie B) \bowtie C$ and $A \bowtie (B \bowtie C)$ shared the n-nodes $A \bowtie B \bowtie C$, A, B and C. A global QDAG is maintained by unifying the QDAG of each query encountered into the global QDAG, again, by merging equivalent n-nodes.

From a QDAG we can derive logical plans for queries. The planner operates in terms of the *global* (aggregate) QDAG, it traverses the QDAG searching for a plan (i.e., a sequence of t-node triggers) that will cheaply materialize the requested n-node(s). This way, the cost of historical queries can be easily considered at every step of the process. The problem of reusing materialized intermediate results is thereby conceptualized as similar to a graph traversal problem.

This technique is heavily inspired by multiquery optimization (MQO) approaches like [20] where they use the AND-OR DAG to solve multiple queries simultaneously, optimizing computation reuse.

QDAGs are different from AND-OR DAGs in a subtle but important way: each t-node may have multiple outputs and can be partially bidirectional. In other words, a t-node, as well as multiple inputs, may have multiple outputs, any number of which may be materialized by it being triggered. A *reverse trigger* of said t-node would materialize inputs provided that all outputs are materialized. Where AND-nodes are similar to functions that when evaluated produce an output given some input, t-nodes correspond to bidirectional relations between sets of input and output relations.

This not only creates opportunities in terms of what plans are possible, but also opens the gate to a whole new regime for how garbage collection is involved in the process of query planning. Any n-node may potentially be garbage collected, even primary tables, as long as the garbage collector can prove that it is reconstructable by triggering or reverse triggering t-nodes. The storage budget is thereby taken into account not only when selecting or rejecting materialized views, but also while planning the query solution itself.

To make this process clearer, as an example, consider a DAG that contains just a selection in figure 3.1 and a more complete example of a QDAG involving multiple selection is presented in figure 3.2

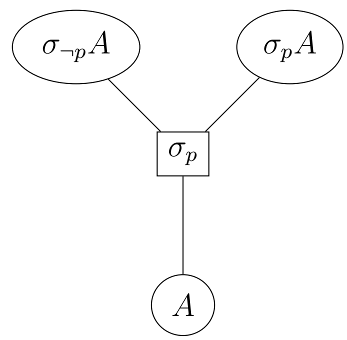


Figure 3.1.: A selection t-node may generate both output n-nodes, $\sigma_p A$ and $\sigma_{\neg p} A$, rendering it a partition operation.

In the figures throughout this work we will denote n-nodes as circular and t-nodes as squares or rectangles. When materializing $\sigma_p(A)$ the planner has the option of also materializing $\sigma_{\neg p}(A)$. If deemed beneficial A can be safely deleted as the t-node can be reverse triggered to produce it. A reverse trigger amounts to the union $\sigma_p(A) \cup \sigma_{\neg p}(A)$. It is generally assumed that any row not selected by p is necessarily selected by $\neg p$. To remind the reader of our RA assumptions, this means that we do not support missing or `NULL` values as they break Boolean excluded middle. Furthermore, relations are taken to be unordered sets of tuples.

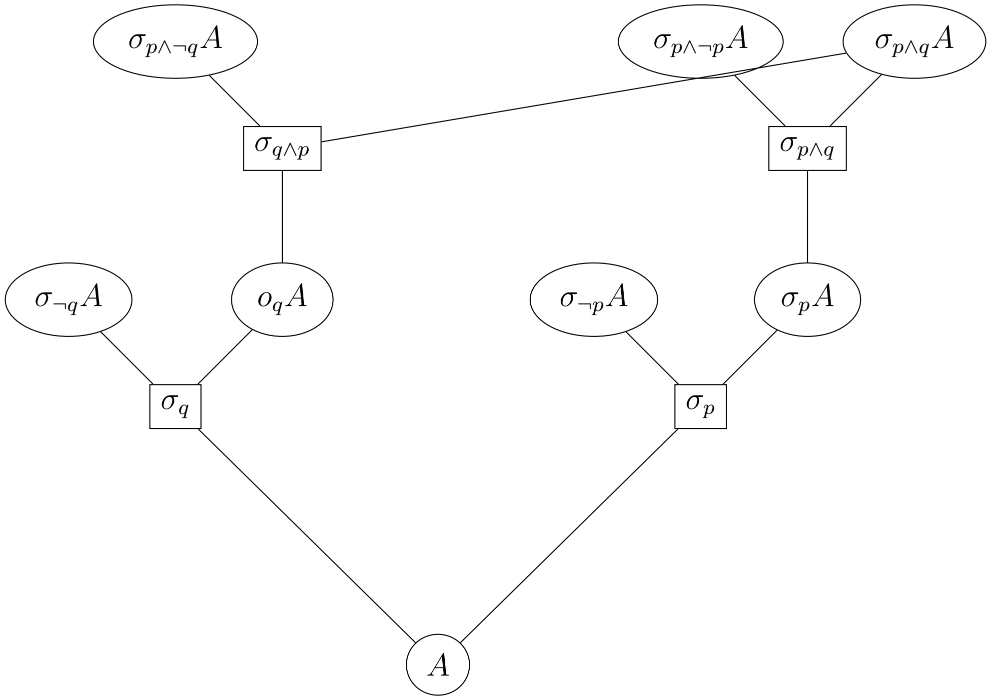


Figure 3.2.: Multiple selections can be composed to create larger networks.

In figure 3.2 if $\sigma_{\neg p}A, \sigma_{q\wedge p}A, \sigma_{p\wedge\neg q}A$ are all materialized, then A is materializable as their union. This way $\sigma_{p\wedge q}A$ is readily available with no space overhead. Instead, the price paid is that in the event that A is required, possibly in a different context, constructing it requires the overhead of unioning all its parts. Furthermore, there are two options for materializing $\sigma_p A$: either by selecting over A, or via the union $\sigma_{q\wedge p}A \cup \sigma_{\neg q\wedge p}A$. Table 3.2 demonstrates the correspondence between relational algebra operators and the equivalence that enables their reverse

Operator	Equivalence
Aggregation ($\gamma_e(X)$)	$\perp$
Sorting $s_e(X)$	$\perp$
Limit ($l_i(X)$), drop ($i(X)$)	$l_i A \cup d_i A \equiv A$
Join ($X \bowtie_p Y$) / Anti-semijoin ($X \cancel{\ltimes}_p Y$)	$\bar{\pi}_{cols(A)}(A \bowtie B) \cup (A \cancel{\ltimes} B) \equiv A$
Selection ($\sigma_p(X)$)	$\sigma_p A \cup \sigma_{\neg p} A \equiv A$
Projection ($\pi_r(X)$)	$\pi_r A \bowtie \pi_{\bar{r}} A \equiv A$

Table 3.2.: Each operation that may end up in the final result and the equivalence that ensures reversibility. $\perp$ values indicate that the operator is not generally reversible.

A concept closer to the functionality of t-nodes and n-nodes than AND and OR nodes, and our very inspiration for this approach, is *propagators* [40]. In short, propagators are hyperedges in a hypergraph, the nodes of which contain partial information about a value. Information about a value can be accumulated from its neighbors. Each node can obtain at least as much information about its value as the most precise neighbor. The example provided in the paper is one of measuring the height of a building by means of a barometer. One approach is timing the barometer's free fall from the top of the building, another is by measuring the length of the shadow of the barometer and comparing with the length of the shadow of the building at the same time of day. One may even use the barometer in its intended function and derive the height of by comparing the difference in air pressure at the roof and at the ground floor. Each of the mentioned methods will give very noisy or partial information about the magnitude in question. In the propagator paradigm, each of these measurements might be represented as a node in a hyper-net. A propagator edge connecting these nodes would combine information from all measurements and improve each of the nodes' accuracy in their perception of the underlying value. A t-node is similar to a propagator in that it does not have a strict direction of information flow and can generate information on either side given information on the other.

3.2.1. Clusters and irreversible edges

Generally, there is no one-to-one correspondence between t-nodes and relational operators. Usually that is the case but there is an exception: the join operator. As we made clear, a t-node can generate any number of its inputs by reverse triggering it as long as all of the outputs are materialized. Then one approach to implementing a join operator might be figure 3.3

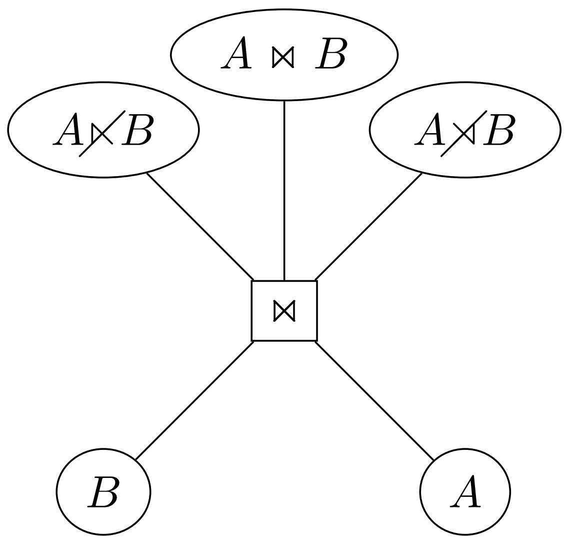


Figure 3.3.: Representing a reversible join as a single t-node.

We could, however be slightly more flexible than that, as we might want to materialize $A \not\rtimes B$ but not $A \not\ltimes B$ to the end of ensuring the materialisability of only A and not B.

In this case neither the entire input nor the entire output are materialized and yet all tables involved are materializable. To overcome the limitation that this poses, we make the graph a bit more complex and a bit more flexible by breaking the join into two stages (see figure 3.4). First, we separate the join operands (A and B) into the part that will be used in the join ($A \ltimes B$ and $A \rtimes B$ respectively), and the part that will not ($A \not\ltimes B$ and $A \not\rtimes B$ respectively) and then we join only the useful parts $(A \ltimes B) \bowtie (A \rtimes B) \equiv A \bowtie B$. Thus, we fundamentally break the 1-1 correspondence between t-nodes and RA operators. Rather, operators correspond primarily to *clusters* of nodes. As we will see in detail in 3.4 the join cluster is the only multi-t-node cluster.

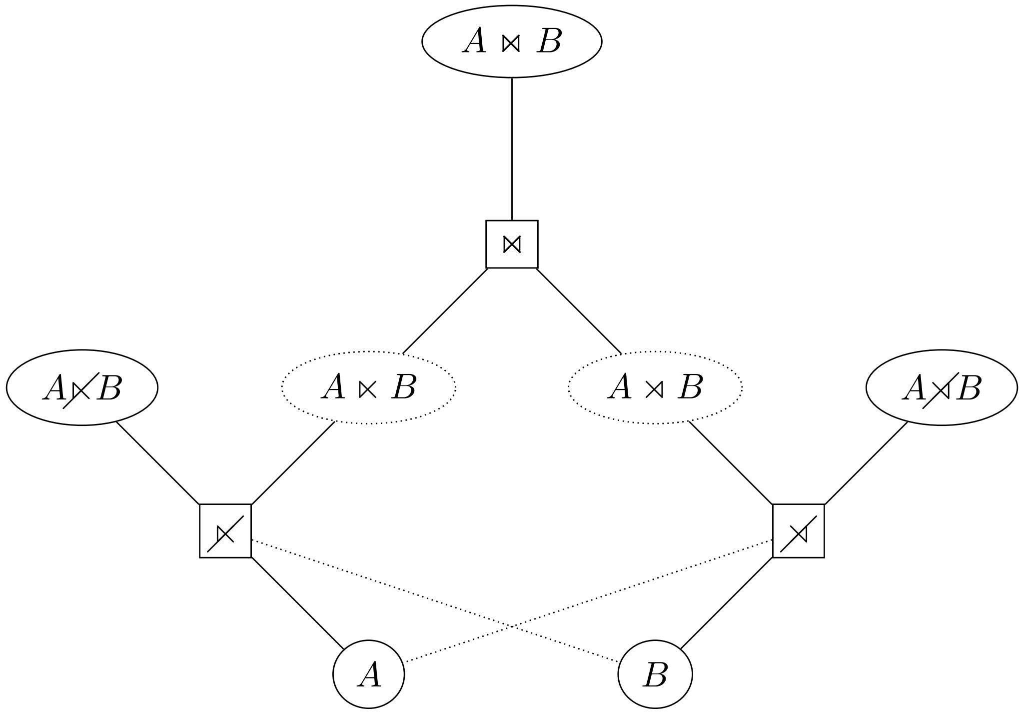


Figure 3.4.: The QDAG corresponding to a join is broken down in multiple t-nodes to facilitate greater flexibility.

The left antijoin node partitions A into $A \ltimes B$ and $A \not\ltimes B$, which is left semijoin in our null-less context. Similarly for the right antijoin t-node which partitions B to $A \rtimes B$ and $B \not\rtimes A$. Joining the semijoin relations $A \ltimes B$ and $B \ltimes A$ is equivalent to joining A and B.

In figure 3.4 we used an antijoin t-node. From the semantics of the antijoin $A \ltimes B$ it is clear that no tuples from B are included in either $A \not\ltimes B$ or $A \ltimes B$ as B acts as a kind of predicate for selecting over A. This means that even though B is an input, there is no natural way of creating B from the t-node's output. For these cases, we introduce *irreversible edges*. Remember that edges are always connections between n-nodes and t-nodes. An irreversible input edge (a connection from an n-node to a t-node) can only be used as an input to the t-node and its n-node can *not* be materialized by reverse triggering the t-node.

Irreversible edges appear in the inputs of anti-join t-nodes but also on the input of operations that cannot be information preserving, notably aggregations. The output of an aggregation can not in general be supplemented with extra information to reconstruct the input, unless the entire input table is replicated. We therefore give up the reversibility of such operations completely and mark the input edge as irreversible. This is generally not a big problem as on the one hand the results of aggregations tend to be small compared to the input relations, and in the case of anti-join t-nodes, which are only found in join clusters, for every anti-join n-node $A \not\ltimes B$ that is unable to generate B by reverse triggering, there is a $A \not\rtimes B$ that will happily generate B albeit refusing to generate A.

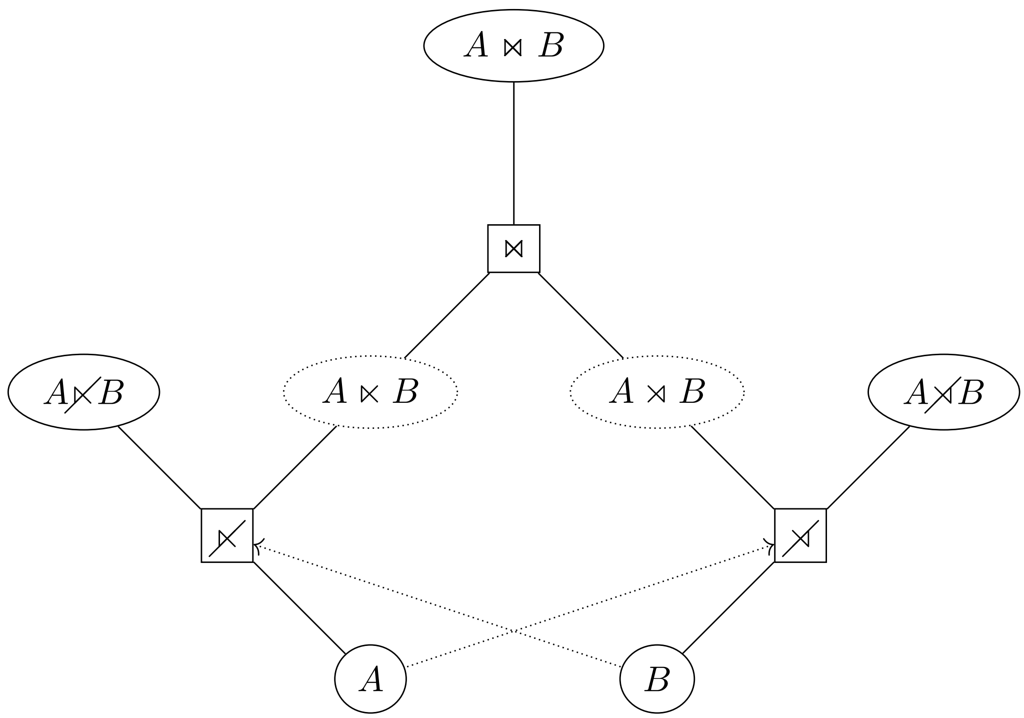


Figure 3.5.: The QDAG corresponding to a join is broken down in multiple t-nodes to facilitate greater flexibility. Notice that the the edges $B \rightarrow \not\ltimes$ and $A \rightarrow \not\rtimes$ are now directed.

3.3. Deterministic query processing

In this section, we will look at the initial steps of what happens when FluiDB receives a query. Initially, the query is accepted in textual form as SQL. Then it is sanitized to adhere to the properties we defined for relational algebra and some trivial optimizations are applied to remove artifacts of the parsing process. Finally, the query is translated to a forest of possible join orderings. These are rewrite-based optimizations that are deterministic and do not take into account the rest of the workload.

3.3.1. Query parsing

We use a custom library for parsing SQL queries into relational algebra based on parser combinator monads [110]. For an introduction to monads see appendix B. Our parser handles both the parsing and decorrelation of the queries. In brief, parser combinators are a functional interface to combining parsers to make larger, more complex parsers. A parser `p` has the form `p a` which means that if the parser is used to parse the matching text, it will return a result of type `a`. For this framework to work, two combinators are required:

- sequence (monadic bind) `(>>=) : p a -> (a -> p b) -> p b` which sequentially tries two combinators and the latter one may use the result of the former.
- the 'alternative' parser combinator `<|> : p a -> p a -> p a` which tries one parser and if that fails tries another.
- Our parser implements `MonadGen e p` which means it can generate a value of type `p e` and does not consume any input. Useful for creating names for unnamed expressions in projections, groups. The function reifying this feature is called `mkSymbol`.

Simple parsers, like the `parseInt` parser combinators, are then combined to create more complex parsers (listing 3.2). This approach is fundamentally not dissimilar to writing yacc files, but it is much more flexible as parser combinators function as a domain-specific language that inherits all the power of the host language, in our case Haskell.

```haskell
parseInt :: MonadParse p => p Integer
parseInt = do
  sign <- fmap (maybe 1 (const $ negate 1)) $ parseMaybe $ char '-'
  void $ parseMaybe $ char '-'
  (* sign) . read <$> readWhile isDigit

```

```haskell
parseSelectProj
  :: (MonadGen e p,MonadParse p)
  => p e -> p (Query e s -> Query e s)
parseSelectProj eM = do
  word "select"
  fmap (Q1 . QProj) $ sep1 (word ",") $ do
  ex <- parseExpr (E0 <$> eM)
  parseMaybe (word "as" >> eM) >>= \case
    Just e -> return (e,ex)
    Nothing -> do
  e <- maybe pop (return . mkSymbol) $ asSymbol ex
  return (e,ex)
```

Listing 3.2.: This parser returns a query modifier that is meant to be applied to a very simple product query generated by the `from` clause. The actual selection parser is complex and handles many different cases, but it is built up from simple fundamental blocks.

3.3.2. Query prepossessing

Immediately after parsing, the queries have several potential inconsistencies with the notion of relational algebra that we described in section 3.1. For that reason, we implement an intermediate stage where we rewrite the query to comply with various properties of the RA, and which remove redundancies. In brief, those are the following:

- Projections and selections are squashed. For example $\sigma_p\sigma_q$ becomes $\sigma_{p\wedge q}$ and $\pi_m\pi_{m'}$ becomes $\pi_{m\circ m'}$.
- Date intervals are translated to dates. This is because the execution engine does not handle dates specially so all dates and date intervals are converted to timestamps and second intervals. For example the expression `date '2020-12-01' - interval '90' day` is immediately turned into `date 2021-03-01`. Note that an expression like `date '2020-12-01' + shipping_time` is not supported as it is not date arithmetic between literals.

- Optimization of the `like` operator: `<VAR> like <STRING_LITERAL>` expressions where `<STRING_LITERAL>` begins or ends with `%` are turned into `suffix_of` and `prefix_of` expressions. For example, the expression `a like "%.txt"` would be turned into `".txt".suffix_of(a)`.

- Substring expressions are turned into 0-indexed ones from 1-indexed as is the case in SQL.

- It is asserted that each intermediate relation has uniquely named columns. For example, there are necessarily name conflicts in $A \bowtie A$.

- It is asserted that sorting appears higher in the query AST than a selection, a join or an aggregation. All three of these operators affect the ordering of their inputs (see chapter 4 on code generation), and sort is the only operation that asserts tuple ordering on its output. Therefore $s(A \bowtie B)$ is valid but $s(A) \bowtie B$ is not.

Once the preliminary sanitisation and simple optimization steps are done we mark each instance of the primary table symbols with the columns and their types. This is simply information derived by the database schema.

Remapping Unique subtuples Each intermediate result, and each relation that we want to be able to reason about needs to have uniquely identifiable rows. For each relation in the QDAG we are keeping track of a set of columns, the unique sub-tuple, the combination of which is unique among each tuple of the same table. It is fairly straightforward to keep track of the unique subtuple for most operations, e.g., the unique sub-tuple of a Cartesian product or join is generally the concatenation of the unique sub-tuples of the input tables, the unique sub-tuple in the case a selection is the same as the unique sub-tuple of the input, in aggregations the unique sub-tuple is the same as the columns on which we aggregate, etc. The main challenge is projections and, in a similar manner aggregations, where it is possible that the operation does not project on the entire unique sub-tuple of the input.

To mitigate this, we perform a pre-processing step to rewrite the projections to expose the entire unique sub-tuple. It is worth mentioning that there are cases where there are more than one combinations of fields that form a unique sub-tuple. We opt to keep track of all of them to have more flexibility in this pre-processing step: ideally we want to modify the projection as little as possible. In other words, we want to project as few extra columns as possible to keep the output table size as small as possible. As a simple example, the query $\pi_{p\_color} part$ would become $\pi_{p\_color,p\_partkey} part$.

Symbol annotation We annotate each symbol referring to a column with the corresponding primary table if there is one (named columns of projections do not correspond to a primary table), and with the type of the column.

This inference is sometimes trivial, for example, in the case of references to columns of primary tables, or when annotating literal values, but when references to columns of projections it is more complex. For example, in the query $\sigma_{p(a)}\pi_{a\mapsto f(X)}$, we need to infer the type of a. FluiDB will apply some very simple heuristics to fake proper type inference and that is usually enough to come up with accurate types as SQL's type system is fairly simple.

Shared unique subtuple annotation on projections As we will see in detail in section 6.4.2 where we will be discussing the implementation details of the projection algorithm and its reverse, it is important that at least one unique tuple of a projection is shared between a projection and its complement. At this stage, it is decided which unique subtuple is to be shared and it is recorded in the RA expression tree.

3.3.3. Possible joins

Join ordering is one of the most detrimental optimizations of a query. It is the selection of the associative order of joins. The relation $A \bowtie B \bowtie C$ can be expressed as $(A \bowtie B) \bowtie C$ or as $A \bowtie (B \bowtie C)$. Individual joins can have a wide range of behaviors. From being similar to Cartesian products, being prohibitively costly, and exploding the cardinality of the result; to being similar to lookups, very cheap, and with a very small result. Optimisers that process queries in isolation can aggressively push down selections and prune the search space of join combinations when they encounter expensive intermediate results. FluiDB, however, needs to keep track even of more expensive intermediate results as the high cost may be amortized in the long run.

For this reason, at this stage we enumerate all possible join permutations building a graph of all possible joins.

- All joins are turned into selection-products, so $A \bowtie B$ becomes $\sigma(A \times B)$
- selections are pulled up and merged, ($\sigma_p \sigma_q A$ becomes $\sigma_{p \wedge q}$)
- products are represented as sets of relational algebraic expressions.

Then all possible permutations of joins are created. For example $\sigma_{a=b \wedge b=c}(A \times B \times C)$ becomes $\{(A \bowtie_{a=b} B) \bowtie_{b=c} C, A \bowtie_{a=b} (B \bowtie_{b=c} C)\}$.

Note that we do not include queries that are only different w.r.t. commutativity, i.e., we don't include both $A \bowtie B$ and $B \bowtie A$. This part of FluiDB depends on the consumer of the data structure to disambiguate between them. Another caveat is that since Cartesian products are very large and are almost never used in the actual plans, an option that disallows product intermediate results is enabled by default. Therefore, in our example we prune relations like $\sigma_{a=b}((A \times C) \bowtie_{b=c} B)$. Furthermore, allowing Cartesian products exponentially explodes the space of possible plans to a degree that makes the resulting data structure virtually unusable.

Once selections are merged, we find product trees, i.e., contiguous parts of the tree that only contain Cartesian products, and combine them into multi-sets of subtrees such that $\sigma(C \times (A \times B) \times D)$ becomes $\sigma(\{A, B, C, D\})$.

Then selection propositions are normalized to conjunctive normal form and into sets. For example $\sigma_{p_1 \vee (p_2 \wedge p_3)} A$ becomes $\sigma_{(p_1 \vee p_2) \wedge (p_1 \vee p_3)} A$, and in fact the conjunctive terms are also represented as sets of subterms, so the final form would be better expressed as $\sigma_{\{p_1 \vee p_2, p_1 \vee p_3\}} A$. This leaves us with a tree of terms of the form $Q[\sigma_{\{p_0, p_1, \ldots, p_l\}}\{Q_0, Q_1, \ldots, Q_k\}]$ where each term p_i refers to columns in one or more Q_j relations and $Q[\cdot]$ is a query tree with $\cdot$ as a subtree. We will refer to such a selection-product pair as an *SP-term*.

To turn this representation into a forest, we will turn each SP-term into a set of regular join trees, the Cartesian product of which will yield the possible joins. We summarize the process as follows:

- We iterate over all pairs of sub-multisets into which the multiset $\{Q_0, Q_1, ..., Q_k\}$ can be split. We follow each of the following steps for each of the resulting sub-multisets.

- We split the p terms in three parts depending on which sub-multiset of relations their symbols refer to. One set are terms where the columns mentioned refer *only* to the left sub-multiset (left p-terms), another set are terms where columns mentioned refer *only* to the right sub-multiset, and the rest (right p-terms) must necessarily have references to columns of *both* sub-multisets. We call the latter category *connector p-terms*.

- We turn each side of the split into an SP-term by associating it with the left and right p-term set. We repeat the process of generating join trees out of each SP-term created and connect them in an all-to-all fashion by creating theta joins using the conjunction of the connector p-terms. In case the the set of connector p-terms is empty, that would mean that the connection of the join trees must be done with a Cartesian product. As mentioned above, we want to avoid inserting Cartesian products in our graph so we reject those cases outright. Optionally, we may even want to reject all non-natural joins altogether, unless that would mean eliminating all possible plans.

The fundamental forest structure of literally collecting each logical plan individually is highly redundant. Therefore, we abstract away the redundancy by representing the forest as a *bipartite tree*.

- One kind of node represents a plan fragment, or a *sub-tree*

- The other kind represents a forest of semantically equivalent subtrees earmarked with a unique hash for them. The fact that the subtrees are semantically equivalent means that they evaluate to the same data. They therefore have the same normal form as discussed in section 3.4 on Query Normal Form (QNF). Finding the normal form hash of one would mean the normal form hash of the rest. This will prove useful when inserting this structure in the graph. We refer to these as *sub-forests*.

In other words, the relational algebra tree is interleaved with OR-nodes of different representations of the subquery. For example $\gamma(\sigma_{a=b \wedge b=c}(A \times B \times C))$ becomes $\gamma\{A \bowtie_{a=b}$

$B) \bowtie_{b=c} C, A \bowtie_{a=b} (B \bowtie_{b=c} C)\}$. This way we mitigate the combinatorial explosion of interleaved nesting joins with other operations.

The algorithm for building the forest out of the pre-processed query is described in listing 3.3.

```
def possible_joins(q):
    # Split a query that is equvalent to
    # sel(p1 and p2 and p3 and ..., prod(q1, q2, q3, ...))
    # into
    # [p1,p2,p3,..], [q1,q2,q3,...]
    and_props,subqueries = as_prod(q)

    # If the query is product just reject it.
    if len(and_props) == 0 and len(subqueries) > 1:
        return

    # if the query is not a join get the possible joins of the subqueries.
    if len(and_props) == 0 and len(subqueries) == 1:
        yield q.map_children(possible_joins)
        return

    # All possible partitions where the partitions are non-empty
    for lqs,rqs in possible_partitions(subqueries):
        # has_free_vars(p,[q1,q2,q3...]) checks if all the atoms in p
        # are either literals or columns in one of the q1,q2,...

        # The props that can be pushed in the left partition
        lps = filter(lambda p: not has_free_vars(p,lqs), and_props)
        # The props that can be pished in the right partition
        rps = filter(lambda p: not has_free_vars(p,rqs), and_props)
        # The props that connect the partitions into a join.
        jps = filter(lambda p: has_free_vars(p,lqs)
                     and has_free_vars(p,rqs),
                     and_props)
```

```

        # If there are no predicates connecting it is product. We
        # disallow products.
        if len(jps) == 0: continue

        # A valid join to be considered while planning!
        lq = sel(lps,prod(rqs))
        rq = sel(rps,prod(rqs))
        yield join(jps,possible_joins(lq), possible_joins(rq))
```

Listing 3.3.: Pseudo-python description of finding all possible joins. For clarity, it is abbreviated to omit sanity checking, memoization, some type conversions, etc.

It is worth stressing that many of the subtrees generated this way will be equivalent. As analyzed in [92] and discussed in section 3.4, in pure functional settings like our, DAGs can have explosive complexity during traversal. Even worse in this case even the space complexity can indeed be needlessly exponential. For that reason we maintain a hash map matching hashes with the corresponding matching subtrees. Before inserting a newly created forest into our structure, we perform a lookup in the hash map. If we find something we throw away the newly created forest for the optimizer runtime's garbage collector to recycle, and we use the one in the hash map instead. Otherwise, we insert our forest both in the hashmap and in the tree. This way we deduplicate our data structure to some degree. Furthermore, in the same vain as [92] we earmark all our forests with their hash so we do not have to traverse them again to compute it. The result of this is that in total we completely traverse the forest once, in order to compute the hashes of each sub-forest and then we use those hashes to test sub-forests for equality.

3.4. Query Normal Form (QNF)

It is imperative that the QDAG has as little redundancy as possible, two subqueries that describe the the same relation should be represented as the same node. Equivalence of RA expressions is infamously NP-complete [1] but we make a best effort to normalize queries such that they are hashable and comparable via simple structural equality. The core of normalization is transforming the queries to Cartesian normal form

$$\zeta_{x_1}(A) \bowtie_{p_1} \zeta_{x_2}(B) \bowtie_{p_2} \zeta_{x_3}(\sigma_{p_3}(C)) \to \zeta_{x1;x2;x3}(\sigma_{p1 \wedge p2 \wedge p3}(A \times B \times C))$$

where $\zeta \in \{\pi, \gamma\}$.

3.4.1. QNF Structure

FluiDB's Query Normal Forms are highly polymorphic data structures separate from RA data structures. The general QNF form can encapsulate one of several special forms, all of which strive to abstract away as many valid RA rewrites as possible, and ideally achieve a one-to-one correspondence between a QNF and a table. While the QNF in its general form (listing 3.4) is specialized to represent several different forms internally to the QNF subsystem, outside of it there are two main objects:

- The normal form of a relational algebra query
- A column of a normal-form query.

```
data QNFQuerySPDCF sel_f prod_f dbg_f col_f f e s =
  QNFQuery
  { qnfColumns :: col_f (QNFProj f e s) (QNFAggr f e s)
    -- ^ What columns kept. Each of these is a column of
    -- QNFProd.
    ,qnfSel :: sel_f (QNFSel e s)
    -- ^ And conjuction of selection.
    ,qnfProd :: prod_f (QNFProd e s)
    -- ^ `QNFProd e s` is a set of multiple rewrites of a single
    -- relation. Essentially {Query (..) (QNFQuery e s)}. Each element
    -- of this set needs to have the same set of QNFQuery's. The binary
    -- operators allowed here MUST expose plans from ONE side. Otherwise
    -- the qnfColumns will be ambiguous. In practice only Product/Join
```

```
    -- expose both sides but if there are more in the future know that
    -- qnf machinery will break.
    ,qnfOrigDEBUG' :: dbg_f (Query e s)
    -- ^ The original query expression for debugging purposes.
    ,qnfHash :: QNFKey e s
    -- ^ Cache the hash. Hashing is done separately for each field so we
    -- don't recompute hashes of unchanged fields after each operation.
  }
```

Listing 3.4.: The QNF datastructure.

In section 3.4.2 on building QNFs, we will look at several specializations that are useful as intermediate. In this section we will focus on each of the fields of the `QNFQuery` constructor.

QNF Product collection

The `qnfProd :: prod_f (QNFProd e s)` field contains the unordered collection of subqueries $\{A_1, ..., A_n\}$ in the equivalent $\pi\sigma(A_1 \times ... \times A_n)$, typically as a hash-set (via the instantiation of `prod\_f` as `HashSet`). Each of the A_i terms may be one of the following:

- A primary table
- A QNF with an opposite kind `qnfCol` (if Q is a projection an A_i may be an aggregation)
- A set of equivalent RA expressions that are not reducible to $\pi\sigma(A_1 \times ... \times A_n)$ form, for example $\{A \ltimes B, B \rtimes A\}$ or $\{l_N(A)\}$

Focusing on the latter case, there are certain equivalent rewrites that are not easily expressible in a QNF. For that reason instead of perpetually extending the QNF system we provide this expensive method of simply enumerating equivalent queries in RA form. These RA queries have at the leaves of the expressions either QNFs or primary tables. The only restriction that we place on these queries is that the schema of the inputs is simply forwarded to the output without modification of the individual fields so that the `cnfColumns` vector can refer directly to them.

QNF Columns, projections and aggregations

The `qnfColumns :: col_f (QNFProj f e s) (QNFAggr f e s)` field indicates which columns are exposed by the query. There are two kinds of column sets: aggregated columns and projected columns. Depending on the context `col_f` is instatiated to a type that allows the column to be *always* aggregation, *always* projection, or either of the two.

Whenever a `QNFQuery` is found outside of the QNF sub-module `col_f` will be instantiated as `Either`, which is to say that a query may be an aggregation (`Right`) or a projection (`Left`). A query with no explicit projection operand at its root is translated to QNF by enumerating the columns into this field.

In each case, the columns themselves are stored in a container of parametric type `f`, which is the main distinguishing factor between QNF columns and QNF queries: a query projection exposes an unordered bag of columns through the projection/aggregation field (`f` is instantiated to `HashBag`) while a QNF column exposes *exactly one* column (`f` is instantiated to `Identity`).

We extend the notion of a column to to define names (see listing 3.5), the atoms of predicates and other expressions (see section 3.1.1). A name in the context of QNFs can be one of three different kinds:

- **A column of a non-primary relation.** This relation *must* be a member of the `cnfProduct` collection of the QNF structure where the name appears. Because there may be more than one equivalent query in the product collection, we index the columns with an integer.
- **A column of a primary table**. These columns have a name and are associated with a table (`s`) which needs to be in the product collection like in the case of non-primary columns. It is indexed much like the non-primary relation.
- **A literal value**

```haskell
data QNFNameSPC sel_f prod_f col_f e s =
  PrimaryCol e s Int
  -- ^ Column of a primary table
  | Column (QNFColSPC sel_f prod_f col_f e s) Int
  -- ^ a column of a QNF.
  | NonSymbolName e
  -- ^ A literal.

-- A column is a QNF that has one element in the projection set
-- (Identity) and no debug query.
type QNFColSPC s p c = QNFQuerySPDCF s p EmptyF c Identity
```

Listing 3.5.: A QNF name may be an unnamed column of a relation, a named column of a primary table or a literal.

In listing 3.6 we present the definition of the projection field of the QNF structure. It departs from the definition we examined in section 3.1 where we described the RA in two important ways, a) no names are provided for the columns and b) the columns are unordered, depending on the definition of `f`. We maintain, however, the definition of each column as an `Expr`. FluiDB makes no attempt to normalize the expressions, so equivalent expressions that differ syntactically will produce different QNFs.

```haskell
type QNFProj f e s = f (Expr (QNFName e s))
```

Listing 3.6.: A QNF projection field is a collection of expressions that refer to QNF names. The particular structure of this collection is parametric. When the collection `Identity` the QNF query is essentially just a column. A normal QNF query would instantiate `f` to `HashBag`, an unordered multiset.

As mentioned, the columns in the QNF name, both primary and of general relations, are indexed in the case that there are more than one equivalent. This is an edge case that can cause problems because different indexing can cause equivalent n-nodes to have different QNF representations. For example, the query `select * from A as A1, A as A2 where A1.a = A2.a` is symmetric in all respects and therefore the way QNF names representing columns `A1.a` and `A2.a` are indexed makes no difference provided that the equality expression `A1.a = A2.a` is ordered consistently.

Consider, however, the query `select * from A as A1, A as A2 where A1.a = A2.a + 1` which is not symmetrical. Indexing `A1` and `A2` differently does not change the semantics of the query, but it changes the QNF representation. This is not commonly a problem, a rare edge case rather, but it is important to keep in mind. In an attempt to mitigate this *all* different combinations of indexing are emitted by the QNF builder (see section 3.4.2).

From the perspective of QNFs, aggregations (defined in listing 3.7) are very similar to projections. However, they only operate on the non-recursive `Aggr` algebra, in contrast to the `QGroup` unary RA operator that incorporates `Expr (Aggr (Expr e))` typed expressions.

QNF moves the inner and outer `Expr` to one level up and one level down respectively, so an aggregation $\gamma_{l \mapsto f(g(h(x)))} S$ would be rewritten to the normalized $\pi_{l \mapsto f(l_1)} \gamma_{l_1 \mapsto g(l_0)} \pi_{l_0 \mapsto h(x)} S$.

```
type QNFAggr f e s =
  (f (Aggr (QNFName e s))
  , HS.HashSet (Expr (QNFName e s)))
```

Listing 3.7.: The QNF aggregation form of the projection field is similar to projection only, much like the `QGroup` constructor, it also includes a `HaskSet` of exprssions on which to group.

QNF Selection

The `qnfSel :: sel_f (QNFSel e s)` field contains an expression of the selection predicate (see listing 3.8). With very few exceptions `sel_f` is instantiated as an unordered hash containing the terms of the Cartesian normal form of the selection. For example a `qnfSel` with value $\{p_0, p_1, p_2\}$ corresponds to the selection predicate $p_0 \wedge p_1 \wedge p2$.

```
type QNFSel e s = Prop (Rel (Expr (QNFSelName e s)))
```

Listing 3.8.: Selection name refers to a version of the current QNF that has all fields erased except the projection.

We went into detail in the previous section how the atoms of the predicates in QNFs, when they refer to columns, take the form of QNF structures themselves. Further, we asserted that the columns in the expressions of projection/aggregations refer to the product. Similarly, selection names refer to columns of the product. This means that in the QNF of the expression $\sigma_{f(a)}(Q_0 \times Q_2 \times ...)$ the atom a is encoded as a column of one of the Q_i subqueries.

This way we can update the fields of the QNF without needing to keep the columns in sync with the columns referenced in `qnfSel`. This eliminates the need for both indexing the columns and for dealing with primary columns in `QNFSelName`. This has to do with the way selections are being created: the RA expression $\sigma_p A$ is translated by first translating A to a named QNF. Then p refers directly to the columns of the query.

QNF Misc fields and hashing

We define two more fields in the generic QNF datastructure we described. `qnfOrigDEBUG' :: dbg_f (Query e s)` is the more straightforward one. It simply keeps track of the query used to create the QNF for debugging purposes and is disregarded by both the equality and hashing operations. Typically `dbg_f` is instantiated to `EmptyF`, an empty container to save on memory.

The other field is `qnfHash :: QNFKey` (see listing 3.9). Because the QNF is a highly self referential data structure equality in a purely functional environment like Haskell traversing it for the purposes of equality or is a costly operation. For that reason, we depend on hashes to detect equivalence, and we cache those hashes to avoid recomputing them, leaning on the purity of Haskell. As the QNF has three distinct parts that are being often updated during building of a QNF `qnfHash` is represented as a tuple of their respective hashes. This way, when one of the fields is updated, we do not have to recompute the hash of each of the others.

```haskell
type QNFKey e s = (Int,Int,Int)
```

Listing 3.9.: A key that uniquely identifies a QNF is made of three separate hashes, one for each part of the QNF structure so that they can be updated independently.

Since QNFs are rarely used as anything other than a token for relating RA form queries FluiDB saves on memory and use `QNFKey` in place of the `QNFQuery` structure (see listing

3.10). On the other hand, in a more pedantic but correct mode, we can avoid depending on the quality of the hash function and define equality as recursively checking between all types (see listing 3.11). In practice, we have never found the former method to yield a false positive for equality.

```
instance Eq (QNFQuerySPDCF sel_f prod_f d col_f f e s) where
  x == y = qnfHash x == qnfHash y
```

Listing 3.10.: A fast and loose definition of equality between QNFs that depends on the quality of equality.

```
instance Eq (QNFQuerySPDCF sel_f prod_f d col_f f e s) where
  x == y = qnfProd x == qnfProd y
    && qnfSel x == qnfSel y
    && qnfColumns x == qnfColumns y
```

Listing 3.11.: A very inefficient but correct equality between QNFs.

3.4.2. QNF Computation

As it became clear while describing the parts of a general QNF in many cases transformation algorithm from RA to QNF can be easily inferred. In this subsection, we will look at a few parts of the QNF building process that are maybe less than obvious.

QNFs are built bottom up from the leaves to the top. Building the QNF we apply the operations incrementally to a named QNFs (NQNF). Applying the operator σ_p to a QNF A we need to be able to to relate the symbols appearing in p with the columns of the QNF A. To mitigate this, we define a *named QNF* which is simply a tuple of a QNF along with a mapping between symbols and columns of that QNF. For example, the name map in the NQNF derived from the query $\pi_{n \mapsto f(a)} A$ would map the symbol n to a column with the elements $\{f(a) | a \in A\}$.

The entire process of computing QNFs works within a monad stack with the following features:

- A `ListT` monad transformer allows for non-deterministic computation in order to account for the different possible column index assignments described in subsection 3.4.1 on QNF projections/aggregation.

- Since the resulting QNF is a highly self-referential structure, it is to be expected that a lot of the computation might be duplicated. The cache object is also shared between different QNF building processes that we know are likely to have have a lot of overlap, for example, when inserting n-nodes into the graph without shared caching between QNF computations, FluiDB would be repeatedly computing the QNF of the same RA subqueries. This takes the form of simply memoizing the interface functions for building the QNF.

- The process of building a QNF can fail when processing malformed RA forms.

These characteristics are encoded in the monad within which we build the QNF (listing 3.12)

```haskell
-- | The monad in which the computation happens. Not all transformations
-- require non-determinism.
type QNFBuild0 e s = StateT (QNFCache e s) (Either (QNFError e s))
type QNFBuild e s = ListT (QNFBuild0 e s)
```

Listing 3.12.: QNF computation monad provides non-determinism, caching, and error handling.

Every QNF building function accepts one or two `NQNFQuery` forms and a unary or binary relational algebra operator to be applied to the query. It returns an `NQNFResult` (listing 3.13) contains

- A named QNF which is the main payload of the QNF building function.

- A mapping between input and output QNF columns. When there is no direct mapping (like in the case of aggregations), this is an empty list.

- The RA operator that was provided as input with the expression atoms translated to QNF names referring to the input or output columns. The way the names in the operator expressions are translated to `QNFName`s is specific to each operator. For example, in the selection operator, all atoms are directly translated using the name map of the input `NQNFQuery`. When the operator is a projection $\pi_{\{n_0 \mapsto K_0, n_1 \mapsto K_1, ...\}}$, the builder function takes as input a QNF Q_{IN} and the parameters of the projection $\{n_0 \mapsto K_0, n_1 \mapsto K_1, ...\}$, and outputs a `QNFResult` containing the QNF Q_{OUT}. In the projection operator output, the left hand side atoms of the projection $\{n_0, n_1, ...\}$ are columns of Q_{OUT} and the atoms in the right hand side expressions $K_0, K_1, ...$ are columns of Q_{IN}. This is for convenience assuming that the RA query that was just normalized is a logical plan. The top level operator can then be used to annotate QDAG elements that relate to the normalized query. The importance of this is analyzed in depth in section on symbol correspondence.

```
data NQNFResultDF d f op e s =
NQNFResult
{ nqnfResNQNF :: NQNFQueryDCF d Either f e s
  ,nqnfResOrig :: op
  -- ^ The operator translated with the symbols translated to
  -- QNFName
  ,nqnfResInOutNames :: [((e,QNFCol e s),(e,QNFCol e s))]
  -- Map the names exposed by the input qnf to names on the output
  -- qnf. Unless qnfResOp is QProj, QGroup each input name corresponds
  -- to 0 or 1 name in the output. Thus, if qnfResOp is QProj or
  -- qnfResInOutNames has no meaning and will be `const Nothing`
}
```

Listing 3.13.: The internal QNF building functions provide some more information that was created during the generation of the QNF, precisely a name map relating column names to QNF names, a map relating input QNF names to output QNF names, and the top level operator with the names translated appropriately to input or output QNF names.

The details of most QNF functions are fairly mundane and easy to infer with the possible exception of products (listing 3.14). When calculating the the product of two NQNFs Q_1 and Q_2 we first find conflicts, where columns of Q_1 can be found in the product collection of Q_2. If none are found, we can simply merge all of their fields and recalculate the hashes. If there is at least one conflicting pair of `QNFName`s (c_1, c_2), non deterministically increment, the index of c_1 or c_2. For example, if we want to compute the QNF of the product $\sigma_{a_0=1}(A_0) \times \sigma_{a_0=2}(A_0)$ we get in QNF-like form $\sigma_{a_0=1 \wedge a_1=2}(A_0 \times A_1)$ and also $\sigma_{a_1=1 \wedge a_0=2}(A_0 \times A_1)$.

```python
def qnf_product_code(nqnf1, nqnf2):
    # The named qnf is a pair of a map names→columns and a QNF.
    nm1, qnf1 = nqnf1
    nm2, qnf2 = nqnf2
    # Find relations in the product set that could cause name
    # conflics.
    conflicts = [c for c in qnf1.qnfProd if c in qnf2.qnfProd]
    for c in conflicts:
        # Python does not support non-determinism but with ListT we
        # can do it in haskell. The `nondet' function non-deterministically
        # selects one of the arguments and returns
        # it. The result of the overall computation is a stream of all
        # possible results.
        #
        # The `increment_indices` function returns the same named CNF
        # with all columns' indices that correspond of the argument CNFProd
        # incremented.
        ncnf1, ncnf2 = nondet(
            (increment_indices(ncnf1, c),ncnf2)
            (ncnf1,increment_indicse(ncnf2, c)))
    return ncnf1 + ncnf2
```

Listing 3.14.: The QNF product algorithm finds name conflicts between the operands and non-deterministically increments one of the sides. In FluiDB non-determinism is handled by the `ListT` monad.

3.4.3. Pragmatism of QNFs

As mentioned, query equivalence is an NP-complete problem. QNF is only a heuristic that does fail to identify many cases of equivalent RA queries. For example, our design of QNFs does not capture the equivalence $\sigma_p A \cup \sigma_{\neg p} A = A$. It captures enough rewrite rules, however, to be useful in most common cases. In particular, it covers the following RA properties:

- $\sigma_p \sigma_q A = \sigma_{p \wedge q} A$
- $\pi_m \pi_n A = \pi_{m \circ n} A$
- $A \bowtie B = B \bowtie A$
- $(A \bowtie (B \bowtie C) = (A \bowtie B) \bowtie C$
- $\sigma_\theta (A \times B) = A \bowtie_\theta B$
- $\pi_m \gamma_{n,g} A = \gamma_{m \circ n, g} A$
- $\gamma_{m,g} \pi_n A = \gamma_{m \circ n, g \circ n} A$
- $A \cancel{\ltimes} B = B \cancel{\rtimes} A$ *(via explicit enumeration in* `QNFProd`*)*
- $A \cup B = B \cup A$ *(via explicit enumeration in* `QNFProd`*)*

3.5. Query cluster internals

Clusters are the elements of the highest organization level of the graph. This section describes the semantic role of the cluster as a set of connected n-nodes and t-nodes. Clusters represent *logical operations* and they are the connection between the basic graph and the relational algebra semantics.

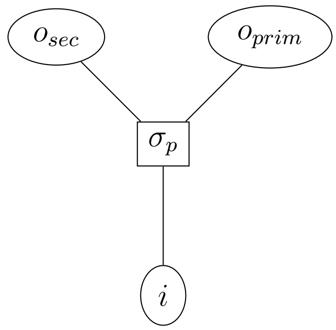
3.5.1. Cluster polymorphism

As alluded to when describing the QDAG in section 3.2, clusters come in different shapes depending on what the underlying operator is. In this section we will take a closer look at the structure of the different cluster types. Each type of cluster has input, output and intermediate slots, and each of the slots typically contains an n-node reference and an RA operator that relates the n-node to the cluster. They also contain slots to accommodate the t-nodes connecting the said n-nodes. In dealing with clusters, it is assumed that input and output n-nodes contain values that may be shared by other clusters. In contrast, intermediate n-nodes and t-node slots are always completely private to each cluster.

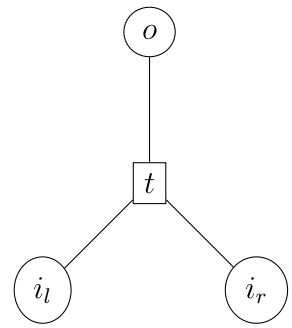
We define are three different kinds of clusters and a unified type `AnyCluster`:

- `NClust` is a cluster containing exactly one slot which corresponds to a primary table.
- `UnClust` a cluster corresponding to a unary operation. It has two slots in the output position, one slot in input position, no intermediate n-nodes and a single t-node slot (figure 3.6a).
- `BinClust` is a cluster corresponding to a non-join binary RA operation. It has one output slot, two input n-node slots and a t-node slot (figure 3.6b).
- `JoinClust` augments the `BinClust` with two intermediate n-nodes, two extra output n-nodes, and two input t-nodes to facilitate the antijoin results of a FluiDB join operation (figure 3.6).

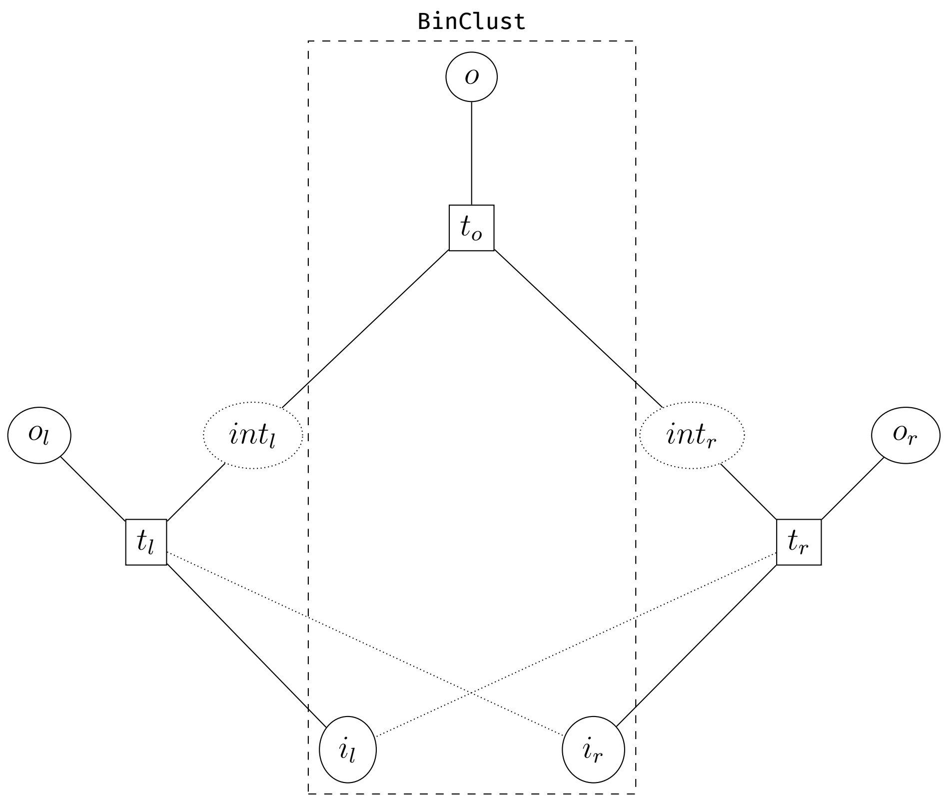
Every cluster must always be hashable and the hash is cached inside the cluster itself.



(a) The structure of a `UnClust`.



(b) The structure of a `BinClust`.



(c) The structure of a `JoinClust` contains a `BinClust` and includes two intermediate and two complementary output n-nodes to facilitate data related to antijoins.

Figure 3.6.: There are four cluster types. The `NClust` is trivial, just a single n-node. The other 3 are `BinClust`, `JoinClust`, and `UnClust` representing binary, join, and unary operations respectively.

3.5.2. Populating the graph with clusters

Once the query is transformed into a forest of possible join orderings as described in section 3.3.3 on enumeration of possible joins, the query is transformed into a forest of plans we can insert it into the tree (listing 3.15). This is an iterative process where each operation in each of the trees in the forest is translated to a cluster which is then inserted into the QDAG. The process is inserting a sub-forest or subtree into the QDAG has the effect of extending the QDAG by the connected clusters corresponding to the operators at the nodes of the forest, and also returns an n-node that corresponds to the subforest or sub-tree relation. However, since the insertion function is idempotent, i.e., inserting the same forest in the QDAG more than once has no effect either on the returned value or on the resulting QDAG, we memoize the function. Since the same function recursively inserts all sub-trees and sub-forests we get for "free" a mechanism of not traversing the same tree twice during insertion.

```haskell
data QueryForest e s =
  QueryForest
  { qfHash :: Int
    ,qfQueries
      :: Either
        [(Query e (QueryForest e s))] -- An actual forest
        (s,RelationShape e s) -- A final leaf
  }
```

Listing 3.15.: The definition of the query forest. The query forest is hashed so that we can avoid traversing the same query forest repeatedly. The query forest is essentially a non-empty of queries with forests at their leafs.

The process of traversing the forest and inserting the clusters in the graph is fairly straightforward. The actual creation of the clusters, however, is slightly more convoluted. First, a cluster of a type corresponding to the query operation at the forest node is created (see section 3.5.1 on cluster polymorphism). That cluster initially contains QNF forms at its edges. The QNF forms are then matched against existing QNFs in the QDAG and the missing ones are generated. If no new nodes were generated and the matched nodes all correspond to the same existing cluster, we infer that the cluster is already in the QDAG and the process finishes. Otherwise, we need to create and insert a cluster and a corresponding propagator (see section 3.6.1). Finally, all new nodes, QNFs, and the cluster are registered and cross-matched so they are easily cross referenced. We memoize the entire process using the `qfHash` to avoid re-traversing the same path.

As an example of what a more realistic QDAG looks like, first run through all 12 queries of the SSB TPC-H benchmark (see chapter 7) produces the graph presented in figure 3.7.

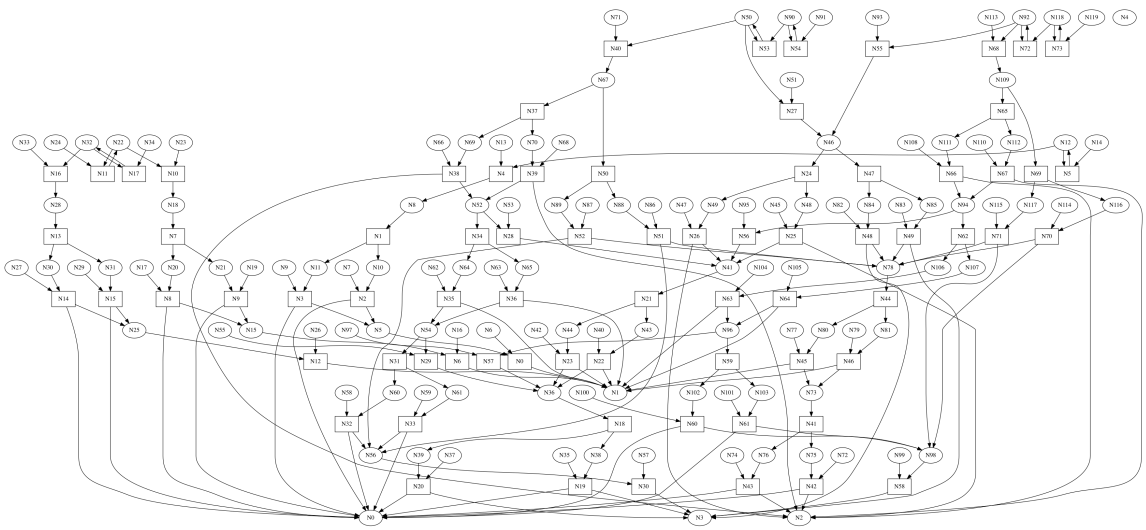


Figure 3.7.: A QDAG accumulated out of every query in the SSB TPC-H. This figure is meant to provide an intuition of the scale of a QDAG.

3.6. Relation shapes

In section 3.4 on QNFs we discussed the need to abstract away information that pertains to the particular structure of a relation in favor of semantic information that is rewrite and data independent. Relation shapes cover the opposite need: they encode the particular cardinality and shape of a relation at a specific point in time.

A relation shape is a data structure that contains information about the shape of the result of a query.

```
data RelationShape' e' =
  RelationShape
```

```
  { rsSchema :: [(e',ColumnProps)]
    ,rsUnique :: NEL.NonEmpty (NEL.NonEmpty e')
    ,rsSize :: RelationSize
  }

type RelationShape e s = RelationShape' (ShapeSym e s)

data ShapeSym e s =
  ShapeSym { planSymQnfName :: QNFName e s
    ,planSymQnfOriginal :: e
  }
  deriving (Show,Generic)
```

Focusing on the field `rsUnique`, a shape contains all combinations of columns that comprise a unique subtuple. This is useful primarily to determine foreign key joins. The expression

$$Q_1 := A \bowtie_{id_{A\to B}=id_B \wedge id_{A\to C}=id_C} (B \times C)$$

behaves like a foreign key join while the join when id_B is the only element of the unique subtuple of B and id_C is the only element of the unique subtuple of C.

The following, however, does not

$$Q_2 := A \bowtie_{id_{A\to B}=id_B} (B \times C)$$

because for the expression $B \times C$ neither id_B nor id_C are unique on their own. Only their combination is. Therefore, the cardinality $|Q_1|$ is at most the size of A because each row of A matches at most one row of $B \times C$.

So the unique subtuples are computed using the following rules:

- The set of unique subtuples of $A \bowtie B$ is computed by concatenating all combinations of the unique subtuples of A and B.

- The unique subtuples of a selection, semijoin, antijoin, and limit are the same as the unique subtuples of the underlying relation.
- The unique subtuples of a projection are the unique subtuples of the input that are entirely exposed. The reader is reminded that FluiDB's notion of RA requires that there is at least one such unique subtuple, and that this requirement is facilitated by the query pre-processor.

The rest of the fields are maintained by *shape propagators*.

3.6.1. Shape propagators

Shape propagators are based on the the idea of the propagator as first introduced in [41] and analyzed more in depth in [96] and mentioned in section 3.2. Very briefly, a propagator network is a hypergraph of cells containing mutable state and hyperedges as multi-directional functions that, when triggered, update the state of the connected cells so they are consistent with each other. The state of each cell represents some information about a value (like a bound or a probability distribution), and the propagator takes into account the state of each cell and propagates that information to the other cells (for example tightening their bound or finding a linear combination of alternative values based on the certainty of each). Kmett [108] formalized the notion of "information about a value" drawing from [62] encoding it as a lattice with $\top$ meaning contradiction, and $\bot$ meaning that there is no information about the value. Then the new values of cells are being joined ($\vee$) with the old ones and the value is updated to reflect the combination of information. The example provided by Kmett is each cell value being embedded in a 3 level lattice where the bottom level is `Nothing`, the middle level is `Just x` and the top level being a Haskell error (ironically denoted as $\bot$ in literature unrelated to propagators).

As the name of our propagator implementation suggests, our idea is to correspond each cluster to a propagator to form a propagator network that computes the shapes of the relations corresponding to n-nodes.

We model a propagator as a partial function that changes the values at the edges of a cluster. Each cluster is equipped with a propagator which, when triggered synchronizes the values at the edges of the cluster. The partiality of the function stems from the fact that it might detect irreconcilable inconsistencies between the cells (see listing 3.16). This conception of the propagator departs slightly form the conception described in the paper in that it clearly separates the *cluster* as a data structure that holds a fixed number of interdependent values, and the function that updates the values in that cluster.

```
type ACPropagator a e s t n =
  EndoE e s (PropCluster a NodeRef e s t n)
type EndoE e s x = x → Either (AShowStr e s) x
type ShapeCluster f e s t n =
  PropCluster (RelationShape e s) f e s t n
type PropCluster a f e s t n =
  AnyCluster' (ShapeSym e s) (WMetaD (Defaulting a) f) t n
newtype WMetaD a f b = WMetaD { unMetaD :: (a,f b)}
```

Listing 3.16.: A propagator matches a cluster with shapes at the edges to the same kind of cluster with the shapes synchronized.

Given a cluster with n-nodes at the edges, we look up the shape of each n-node and add that to the cluster edge. Once a cluster is triggered and the values synchronized, those values are checked against the old ones and for the ones that were updated we find the clusters in which they participate and run the same process. The convergence of this process is justified in [62].

3.6.2. Defaulting functor

As mentioned, the cells of a propagator involve more structure than raw values. Indeed, FluiDB wraps these values in a functor that we dub as *defaulting functor (DF)* to reason about the consistency of the relation shape. In particular, we require that the cell can handle cases where

- The shape is not computed yet, i.e., there is no information about the value.
- The shape is inferred via the propagator network and is therefore subject to change and

- The shape stored in the cell corresponds to a materialized n-node and other parts of the FluiDB's internal state depend on it.

The latter case is salient. Different logical plans can lead to different, yet semantically equivalent as per the FluiDB RA.

The DF may hold up to two values (see listing 3.17).

- An empty DF means we have no information about the value.
- A single value (the "default" value) corresponds to a derived shape that was computed using the propagator network. The default value is updated when propagators are triggered.
- A full value exists alongside a default value and is constant through its lifetime as it matches the value of some state that is external to the propagator network, in our particular application it represents the shape of a materialized relation. Until that relation is deleted, and the full propagator is demoted to a defaulting one, the registered shape is not allowed to change.

```
data Defaulting a =
  DefaultingEmpty
  | DefaultingDef a
  | DefaultingFull a a
```

Listing 3.17.: The defaulting functor definition.

Since instances of the DF are meant to accommodate propagator values, adhering to Kmett's approach to cell value maintenance, we define a semilattice over the DF and define it in terms of a monoid (listing 3.18).

Semigroups are not commutative, so we afford meaning to the order of the operands. The left operand of the semigroup operator is the newer value and the right one is the older value. Therefore, the DF does not require all the properties of the semilattice which is required to commutative and associative. In general, the semigroup implemented by the underlying type of the DF is taken to mean $better \diamond backup$, i.e. keep as much of the left operand as possible unless the right operand is found to be "better", for some definition of better. For example, when calculating the cardinality we include a value of certainty between $[0, 1]$. A more certain right-hand cardinality is preferred over a less certain left-hand side.

Bringing our attention to the implementation of the `Def <> Full` combination in the semigroup definition 3.18 it is worth noting that it inverts the order of the combination of the default values. The reason we do this is to give priority the the full valued (materialized) n-nodes when propagating to their immediate neighbors. We need this when committing an operator to the code generation.

```haskell
instance Semigroup a => Semigroup (Defaulting a) where
  DefaultingEmpty <> a = a
    a <> DefaultingEmpty = a
  DefaultingDef a <> DefaultingDef a' = DefaultingDef $ a <> a'
  DefaultingDef a <> DefaultingFull a' b' =
    DefaultingFull (a <> a') b'
  DefaultingFull a b <> DefaultingDef a' =
    DefaultingFull (a <> a') b
  DefaultingFull a b <> DefaultingFull a' _ =
    DefaultingFull (a' <> a) b

instance Semigroup a => Monoid (Defaulting a) where
  mempty = DefaultingEmpty
```

Listing 3.18.: The monoid define over the DF is the right semilattice. A DF contains up two values, the "default" and the "full" value. The "full" value refers to some property external to the defaulting functor, namely a materialized relation, and therefore is not subject to change. The "default" value is gradually refined through monoidal combination. Combination of DFs is associative but not commutative: the right operand is expected to be the more recent one. Therefore we keep the left hand side "full" if we are combining two of them.

When committing to a query plan, we are also committing to full values for the defaulting values in the propagator cells. Every operator of the plan corresponds to a relation shape propagator. When committing an operator to the plan by translating it into C++ code, we promote all the defaulting values at the edges. It is clear that the generated code expects a certain data layout and therefore the *full parts of the values must be structurally consistent with each other*.

As we went over in the previous sections, however, the default values are consistent with other default values of the same cluster w.r.t. the amount of information they hold, but not w.r.t. the order of the columns in the relation shape. Therefore, before promoting the default functors to full functors by duplicating the default value to fill both fields of the `DefaultingFull` constructor, we trigger the propagator to enforce full consistency between the `DefaultingDef` values.

The full part and the default part of the the defaulting value are not necessarily structurally synchronized. We *internally synchronize* a cell before triggering the propagator to make sure we maintain the consistency of the newly promoted cells with the already existing full cells. It is notable that there is no formal guarantee that the propagator will be able to create structurally consistent cells. In that case the propagator will fail. Handling of this failure gracefully is beyond the current scope of this work.

When an n-node is to be materialized, the corresponding defaulting functor containing the value is *promoted*. Promotion of propagators is copying the default value of a non-full propagator to the full state (listing 3.19).

```
promoteDefaulting :: Defaulting a → Defaulting a
promoteDefaulting = \case
  DefaultingEmpty        → DefaultingEmpty
  DefaultingDef x        → DefaultingFull x x
  d@(DefaultingFull _ _) → d
```

Listing 3.19.: Promoting of defaulting functor happens during code generation when an n-node is materialized.

3.7. Summary and conclusion

In this chapter we detailed how FluiDB processes queries at the logical level. From parsing to generating a graph of queries (QDAG) and inferring the shapes of the relations in that graph. We also discussed different conceptions of relational algebra that are used in FluiDB (normal RA and QNF) and reverse operations. This will serve as a solid bedrock on which we can build the query planner and on top of the code generator.

There are a few shortcomings to this model that we have not yet addressed but we believe do not bar FluiDB from being a complete system. The most important ones being:

- As things stand, the QDAG will keep growing infinitely as new queries arrive and there is no obvious way to prune it while guaranteeing the materializability of all n-nodes.
- There is no obvious path to supporting updates in the primary tables. There has been a lot of work on materialized view maintenance but most of it assumes that the primary tables are already materialized.

CHAPTER 4

Physical planning

We may note in passing, one peculiarity in regard to all the final resolutions taken by [Raskolnikov] in the matter; they had one strange characteristic: the more final they were, the more hideous and the more absurd they at once became in his eyes. In spite of all his agonising inward struggle, he never for a single instant all that time could believe in the carrying out of his plans.

(F. Dostevski – Crime and Punishment)

Chapter summary

- We introduce a monad-based functional method of weighted backtracking search that supports *once* and *fallback*, two cut-like operations.
- The FluiDB query planner follows an $A^{\star}$-like method for searching plans are searched in a top-down fashion, guided by a cost model that takes into account the cost historical queries.
- The query planner can make use of *forward* or *reverse* triggering of operators.
- The garbage collector clears up space on demand maintaining the materializability of all n-nodes in the graph.

This chapter goes in detail about the architecture of the logical planing of a query. Query planning is based on an $A^{\star}$-like search algorithm through the space of partial plans. We initially describe the fundamental computational structures that facilitate this search and their unique properties that allow us the necessary flexibility. Then we discuss the basic backtracking algorithm and the way the graph is traversed. We move on to discussing the garbage collection that generates plan fragments that allow the plan to free up space in the underlying storage. Finally, we talk about the cost model that guides the plan search process.

In this chapter, we introduce `HCntT`, a new monad for implementing branch and bound search. While abstracting search algorithms using monads has many benefits like composability and low boilerplate allowing the implementation of complex heuristics with low engineering effort. As we discussed there has been little work towards encapsulating best-first traversal in a monad. The `HCntT` monad transformer fills this gap by combining `MonadPlus` with soft-cut and the `halt` operation that will describe in section 4.1.2.

4.1. HCntT logic monad

The FluiDB planner is designed in terms of a backtracking search algorithm that searches in the space of subplans for an optimal plan. Because the algorithm involves a lot of complicated heuristics it is important that a powerful underlying framework is deployed that matches the purely functional infrastructure in which FluiDB is implemented. In particular, we require a monad that supports both weighted search and soft cuts. Because none of the solutions we found in the literature support both we developed the `HCntT` backtracking monad that is described in this section.

4.1.1. Background

Monads for backtracking in a functional context have been proposed in various incarnations. The most common monad that encapsulates this functionality is the `ListT` monad transformer `ListT m a = forall b . (a → m b → m b) → m b → m b`, also dubbed the Church encoding of lists or an application of the Cayley theorem on lists. It was first proposed (in an untyped form in scheme) in [3] where Haynes explicitly uses it to implement a logic programming framework back in 1987, but most authors in the field cite [18] as the seminal work on the concept where the authors seem to have independently re-discovered the application of the concept in 2000 this time in strongly typed ML, and also [107] where the authors focused on a notion of fairness in backtracking. As shown in [97] this representation has $O(n^2)$ BFS complexity.

The other common representation of a list monad transformer is the more straightforward

```
newtype ListT m a = ListT (Maybe (a,ListT m a))
```

Which mirrors precisely the idea behind `MonadLogic` from [107]. As demonstrated in listing 4.1 the `MonadLogic` able to `cons` (dubbed `reflect` which directly corresponds to the `ListT` constructor) and `uncons` (dubbed `msplit` which directly corresponds to the underlying type wrapped by `newtype`).

```
class MonadPlus m ⇒ MonadLogic m where
  msplit :: m a → m (Maybe (a,m a))
  -- ... other methods are defined in terms of msplit

reflect :: Maybe (a,m a) → m a
reflect Nothing = mzero
reflect Just (a,as) = pure a <|> as

-- msplit >=> reflect ≡ return
```

Listing 4.1.: The logic monad typeclass

In [107], on top of this framework the authors build an `interleave` and a monadic bind (>>-) operator that works with it to make computation be slightly more "fair". In this context, fair means that the branches share computation resources, as opposed to an "unfair" approach where a branch is evaluated to completion or failure before other branches are considered. Interleaving is fair between two computations, but when more than two are involved, half the time goes to the first computation, out of the half that is left, half ($1/4$ of the total) goes to the second, out of the $1/4$ left, half ($1/8$) goes to the third and so on. While this power-series that describes the way the way resources are allocated to branches may seem arbitrary, in practice, it may mean the difference between terminating and non-terminating computations and is thus fair in the temporal model-theoretic way. For example, the code in listing 4.2 can be translated to something like the code in listing 4.3 which does terminates due to the fact that while it does not give the same chance to all the branches it does not completely starve any of them.

```haskell
nonTerm :: [(Int,Int,Int)]
nonTerm = do
  (a,b,c) <- (,,) <$> genNaturals <*> genNaturals <*> genNaturals
  guard $ a + b - c == 10
  return (a,b,c)
```

Listing 4.2.: Using a simple list to drive non-determinism is implicitly equivalent to a DFS algorithm which in many useful cases does not terminate.

```haskell
interleaveTest :: [(Int,Int,Int)]
interleaveTest = runLogic @[] $ do
  (a,b,c) <-  (,,) <$$> genNaturals >*< genNaturals >*< genNaturals
  guard $ a + b - c == 10
  return (a,b,c)
```

Listing 4.3.: Interleaving (in this example `>*<`) is not *actually* fair in the sense that it does not give all the processes

Can we do better than avoiding non-termination due to branch starvation? Sometimes we can with *weighted search*. Weighted search refers to the backtracking search where branches are weighted, or *prioritized*, and branches with higher priority are scheduled before branches with lower priority. A sketch of the API is demonstrated in listing 4.4. The backtracking monad implements the `halt` operation (called `tell` in [97], presumably to echo the `MonadWriter` interface) which accepts a value indicating the priority of the current branch and yields control to the scheduler. The scheduler then passes control to the branch with the highest priority. The value passed to `halt` must implement a monoid such that the priority of a branch is the concatenation of all values passed to `halt` in that branch up to that point.

```haskell
-- | Non-deterministically return an integer but before that pass
-- control to the scheduler which prioritizes branches based on a
```

```haskell
-- total sum which it tries to minimize.
stream :: (HeapKey h ~ Sum Int,IsHeap h,Monad m) => HCntT h r m Int
stream = go 0 where
  go i = do
    halt $ Sum i
    return i <|> go (i+1)

-- | Non-deterministically choose three numbers and only keep the
-- branches that satisfy a + b - c == 10. Avoid diverging by preferring
-- branches for which the sum of the numbers is minimum.
test2 :: IO [(Int,Int,Int)]
test2 = takeListT 5 $ dissolve @(CHeap (NonNeg Int)) $ once $ do
  (a,b,c) <- (,,) <$> stream <*> stream <*> stream
  guard $ a + b - c == 10
  return (a,b,c,x)
-- > test
-- [(5,5,0),(6,4,0),(5,6,1),(6,5,1),(6,6,2)]
```

Listing 4.4.: Prioritise branches that we want to be executed first.

None of the work we could find easily implements all the required features simultaneously, so we implement yet another backtracking monad transformer, `HCntT`. With that in mind, for the FluiDB planner we require that our logic framework supports the following features:

Weighted search Not all plans that match our criteria, i.e. that solve the query within the space are equally admissible. We need to find as good plans as possible and we do not want the planner to spend time looking into plans that are unlikely to be efficient. For that reason neither breadth-first nor depth-first traversals are ideal for our purpose. We need a robust way to search in a *weighted* or *best first* manner.

Soft-cut/fallback As we will see in more detail in section 4.2, he planner is initially optimistic about being able to materialize a n-node until it hits the budget limit. If the required n-node can not be materialized within the available budget, the branch fails and a new branch tries running the garbage collector first and then trying to materialize the n-node. If the budget turns out to be large enough, FluiDB should completely disregard the the latter branch. The reason is that the later a GC pass happens the more options it will have and therefore the better job is likely to do w.r.t. deleting less useful relations [1]. To achieve this we implement the operator `<//>` (pronounced *fallback*) which is similar to prolog's soft-cut. Unlike other similar approaches our conception of the fallback operator refers to the *continuation* of the left-hand operand rather than the content of the operand itself, which is to say that the right hand side operand is considered if the current branch fails taking into account the value on the left hand side, *not* if the left operand fails to yield a value.

Once In the context of non-weighted search it is fairly straightforward to implement an operator that demands that a sub-computation yields no more than one value (prolog's `once`). Simply run the entire computation in-place requesting one result and if it does not fail, return that result. This approach can still work with weighted but we would like the `halt` calls inside the computation to have global effect for the scheduler. In other words, we want the scheduler to be able to interleave the current branch with other branches while preserving the semantics of `once`.

4.1.2. The `HCntT` monad

The `HCntT` monad uses delimited continuations and, similarly to [97] it traverses the search space in a best-first manner by running the highest priority continuation available (see listing 4.5).

```
type HCntT h r m = ContT (HRes h m r)
  (ReaderT (HeapKey v)
   (StateT (CompState h r m) m))

newtype HRes heap m r = HRes (heap (Brnch heap r m),[r])
```

[1] This assumption depends heavily on the quality of the garbage collection heuristics. As we will see in 7 it is not always the case that this will lead to better results.

```haskell
type Brnch h r m = ReaderT (HeapKey v)
  (StateT (CompState h r m) m) (HRes h m r)
```

Listing 4.5.: The HCntT monad transformer allows continuation based non-determinism that allows switching between branches.

We define a recursive relationship between the monad within which the computation takes place – HCntT – and the value returned to the scheduler at each step – HRes. In particular HRes contains a priority queue (heap) of continuations and a set of final results. The heap type is parametric to allow flexibility with respect to how to best handle the particular key types (see listing 4.6)

- is required to be stable, i.e. items with the same key are returned in the order they were inserted.
- The HeapKey mempty must be higher priority than all other HeapKeys.

```haskell
class (forall v . Monoid (h v),Functor h,Monoid (HeapKey h),Ord (HeapKey h))
  ⇒ IsHeap h where
  type HeapKey h :: *

  popHeap :: h v → Maybe ((HeapKey h,v),h v)
  singletonHeap :: HeapKey h → v → h v
  maxKeyHeap :: h v → Maybe (HeapKey h)
```

Listing 4.6.: We parameterize the type of heap to allow the user to decide an efficient priority queue for the branches.

HCntT values are built using the combinators we mention and which we will describe in more detail an it needs to be run or *dissolved* to run the actual search. Dissolution (listing 4.7) is the process of turning an HCntT value to a ListT value. The ListT object will lazily produce just as many results as are required, and will produce just as many side-effects[2] as required. This indicates that the process of building computations is composable but not incremental. The price of the <//> combinator is that, unlike in other non-continuation

[2]A value of type ListT m a can produce values of type a and the production of each value may involve side-effects defined by the monad m.

based logic monads, once we start drawing results from the computation we can not apply further constraints on the rest of the computation, at least not in terms of the `HCntT` object. Contrast that with the case of `ListT` where we can draw the first couple of results and then use the rest in a different computation.

```haskell
dissolve :: (IsHeap h,Monad m) => HCntT h r m r -> ListT m r
```

Listing 4.7.: Dissolution is the process of turning an `HCntT` computation into a `ListT`.

The way dissolution works then (listing 4.7), is to first commit to the computation constructed by applying the continuation and obtaining an `HRes` object. If that object contains concrete results they are yielded one by one into the resulting `ListT`. Then, if the heap is not empty, the highest priority branch is popped and scheduled to run until it yields a new `HRes`. The concrete results are yielded into `ListT` and the new heap is combined with the old one. Scheduling a branch entails running the reader layer of the monad transformer using its previous value.

As mentioned, when describing the heap, a valid heap for `HCntT` must be stable. While this is a weighted search, it remains a question how the branches that have the same priority should be ordered. To avoid complicating things, up to this point, we have been implying that continuations are accumulated in a single heap. However, with just one heap, appending the newly produced heaps to the left would make for a depth first traversal of the *same-priority search space* while appending on the right would result in a breath-first traversal of the same-priority search space. We want the operators to have the flexibility to decide on which side each branch should be appended. For that reason the `HRes` actually contains a *pair* of heaps: one appended to the left (DFS), and one to the right (BFS) of the scheduler's (priority) queue (see listing 4.8). This is of particular importance to the correct implementation of <//> as we will see shortly.

```python
def dissolve(branches):
    while len(branches) > 0:
        priority,best_branch = branches.pop()
        (sub_branches_left,sub_branches_right),results = best_branch.run()
        branches = sub_branches_left + branches + sub_branches_right
        for r in results:
            yield r
```

Listing 4.8.: The dissolution algorithm in pseudo-python

In the following, we will briefly describe the implementations of the various combinators of `HCntT`.

Alternative/MonadPlus

The most fundamental combinator for any backtracking monad is the one spawning branches, the implementation of `Alternative` or, equivalently in our case, `MonadPlus`. These typeclasses require that we can compose non-deterministic values (`a <|> b` or `mplus a b` that non deterministically takes the value of `a` or `b`) and that we can make branches fail (`empty` or `mzero`). The implementation for `HCntT` is fairly straightforward (listing 4.9): for `empty` or `mzero`, which indicates the failure of a branch, simply disregards the continuation and returns an empty `HRes`. The `mplus` or `<|>` simply returns an `HRes` with only the two alternative branches having maximum priority (i.e. `mempty :: HeapKey h`) and bounded to the current continuation. Both those branches are pushed into the left so that they are scheduled before other same-priority branches.

```haskell
instance (IsHeap h,Monad m) => Alternative (HCntT h r m) where
  ContT m <|> ContT m' = ContT $ \f -> return
    $ HRes (Pair (h (m f) <> h (m' f)) mempty,[])
    where
      h = singletonHeap mempty
  empty = ContT $ const $ return emptyHRes
```

Listing 4.9.: The implementation for `Alternative` is the same as the implementation for `MonadPlus`.

Halt: passing control to the scheduler

We mentioned the special `halt` process that is fundamental to `HCntT`. `halt` accepts a value indicating the updated priority of the current branch and passes control to the scheduler.

The provided priority is offset by the previous priority of the branch using the `ReaderT` monad transformer that is internal to `HCntT`.

Because we want to be able to transform the `HCntT` monad we define the class of `MonadHalt` that can support this operation. The most common monad transformers of `HCntT` (like `ReaderT`,`StateT`, etc) can trivially support `halt`. The implementation of `halt` for `HCntT` itself is demonstrated in listing 4.10.

```
class Monad m => MonadHalt  m where
  type HaltKey m :: *
  halt :: HaltKey m -> m ()

instance (Monad m,IsHeap h) => MonadHalt (HCntT h r m) where
  type HaltKey (HCntT h r m) = HeapKey h
  halt v = ContT $ \nxt -> do
    v' <- asks (v <>)
    -- A singleton heap that will be appended to the left
    -- of the queue and contains the continuation of the branch.
    let leftHeap = singletonHeap v $ nxt ()
    -- return a value-less result that appends the branch to the left.
    return $ HRes (Pair leftHeap mempty,[])
```

Listing 4.10.: The priority of the branch being halted is updated by the provided value as control is yielded to the scheduler.

Soft-cut/fallback

The `<//>` (pronounced fallback), `left <//> right` runs the left hand side operand (primary) and if no values are produced in the *entire* computation based on that, only then does it try to evaluate the right hand side (fallback). `HCntT` does not guarantee that the right hand side will be scheduled immediately after the primary branch completely fails, but it does guarantee that it will be scheduled before it moves on to a new priority value. In other words it is only guaranteed that the priority of the fallback branch will tie the last failing branch of the left hand side in terms of priority, but if there are other branches in the queue that tie, there is no guarantee of how those will be scheduled.

There are two challenges that our particular feature set imposes to implementing this:

- We want <//> to operate on the continuation, unlike [107] that makes the decision on whether to run the fallback solely based on whether the left hand side returns a value. For example, in listing 4.11 none of the operands of the fallback operator fail individually but the entire branch fails for one of the two.
- In the context of a weighted search, control needs to be able to escape a branch that passes through the left hand side operand before it is exhausted. Therefore, we can not simply dissolve the left hand side and decide based on the number of results obtained.

```
comp = do
  -- The left computation succeeds on its own  but the scheduler will
  -- move on to the right branch because the branch fails later on.
  x ← return 0 <//> return 1
  guard (x > 0)
  return x
```

Listing 4.11.: This computation will evaluate to `1` because, while the computation `return 0` always succeeds the branch fails.

We solve these problems with the use of a special kind of branch we call a *marker*, and a global state that keeps track of all markers in the branch heap. Since branches are arbitrary processes we equip them with access to global state [3], which is a lookup table (`CompState`) full of fallback branches (see figure 4.1). The main idea is that a fallback branch corresponds to an entry in the lookup table. This entry is modified accordingly by the children of the right hand side. When <//> is evaluated, we push a marker into the heap with priority lower than the child branches so that when it is scheduled, it will itself schedule the fallback function if all the children branches have finished without yielding a concrete result. At any time there is exactly one valid marker per active fallback and it is denoted as such by the fallback entry in the lookup table. The rest of the makers are invalid and should be equivalent to `noops`.

[3] *global* here means that it is shared between the branches of the computation. The state is visible from different schedulers

More precisely, about the internals of the marker processes, each one refers to a location in the lookup table via its closure. Also each marker is uniquely identifiable, so each valid entry in the lookup table references the lowest priority one that corresponds to it. When a marker is scheduled, it looks up the fallback branch entry in the table and checks if the entry also refers to that marker. There are 3 possible scenarios that may play out at this point:

- The fallback entry in the table has been invalidated by a branch that yielded a result. In this case, the marker is invalid and just returns.
- The fallback entry is valid but does not correspond to the scheduled marker. This means due to the left hand side branch spawning low priority subbranches, another marker has been inserted to (possibly) trigger the fallback at some time in the future. The current marker is invalid
- The fallback location is valid and corresponds to the scheduled marker. This means that the fallback process needs to be run and removed from the table.

What is the life cycle of the fallback entries? For a visual aid to the following see figure 4.1: When an expression `A <//> B` appears we create a new fallback entry in the lookup table containing `B` and recursively "infect" all spawned sub-branches of `A` to perform the following actions immediately after they generate new branches and results:

- If there is *at least one valid concrete result*, invalidate the fallback in the lookup table and stop infecting child branches with the currently described hook.
- If the *fallback is invalid* it means there have already been valid results that rendered the fallback obsolete. Stop infecting sub-branches.
- If *none of the above* happened, check the *priority* of the last marker corresponding to the fallback (registered in the lookup table entry).
  - If it is strictly lower than the lowest priority subbranch do nothing because there is a well-placed marker to handle it.
  - Otherwise, create a new marker of the same priority as the lowest priority subbranch and put it in the *right-append heap* (see section 4.1.2 on the pair

of heaps). This way we know it will be scheduled *after* the last branch relating to the fallback but not too long after that.

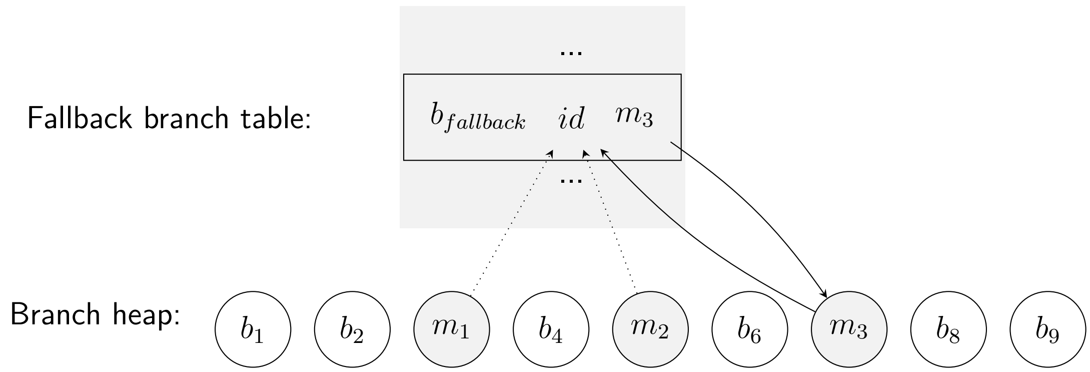


Figure 4.1.: Marker "branches" are injected in the heap of prioritized branches to possibly trigger the fallback branch. Each marker gives the scheduler the opportunity to run a particular fallback branch. The entry of the fallback branch entry references the last marker so that only that one actually trigger the fallback branch. A child branch that succeeds removes the fallback entry so the final marker also fails to trigger the fallback.

There are a few optimizations that can be implemented to avoid too many lookups in the fallback table in the case of deep fallback nesting that take advantage of the fact that new markers for outer fallbacks may only be created when new markers for inner fallbacks are created. However, for FluiDB we did not find `HCntT` to be a performance bottleneck so we leave it for a future iteration.

Once

The `once` operator runs the argument computation and stops once it returns the first result. FluiDB uses `once` to run the garbage collector without exploding the search space, as the GC may have to search an exponentially large space of plan fragments. We prune that space by settling for the first result the planner can find and carefully crafting the GC such that it tries more robust plans first (see section 4.2.1).

The `once` operator is built on top of a concept we call a *nested scheduler* which is exactly what it sounds like. The nested scheduler is built from an `HCntT` value (the *nested computation*) and needs to know what to do in case of a success and in case of complete failure (no more branches to run), implemented in the general form as `nested` (see listing 4.12).

The nested scheduler then is a process that *always* returns a single subbranch which has the priority of the highest priority subbranch of the nested computation. When scheduled by the outer scheduler, the inner scheduler internally schedules the next branch of the nested process. When the process yields results or completely fails, the corresponding hook is run. `once` implements these hooks as "return the result and stop" and as "propagate the failure" respectively. Since the implementation of `once` based on the nested scheduler(`nested`) is fairly small it is provided along with the types in listing 4.12.

```
nested
  :: forall h m r a .
  (Monad m,IsHeap h)
  ⇒ (Pair (h (Brnch h r m)) → r → [r] → HRes h m r)
  → HRes h m r
  → HCntT h r m a
  → HCntT h r m a
nested success fail c = …

once :: forall h m r a . (Monad m,IsHeap h) ⇒ HCntT h r m a → HCntT h r m a
once = nested (\_h r _rs → HRes (Pair mempty mempty,[r])) emptyHRes
```

Listing 4.12.: The nested scheduler runs a subprocess within a single branch. Once is built on top of that to make sure the process stops once a concrete result is returned.

4.2. The planner

The planner is the subsystem of FluiDB that given the state of the QDAG and a target n-node to be materialized, produces a plan that will materialize the query. Our notion of a plan is slightly more specific than what is commonly considered, i.e. the RA representation of the query. In our case it is a sequence of *transitions* that are to be transpiled to C++ by the code generator. There are three kinds of transitions:

- The *t-node trigger* that assumes the input n-nodes are materialized and produces a subset of the output n-nodes.
- The *t-node reverse trigger* that assumes that the output n-nodes are materialized
- The n-node *deletion*

The planner operates by backtracking using `HCntT` monad. Each branch maintains some branch-internal effects that are reified as monad transformers on top of the `HCntT` defining the `PlanT` monad transformer (listing 4.13).

```
type PlanT t n m =
  StateT
    (GCState t n)
    (ReaderT (GCConfig t n)
     (ExceptT (PlanningError t n)
      (HCntT PlanHeap () m)))
```

Listing 4.13.: The monad that defines all the useful effects used by the planner. `GCConf t n` is an immutable, from the perspective of the planner, configuration that includes the QDAG, the n-node estimated sizes, etc. `GCState t n` is state that is mutated and private to each branch of the planner like the materialized status of the n-nodes, the set of transitions registered so far and various caches. The `PlanningError t n` is a planner specific type of error. The entirety of the result of planning is accumated in `GCState` so the result of backtracking is just the value of unit type (`()`).

We maintain the partial plan (sequence of transitions) as part of the state of each of the planner's branches. *Registering a transition* means that a transition is added to the partial plan.

It is important to clarify the way we use the term "materialized n-node" (`Mat` as opposed to "not materialized" – `NoMat`) from the planner's perspective. A mapping of n-node states is passed to the planner at the beginning of the planning process. This mapping indicates which n-nodes are initially materialized. As transitions get registered, the mapping is updated to reflect the effect that the plan so far would have on the materialized relation set in storage. Since the planner operates via backtracking (using the `HCntT` monad described), each branch maintains its own mapping of n-node states and therefore considers a different set of materialized n-nodes.

The main loop of each branch is a recursive process of asserting that n-nodes are materialized. If the n-node is already materialized the assertion succeeds, if not the planner attempts to trigger or reverse-trigger a neighboring t-node that would lead to the n-node being materialized, asserting first that the input n-nodes of that transition are materialized. In practice, due to intermediate n-nodes being auxiliary and not corresponding to real n-nodes (see section 3.5 on clusters), we know that t-nodes are organized in sequences that need to be triggered entirely or not at all. For example (figure 4.2) the intermediate $\ltimes$ n-nodes are not actually materialized as part of the join operator. This means that a plan triggering $\cancel{\ltimes}$ t-nodes but not the corresponding $\bowtie$ t-nodes is invalid. In other words, within a plan, all intermediate n-nodes must be used as input exactly once for every time they are materialized. To ensure this, we organize the t-nodes into `MetaOps` (listing 4.14) that can atomically be triggered or reverse triggered. `MetaOps` also abstract the distinction between triggering and reverse triggering as they expose a set of input n-nodes, a set of output n-nodes, and a process that registers the correct transitions once the `MetaOp` is triggered. The process of asserting asserting an n-node being materialized then becomes the process of non-deterministically selecting a `MetaOp` with the n-node in question in the output set. Before actually splicing the `MetaOp` computation we assert that the input set is materialized.

```haskell
data MetaOp t n = MetaOp {
  metaOpIn     :: NodeSet n,
  metaOpOut    :: NodeSet n,
  metaOpInterm :: NodeSet n,
  metaOpPlan   :: forall m' . Monad m' ⇒ PlanT t n m' [Transition t n]
  }
```

Listing 4.14.: A `MetaOp` refers to input, output, and intermediate n-nodes that are involved in the set of operations it abstracts. Furthermore, it contains a computation that registers and returns the transitions involved in the `MetaOp`.

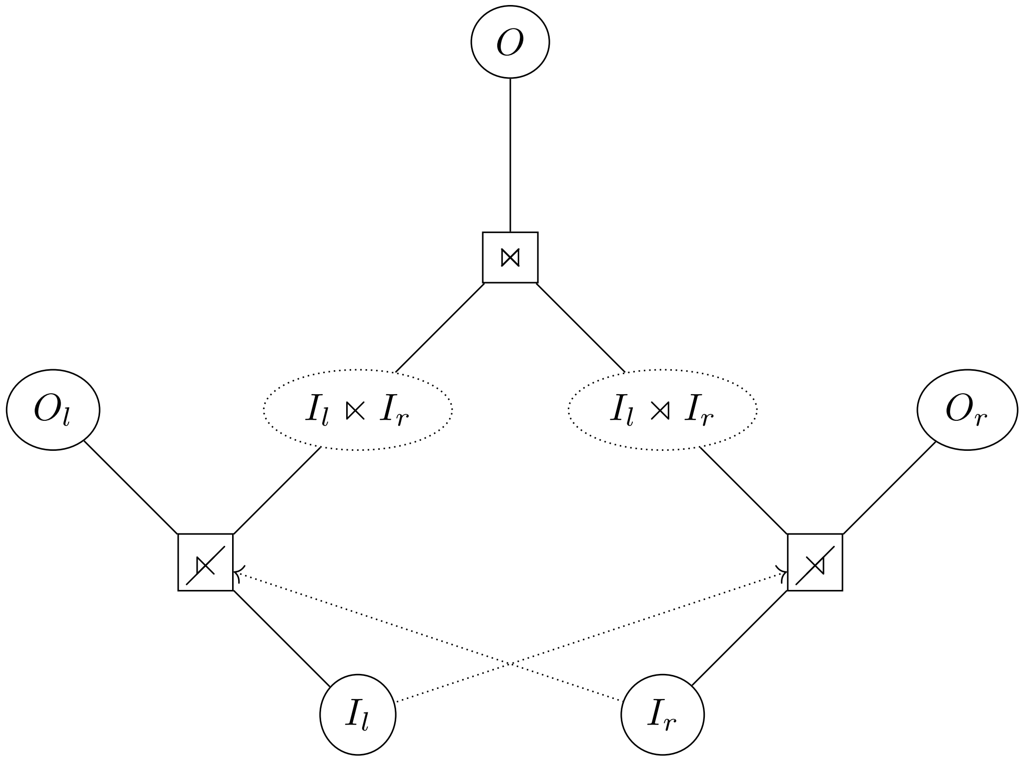


Figure 4.2.: starting at n-node O, the output of a join operation, we can derive three MetaOps that can materialize it. $MetaOp\{in : \langle I_l, I_r\rangle, out : \langle O\rangle\}$, $MetaOp\{in : \langle I_l, I_r\rangle, out : \langle O_l, O\rangle\}$, $MetaOp\{in : \langle I_l, I_r\rangle, out : \langle O, O_r\rangle\}$, $MetaOp\{in : \langle I_1, I_2\rangle, out : \langle O_l, O, O_r\rangle\}$. Because trying all combinations of outputs explodes the search space we always go for the largest and then let the garbage collector deal with the possible repercussions. On the other hand to materialize I_l there is only one `MetaOp` that relates to this cluster $MetaOp\{in : \langle O, O_l\rangle, out : \langle I_l\rangle, interm : \langle I_l \ltimes I_r\rangle\}$

Two questions should arise from the above description: a) how does the planner avoid cycles where it recursively tries to materialize the parents and then the children? and b) how does it know not to materialize an n-node twice? Both those problems are addressed by refining the possible states of the n-nodes (listing 4.15). Rather than being just `Mat` or `NoMat`. We also disambiguate between `Initial` and `Concrete` n-nodes. N-nodes in the `Initial` state are allowed to change their materialization status. In contrast `Concrete` n-nodes have their state fixed with very few exceptions. Then, when an n-node is asserted to be materialized, its status is first checked and following scenarios are possible:

- The n-node's status found to be `Concrete` and `NoMat` and the branch immediately fails as it is not allowed to be set to materialized.
- If it is `Concrete` and `Mat` the assertion simply succeeds
- If it is `Initial` and `Mat` it is turned into `Concrete` and `Mat` and the assertion succeeds.
- If the n-node's status is `Initial` and also `NoMat`, two things need to happen: first, the n-ode is set `Concrete` and `NoMat` in order to avoid cycles, and the aforementioned process of finding a `MetaOp` is carried out recursively in order to materialize the n-node.

```
data IsMat = Mat | NoMat
data NodeState =
  -- Concretified states are allowed to change only within the same
  -- lexical scope.
  Concrete IsMat
  -- Initial states are subject to change when encountered as a
  -- neighbor or by the GC.
  | Initial IsMat
```

Listing 4.15.: The different states that an n-node is allowed to be in.

Once an n-node's is materialized and we start materializing its siblings, it is important that the garbage collector (which we will be discussing in section 4.2.1) does not delete the said n-node. To avoid this, n-nodes materialized are set to `Concrete Mat` until all their siblings, that are inputs to the same `MetaOp`, are materialized. Once the `MetaOp` is triggered and its output n-nodes are set to the `Concrete Mat` state the input n-nodes are set to `Initial Mat` as it is now safe for the garbage collector to remove them.

It is worth highlighting the importance of each n-node in a dependency set of an n-node being *completely* materialized before the process of materializing the next one commences, i.e., that no two sibling n-nodes are simultaneously in the process of being materialized. The reason is that at any time a single trail of parent n-nodes is marked as `Concrete` and `NoMat`, otherwise we cannot be sure whether a `Concrete NoMat` n-node that renders a `MetaOp` non-triggerable is actually in the process of becoming materialized, meaning that when that process succeeds the `MataOp` under consideration will be triggerable.

4.2.1. Garbage collection

One of the fundamental design decisions of FluiDB is that she is committed to materializing all intermediate n-nodes, a la MonetDB [58]r, and keeping them around for as long as possible. On the one hand, the plan selected is crafted so that the intermediate query results are maximally useful for the overall workload, on the other, when the available storage runs out, the *garbage collector* (GC) mechanism selects the least useful n-nodes to be deleted, freeing the pages required for planning the query. In this section we focus on the latter.

The garbage collector is triggered right before a `MetaOp` is to be triggered. It calculates the space required for materializing the output n-nodes and the space available to decide whether inserting a plan fragment generated by a GC run is required. Then it selects a subset of materialized n-nodes to delete in order to make room for the new results and inserts `DelNode <n-node>` transitions for each one into the plan. The selection process has two hard constraints:

- The n-nodes being deleted must be *deletable*, i.e. the n-node's state is `Initial` and if the n-node is deleted that it can be reconstructed from the remaining materialized n-nodes. [4]

[4] There is a caveat to our conception of deletability of nodes, namely that the concept does not take into account budgetary constraints, only that there is no loss of information resulting from the relation's

- The n-nodes being deleted must not be required for the continuation of the current plan. We call these n-nodes *protected*. For example, say we are planning for the query $A \bowtie B$ and we have materialized A already but while materializing B the GC is triggered. A should not be in the repertoire of n-nodes the GC can delete under any circumstances.

Selecting a subset of n-nodes for deletion is not an easy problem, so we follow a simple heuristic (see listing 4.16). Large n-nodes are costly to create and therefore the larger the n-node, the less inclined the planner is to create it, and the more useful it is likely to be. With that in mind, the heuristic we follow is for the garbage collector to prioritise deleting small n-nodes over deleting larger ones.

The first order of business for the GC the is to find the set of deletable n-nodes and sort them by size. It tries to delete them one by one starting from the smaller ones and working its way up to the larger ones. Every time a n-node is deleted, it is possible that other n-nodes that were previously established to be deletable lose that property. For example, if n-nodes A, $\sigma_p A$, and $\sigma_{\neg p} A$ are `Initial` and materialized and not connected to other n-nodes but to each other, each one of them can be materialized from the others and therefore all of them are marked as deletable. If the GC deletes A first, the other two are no longer deletable as neither would be materializable were they to be deleted.

N-nodes that are established as non-deletable in this way are switched from `Initial` to `Concrete` to avoid re-calculating their materializability.

If after this process not enough space is created, the GC resorts to starting a new *planner epoch*. The function of a planner epoch change is for the planner to revisit the assumptions that it has made in the form of setting n-nodes to `Concrete Mat`. That is to say, when a new epoch begins, the unprotected n-nodes materialized in the current run become available for deletion and all the n-nodes states and transitions that were recorded since the last planner epoch started are stashed into the epoch stack. The epoch stack resides in the branch-local state (`GCState`) and contains a map of n-nodes to their states at the end of the epoch and a sequence of transitions. When a new epoch is pushed into the stack, a new mapping of states is created according to the following rules:

deletion.

- The materialization status (`Mat` or `NoMat`) does not change between epochs: All materialized n-nodes from the previous epoch are materialized in the new one materialized and all non-materialized n-nodes are still not materialized.
- All non-protected `Concrete` n-nodes become `Initial` n-nodes.

The sequence of transitions for the new epoch is empty. The reason we keep separate lists of transitions is, as we will see in more detail in the code generation chapter, that the subset of output n-nodes that a physical operation generates is established at the time of physical planning based on the materialized n-nodes according to the epoch corresponding to the transition.

An epoch contains important information about the state of the planner. For this reason, when an epoch is inserted in the stack it is checked for equality against the other epochs in the stack. If two equivalent epochs are inserted in the stack the branch fails permanently as it means that there is a cycle.

This enables another optimization, the *free-to-delete* n-nodes. As mentioned previously, the planner prioritizes the version of `MetaOps` that materializes all outputs for example it will prioritize the branch that follows $MetaOp\{in = \langle A \rangle, out = \langle \sigma_p A, \sigma_{\neg p} A \rangle\}$ over the one that follows $MetaOp\{in = \langle A \rangle, out = \langle \sigma_p A \rangle\}$. It is rarely the case, however, that all the outputs are required. In this example, triggering the former `MetaOp` and then garbage collecting $\sigma_{\neg p} A$ without it being used is never desirable. For that reason we mark the n-node $\sigma_{\neg p} A$ as *free-to-delete* as soon as it is created based on the fact that it is not required. When the garbage collector deletes free-to-delete n-nodes it does so without registering a deletion transition. This way the physical planner will generate a plan where $\sigma_{\neg p} A$ is never created in the first place. All n-nodes lose their free-to-delete status upon the creation of a new epoch.

```
gc reqSize = do
  -- Try the current epoch and if that fails retry with a new epoch.
  return () <//> newEpoch
  -- find the deletable n-nodes and sort them by size
  deleteables ← sortOnM getNodeSize =<< filterM isDeletable =<< getAllNodes
  -- Try deleting each n-node and stop deleting when amassing enough
  -- free pages.
```

```
  forM_ deletables $ \n → do
    freePgs ← getFreePages
    when (freePgs < reqSize) $ tryDelete n <//> markAsConcrete n


-- | Try to delete an n-node.
tryDelete node = do
  -- check that the n-node is still deletable
  guardM $ isDeletable node
  -- If it is free-to-delete do not register the DelNode trigger
  isFreeToDel ← getIsFreeToDel node
  unless isFreeToDel $ register $ DelNode node
  -- Mark the n-node as not materialized.
  setStatus node (Initial NoMat)


-- | Check that an n-node is deletable
isDeletable n = do
  st ← getNodeState n
  case st of
    Initial Mat → do
      -- Only nodes in Initial Mat state might be deletable. Try deleting
      -- and check that it is materializable.
      setNodeState n (Initial NoMat)
      ret ← isMaterializable n
      setNodeState n (Initial Mat)
      return ret
    _ → return False

-- | An n-node that was not deletable should not be considered again
-- for deletion and is therefore marked as concrete.
markAsConcrete n = do
  st ← getNodeState n
  case st of
```

```
      Initial m → setNodeState (Concrete m)
      _ → return ()
```

Listing 4.16.: A sketch of the garbage collector algorithm in pseudo-haskell.

It is hopefully clear at this point that the process of garbage collection involves a large search space. We sacrifice some of our plan repertoire by running the entire process wrapped in a `once` operator that we described in the previous section.

Finally, a word about protected n-nodes. N-nodes are protected within the context of a branch from the time they are established as part of the input set of a `MetaOp` we are making triggerable until the time said `MetaOp` is actually triggered during the normal planning (i.e. not the GC). Because an n-node may be the input of more than one simultaneously considered `MetaOps`, protection of a n-node is not a boolean value that is set when the n-node is encountered as `MetaOp` input and unset when the `MetaOp` is triggered, but rather an natural number variable that is incremented and decremented respectively. When that value is zero, the n-node is not considered protected.

4.2.2. Order of traversal

We mentioned that the `MetaOps` are selected non-deterministically. In this section, we will go in depth on the FluiDB planner's strategy on the order in which it considers its options using the `HCntT` framework, making heavy use of the `halt` operator to decide branch priorities. A branch's priority is dependent on the particular frontier at the time of a halt and is determined by four factors:

- The cost of the `MetaOp`s that the branch is in the process of making triggerable.
- The cost of the `MetaOp`s that have *already been triggered*.
- The sum of the estimated costs of *each n-node in the frontier*.
- A weighted sum of the *stochastic cost* of historical queries based on the current frontier.

The final two factors are values on which we will focus in this section. They are computed often and are dependent on the set of materialized n-nodes. Since the set of materialized n-nodes does not change in a completely arbitrary way from computation to computation a naive approach to calculating those values would involve a lot of duplicate work. For this reason, we use Antisthenis which will be expanded in a chapter 5 to minimize the amount of work required.

We define a very rough way of estimating cost materializing n-nodes. We incrementally estimate the cost of each n-node of the frontier using the following formula:

$$c_{nomat}(n) = \min_{op \in \text{metaops with } n \text{ output}} \left\{ cost(op) + \sum_{i \in inputs(op)} c(i) \right\}$$
$$c_{mat}(n) = 0$$

Where the cost $c(n)$ of n-node n is $c_{mat}(n)$ if the n-node is materialized and $c_{nomat}(n)$ otherwise. In plain English the cost is recursively estimated as the cheapest combination `MataOp` plus the cost of materializing the input n-nodes.

This approach has several problems like the fact that n-nodes that are used more than once are also added more than once and that it does not take at all into account the budget constraints. It is, however, a good enough heuristic for prioritizing branches.

The final factor, which takes into account the historical queries is a bit more tricky. The planner keeps track of the last couple of queries requested and the corresponding n-nodes, and tries to estimate how beneficial following the current branch would be in the event where a query similar to those is requested in the future. We estimate that by summing up a slightly different notion of cost for those queries.

A naive approach would be to just use the the same algorithm as we did for the frontier n-nodes. However, this approach is prone to getting stuck behind materialized n-nodes: past queries, especially recent ones, are likely to still be materialized and therefore their cost is likely to be 0. Even if the GC has already got around to deleting them, their near dependencies are likely to be materialized and block the cost estimator from traversing the graph up to the nodes affected by the current plan. This makes the estimation quite bad for two reasons:

- N-nodes found to be materialized when prioritizing branches may have been deleted by a GC pass by the time they are actually requested.
- We do not care about the cost of the particular n-nodes, but rather about *similar* n-nodes. For that reason we want to avoid depending too heavily on a particular materialized n-node being materialized.

A slightly better way of calculating the expected cost that an n-node will have at a future point in time, which we heuristically attempt to approximate, would be, instead of considering the cost of materialized n-nodes to be zero, to calculate the likelihood that a materialized n-node will still be materialized when we encounter it again. This is related not only to an estimation of how many page writes separate the moment of cost estimation and the actual plan the cost of which is being estimated, but also all the decisions that the garbage collector will make in that time. After FluiDB's budget is exhausted for the first time, for every page write a page needs to be garbage collected. We considered a couple of options for stochastically modeling the behavior of the GC like assuming that it chooses random pages or random tables to delete, but we could find none that was both a mathematically justifiable approximation and computationally viable.

For this reason, we decided to follow a pragmatic approach to the problem and simply assume that for every materialized n-node there is a constant probability that it will not still be materialized when we need it, scaling the cost c_{nomat} of that n-node by that factor to get the stochastic cost. Thus the cost formula h for the n-nodes now is:

$$
\begin{aligned}
h_{nomat}(n) &= \min_{op \in \text{metaops with } n \text{ output}} \left\{ h(op) + \sum_{i \in inputs(op)} cost(i) \right\} \\
h_{mat}(n) &= \lambda \cdot h_{nomat}(n)
\end{aligned}
$$

Where $\lambda \in [0, 1]$ is the estimated probability that the n-node is no longer materialized when needed. As before, materialized n-nodes have cost c_{mat} and non-materialized n-nodes have cos c_{nomat}

4.3. Conclusion

In this chapter we saw two distinct concepts: a weighted backtracking framework and a system implemented on top of that framework, namely the FluiDB's physical planner. These two combined enable FluiDB to search for quality plans within the QDAG in a flexible way.

CHAPTER 5

Antisthenis

I can see a horse, and I can see a man; but humanity and horsehood I cannot see.

(Antisthenes)

You have the eye which sees a horse and a man; but the eye which can see horsehood and humanity you have not.

(Plato)

Chapter summary

- Antisthenis is a framework for building systems for *incremental* evaluation of algebraic expressions using heuristics to optimize the *order* in which subexpressions are evaluated and to calculate some classes of *self referential* computationns.
- *Mealy arrows* are a construct to implement adaptable computations that can can produce a value and a new, semantically equivalent computation. Each iteration adapts to the previous caching values and changing the evaluation plan to be more efficient. Antisthenis computations are implemented as network of mealy arrows that compose larger adaptive (incremental and partial) computations.
- These Antisthenis processes return concrete values, errors (uncomputable values) or partial values (unfinished computations). Based on the context of the computation, all kinds of values have a lifespan after which computation resets and partial values in particular are refined into more concrete kinds of values.
- Machines are named Antisthenis processes that can be referred to by other machines. Antisthenis provides tools for handling some classes of self-referential computations.
- We implement Antisthenis operations to cover requirements of the FluiDB physical planner:
  - calculate min/sum of natural numbers – used to compute *cost estimations*
  - boolean functions – used determine *node materializability*
  - calculate min/sum of numbers annotated with certainty metrics – used to calculate the estimated cost of materialization of n-nodes give nodes that *might* be materialized.

This chapter goes in depth describing the various parts that compose Antisthenis, a framework for building incremental computation systems we developed to assist the FluiDB planner in incrementally and efficiently computing costs and checking if nodes in the QDAG are materializable during the evolution of the inventory.

The chapter starts with an introduction about the motivation for developing Antisthenis as a collection of synchronously communicating processes performing incremental computation. Next, we look at our conception of a Mealy arrow that enables the construction of Antisthenis processes. In the following section we describe the core of Antisthenis, which is the structure of a single Antisthenis process. From there we move on to describe some specific operation implementations that are useful for FluiDB and then describe the systems that facilitate intercommunication and high level organization of Antisthenis processes. Finally, we conclude with some caveats and some future future work that can be done to make Antisthenis easier to apply to solutions other than FluiDB.

The name Antisthenis is a reference to the proto-empiricist and proto-nominalist philosophy expressed by Socrates' pupil Antistenes. Antisthenes promoted particular objects as preceding universal concepts, encapsulated in his famous declaration that one can only interact with a horse but not with the abstract concept of "horseness". Antisthenis-the-framework can compute correct values taking advantage of properties that are highly specific to the particular computation, potentially even making use of unfinished intermediate computations. All that matters to Antisthenis is the computation at hand.

Incremental computation has traditionally been dealing with the tradeoff between incrementalizing general algorithms but being oblivious to particular properties of the functions that comprise the computation, and creating highly effective techniques for incrementalizing an algorithm but these techniques have low transferability. Antisthenis is positioned firmly on the latter side of this spectrum aiming to incrementalize the computation of logical plan cost estimation under changing materialized views. Antisthenis' novelty lies in its ability to take into account the particular properties of the operations involved to efficiently and incrementally compute expressions that may be self referential. This problem has not been directly studied to our knowledge but the need for a good solution to it is central to FluiDB's operation.

5.1. Introduction

The order in which subexpressions of an algebraic expression are evaluated can be important not only for the performance of the expression evaluation but, in the context of infinite expressions, for the very termination of the evaluation itself.

Expressions are typically modeled as abstract syntax trees or computation graphs (DAGs), and evaluation is commonly modeled as a reduction of that tree or DAG. Depending on the evaluation strategy each operand is either fully or partially evaluated. Typically, the strategy for evaluating a subexpression operand , however, is oblivious to the parent expression and a partially evaluated subexpression (thunk) is opaque from the perspective of the parent expression. Breaking this assumption provides some opportunities for incremental computation and efficient evaluation in the presence of non-total arguments.

We built *Antisthenis* around this principle to achieve three important goals:

- To *incrementally evaluate* an expression, i.e. to only re-evaluate the parts of it that depend on parameters that were updated with respect to the last time the expression was evaluated.
- Take advantage of absorbing elements and other operator-specific strategies for *early stopping* the evaluation.
- Often sub-expressions are not total, i.e. they may not be computable. A common reason for that is the expression being self-referential. These errors may be detrimental to the evaluation the top level expression, but often, as we will see, they are not.

Incremental computation is a tool for *efficiently* evaluating expressions of variables that change over time. The area has received some attention recently [52, 66]. An incremental evaluator will typically accept a directed acyclic graph (DAG) of interdependent computations like the following one

$$A = a + B + C + D$$
$$B = C \times b$$
$$C = D + c$$
$$D = 0$$

Which can be represented as a dag in figure 5.1.

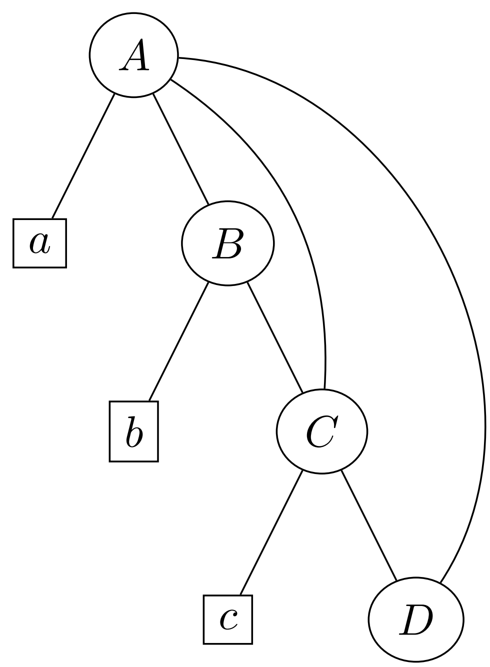


Figure 5.1.: A computation DAG. Circular nodes represent operations and square nodes represent parameters that are subject to change.

Where A, B, C, and D are expression nodes in the DAG and a, b, and c are parameters that change over time. A non-incremental evaluation system would evaluate all four nodes every time a value were requested, regardless of whether and which parameters have changed. Incremental evaluation systems keep track of which variables change and only re-evaluates the nodes of the DAG that have been updated. In our example, if only the parameter b has been updated since the last time A was evaluated, an evaluation of A would require only A and B to be re-evaluated.

Incremental evaluation systems employ a wide range of tricks from simply memoizing the functions [6] to more complex strategies like Adapton [66] where changes in parameters are propagated in the form of "dirtying" DAG nodes and more explicitly breaking the computation into reusable parts.

To our knowledge, none of the existing approaches to incremental computation take into account the presence of either absorbing elements or non-computable subtrees (i.e., recursive values) in their scheduling the evaluation order. Scheduling is either done naively or left up to the user. However, it is a hard requirement of the FluiDB query planner for the incremental computing framework to automatically exploit the properties of the computation at hand in order to incrementally compute node materializability and query costs in the presence of different materialized query sets. Furthermore, in all the approaches we are aware of, the network of sub-expressions was assumed to be well-behaved, in that all nodes can return a valid value and there are no cyclical references.

The main principle behind the architecture of Antisthenis is to manipulate the order in which subexpressions are evaluated in order to exploit properties of the parent operator and avoid fully evaluating each of the subterms. For example, when estimating the cost of a database query, we take it to be the minimum cost of each of the possible logical plans. It is unlikely that one would need to come up with a full estimation of each plan, especially the more expensive ones, before realising the lowest cost. Instead, we would like to accumulate a lower bound for the cost of each plan.

Another prime example of an opportunity pruning the expression tree is the exploitation of absorbing elements. In the general case, in a non-lazy evaluation strategy, to compute the value of a function $f(A, B, C)$ where A, B and C are expressions, we would need to fully evaluate *all* three arguments first. Since the absorbing element of multiplication in the reals is 0, for $f(a, b, c) := a \times b \times c$, a result of 0 for the first argument A would render the evaluation of the rest of the arguments redundant.

As alluded to, a lazy evaluation strategy paired with a "Sufficiently Advanced Compiler", can take steps towards mitigating this problem, but it can only go so far. Consider the following example:

$$A = B \times C \times D$$
$$B = \sum_i i$$
$$C = 10 - 10$$
$$D = \sum_i i$$

Here, when evaluating A, a traditional evaluation system would look at the first argument first, namely B. B is very expensive to evaluate and very soon a human would figure out that it will never evaluate to zero. C, however, does evaluate to zero, and once that is evaluated, the evaluation of A is complete[1]. In a complex set of automatically generated such expressions it is important that the evaluator does not keep trying to evaluate a complex expression when evaluating another simpler one could yield an absorbing value.

Early stopping a computation is useful not only to avoid extra work. As we mentioned, many incremental computation systems assume that all sub-computations terminate successfully. Due to reversible operators, and therefore the QDAG being inherently undirected, the computations we are interested in will invariably be self referential. Therefore, we need to take seriously the case where a sub-expression will fail to terminate. In this case, the parent expression needs to exhaust all possibilities that could lead to a value.

$$A = min(B, C, D)$$
$$B = b_1 + b_2 \cdot D$$
$$C = c_1 + c_2 \cdot A$$
$$D = d_1 + d_2 \cdot B$$
$$b_1 = b_2 = d_1 = d_2 = 1$$
$$c_1 = 3$$
$$c_2 = 0$$

[1] Barring domains where infinity needs to be considered in any way.

Here B and D are clearly not computable but in the context where these values represent the cost of evaluations for nodes A should evaluate to 3 and A should evaluate to $C = 3$. Furthermore, A should depend *all* parameters for the validity of its value while C should depend only on c_1 and c_2.

5.2. Background

In this section we describe some concepts used to construct the Antisthenis processes. Here we assume some familiarity with the common building blocks of effectful computation in Haskell: functors, applicatives, and monads. They are described in some detail in appendix B.

5.2.1. Arrows and profunctors

The basic element of computation in the purely functional world is usually related to the troika of *functors*, *applicatives*, and *monads* [42], in the interfaces of which we find some recurring substructures that relate their modification.

- `a → b` the normal pure function type. We take this for granted as it is provided and managed by the language, in our case Haskell.
- `f a → f b` A functor morphism. The standard functor stack we saw so far focused on producing such morphisms from simpler arrows.
- `f (a → b)` is known as the Cayley morphism
- `a → f b` which for an `f` being a monad is the now familiar Kleisli arrow.

We would like to generalize this notion to an parametric object that we will denote as $\rightsquigarrow$ (occasionally denoted as `p`). We follow the arrow definition from Hughes [25] which dictates that an arrow must be a **category** (i.e. arrows must compose in a commuting way and there must be an identity arrow) and must also satisfy the following:

- It must commute with tuple (i.e., must implement `first` and `second`).

- The Hask category must be embeddable in the category defined by the arrows (i.e. via the `arr` function)

In more concrete terms, we depend on the Haskell typeclasses presented in listing 5.1. An `Arrow` can act on the first or second element of a tuple, and an `ArrowChoice` can act on the the left or the right case of an `Either`.

A related concept to arrows is the profunctor. Profunctors are not necessarily directly composable like arrows nor do they commute with tuple or `either`. They only provide `dimap`. `dimap` is of course implementable in terms of arrow combinators but opting for dimap can produce much more efficient code.

```
class Category p where
  id :: p a a -- Maps a value to itself with no effects
  (.) :: p b c -> p a b -> p a c -- composition

class Category c => Arrow p where
  arr :: (a -> b) -> p a b -- arrows do at least what functions can do.
  (>>>) :: p a b -> p b c -> p a c -- also composition
  first :: p a b -> p (a,c) (b,c)
  second :: p a b -> p (c,a) (c,b)
  (***) :: p b c -> p b' c' -> p (b,b') (c,c')
  (&&&) :: p b c -> p b c' -> p b (c,c')

class Arrow p => ArrowChoice p where
  left :: p a b -> p (Either a c) (Either b c)
  right :: p a b -> p (Either c a) (Either c b)
  (+++) :: p a b -> p b' c' -> p (Either a b') (Either b c')
  (|||) :: p a c -> p b c -> p (Either a b) c

class Profunctor p where
  dimap :: (a' -> a) -> (b -> b') -> p a b -> p a' b'
```

Listing 5.1.: Haskell typeclasses related to the notion of `Arrow`.

All implementations of the interfaces in listing 5.1 are expected to conform to some laws which formalize the intuitions described about the meaning of arrows. We describe the laws required for each typeclass to be valid in listings 5.2

```
-- | Laws that need to hold for any valid category implementation.
-- Right identity
f . id = f
-- Left identity
id . f = f
-- Associativity
f . (g . h) = (f . g) . h


-- | Laws that need to hold for any valid arrow implementation.
arr id = id
arr (f >>> g) = arr f >>> arr g
first (arr f) = arr (first f)
first (f >>> g) = first f >>> first g
first f >>> arr fst = arr fst >>> f
first f >>> arr (id *** g) = arr (id *** g) >>> first f
first (first f) >>> arr assoc = arr assoc >>> first f

where

assoc ((a,b),c) = (a,(b,c))

-- | Laws that need to hold for any valid arrow choice implementation.
left (arr f) = arr (left f)
left (f >>> g) = left f >>> left g
f >>> arr Left = arr Left >>> left f
left f >>> arr (id +++ g) = arr (id +++ g) >>> left f
left (left f) >>> arr assocsum = arr assocsum >>> left f

where

assocsum (Left (Left x)) = Left x
```

```haskell
assocsum (Left (Right y)) = Right (Left y)
assocsum (Right z) = Right (Right z)

-- | Laws that need to hold for any valid profunctor implementation.
dimap id id ≡ id
dimap (f . g) (h . i) ≡ dimap g h . dimap f i
```

Listing 5.2.: Laws for the typeclasses related to arrows.

Examples

The definition and highlighting of important properties of arrows that we have done so far is probably not extremely helpful to understand how computation is defined in such terms. To make the concepts of arrows and profunctors more tangible, consider the following examples:

The Kleisli arrow The `Kleisli` arrow [32] as we mentioned earlier, is equivalent to the arrow `Monad m ⇒ a → m b`. In Haskell it is defined as:

```haskell
newtype Kleisli m a b = Kleisli (a → m b)
```

So the arrow `Kleisli (State s) a b` is an arrow that when mapping values from `a` to `b` also operates on a state value of type `s`. Composing two Kleisli arrows is equivalent to binding two `a → m b` objects using ⟹. The identity of the Kleisli category is equivalent to the monadic `return`.

The state arrow We already saw the `State` monad in the previous section. If we expand the `Kleisli (State s) a b` arrow:

```haskell
Kleisli (State s) a b
  ~= a → State s b
  ~= a → (s → (b,s))
  ~= (a,s) → (b,s)
```

The final way of putting it skips a few intermediate steps, we define a `StateArrow` then as

```
newtype StateArrow s a b = StateArrow ((s,a) → (s,b))
```

Which is equivalent to the Kleisli arrow we created before: an arrow that when mapping values from `a` to `b` also operates on a state value of type `s`.

The `StateArrow` is not unique in its equivalence to a Kleisli arrow. This is a property that a couple of useful arrows have and that we will be exploiting in the following sections. Below is the interface we provide to describe the Kleislifiable arrow (named `ArrowFunctor` below) and a few examples

```

-- | Some arrows correspond to Kleisli arrows. We should be requiring
-- a monad functor but
class Functor (ArrFunctor c) ⇒ ArrowFunctor c where
  type ArrFunctor c :: * → *

  toKleisli :: c a b → a → ArrFunctor c b
  fromKleisli :: (a → ArrFunctor c b) → c a b

-- | A kleisli arrow is trivially a kleisli arrow
instance Functor m ⇒ ArrowFunctor (Kleisli m) where
  type ArrFunctor (Kleisli m) = m
  toKleisli (Kleisli c) = c
  fromKleisli = Kleisli


-- | A state arrow is a state arrow with a bit of rearrangement.
instance ArrowFunctor c ⇒ ArrowFunctor (StateArrow s c) where
  type ArrFunctor (StateArrow s c) =
    (StateT s (ArrFunctor c))
  toKleisli (StateArrow c) a = StateT $ \s → swap <$> toKleisli c (s,a)
  fromKleisli f = StateArrow $ fromKleisli $ \(s,a) → swap
```

```
    <$> runStateT (f a) s

-- | A writer arrow is also a kleisli arrow on top of a writer monad
-- with a bit of rearrangement.
instance ArrowFunctor c => ArrowFunctor (WriterArrow w c) where
  type ArrFunctor (WriterArrow w c) =
    WriterT w (ArrFunctor c)
  toKleisli (WriterArrow c) = WriterT . fmap swap . toKleisli c
  fromKleisli c = WriterArrow $ fromKleisli $ fmap (fmap swap . runWriterT) c
```

5.2.2. Mealy arrows: processes that remember

The concept that we will dub a *mealy arrow* or *mealy process*, is kin to many different ideas: transducer, automaton, and a coroutine to name a few. A mealy arrow (listing 5.3) is a function that along with the result gives a new version of itself. Like an *automaton* after every iteration it moves to a new state ready to continue the process. Like a *transducer* every element of a stream consumed changes a hidden state to be taken into account when consuming the next one. Like a *coroutine* it can be conceptualized as a process that can yield computation to be resumed by the caller.

```
newtype MealyArrow a b =
  MealyArrow (a -> (MealyArrow a b,b))
```

Listing 5.3.: Haskell definition of a mealy arrow.

The mealy arrow is at the heart of Antisthenis as it represents the basic building block of computation. We call the arrow returned by a Mealy arrow along with the value an *iteration*. Assuming that a mealy arrow represents an incremental computation, every iteration is produced in such a way as to exploit information about the structure of the problem based on the previous computation.

For example, a mealy arrow calculating a multiplication of several sub-arrows may want to try the ones that evaluated to zero in the previous iteration to exploit possible domain knowledge that nodes that are zero are likely to remain zero in future iterations.

Mealy arrow transformer

Like monads [9] , arrows can be composed to form composite arrow types when they are represented as *arrow transformers* For brevity we will only talk about mealy arrow transformers but the concept is generalizable to many different kinds of arrows [88].

The motivation behind "transformizing" the mealy arrow is that arrow evolution as in the plain mealy arrow we presented must be pure (due to it being facilitated by →). As we will see in detail when describing the internals of Antisthenis, as subrocesses evolve they need to interact with external data structures, may throw irrecoverable errors, need to run monadic computations from external libraries etc. None of that can be done based on a pure Haskell function. For that reason, we change the mealy arrow definition slightly (listing 5.4)

```
newtype MealyArrow c a b =
  MealyArrow (c a (MealyArrow a b,b))
```

Listing 5.4.: A MealyArrow can take on the properties of other arrows by swapping out the function → type for a parametric one.

Here the mealy arrow is parameterized by an arbitrary arrow `c` which may be a Kleisli arrow or any other arrow which will define what side effects are supported during the evolution of the process.

Building mealy arrows

While arrows are powerful and general, they can be slightly awkward to work with even with the Haskell `proc` syntactic sugar [50]. Transferring state between mealy iterations via their closures even more so. Capitalizing on the parallels between an arrow and a coroutine, and taking advantage of the rich Haskell ecosystem around monads we define a monad `MB a b m` (standing for *Mealy Builder*) to help us build mealy arrows more easily. The interface to that monad is presented in listing 5.5. The `MB a b m x` typed value, as we will see, can run a computation in `m` and return either a value of type `x` or a new `MB a b m x`. Thereby, since `Void` is an uninhabited type, `MB a b m Void` runs a computation in `m` that *must always* return a new `MB a b m Void`. Just like the mealy process, only `MB a b m Void` does not expect an argument at each iteration..

```
yieldMB :: Monad m ⇒ b → MB a b m a
mkMealy :: (ArrowFunctor c,Monad (ArrFunctor c))
        ⇒ (a → MB a b (ArrFunctor c) Void)
        → MealyArrow c a b
```

Listing 5.5.: .The `MB a b m` monad can be used as a convenience to implement mealy arrows using a conroutine-like interface.

The important part is that we want to do something before each iteration, something after, and we want to feed some state to the next iteration like presented in listing 5.6, where it is how how we can transfer variables between iterations via the closure of the `do`-block.

```
foo :: ArrowFunctor c ⇒ MealyArrow c a b
foo = mkMealy go where
  go :: a → MB a b m Void
  go a = do
    b ← mkResult a
    -- Now we got a result we return it to the user and expect the
    -- argument for the next iteration of the mealy arrow
    a' ← yieldMB b
    -- Now both the next argument (a') and the previous result (b) are
    -- in scope. We can use them both. Whatever we do we must
    -- can't simply end the computation. The mealy arrow must
    -- go on.
    go a'
```

Listing 5.6.: An example of the usage of the MB functor to generate mealy arrows.

This involves `toKleisly` in reference to the "Kleislifiable" arrows we saw in the previous section and lifting to `MB a b m` arrow which is precisely defined in 5.7. The basis here is the functor `MealyF a b` which structurally looks like a non-recursive version of a mealy arrow we saw earlier. That is if `x` is replaced with `MealyF a b _` we get a mealy arrow with an extra value. Unfortunately, there is no direct way to make a law abiding monad out of the `MealyF` type but there is a trivial functor for it, which allows us to use the free monad [36] to get the monad we want. Note how the Mealy arrow is *not* kleislifiable itself. The `MB a b m` monad can't produce any mealy arrow. We can produce only arrows that are from `a` to `b` from `MB a b m Void`.

```haskell
-- | Constructing monadic xmealy arrows
newtype MealyF a b x = MealyF (a → x,b)
type MB a b = Free (MealyF a b)
```

Listing 5.7.: Definition of the MB monad transformer.

5.2.3. Zippers

Now that we have a fairly coherent view of computations, we need one more piece of background to start putting together Antisthenis: the zipper data structure. A zipper [13] is an auxiliary data structure that is similar to (but not exactly the same as) a cursor.

For our purposes a zipper is a data structure that represents an alternative configuration of a container such that it focuses on a specific location in that container, allowing an interface for reading and modifying the element at that location. A zipper should also be able to shift focus to adjacent elements.

Zippers can be very complex data structures with interesting categorical properties arising from the fact that they can implement a command interface [27] but for our purposes we only need a rudimentary understanding of it.

As an example, consider the structure `ListZipper`:

```haskell
data ListZipper =
  ListZipper
  { lzLeft :: [a]
   ,lzCur :: a
   ,lzRight :: [a]
  }
```

A list zipper breaks the list in three parts: a left part `lzLeft`, a focused element `lzCur` and a right part `lzRight`.

```haskell
-- Make a zipper from a non-empty list
mkZipper :: [a] -> Maybe (ListZipper a)
-- get a list from the zipper
getList :: ListZipper a -> [a]
-- Focus on an element on the left if there is one
moveLeft :: ListZipper a -> Maybe (ListZipper a)
-- Focus on an element on the right if there is one
moveRight :: ListZipper a -> Maybe (ListZipper a)
-- Modify the current element
modCur :: (a -> a) -> ListZipper a
-- Read the element in focus.
getCur :: ListZipper a -> a
```

These are fairly self-explanatory, a zipper can shift focus from the element in question to the neighboring elements.

5.3. Antisthenis core

The parts that comprise Antisthenis are roughly split in two categories.

- The *core components* that are related to building the Antisthenis processes, the cells of computation, and the interfaces that need to be implemented to specialize them to specific operations.
- and the *external components* that generally relate cells to each other and the systems that specialize proecesses to implement specific operators.

While in this section we focus on the core components and we will frequently allude to external components, which we will see in detail in sections 5.4 on operators and 5.5 on named processes.

5.3.1. Processes

An Antisthenis process is a node in the evaluation graph and it is built using the concepts described so far in order to implement the following properties:

- *Parameterize* the computation by a cascaded configuration (as we will see `Conf w`)
- *Adapt* the computation between runs using Mealy arrows.
- Return metadata about the result *deliniating its validity* (as we will see `CoEpoch w`)
- Disambiguate betweeen an *uncomputable*, a *partial*, and a *concrete* result (as we will see `BndR w`)

As we described in section 5.2.2 mealy arrows define the basis of an Antisthenis process. Indeed, our definition of an Antisthenis incremental cell is a composition of `MealyArrow`, `WriterArrow` and `Kleisli` presented in 5.8

```
type ArrProc w m =
  MealyArrow (WriterArrow (ZCoEpoch w) (Kleisli m))
  (Conf w) (BndR w)
```

Listing 5.8.: The type of an Antisthenis process is a Mealy arrow paired with a kleisli arrow.

The `w` parameter is simply a token related to the operation that the cell is evaluating that we use helps us parameterize the types involved in a process by leveraging the type family feature of Haskell's type system [103]. We call it a `ZipperParams` tag because, as we will be seeing throughout the chapter, it mainly parameterizes the evolution of the internal state of the Antisthenis process, the `Zipper`.

We will describe every type involved in the `ArrProc` type separately, but to do that we first provide a general intuition of what these types mean. The basis is a mealy arrow that emits a monoidal value `ZCoEpoch w` via `WriterArrow` that expresses metadata, along with the result of the computation (`BndR w`). It is a datastructure that summarizes the provenance of the value returned described in detail in section 5.3.3 on Antisthenis caps and bounds. It is used to determine when the value is no longer valid, for example, due to changed parameters.

The `MealyArrow` and `WriterArrow` combined transform an arbitrary Kleisli arrow which is used to embed the computation of Antisthenis processes into parent monadic computations, which in the case of FluiDB is the `PlanT t n m` monad as we saw in section 4.2. In the FluiDB case, that would be the monad computation associated with planning. This way we give the Antisthenis processes the means to lookup values, throw errors, or generally have the effects of any computation we require.

The output side of the `ArrProc` is a `BndR w` type, the returned value, which is parametric to the operation but has some common structure (listing 5.9). This indicates that every Antisthenis process has three options for the value they can return all of which are parameterized on a per-operation basis (ie `ZErr`, `ZBnd` and `ZRes` are type families):

- An error indicates that the value is not computable.
- A final and precise result.
- A partial result, a convex bound indicating that calling the iteration of the arrow will get us closer to ax‘ final result. We will have a chance to see the bounds in detail in the section 5.3.3 on caps and bounds.

```haskell
data BndR w
  = BndErr (ZErr w)
  | BndRes (ZRes w)
  | BndBnd (ZBnd w)
```

Listing 5.9.: The definition of the return value of an Antisthenis process. It may be a final result, an error indicating that a final result is non-computable, or a bound for the final value.

Indeed, the iteration of the process has a very different meaning depending on whether the previous returned value is a bound or an error/result. In the former case the iteration continues a previous computation, in the latter there are two options.

- The previously returned value is still valid and is therefore returned,
- If it is not and the computation is reset.

To reflect this fundamental difference between a paused and a finished computation we will call the former simply an *iteration* and the latter *coiteration*.

Finally, we turn our attention to `Conf w` which represents the configuration for running the next step of the computation. Much like `BndR` the configuration is parameterized w.r.t. `w` but has a basic structure presented in listing 5.10

```
data Conf w =
  Conf { confCap :: Cap (ZCap w)
        ,confEpoch :: ZEpoch w
       }
```

Listing 5.10.: The type definition of a conficuration. It is a tuple containing information that can be used to derive whether a value is valid.

The configuration for running a computation includes an *epoch* that is used to deprecate computed values that are no longer valid (discussed in section 5.3.2) and a *cap* that is used to decide when a partial result should be returned (discussed in 5.3.3). The configuration is passed as input to each computation and propagated to the children computations. If the process is iterating (as opposed to co-iterating, i.e., it has not returned a concrete result or an error), the cap is compared against the previously yielded bound, if the cap is less than the bound returned then the process must keep processing until it finds a bound that exceeds that cap. This way we can control how much work a process is allowed to do before switching to one that might help Antisthenis to cut the overall computation short. This way, Antisthenis avoids falling into computational rabbit holes, where it evaluates a longer computation when switching to a shorter one would help arrive sooner to a final result.

The *epoch* represents state external to Antisthenis like parameter versions. When the epoch indicates that a parameter on which the process depends has been updated, the process needs to be reset and start the computation from scratch (see section 5.3.6 on process resetting).

5.3.2. Epochs and coepochs

As mentioned in the previous section, the interplay between epochs and coepochs determines the validity of the value of each Antisthenis process. When the returned value is no longer valid the process needs to be reset; in particular, the epoch mainly indicates the "version" of the computation parameters. This might represent the full external state, but the purpose of the epoch is to be used as an increasing value that can be used by the processes to infer whether their progress is based on out-of-date assumptions. For example, the epoch may be a map of a natural number per computation parameter indicating the number of times that parameter has been updated. Or it might simply be the actual value of each parameter if it is cheap the compare against. Each process then can compare the version or value of each parameter of the epoch to the ones it used to build its internal state. When a discrepancy is found the process knows its state is out of date and must reset.

It is clear that not all parameters are relevant to all processes. Most processes only depend on a subset of parameters, or in Antisthenis' terms, most processes only ever depend on parts of the epoch. This reference to a subset of any epoch is reified by the *coepoch*. The coepoch is the monoidal type parameter of the writer arrow transformer of `ArrProc`. In practice this means that due to the writer semantics, the coepochs of evaluated subproecesses are concatenated to produce the coepoch of the parent process. A special case for this are the commutative operations that involve absorbing elements ($\wedge$, $\vee$, and $\times$), where only the coepoch of the subprocess that yields an absorbing element is returned. In short, the coepoch of a subprocess is included in all parent processes that are not constant with respect to the value of that subprocess.

In principle, the function that the `ZipperParams` tag needs to implement to facilitate the epoch/coepoch interplay is a function that has a type equivalent to listing 5.11.

```
combEpochCoepoch0 :: ZEpoch w -> ZCoEpoch w -> Bool
```

Listing 5.11.: The type of a naive function checking the validity of a value based on epoch and coepoch.

That will indicate whether the value needs to be pushed down. In practice we can do slightly better at that by enforcing the following law on the `combEpochCoepoch0` function:

$$\mathsf{combEpochCoepoch}_0(e, c_0 \diamond c_1) \Rightarrow$$
$$\mathsf{combEpochCoepoch}_0(e, c_0) \wedge \mathsf{combEpochCoepoch}_0(e, c_1)$$

Where $e \in ZEpochw$, $c_0, c_1 \in ZCoEpochw$ and $\diamond$ is the monoidal merging operation on coepochs.

We therefore have the option of filtering the epoch into a smaller subset of itself that is only relevant to the subprocesses that contributed to the generation of the coepoch being checked. Thereby the combination function can optionally return a new epoch, rendering the type of the function to be something similar to listing 5.12. It is worth noting that if by this process we determine that a reset is required, a new coepoch is also required and therefore there is no way to constrain the epoch to run the reset process.

```
combEpochCoepoch :: ZEpoch w → ZCoEpoch w → Maybe (ZEpoch w)
```

Listing 5.12.: The final function for epoch and coepoch combination.

5.3.3. Antisthenis caps and bounds

In section 5.3.2 regarding epochs and coepochs we described how Antisthenis deals with the validity of values returned by a process. In this section we will focus on how the children processes avoid falling into avoidable computational rabbit holes using the interplay of *caps* and *bounds*. In the example discussed in the introduction of this chapter (5.1), we saw that there are cases where, as soon as a subprocess can prove that its value is going to exceed a certain threshold (*cap*) any further work on its part is likely futile. It then returns a *bound* which is propagated up the chain of parent processes and handled at the point where the cap was imposed. The reader is encouraged to maintain a mental model where the cap is an threshold and the bound is a lower bound.

We expect that the bound of a function satisfies the *mototonicity criterion*:

$$C \dot{<} B[f(a_0, ...)] \Rightarrow C \dot{<} f_B(B[a_0], ...)$$

Where C is the cap, $B[E]$ is the bound of a process expressed by the expression E by only taking into account the top level terms. f is the function of the operator and f_B is a function that combines the bounds of the arguments to come up with a bound for f. $C \dot{<} b$ means that the bound b exceeds the cap C. In simpler terms, this means that if a bound exceeds the cap, evaluating the arguments and coming up with tighter bounds will still exceed the cap.

There may be more requirements of the properties of a cap w.r.t. a particular operator. For example, the minimum operator requires that bounds be translatable to caps and that if a bound b is translated to a cap c and $b' \dot{<} c$ then $b' < b$. This requirement is only related to the way the particular operator is implemented and is not necessarily a requirement for any other operator.

The `ZipperParams` tag `w` needs to provide a way for comparing caps to bounds. Unlike the case of coepoch and epoch combination function this one is much more straightforward:

```
exceedsCap :: ZCap w → ZBnd w → Bool
```

Simply given a cap and a bound, check if the bound exceeds the cap. A special case of a cap is `ForceResult` for which `exceedsCap ForceResult _ = False`. As the name suggests `ForceResult` is used when we want a process to necessarily return a concrete result or an error, as opposed to a bound.

Now that we have seen how the cap is handled on the input side we should discuss how the parent process passes a cap to its child processes. The cap is created in an operator-specific way (i.e., via the overloading of a function based on the tag `w`). In particular an operator defines a function that "localizes" the entire configuration.

```
data MayReset a = DontReset a | ShouldReset
zLocalizeConf :: ZCoEpoch w → Cong w → Zipper w p → MayReset (Conf w)
```

The `zLocalizeConf` function accepts a coepoch, the configuration that the parent process receives and returns the configuration passed. The `zLocalizeConf` function also decides whether the process needs to be reset by calling the `combEpochCoepoch` function that we discussed in subsection 5.3.2 on epochs and coepochs.

To make all this more tangible, a process that computes the sum of its subprocesses might implement `zLocalizeConf` roughly like shown in listing 5.13. Since the process knows it will be evaluating the cursor, it subtracts the partial sum so far from the cap to come up with the cap to pass to the subprocess.

```haskell
zLocalizeConf coepoch conf z =
  combEpochCoepoch coepoch (confEpoch conf)
  $ conf { confCap = newCap
         }
  where
    -- zRes is the partial result so far
    newCap = case zRes z of
      SumPartErr _ -> error
        $ "The partial sum is an error: this thunk "
        ++ "should never be reacahble because "
        ++ "no subprocesses should be called."
      SumPartInit -> confCap conf -- No subprocesses have been evaluated.
      SumPart partRes -> case confCap conf of
        -- partRes is the total sum so far. We offset the global cap
        -- by the sum so far so that the local cap ensures that the
        -- cursor process, when ran, does not cause the overall
        -- bound to exceed the global cap. This may generate weird
        -- caps like negative values. That is OK as it should be
        -- handled by the evolutionControl function. Note again that
        -- zRes does not include the value under the cursor.
        CapVal cap -> CapVal $ subCap cap partRes
        ForceResult -> ForceResult
```

Listing 5.13.: A sample implementation of the function that transforms the configuration received by a parent process into one suitable for the child process. Checks if the parent process needs to be reset and uses the partial result to constrain the cap. This particular implementation is taken from our implementation of the sum operator.

5.3.4. Antisthenis zipper

We saw in the introduction of Antisthenis (section 5.1) what a zipper is in general. Here we will specialize the notion for the specific case of the internal state of the computation. A single process is defined by an operator, the subprocesses and a partial result for the computation. The process evolves by evaluating a subprocess at a time. We define a zipper structure that:

- Focuses on the particular process to be evaluated next.
- Arranges a reset process that restarts the evaluation from a state that possibly takes into account information about the previous evaluation, for example, by starting evaluation from a subprocess that previously yielded an absorbing element.

With this in mind, we define the zipper data structure keeps track of the state of the node (see figure 5.2).

- A set of *initial* processes that have not been evaluated yet or that have yielded deprecated values (see section 5.3.2 on epochs and coepochs).
- A set of *iteration* processes, processes whose latest evaluation has yielded a result bound is represented by a data structure that associates the iteration processes with initial processes bounds returned. Since these processes that have yielded a value bound and need to be rerun with a different cap (see section 5.3.3). It should be stressed that this still acts like a heap of subprocesses where the internal properties of the data structure decide the top element that is to be popped. Since we have some information about the final result of these processes, namely the bound, the parent process can be smart about the order in which they are evaluated. As the particular strategy depends on the operation implemented by the parent process, the particular data structure used is also dependent on said operation. For example, a process implementing a sum between the subprocesses stores them in a list as there is no beneficial order in which to evaluate the subprocesses, while minimum operation benefits from evaluating the processes that have lower bounds first (for more details on the minimum operation see section 5.4.1).
- *Coiteration processes* are processes that have been evaluated and yielded a concrete

value (error or result), i.e., a value that is predicated on the epoch but can not be refined by re-evaluating the process with a more permissive cap. Upon reset the coiteration stack is ordered in an operator-specific way and moved to the init stack.

- A *cursor* that is the next process to be triggered along with a way to reset it and the previous value.
- A *partial value* to which values can be inserted or removed (for example the sum accumulated up to that point) which represents the progress of the process so far. As values, concrete or partial are extracted from the subprocesses the partial value is updated. The partial value does *not* include the value stored in the cursor.

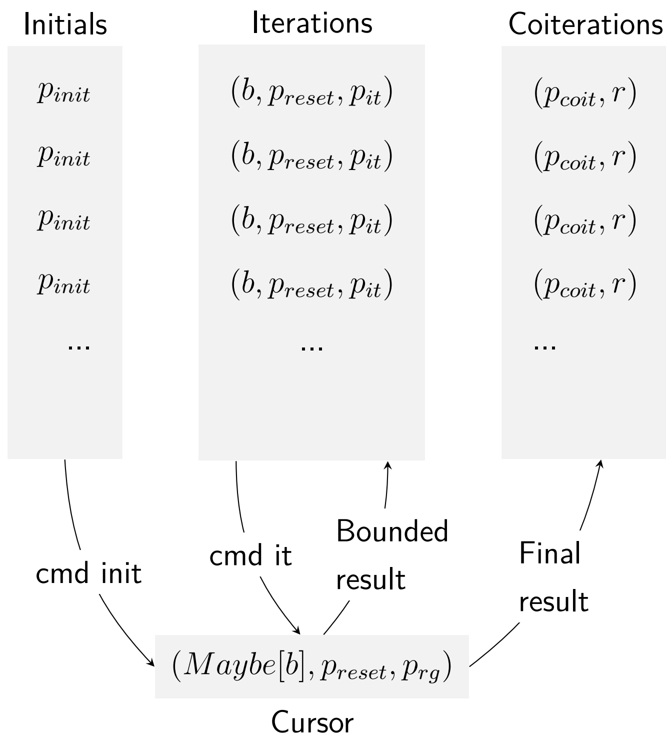


Figure 5.2.: The zipper splits the sub-processes that cooperate to compute values for the owning process in four categories a) initials that have not yielded a currently valid value b) iterations that have not yielded a valid partial value c) coiterations that have yielded a full valid value d) the cursor that is the next sub-process to be evaluated. Depending on the evaluation strategy of the operator once the cursor process is evaluated it is replaced with an init process or an iteration process, and depending on the value the cursor process yields it is pushed to the iterations or the coiterations of the zipper.

During computation, the subprocesses owned by the parent process are evaluated, and their iterations are then moved from the *initial* process set to the *coiteration* process set, possibly with an intermediate stop in the *iterations* set according to the values and strategies of the operator implementation which is further elaborated in section 5.4 on Antisthenis operations. The structure of *iterator set* is highly operator dependent and primarily geared towards efficiently evaluating the parent process, but fundamentally it must not compromise on the correctness of the operator. For example, a process is combining elements of an Abelian group (commutative, associative, invertible) and the bound has the same type as the result – as it is the case with addition – the structure of the zipper is fairly straightforward:

- The partial result maintains the aggregation of all values and bounds encountered.
- When evaluating the cursor we add the result or returned bound to the group and remove the bound of the next cursor if we are popping from the iteration set.
- The bound of the final value at any moment is the partial result.

At the other end of the spectrum, a process evaluating a magma (a set with a closed binary operation), where the binary operator has no properties besides closure, must never push anything to the iterations set. The set is equivalent to unit (`()`), and the cursor is forced to yield a concrete result or an error by being run with a `ForceResult` cap.

To make the concept tangible, below are some examples of what the iterator set is defined as for different operations (all numerical operations are assumed to operate on non-negative numbers, in particular query costs):

- Addition is the simplest case: the iteration set is an always-empty container since we always evaluate the cursor until it is sent to the coiteration list. In the context of addition there is no heuristic to predict which subprocess is more beneficial to evaluate.
- For multiplication over positives, we evaluate the ones that have a zero bound first hoping that they will turn out to evaluate to absorbing elements. Therefore, the iteration set is represented as two lists: one where the zero-bounded iterations are stored and one for the rest.
- In the case of the minimum operator the iteration set is a heap. In each step we evaluate the one with the minimum bound. When the minimum bound exceeds the minimum concrete result encountered, the min concrete result is the final result.

For completeness, we provide the definition of the `Zipper` in listing 5.14

```
data ZipState w a =
  ZipState
  { bgsInits :: [InitProc a]
```

```
    ,bgsIts :: ZItAssoc w (InitProc a,ItProc a)
    ,bgsCoits :: [(Either (ZErr w) (ZRes w),CoitProc a)]
   }

data Zipper' w cursf (p :: *) partialRes =
  Zipper
  { zBgState :: ZipState w p
   ,zCursor  :: cursf (Maybe (ZBnd w),InitProc p,p)
   ,zRes     :: partialRes -- The partial result without the cursor.
  }
type Zipper w p = Zipper' w Identity p (ZPartialRes w)
```

Listing 5.14.: The definition of the zipper.

Finally, each operator defines the type of the partial result (`ZPartialRes`) maintained in the zipper, as well as functions for putting and replacing values in it. When the subprocess of the cursor is evaluated the result needs to be either "appended" into the partial result if the new cursor is drawn from the initials set, or to replace the corresponding bound value if it is replaced with a value from the iterations set.

5.3.5. The Cmds functor

As indicated in section 5.3.4 on Antisthenis zippers the process evolution is guided by shifting the zipper focus to different subprocesses and then evaluating them.

A process is internally represented as a Mealy arrow (introduced in section 5.2.2) describing the evolution of a `ZProc` which is transformed into the Antisthenis process described in section 5.3.1 by following the strategy in which the `Zipper` is meant to evolve. An operator is allowed to follow a strategy defined specifically its `ZipperParams` tag, but the function for evaluating said strategy applies to the `ZProc` object. There are two important aspects to be noted about `ZProc` (listing 5.15).

- The return value is a zipper full of processes `Zipper w (ArrProc w m)`
- The Kleisli arrow is a free functor of commands `FreeT (Cmds w) m`.

```haskell
type ZProc e m =
  MealyArrow
    (WriterArrow (ZCoEpoch w) (Kleisli (FreeT (Cmds w) m)))
    (LConf w)
    (Zipper w (ArrProc w m))
```

Listing 5.15.: An internal representation of the process evolving the internal representation of a process: the zipper. Note the use of `FreeT Cmds` in the Kleisli arrow.

`Cmds`, used in type parameter of the Kleisli arrow in `ZProc`, is a union type encapsulating different directions to which the zipper can be evolved. Its definition is provided in listing 5.16. The point of `Cmds` is to separate the logic of *how* each possible zipper transition is carried out which is largely operator agnostic, from the logic of *which* transition should be carried out which is highly operator dependent. `ZProc` is therefore a process that simultaneously encapsulates all possible evolution paths for the zipper. The command constructors reflect the different evolution steps that can be taken at each time:

- It can always be reset the process
- It can pop a process from the initial to the cursor to be executed
- It can pop a process from the iterator set if it is not empty. Which one is to be popped is decided by the parameters
- The may have reached a final value where it can't be evolved anymore.

```haskell
data Cmds' r f a =
  Cmds { cmdReset :: ResetCmd a
        ,cmdItCoit :: ItInit r f a
       }

data ItInit r f a
  = CmdItInit (ItProcF f a) (InitProc a)
  | CmdIt (ItProcF f a)
```

```
  | CmdInit (InitProc a)
  | CmdFinished r -- when a process finishes it should stick to a
                  -- value until the epoch/coepoch pair requests a
                  -- reset.
```

Listing 5.16.: Definition of the commands functor that provides different branches of evolution for zipper.

`ZProc` internally takes care of the logic related to pushing the evaluated process of the cursor into the appropriate set. The `Cmds` functor facilitates a tree of actions in conjunction with a free monad. A free monad of functor `f` is actually many `f` nested like `f (f (f ... f (f a)))`. This that `FreeT Cmds m` is an arbitrary depth tree of `Cmds`. This tree is navigated in an operator-specific way guided by the generic function `evolutionStrategy` the type of which is presented in listing 5.17.

```
evolutionStrategy
  :: forall x .
  FreeT (Cmds w) m x
  → m (Maybe (RstCmd w m x),Either (ZCoEpoch w,BndR w) x)
```

Listing 5.17.: A function that each Antisthenis operator needs to implement, that decides the traversal of the tree created by the different possible evolution paths of zipper. The implementation may also deem that a good reset point has been discovered.

By means of evaluating the `FreeT` monad transformer, this function navigates the tree in the way that is most beneficial to the particular operator and comes up with a pair of values:

- A reset command to be used if reset is required in the future (this will be discussed in the section about 5.3.6 resetting)
- When encountering a `CmdFinished` value return the result of the finished operation, otherwise traverse the tree to the preferable leaf and return the leaf.

An abridged example is presented in figure 5.3.

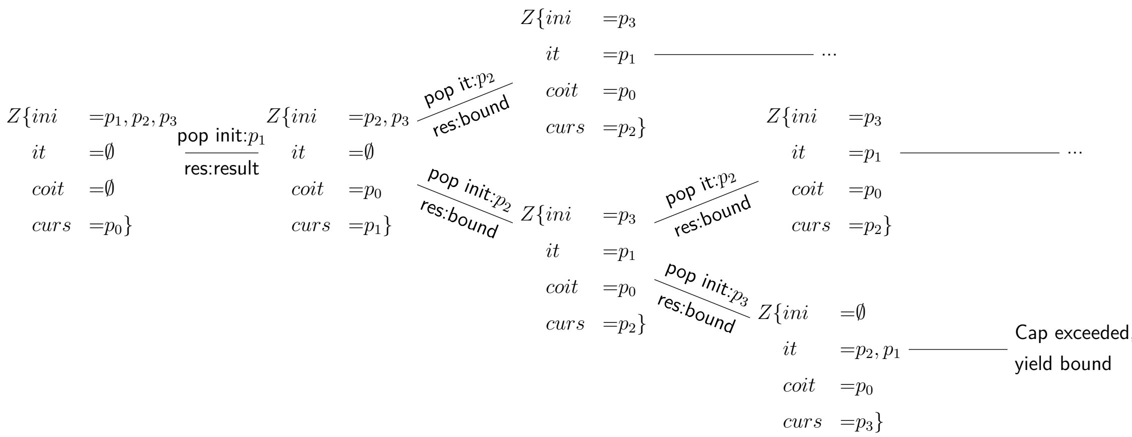


Figure 5.3.: A tree of different evolution paths a zipper may undergo during between the calling of a process and the accumulated bound exceeding the cap. p_0, p_4, p_3 and p_4 are subprocesses. The values returned, the specific iteration subset data structure are not presented for brevity and to shift focus to the movement of the subprocesses within the zipper. Nodes in this tree that have two children correspond to the `ItInit` constructor, nodes with one node correspond to the `CmdIt` or `CmdInit` constructor depending on which process set the process in the next cursor is drawn from. When a concrete value is reached or the cap is exceeded the process stops and the free monad that wraps `Cmds` takes the value `Pure`. A tree like this is reified by the `Free Cmds` monad, the traversal of the tree is a completely separate logical process that is operator specific.

5.3.6. Resetting processes

In subsection 5.3.5 discussing the `Cmds` functor we mentioned that the operator needs to always be able to reset the process. Why create a reset process when reusing the initial process would reset the computation just as well? In every case, a reset moves all the init parts of the elements of iteration set as well as the coiterations into the init set of the zipper. But in what order? From previous runs it is often apparent that some execution strategies are more likely to lead to a quick result than others. For example, under the assumption that processes change their values less often than they maintain them, while computing a logical AND operation ($\wedge$) we likely want to start the computation from the processes that previously yielded `False` to take advantage of early termination due to absorbing elements.

To accommodate this, the `Cmds` functor can yield a reset point. While running a process we maintain in the lexical scope of the mealy arrows computation the latest reset process to switch to when the epoch/coepoch comparison requires as much. When generating a `Cmds` functor, the operator has the option to provide a reset process that resets a running zipper that is at an arbitrary state.

5.4. General operators

While describing the core concepts of Antisthenis we often dealt with functionality that is operator specific. We reify operators via uninhabited types that instantiate the `w` parameter of many types we mentioned so far (`BndR w`, `Zipper w _`, `ArrProc w m`, etc). We refer to these types as *tags*. We have implemented a few tags in Antisthenis to cover the needs of FluiDB (see listing 5.19).

```
-- Result types
class BndRParams w where
  -- The result type when the value is non-computable
  type ZErr w :: *
  -- The partial result.
  type ZBnd w :: *
  -- The full result
  type ZRes w :: *

-- Zipper internal types
class BndRParams w => ZipperParams w where
  -- The type of the process cap
  type ZCap w :: *
  -- The type of the process epoch
  type ZEpoch w :: *
  -- The type of the coepoch
  type ZCoEpoch w :: *
  -- The type of the iteration set
  type ZItAssoc w :: * -> *
```

```
  -- The type of the partial result
  type ZPartialRes w :: *
```

Listing 5.18.: Operator specific types that need to be implemented by every operator.

```
-- The witness type for the sum operator
data SumTag p
-- The witness type for the minimum operator
data MinTag p
-- The witness type for multiplication
data MulTag p
-- The witness type for logical And/Or
data BoolTag op
```

Listing 5.19.: Tags are phantom types that witness the Antisthenis operations. The tags themselves may be parameterized using type parameter `p`. For the needs of FluiDB we define operations for addition, subtraction, multiplication, and Boolean operations.

The parametric part of the tags, especially for `SumTag` and `MinTag`, is due to the fact that we want to be able to further parameterize them for concrete cost calculation (using sum and min operators normally) and for stochastic cost calculation (see section on 5.5.4 historical cost). These two both utilize minimums and sums, but they use different types for return values as well as caps/bounds and epochs/coepochs.

Since all types and implementations can be derived from the `ZipperParams` type `w`, the Antisthenis process creation is a completely generic process `mkProc` (see listing 5.20) which combines a list of subprocesses into a parent process that evaluates the operator described by the tag `w` without requiring any further information other than the implementation of `ZipperParams`.

```
mkProc
  :: (Monad m,ZipperParams w,Eq (ZCoEpoch w)) ⇒ [ArrProc w m] → ArrProc w m
```

Listing 5.20.: The type `w` fully defines the operator so combining subprocesses into a process is unambiguous.

Compatibility between heterogeneous processes

The astute reader has likely noticed that `w` is the parameter of both the subprocesses and the process even though they may be different operators, for example, the operands of a min are themselves sum expressions in FluiDB cost estimation. The reason is that a process expects a specific type of coepoch and bound to be returned by its subprocesses, while the subprocess must be able to deal with the epoch and cap that the parent process is defined for. To do that we avoid defining an $N \times N$ correspondence between all the operators, but rather we define an ad-hoc record that facilitates the conversion, the type of which is presented in listing 5.21. This way we can compartmentalize the translation among operators from the internal logic of the operators.

```
data Conv w w' =
  Conv
  { convEpoch :: ZEpoch w' -> ZEpoch w
   ,convCoEpoch :: ZCoEpoch w -> ZCoEpoch w'
   ,convCap :: ZCap w' -> ZCap w
   ,convRes :: ZRes w -> ZRes w'
   ,convBnd :: ZBnd w -> ZBnd w'
   ,convErr :: ZErr w -> ZErr w'
  }
convArrProc :: Monad m => Conv w w' -> ArrProc w m -> ArrProc w' m
```

Listing 5.21.: An object of type `Conv w w'` can act as an interface between parent processes of type `w` to subprocesses of type `w'`.

5.4.1. Basic cost estimation operators

General cost estimation involves minimum and sum operators with some special semantics:

- Non-computable values are larger than any value from the perspective of minimum.
- All values are positive, therefore all values are implicitly bounded by $-\lim_{\epsilon \to 0} \epsilon$

Sum

We do not implement a general sum operator but rather a sum operator that assumes the properties of query costs as the values it deals with. The sum operator is probably the simplest operator implemented. The sum zipper evaluates each initial subprocess under a cap $c_{subproc}$ that is

$$c_{subproc} := c_{parent} - r_{partial}$$

where $r_{partial}$ is the partial result omitting the cursor, i.e., the current subprocess and c_{parent} is the cap that the parent process is operating under. If the parent cap is `ForceResult` all arguments are also evaluated under `ForceResult`. Of course if $c_{parent} > r_{partial} + b_{cursor}$ then $r_{partial} + b_{cursor}$ is returned as the result bound.

Minimum

Much like the sum operator, the minimum operator implementation also assumes that it is manipulating cost values, and it is used in conjunction with the sum operator to define our cost evaluations. The operation of the minimum Antisthenis operator depends heavily upon the iteration set which has the interface of a heap. The zipper evolution strategy is described by the pseudo-python code in listing 5.22

```python
def exceeds_cap(cap, zipper):
    """A cap is exceded by the current state of a process computing
    a minimum value when both the best (minimum) concrete result and the
    best bound exceed it.
    """
    return zipper.iter_procs.min().bound > conf.cap \
        and zipper.coiter_procs.min().result > conf.cap \


def minimum_zipper_evolution(conf,zipper):
    # Run each initial process with a zero cap
```

```
    res,nxt_proc = zipper.cursor.run(conf{cap = 0})
    while len(zipper.initial_procs) > 0:
        # We cant hope to get a cost lower than zero
        if res == BndRes(0):
            # Until the epoch is updated
            return BndRes(0)

        # Push it to the correct iter set.
        if res.is_bound():
            zipper.iter_procs.push(res,nxt_proc)
        else:
            zipper.coiter_procs.push(res,nxt_proc)

        zipper.iter_procs.push(zipper.cursor)
        zipper.cursor = zipper.initial_procs.pop()
        res,nxt_proc = zipper.cursor.run(conf{cap = 0})

    # Run the iterating processes
    while True:
        # If the result is better than the best iteration, it's the final result
        if zipper.coiter_procs.min().result < zipper.iter_procs.min().bound:
            return BndRes(zipper.coiter_procs.min().result)

        # If there are two iter processes there's work to do
        if not exceeds_cap(conf.cap, zipper):
            # Run the best iterating process until it exceeds the
            # second best iterating process or the final result
            cap = min(zipper.iter_procs.second_min().bound,
                      zipper.coiter_procs.min().result)
            res,nxt =  zipper.iter_procs.min().run(conf{cap = cap})
            # Push it to the correct iter set.
            if res.is_bound():
                zipper.iter_procs.push(res,nxt_proc)
            else:
```

```
                zipper.coiter_procs.push(res,nxt_proc)
        else:
            # Otherwise yield the bound and get a new conf
            conf = yield zipper.iter_procs.second_min().bound
```

Listing 5.22.: Pseudo-python for the algorithm of evaluating a process up to a threshold defined by the cap. For brevity we omit sanity checks and the reset handling code.

Boolean conjunctions and disjunctions

Antisthenis implements operators for Boolean conjunction and disjunction to answer the question of whether a node is materializable in the presence of a specific inventory of materialized nodes. Booleans are different from the other operations in how they handle caps. Boolean $\wedge$ and $\vee$ do not use the same type for caps and bounds as for results. Instead, the bound is a two dimensional value that represents the minimum number of steps required to arrive to each possible value (`True` or `False`).

To efficiently evaluate Boolean algebra expressions we lean on the existence of absorbing elements. Consider the evaluation of a Boolean expression

$$Q := A \wedge B \wedge C$$

where A, B and C are subexpressions. Obviously if any of the expressions A,B or C evaluates be `False` the entire expression would consequently evaluate to `False` as `False` is the absorbing element for the group of Booleans over $\wedge$. Furthermore without taking into account the structure of each of the subexpressions A, B and C we can tell that the amount of work for evaluating it depends on the result. If the result of this expression is `False` then the best case scenario in terms of amount of work we would need to do is for A to be the expression `False`. Or more succinctly

$$w(Q|Q = False) \geq \min[w(A|A = False), w(B|B = False), w(C|C = False)]$$

Where $w(E|E = k)$ is the amount of work required for calculating E if E turns out to be k. In this case, we would be able to evaluate the expression with just one dereference. If the result is `True` then the best case scenario is for A, B and C to all be `True` and we would be able to come to a result with three dereference steps. Or equivalently

$$w(Q|Q = True) = w(A|A = True) + w(B|B = True) + w(C|C = True)$$

Now let us consider the compound expression

$$X := (A_1 \vee A_2 \vee A_3) \wedge (B_1 \vee B_2) \wedge (C_1 \vee C_2 \vee C_3)$$

Where A_i, B_i and C_i are expressions. How would we navigate this expression to minimize the number of operations? We could completely expand each term and hope that we are lucky, and we encounter absorbing elements early on. As explained in this chapter's introduction, especially for terms automatically generated this strategy is unlikely to be efficient.

Instead, at each expansion we track the minimum number of steps required for each value. The best case scenario if $X = False$ is $B_1 = False$ and $B_2 = False$, in which case the expression is evaluated in a minimum of 4 steps (dereferences):

- Evaluate $B_1 = False$ in a single step
- Evaluate $B_2 = False$ in a single step
- Evaluate the subexpression $False \vee False$
- Evaluate $X = (...) \wedge False \wedge (...) = False$

For a $X = True$ we need 7 steps:

- Evaluate $A_i = True$ in a single step for any i
- Evaluate the subexpression $True \vee ...$

- Evaluate $B_i = True$ in a single step for any i
- Evaluate the subexpression $True \vee ...$
- Evaluate $C_i = True$ in a single step for any i
- Evaluate the subexpression $True \vee ...$
- Evaluate $X = True \wedge True \wedge True$

A Boolean expression in Antisthenis, then, takes the bound value of the minimum cost for $True$ values and $False$ values. If this expression were evaluated with a cap of $\{True : 6, False : \infty\}$ it would return a bound of value $\{True : 7, False : 4\}$. The meaning of this bound value in plain English is "re-try to evaluate this if your other options are more expensive than 7 steps for `True` and 4 for `False`". It could be seen as a metric for the size of the tree.

To demonstrate the derivation of caps we present the following expression:

$$X = X_1 \wedge X_2 \wedge X_3$$

Any of the X_i terms has a minimum bound of $\{True : 1, False : 1\}$. Since the absorbing element for $\wedge$ is `False` expanding any of the X_i terms is bounded by $\{True : \infty, False : 1\}$ so an expansion of

$$X_1 = A_1 \vee A_2$$

would cause the subprocess to immediately yield $\{True : 2, False : 3\}$ passing control to the parent process. Thus, a symmetrically growing tree would be traversed in a breadth first fashion and any asymmetries would cause Antisthenis to try to greedily exploit the smaller parts of the tree.

5.5. Node machines

A referable machine, or node machine, or simply machine is, in the context of Antisthenis, a process attached to a name or node. That can be a subprocess to more than one other processes. In FluiDB a machine name corresponds to a QDAG node since the Antisthenis infrastructure is utilized to provide incremental computation of properties on nodes (materializability, cost, stochastic cost). The term node is used in reference to the nodes of a computation DAG or graph. In this section we look into the the layer of Antisthenis that takes into account the dynamics of referable machines.

5.5.1. Antisthenis machine tape

The *machine tape* is a dynamic directory that maps nodes to machines. The meaning of a node is dependent on the implementation, in the context of FluiDB nodes in this context correspond to the FluiDB internal graph n-nodes. The machine tape is part of the computation context and is updated by new and temporary values of machines (see section 5.5.3 on cyclical machines).

Not all processes involved in a computation can also be found in the machine tape. Each machine in the tape is composed by a hierarchy of processes that are inaccessible from outside that machine. In fact a machine is different to a process solely by virtue of the fact that it can be referenced via the tape.

Since machines are referable by different processes it means that we need to be careful that more than one process do not process their own diverging copy of the same machine. To avoid this, processes referring to machines do so indirectly via a *machine wrapper process*. A machine wrapper is a process that always does the following:

- Look up a name in the tape.
- Temporarily replace it with a cyclical machine (see section 5.5.3).
- Evaluate the machine that was just removed from the tape.

The machine tape state is managed via a state monad in the underlying Kleisli arrow.

Thereby, a recursive reference is formed between the tape and the machine.

5.5.2. Process stack

Before we go further on the details of how exactly named machines are handled, it is important to make the distinction between a process and a computation. In the context of Antisthenis a process is the mealy machine that evolves during evaluation and survives between evaluations. On the other hand, a computation is the part of the evaluation itself that refers to the particular machine. In a more abstract sense, a computation is the *relation* between the process and the value produced, regardless of whether that value is an actual result, a bound for a potential concrete result, or a witness to the non-computability of the result (error).

With this distinction in mind, we introduce the notion of a process stack which is similar to the call stack in most programming languages. The process stack contains all processes that participate in the computation at any given moment. When a process is called, it is pushed to the process stack and when a result is returned it is popped from the process stack. The process stack is completely ephemeral, acyclic and should be completely empty between computations. In this sense, it is part of the *computation context*, the set of effects that affect each computation separately. The computation context may be implicit (like in the case of the process stack as we will see when discussing cyclical machines in section 5.5.3) or explicit in the process' underlying Kleisli arrow.

5.5.3. Cyclical machines

By virtue of machines being referable by arbitrary other machines, it is possible, and certain in the case of FluiDB, that the Antisthenis machines will form referential cycles. Such cycles in most computational frameworks, especially ones that deploy a non-lazy order of computation, will cause the computation to either fail or to not-terminate. Antisthenis is explicitly designed around allowing the computation to handle such self-referential cases where possible.

We already discussed that machines are stored in a data structure called a tape that allows them to be referenced by other machines, and that in the period from the time a value is requested from a machine and the time a value is returned (including an error or a value bound) the machine is said to be in the computational stack (section 5.5.2). During its residence in the stack, a machine is in a state where it is unable to produce a value. A cycle occurs when a machine is referenced while in the computational stack.

This works for some case but is problematic in general as the process stack is an ephemeral property (i.e. a property specific to the current computation) and yet it affects the return values that are non-ephemeral, (i.e. they survive the computation). The only solution for this is to lift information about the cycles to the non-ephemeral plane. Indeed, what is needed is for values that are deemed non-computable to *reset* when accessed with a different stack. Coepochs fill the bill precisely. We extend the coepoch to include, along with the parameter versions used to derive a particular result, also *a set of machines that need to be non-computable in order for the value to be valid.*

To accomplish this, when a machine is looked up, before being evaluated it is removed from the tape and in its place a *cyclical machine* is placed. This machine always returns the same $\perp$ result that indicates that the value is non-computable and sets to the coepoch that it is itself not computable. A machine named A while being computed is replaced in the tape with a cyclical machine that returns $\perp$ predicated on A being uncomputable, denoted $\perp_A$. This coepoch property is propagated to machines that call the cyclical machines, effective making their value also depend on A being uncomputable.

When the original machine comes up with a value, it replaces the cyclical machine with the iteration or coiteration of the process and the coepoch is *censored* to remove the predicate that the current machine must be $\perp$. This way whenever any machine tries to evaluate another non-computable machine it finds a cyclical machine which handles the situation automatically: Upon evaluating the cyclical machine, the coepoch of the caller is infected with the predicate that the cyclical machine must be $\perp$ (see listing 5.23).

```python
def cycle_machine(node):
    # Return a process that returns non-comp and a predicate nofifying
    # that it is itself not computable.
    class CycleProc:
        def run(conf):
           return (lookup_node(node),non_comp,Coepoch(predicates=[node]))
```

```

    return CycleProc()

def run_submachine(node,conf):
    # Replace the real process with a dummy cycle process.
    p = lookup_node(node)
    insert_node(node,cycle_machine(node))
    nxt,res,coepoch = p.run(conf)
    insert_node(node,nxt)
    # Censor the current node from the predicates. If the result is
    # computable then the predicate is already falsified, if not it is
    # confirmed.
    return (nxt,res,coepoch{predicates=coepoch.predicates.delete(node)})
```

Listing 5.23.: The algorithm for creating predicates.

At the end of the computation, Antisthenis goes through all the non-computable predicates are carried by the value emitted and asserts that they are indeed non-computable. The ones that turn out to actually be computable are collected and set as *copredicates* in the epoch. The node is then reevaluated with the new epoch. A value depending on a predicate is valid as long as there is no corresponding copredicate in the epoch. By setting a name as a copredicate in the epoch, Antisthenis invalidates all values that associated with that name as a non-computability predicate.

Due to the incremental nature of Antisthenis only the parts of the graph that are dependent on the predicates are being reset by this process. The process is described in listing 5.24

```
def safe_run(n,conf):
    # To check if new copredicates were added
    initial_copred_num = len(conf.copredicates)
    # Run the process
    coepoch,res,coit_p = lookup_node(n).run(conf)
    # Remember: machines censor their own reference from the predicate
    # and no predicates.
    assert n not in coepoch.predicates
```

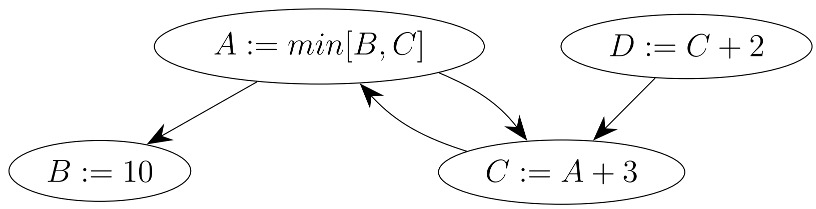
```
    assert len(intersection(conf.epoch.predicates,coepoch.predicates)) == 0
    # For efficiency any result we find we register all resutls we
    # find that may be encountered later as predicates.
    conf.copredicates.push(n)
    # For each non computable predicate
    for noncomp_node in coepoch.predicates:
        # Run the predicate to check if indeed it is non-computable
        res = safe_run(lookup_node(noncomp_node), conf)
        if is_computable(res):
            # If it is actually computable then register it a copredicate.
            conf.copredicates.push(noncomp_node)

    if len(conf.copredicates) > initial_copred_num:
        return lookup_node(n).run(conf)
    else:
        return res
```

Listing 5.24.: Algorithm for handling the predicates.

Another way to look at predicates is as deferred parts of the coepoch and to get the full coepoch we also need the coepochs of the predicates. The copredicates then are a forcing mechanism where Antisthenis assumes that some of the deferred parts are actually invalid. Antisthenis is lazy with these coepochs first and foremost because it depends on them to handle cycles but also because coepochs may be inhibited by absorbing elements, and they may not need to be evaluated at all. But if a predicate survives both the process stack based censoring that we just described, and being inhibited by the operators, Antisthenis has another trick up his sleeve: because it has a completely relativistic view of values, where they are only valid for particular epochs, the only requirement at every moment is that the final result of the computation be correct. Furthermore, the value depends on the machines in the predicate being $\perp$. Therefore, it is enough to check that they are indeed $\perp$ certifying the predicates. The predicates that are falsified in this way are the ones for which there is no other option but to force their value be taken into account via the epoch's copredicates.

Consider the example presented figure 5.4a. In that example, first a node is computed returning a valid result. The internal nodes of the graph, however may not have correct values.



(a) A recursive cost graph.

$$\left.\begin{array}{l} comp[A]: \\ \quad comp[B] \Rightarrow 10 \\ \quad \left.\begin{array}{l} comp[C]: \\ \quad comp[A] \Rightarrow \perp_A \end{array}\right] \Rightarrow \perp_A \end{array}\right] \Rightarrow 10$$

(b) Uncomputable values count as ∞ in our algebra. First A is computed in figure 5.4a. In that computation C is declared uncomputable due to A (predicate). This predicate would propagate to the result of A but it is censored: the value of A is never predicated on A itself.

$$\left.\begin{array}{l} comp[D]: \\ \quad comp[C] \Rightarrow \perp_A \end{array}\right] \Rightarrow \perp_A$$

(c) A first attempt at computing D yields a result predicated on A being uncomputable. Immediately checking the value of A falsifies the predicate.

$$\left.\begin{array}{l} comp[D]: \\ \quad \left.\begin{array}{l} comp[C]: \\ \quad \left.\begin{array}{l} comp[A]: \\ \quad comp[B] \Rightarrow 10 \\ \quad comp[C] \Rightarrow \perp_A \end{array}\right] \Rightarrow 10 \end{array}\right] \Rightarrow 13 \end{array}\right] \Rightarrow 15$$

(d) Computation for D with $\{A\}$ as the copredicate. This causes the value of C as $\perp_A$ to become invalid.

Figure 5.4.: First A is computed in figure 5.4a and Antisthenis is immediately happy with the unpredicated result it emits. Then D is requested and that is done in two steps. First (figure 5.4c) it comes up with $\perp_A$ meaning that D is uncomputable as long as A is uncomputable. Antisthenis must check that A is $\perp$ and immediately finds that it is not, meaning that the value of D is invalid. It re-evaluates D (figure 5.4d this time asserting that A is computable (copredicate)).

Predicate censorship: incorrect arguments, correct results

Censorship of the predicates after the evaluation of a machine is based on the observation that for some computations evaluating the correct value of a process does not *always* require that all subprocesses evaluate to the correct values. Consider the self-referential machine assignment $A := E[A]$, $E[A]$ is an expression that depends on the assigned to the name A. When evaluating this expression we set A to $\perp_A$. It is not necessary, however, that $E[\perp]$ and therefore A will *iteslf* evaluate to $\perp$. We do however censor A which is effectively us saying "regardless of whether we actually used the cyclical machine assumption that $A = \perp$, the value we just retrieved is not beholden on that assumption". How can we trust that the value is actually correct even though it is based on a false assumption? In general, for censorship to be valid we *require* that:

$$A := E[A] \Rightarrow E[E[\perp]] = E[\perp]$$

We sketch a proof of this in the case of min/sum algebra of costs: We assume that all values are non-negative and that $A := A+K \Rightarrow A = \perp$ and that $A := \min(b, A+k) \Rightarrow A = b$. We can prove the validity of censorship by using associativity of $\min$

$$\min(\min(X_1, \ldots, X_k), \min(Y_1, \ldots, Y_l)) = \min(X_1, \ldots, X_k, Y_1, \ldots, Y_l)$$

and distributivity of addition over $\min$

$$b + \min(X_1, \ldots, X_k) = \min(X_1 + b, \ldots, X_k + b)$$

we can derive that any expression $E[A]$ that is built by a combination of min/sum can be rewritten to

$$E[A] = \min(a_1 A + b_1, ...)$$

where $a_i \in \mathbb{N}$ and $b_i \geq 0$.

We then can distinguish between two cases:

- The first case is that $\forall i, , a_i > 0$ which means $a_i - 1 \in \mathbb{N}$ and therefore we can rewrite the normalized $E[A]$ into

$$A := E[A] = A + \min((a_1 - 1)A + b_1, ...) \Rightarrow A = \bot$$

  So $E[\bot] = \bot \Rightarrow E[E[\bot]] = E[\bot]$, the proposition we were looking for.

- The other case is that there exists a subset of elements in the series a_i for which $a_i = 0$. Without loss of generality we assume it's the first k, i.e., $\forall i < k, , a_i = 0$. This means that the normalized $E[A]$ can be rewritten as

$$A := E[A] = \min(b_0, \ldots, b_{k-1}, a_k A + b_k, \ldots) = \min(b, \min(a_k A + b_k, \ldots))$$

  Where $b = \min(b_i, \ldots, b_{k-1})$. Similarly to above, since $\{a_k, ...\}$ are all positive naturals, we can infer that

$$\min(a_k A + b_k, \ldots) = A + \min((a_k - 1)A + b_k, \ldots) = A + K$$

  where $K = \min((a_k - 1)A + b_k, \ldots) \geq 0$. Putting it all together we get

$$E[A] = \min(b, A + K) \Rightarrow E[\bot] = b$$

  and

$$E[E[\bot]] = E[b] = b$$

  Therefore $E[E[\bot]] = E[\bot]$

5.5.4. Calculating historical query costs

As we saw in section 4.2 on the query planner, the planner depends on the cost of past nodes to make decisions about the overall usefulness of a plan. A naive approach to this would be to just use the normal min/sum operation set to calculate historical costs. As explained in more detail in that chapter, this approach is problematic. We need a way of convincing Antisthenis to "look behind" those materialized nodes a) without completely ignoring the materialized nodes and b) by handling referential cycles more gracefully than declaring nodes non-computable. For this reason we develop the *stochastic costs model*.

The correct way of calculating the expected cost of a node in the future, which we very informally and heuristically attempt to approximate, would be to instead of considering the cost of materialized n-nodes to be zero, to calculate the likelihood that a materialized n-node will still be materialized when we encounter it again. This is related not only to an estimation of how many page-writes separate the moment of cost estimation and the actual plan the cost of which is being estimated, but also all the decisions that the garbage collector will make in that time. After FluiDB's budget is exhausted for the first time, for every page-write a page needs to be garbage collected. We considered a couple of options for stochastically modeling the behavior of the GC like assuming that it chooses random pages or random tables, but we could find none that was useful and computationally viable.

For this reason, we decided to follow a pragmatic approach to the problem and simply assume that for every materialized n-node there is a constant probability that it will not still be materialized when we need it, scaling the cost by that factor to get the stochastic cost. Furthermore, under this regime, Antisthenis would encounter many more cycles, rendering virtually all machines non-computable. For that reason we make all values *semi-computable* by introducing a $[0, 1]$ scale where 0 would mean that the value is fully computable and 1 would mean that the value is completely non-computable. We will refer to this value as κ. In short:

- When a materialized n-node is encountered the first step is to compute the stochastic cost as if it were not materialized. The derived cost, be it a concrete result or a bound, and κ are scaled down by a constant factor λ (in the default configuration $\lambda = 0.5$). $r = \lambda \times r_{orig}$.

- When calculating the stochastic cost of a materialized node, the cap is increased to reflect the scaling that takes place $c = \frac{c_{orig}}{\lambda}$. This is necessary because when the parent process requests a result under a particular cap c, the subprocess *must never* return a bound lower than the requested cap. Scaling the result by λ could break this rule.

- Circular nodes return a constant value that represents 0 with $\kappa = 1$ rather than $\perp$.

- Once a certain (configurable) number of materialized n-nodes are in the stack (*mat-depth*) the materialized nodes are treated as normal materialized nodes (cost 0).

- Each value is valid for a particular mat-depth and the mat-depth is a non-ephemeral value. For that reason it is kept track of at the level of epochs and coepochs so that machine resets are handled automatically.

Fortunately, this change does not require a complete overhaul of the operator implementations for sum and min, only the definition of more complex types for cap, bound and result values. Particularly, we qualify each value with κ.

The cap (listing 5.25) is similar to the result type, only it is further qualified by the number of machines corresponding to the materialized n-nodes that are currently in the computation stack, i.e., machines that would have otherwise yielded zero cost.

```haskell
data HistCap a =
  HistCap
  { hcValCap :: Min a
   ,hcNonCompTolerance :: Double
  }
```

Listing 5.25.: Definition of the type used for capping the cost of historical queries.

```haskell
data HistVal a =
  HistVal
  { hvComp :: Double  -- [0,1] computability metric, κ.
   ,hvRes :: a
  }
```

```

extExceedsCap _ HistCap {..} HistRes {..} =
  maybe False (hrRes >) (unMin hcValCap)
   || hrComp bnd > hcNonCompTolerance
  where
    maxMatTrail = 5 -- actually this is part of a global config.
```

Listing 5.26.: Comparison between bounds and between bound and cap are different. Between bounds we need to account for the semi-computability metric. The cap on the other hand defines a three-dimensional bound () that the bound must fall within in order to not exceed it.

5.6. Conclusion

In this chapter, we described in detail the framework for building incremental evaluation systems we developed for FluiDB that we call Antisthenis as well as a brief summary of the way we use it in FluiDB. The main goals of Antisthenis are to temporally reuse computation, to gracefully handle computational cycles when possible, and to order the operations in each operator in a specific way such that the overall computation finishes as soon as possible.

There are two major shortcomings of Antisthenis:

- Extending it with new operators, and even using the existing operators, involves a lot of boilerplate code. This is an immediate effect of the current version of Antisthenis being highly specific to the FluiDB planer.
- Our current implementation relies heavily on the higher order functional capabilities of Haskell that are efficient for what they are, but they stress the garbage collector and are hard for GHC to optimize.

Both of these could be mitigated in a future version of Antisthenis that would be detached from FluiDB and that would replace the Antisthenis process with a more “traditional” data structure.

Further work could also be done to parallelise Antisthenis. Some operations like `min` rely on the operands being evaluated serially to take advantage of early stopping but addition is much more flexible with the order of evaluation. It is clear that parallelism would also be implemented in an operator specific way.

Execution engine

> The concrete conditions for realizing the truth may vary, but the truth remains the same and theory remains its ultimate guardian.
>
> *(Herbert Marcuse)*

Chapter summary

- Tables are stored in a memory file system and records are stored as POD binary objects organized in 4K pages.
- The extent of each relation involved in a plan is converted to C++ `struct` and each predicate is converted to a C++ callable class.
- These parameterize the template-based operator implementations to allow the compiler to generate highly specialized code.
- In this chapter we discuss the implementation of important operations and their reverses.

Code generation is becoming increasingly common in RDBMSs. Used mainly in in-memory databases, where disk IO does not dominate the runtime, it aims to minimize the overhead of data access function calls, to optimize the predicates and numerical expressions and to avoid indirection in tight loops. Approaches to code generation fall generally on a spectrum between two extremes:

- Transpilation of the physical plan to a low level programming language like C or C++ for every query
- JIT compilation of small parts of the plan as the query executes.

FluiDB follows the approach of Krikellas et. al. [49] falling far to the former end of the spectrum. We generate very specialized, template-heavy C++ code for every query and we use an off-the-shelf compiler to generate highly optimized machine code.

In this chapter we will describe in detail all aspects of code generation, the form of data the generated code manipulates and the *FluiDB C++ Library (FCL)* that supports the generated code by implementing operators, utility functions and types.

This chapter's contribution to the state of the art revolves around an efficient approach to generating efficient query specific C++ code as well as describing the underlying data structures and algorithms that FluiDB to leverage compiler technologies.

6.1. Storage memory management

We opted for delegating the task of memory management to the OS and use use use tmpfs as a storage layer to our database. The tmpfs filesystem depends on the the *shmem* module (as of Linux v5.13) for handling file operations, a resizable virtual memory filesystem for linux. Where a typical persistent filesystem stores files in a block device and caches pages in memory for efficiency, shmem keeps files exclusively as pages in the page cache. The OS tries to keep all pages in memory but when resources start running out, it writes pages in swap.

Assuming that the pages are not in swap, normal reads for shmemfs using `read()` are equivalent to copying pages from the page cache to the user space, while shmem `writes()`

operate directly to the pages. We can mitigate the copying overhead of reads using `mmap` which will remap the page to the address space of the application. The problem with this approach, however, is that, while it allows us to minimize copying, we still need to run system calls in tight loops, which can be very computationally expensive. This can't be completely mitigated unless we move the "storage" layer to the userspace, re-implementing the memory management that we get for "free", in terms of engineering effort, so we leave that for a future version of FluiDB.

On the other hand, assuming a normal file system layer allows FluiDB to easily be adapted to operate over any filesystem backed by different storage technologies like NVM.

6.2. Data layout

Before we get into the details of the actual physical plan (in the form of C++ code) we need to state some assumptions about the layout of the code and the primary data.

FluiDB is a *row store system* which stores tables as files, each of which is a collection of pages, each of which is a collection of records (rows). We use one file per table or intermediate result, and each file is simply a sequence of records organized into pages. For now, FluiDB does not support any kind of indexing or compression. FluiDB can, of course, be run over any filesystem, but non-memory based file systems diminish the benefit of code generation, making performance IO bound.

With this in mind, there are three parts to understanding the principles of FluiDB storage:

- The format in which primary data is inserted into the database.
- The layout of the data within the database
- The transformation from the former to the latter.

6.2.1. Initial data conversion

Starting with the initial data, for FluiDB to be adapted to a particular dataset, it requires the primary data in CSV format and some Haskell configuration code describing the shape of the data and the database configuration. Our experiments so far have been revolving around the SSB TPC-H benchmark, so the format expected is the plain text format that dbgen [113] generates. This comprises of two steps: first a Haskell program parses the CSV records into standard-layout binary objects that can be directly cast to C/C++ standard-layout structs. These binary objects are stored one after the other in a flat binary file. The end result of what we want to do to be able to execute code similar to the one presented in listing 6.1.

```
template<typename R, size_t batch_size=2000>
void flat_to_dat(const std::string& flat_file, const std::string& dat_file) {
  int fd;
  size_t read_bytes;
  std::array<R, batch_size> batch;
  // Open the file in binary mode
  fd = ::open(flat_file.c_str(), O_RDONLY);
  // The writer is the object used by all operator implementations
  // and it will write each record in pages. Using it here asserts
  // that the data will have the correct format.
  Writer<R> w(dat_file);
  do {
    // Read a batch of data,
    read_bytes = ::read(fd, batch.data(), sizeof(batch));
    // Use the writer object to write each record.
    for (size_t i = 0; i < read_bytes / sizeof(R); i++) {
      w.write(batch[i]);
    }
    // Keep reading until a batch is cut short.
  } while (read_bytes == sizeof(batch));
  // Wrap up.
  w.close();
  close(fd);
```

```
}
```

Listing 6.1.: For standard FFI communication C++ structs that do not contain fancy copy constructors, virtual methods, etc (standard layout) can be used directly. We take advantage of that to store structs directly into files as contiguous binary objects.

For this to work we refer to the C++ standard [102]. The types that represent table rows must be standard layout[1]. According to the standard:

> A class S is a standard-layout class if it:
>
> - has no non-static data members of type non-standard-layout class (or array of such types) or reference,
> - has no virtual functions and no virtual base classes
> - has the same access control for all non-static data members,
> - has no non-standard-layout base classes,
> - has at most one base class subobject of any given type,
> - has all non-static data members and bit-fields in the class and its base classes first declared in the same class, and
> - has no element of the set M(S) of types as a base class, where for any type X, M(X) is defined as follows. [Note: M(X) is the set of the types of all non-base-class subobjects that may be at a zero offset in X.]
>   - If X is a non-union class type with no (possibly inherited) non-static data members, the set M(X) is empty.
>   - If X is a non-union class type with a non-static data member of type X_0 that is either of zero size or is the first non-static data member of X (where said member may be an anonymous union),

[1]In C, standard layout is commonly known as POD (Plain Old Data). This term is deprecated as of C++20 in favor of standard layout classes which incarnate the data layout aspect of POD, and trivial classes which incarnate the behavioral aspect of POD.

the set $M(X)$ consists of X_0 and the elements of $M(X_0)$.

– If X is a union type, the set $M(X)$ is the union of all $M(Ui)$ and the set containing all U_i , where each U_i is the type of the i th non-static data member of X.

– If X is an array type with element type X_e, the set $M(X)$ consists of X_e and the elements of $M(Xe)$.

– If X is a non-class, non-array type, the set $M(X)$ is empty.

The records we generate certainly conform to these requirements. An example is the supplier row in 6.2. Therefore the object is trivially copyable (has a default copy constructor) and occupies contiguous bytes of storage. This means that we can safely write each record `R` as `sizeof(R)` contiguous binary data to a file and expect to find the same value of `R` when we read it. Based on this, we can safely copy binary record objects from memory to disc and vice versa.

```
class Record {
public:
  Record(unsigned __s__suppkey, fluidb_string<25> __s__name,
         fluidb_string<40> __s__address, fluidb_string<16> __s__city,
         fluidb_string<16> __s__nation, fluidb_string<13> __s__region,
         fluidb_string<15> __s__phone)
    : s__suppkey(__s__suppkey),
      s__name(__s__name),
      s__address(__s__address),
      s__city(__s__city),
      s__nation(__s__nation),
      s__region(__s__region),
      s__phone(__s__phone) {}
  Record() {}
  std::string show() const {
    std::stringstream o;
    o << s__suppkey << " | " << arrToString(s__name) << " | "
      << arrToString(s__address) << " | " << arrToString(s__city) << " | "
```

```
      << arrToString(s__nation) << " | " << arrToString(s__region) << " | "
      << arrToString(s__phone);
    return o.str();
  }
  bool operator==(const Record& otherRec) const {
    // compare each field ...
  }
  bool operator!=(const Record& otherRec) const {
    // compare each field ...
  }
  unsigned s__suppkey;
  fluidb_string<25> s__name;
  fluidb_string<40> s__address;
  fluidb_string<16> s__city;
  fluidb_string<16> s__nation;
  fluidb_string<13> s__region;
  fluidb_string<15> s__phone;
};
```

Listing 6.2.: The supplier row representation in the generated C++ code. The `fluidb_string` type is a constant size array of characters.

However, in the case of parsing, we need to take great care with byte alignment which is compiler dependent. Fortunately, Clang and GCC informally agree on the algorithm for `alignof(<cls>, <member>)` for standard layout objects. The algorithm is presented in 6.3

```
schemaPostPaddings :: [CppType] -> Maybe [Int]
schemaPostPaddings [] = Just []
schemaPostPaddings [_] = Just [0]
schemaPostPaddings schema = do
  elemSizes <- sequenceA [cppTypeSize t | t <- schema]
  spaceAligns' <- sequenceA [cppTypeAlignment t | t <- schema]
  let (_:spaceAligns) = spaceAligns' ++ [maximum spaceAligns']
  let offsets = 0 : zipWith3 getOffset spaceAligns offsets elemSizes
  return $ zipWith (-) (zipWith (-) (tail offsets) offsets) elemSizes
```

```
  where
    getOffset nextAlig off size =
      (size + off)
      + ((nextAlig - ((size + off) `mod` nextAlig)) `mod` nextAlig)
```

Listing 6.3.: Algorithm to infer the padding of members according to the Itanium ABI.

Once the flat files are generated, C++ code is generated for parsing the file calls into the C++ function `flat_to_dat` that is parametric to the type of the object being and uses the FCL library to write objects, thus making sure that the data is readable by the operators. `flat_to_dat` simply reads the input `.flat` file as a stream of records of size `sizeof(Record)` casting the bytes with `reinterpret_cast<Record*>` into the record. It then use the FCL record writing facility `Writer<R>::write` that takes care of organizing the record into pages. Thus, the final *data file* is created that is ready for use by the generated code.

```
#include <bamify.hh>
class Record {
  // ...
};
int main(int argc, char* argv[]) {
  flat_to_dat<Record>("supplier.flat","supplier.dat");
}
```

Listing 6.4.: Convert a flat file to a paged data file.

6.2.2. Pages

We translate the flat binary files into `.dat` files that contain the table data in its final format. The basic block of the file is the `Page`, and the file is simply a raw sequence of pages. A page is constant-size data structure that contains up to $\left\lfloor \frac{S_{rec}}{S_{pg}} \right\rfloor$ *whole* records where S_{rec} is `sizeof(Record)` and S_{pg} is the size of the page which typically is 4KB. All pages must contain as many whole records as can fit except the last one.

All transactions on the storage are made at the page level: we either read or write only entire pages. These operations are abstracted by the `Reader` and `Writer` classes. We use one more level of abstraction for convenience, the higher order `eachRecord` function. To demonstrate what the interface looks like, we present the implementation of `eachRecord` 6.5.

```
// Fn could be instantiated to std::function<void(const R&)> but that
// will *always* forbid f from being inlined.
template <typename R,typename Fn>
inline void eachRecord(const std::string& inpFile,Fn f) {
  Reader<R> reader;
  size_t i = 0;
  reader.open(inpFile);
  while (reader.hasNext()) {
    i++;
    f(reader.nextRecord());
  }
  reader.close();
}
```

Listing 6.5.: `eachRecord` is a utility that simply iterates over all the records in a file.

As alluded to in the previous section, both the `Writer` and `Reader` use `reinterpret_cast` to "serialize" and "deserialise" the data respectively.

6.2.3. C++ row iterators

It is important for the planner to have full control of the storage budget and assume that no significant memory is required for evaluating each operator. For that reason, we require that all operators' algorithms have constant space complexity. This may mean major compromises for some algorithms with respect to the time complexity like join and aggregation. Fortunately, we can mitigate that by taking advantage of the set semantics of FluiDB relational algebra and sorting the input tables in-place before running joins or aggregations. Our particular implementation of in-place sorting hinges on the `RecordMap` type that provides C++ random access iterators to the records of a file, abstracting the page reads and writes. We pass these iterators to `std::sort` (as demonstrated in listing 6.6) which runs insertion sort for small ranges to take advantage of the processors reorder buffer and quicksort for longer ones. An easy optimization for the next version of FluiDB would be to use a custom in-place sort optimized for the page size and which respects the page boundaries.

```
RecordMap<size_t> fs("/tmp/removeme.dat");
std::sort(fs.begin(), fs.end());
```

Listing 6.6.: Using a `RecordMap` to sort the records of a file by providing an iterator range to `std::sort`.

The operation of `RecordMap` is very simple. Each iterator is paired with the page that contains the record it points to, when an iterator is the only on pointing inside a page and moves out of the page, said page is written back to the file and the new page is read. When the only iterator pointing to a page is deleted the page is written to the file. Multiple iterators pointing to the same page do not maintain different copies of that page and the last one to be destroyed or to leave the page triggers the page to be written back to the file.

6.3. Physical planning

The fundamental logic of the code generator is fairly simple: The input of the code generator is a a list of transitions generated by the FluiDB planner. The reader is reminded that each transition has one of three kinds:

- Trigger of a t-node. The input n-nodes at the time of the trigger are materialized and the trigger itself materializes a subset of the output n-nodes
- Reverse trigger of a t-node, which represents the materialization of a subset of the input n-nodes from materialized output n-nodes
- The deletion of an n-node.

Transitions are meant to be executed in the order they appear in the received sequence to preserve correctness. However, there isn't necessarily a one-to-one correspondence between operators and transitions. Instead, as we saw in section 3.5, each operation corresponds to a cluster of connected t-nodes. The first step of the code generator, therefore, is to group the low-level transitions received by the planner into higher level batches that correspond to exactly one operation. This process is driven by intermediate n-nodes, i.e. helper n-nodes that do not correspond to materializable relations, but are rather part of the graph to reify the valid combinations that the planner is allowed to materialize. The constraint we are trying to preserve while batching the low level transitions such that the cluster level transitions do not materialize any intermediate n-nodes.

Since intermediate n-nodes are always internal to clusters and no cluster shares a t-node with another cluster, each batch of transitions corresponds to exactly one cluster, except for deletion transitions which are standalone. Furthermore, each cluster corresponds to exactly one relational operator, which we also include in the higher order transition.

6.4. Generated C++

The C++ AST is expressed as a tree of Haskell algebraic data types. They do not capture the entire C++ language, only the following concepts concepts:

- Function declaration
- Function application and arguments
- Expressions
- Code symbols
- Assignment
- Includes
- Literals

- Classes
- Member declarations
- Global declarations

The code generated version of each operator is parameterized by the record types of its inputs and outputs, and highly specific code is generated by the C++ templating system.

All operator implementations live in the FCL library which branched out from [49]. The FCL library makes heavy use of templates and `constexpr` to generate very query-specific machine code for each operator.

6.4.1. Code structure

The FluiDB code generator generates a couple of different kinds of C++ components to construct the final file. In this section we will discuss the kinds of C++ structs that FluiDB generates in order to parameterize the FCL operators at compile time, leveraging the C++ template system.

In the following subsections when we say *compile time* we refer to the compile time of the generated code, not the compile time of FluiDB.

Maybe types

Maybe types are used to indicate optional outputs of operators. For example an operator σ_p may yield $\sigma_p A$ or $\sigma_{\neg p} A$ or both. On the one hand, the precise outputs that are required are important for performance and memory budget management, and on the other any combination of these outputs can and should be generated through one pass over the input. Furthermore, information about which outputs are required is known at compile time so the generated code should be specific to the combination of outputs that are required.

To address this, instead of passing in a simple `std::string` as a path, we pass one of two types `Nothing` or `Just` (listing 6.7) that wrap the file path. Both these types contain an `isNothing` member of `static constexpr bool` type that can be used by `if constexpr ( ... )` expressions in the selection operator to help the compiler generate highly specialized machine code.

```
template <typename T>
struct Just {
  Just(T t) : value(t) {}
  T value;
  constexpr bool operator==(Just<T> const& j) const {
    return this->value == j.value;
  }
  static constexpr bool isNothing = false;
};
template <typename T=std::string>
struct Nothing {
  Nothing(T s) {}
  Nothing() {}
  static constexpr bool isNothing = true;
};

int main () {
  // At compile time the select algorithm can be constructed
  // with constexprs to only emit records to "prim_out.dat".
  Select<...> op(Just<std::string>("prim_out.dat"),
                Nothing<std::string>, // secondary out
               "input.dat");
  op.run();
}
```

Listing 6.7.: The type level maybe and an example.

Record types

As mentioned, FluiDB is a row store engine and handles table rows by generating record types specific to each table, where objects of that type are rows of the corresponding table. We went over some aspects of the the generated record types when discussing the conversion of primary data to FluiDB specific binary data. Record types directly or indirectly parameterize all operators. A selection operator σ_p instance must be parameterized by the record type of its input which will be the same as the output. A projection π is parameterized by the records of both the input and the output type. A generated record type must have some structural properties to function properly with FCL and the rest of the generated types:

- First and foremost, it must be standard-layout type so it can be trivially copied from and to binary files. This means that there should be no virtual members, the destructor must be trivial and no static members.
- They should be comparable by equality ($=$), inequality ($\neq$) and they should be ordered ($<$). The actual semantics of the ordering are not important as long as there is a deterministic way of ordering records of the same type.
- Finally, every record type must implement an `std::string show()` function that serializes the contents into a human readable string.

Predicate types

Predicate functions have the type of n-ary Boolean functions $A_1 \times ... \times A_k \rightarrow \{T, F\}$ useful to generate code for selections and θ-joins so in practice they are unary or binary. They are passed as template arguments to the operators, so the compiler has the chance to inline them to avoid the indirection. To avoid too much code clutter in the generated code they are expected to provide the types of the function domain $A_1, ..., A_k$ (demonstrated in listing 6.8) in order to minimize the number of template parameters and keep the generated code relatively human readable.

```
class Predicate3421 {
  typedef Record123 Domain0;
  typedef Record32 Domain1;
  bool operator() (const Domain0& rl, const Domain1& rr) {
    return rl.__field1 < rr.__field2;
  }
```

```
}
```

Listing 6.8.: Example of generated predicate function corresponding to the predicate $field_1 < field_2$.

Record transformation types

Record transformations, much like predicates are callables representing pure functions, only the codomain is a record type, instead of a Boolean. They are used by joins to combine the matching records, by projections and aggregations to produce new records from the inputs, by equi-joins to extract the subtuples to be checked for equality, etc. Much like predicate functions, they can be queried for their domain and codomain to reduce the number of template arguments in operators (see listing 6.9)

```
class Transform {
  typedef Record123 Domain0;
  typedef Record32 Domain1;
  typedef Record10 Codomain;
  Codomain operator() (const Domain0& l, const Domain1& r) {
    return Record10(l.__key,l.__field1, r.__key, r.__field2);
  }
};
```

Listing 6.9.: A record transformation type defines objects with no internal state that are callable.

Operators

Operators are not generated, but they are parameterized by all the kinds of gernerated code we mentioned. They are classes that are constructed using maybe-filenames, record transformers, record types, and predicates. Operators must implement the `run()` method which actually runs the internal code. For demonstration an abbreviated version of the σ_p operator, which highlights the shape of an operator class is presented in listing 6.10.

```
template <typename Predicate,
          typename PrimaryOutType,   // Maybe(std::string)
          typename SecondaryOutType  // Maybe(std::string)
          >
class Select {
  typedef typename Predicate::Domain0 Record;

public:
  Select(PrimaryOutType prim, SecondaryOutType sec, std::string in)
    : primary_file(prim), secondary_file(sec), infile(in) {
    static_assert(!PrimaryOutType::isNothing || !SecondaryOutType::isNothing,
                  "Both primary and secondary output files are Nothing.");
  }

  ~Select() {}

  void run() {
    // ...
  }

  void print_output(size_t n) {
    // Prints the first n records from each output table.
  }

private:
  PrimaryOutType primary_file;
  SecondaryOutType secondary_file;
  std::string infile;
  static Predicate predicate;
};
```

Listing 6.10.: The selection operator. It is parameterized by the predicate and the primary and secondary output types. Enough information about these values is known at compile time such that the compiler can generate highly specialized code.

Unfortunately, even recent versions of the C++ standard do not include template type inference for classes and structs, therefore, to make the generated code simpler we wrap the constructor into a function like shown in listing 6.11. This allows the generated code for selection to look like the one presented in listing 6.12.

```
// C++20 can only infer typenames (primaryout secondaryout) in
// function templates.
template<typename Predicate,
         typename PrimaryOutType,   // Maybe(std::string)
         typename SecondaryOutType  // Maybe(std::string)
         >
auto mkSelect (const PrimaryOutType prim,
               const SecondaryOutType sec,
               const std::string& in) {
  return Select<Predicate,
                PrimaryOutType,   // Maybe(std::string)
                SecondaryOutType> // Maybe(std::string)
    (prim, sec, in);
}
```

Listing 6.11.: The C++ declaration of the select.

```
int main() {
  // ...
  {
    auto op = mkSelect<Predicate32>(Just("data123.dat"), Just("data53.dat"),
                                    "lineitem.dat");
    op.run();
    op.print_output();
  }
  // ...
  return 0;
}
```

Listing 6.12.: A block representing a particular operator.

6.4.2. Operator implementations

In the following subsections we will take a tour to the implementation of some relational operators in FCL. We do not mention every operator supported for brevity as many are trivial.

Select algorithm

The selection algorithm is likely the simplest of the implemented ones because FluiDB does not support indexes and makes no assumptions about the ordering of the data. In the forward variety, it is implemented as either a partition or a selection depending on which of the outputs it is materializing. In its backward variety it is essentially a union $A \equiv \sigma_p A \cup \sigma_{\neg p} A$. As we discussed already (listing 6.11) the FCL library expects only a predicate class as a template parameter and three filenames as its arguments.

It is important to remind the reader that no order is expected to be preserved by union, which is to say that the transition $A \Rightarrow \{\sigma_p A, \sigma_{\neg p} A\} \Rightarrow A$ does not preserve the order of the rows. The reader is also reminded that FluiDB does not support `NULL` values to preserve the correctness of the reverse select operation.

Projection algorithm

The implementation of the projection algorithm is parameterized by two template arguments that extract complementary subtuples from an input relation, affording each one with a common unique subtuple to facilitate the inverse operation. The inverse of a projection then is simply an equijoin between the two produced slices based in that shared unique subtuple.

While the FCL side of the calculation is fairly straightforward, on the Haskell side (code generation) it is slightly more complex. The first piece of the puzzle is the representation of the projection. When the query is initially processed so that projections are augmented to expose a unique subtuple we actually change the representation of projections from `QProj [(e,Expr e)]` to `QProjI [e] [e] [(e,Expr e)]`. The two extra parameters typed `[e]` represent the complement of the projection (the columns from the input not exported by the projection) and a unique subtuple that the complement and the projection must have in common, while `[(e,Expr e)]` is the normal projection. We make this transformation because the easiest and cheapest place to calculate the first and second parameters (complement and correspondence) is during the projection augmentation phase that happens immediately after parsing. Therefore, we have all projections carrying information about the complement at the level of the RA operator.

In particular, this process is combined with remapping of unique subtuples into the projection. For example, during the unique column exposure process the query `select p_color from part` is found not to be valid because `p_color` is not unique for each row. For that reason, during preprocessing we remap the `p_partkey` column since `p_partkey` is the only unique column of `part` this way the constraint that every valid relation in FluiDB needs to have at least one subtuple that is unique within that relation is satisfied and the query becomes

```
select p_partkey,p_color from part
```

To be able to reconstruct the input we require that at least one unique subtuple is shared between the primary projection relation and the complement. In the example demonstrated, this means the `p_partkey` column is duplicated in both output tables of the operator. The particular valid projection operator that would be generated the `QProj` is presented in listing 6.13.

```
op =
  QProjI
    -- All keys except p_color
    [p_partkey,p_name,p_mfgr,p_category,p_brand1,p_type,p_size,p_container]
    -- The unique subtuple (just one column)
    [p_partkey]
    -- The the actual projection
    [(p_partkey,E0 p_partkey),(p_color,E0 p_color)]
```

Listing 6.13.: The projection operator produced from the SQL query `select p_partkey,p_color from part`.

The algorithm for reverse projection is almost a standard join. The only difference is that in our semantics of join, the relation $A \bowtie_{a=b} B$ contains both the column a and the column b, while when joining a projection with it's complement we only create a single copy of the shared subtuple. We will see more details about join in the next section.

The join operator

We distinguish between two kinds of join operators: the equijoin and the general θ-join. The former is specific to cases where subtuples from each table are compared and the implementation is simply a merge join, while the latter is used for general predicates and is essentially a pipelined $\sigma(A \times B)$. The important detail worth noting about join algorithm implementations in FluiDB is that they must be able to produce antijoin relations $A \not\ltimes B$, $A \not\rtimes B$ simultaneously with the join relation $A \bowtie B$.

It is important for the join algorithm to be space efficient, and particularly constant-space, due to the assumption made by the planner that it has full control over memory. For general θ-joins we use nested loop joins. For equijoins we take advantage of the assumption that the tables are unordered and use merge join after sorting the input tables in-place. A sketch of the algorithm is presented in listing 6.14

```python
# This is our pythonic way of detecting which input stream finished.
class LeftFinished:
    pass
class RightFinished:
    pass


def lnext(it):
    """If iteration finishes throw LeftFinished"""
    try:
        return next(x)
    except:
        raise LeftFinished

def rnext(it):
```

```
    """If iteration finishes throw RightFinished"""
    try:
        return next(x)
    except:
        raise RightFinished

@template(extractl,extractr,combine)
def merge_equijoin(in_l,in_r,anti_l,out,anti_r):
    # In place sort each of the inputs
    sort(in_l,key=extractl)
    sort(out_l,key=extractr)
    # Define iterators and current values for each of the inputs.
    it_l = iter(in_l)
    val_l = next(it_l)
    it_r = iter(in_r)
    val_r = next(it_r)

    try:
        # The actual merge join algorithm
        while True:
            # Gather blocks of equal records from left and right and write
            # out their product.
            if extractl(val_l) == extractr(val_r):
                # Gather a sequence of equal-key values from the left
                val_ls = []
                tmp = val_l

                while extractl(val_l) == extractl(tmp):
                    val_ls.append(val_l)
                    val_l = lnext(it_l,None)

                # Gather a sequence of equal-key values from the right
                tmp = val_r
                val_rs = []
```

```
                while extractr(val_r) == extractr(tmp):
                    val_rs.append(val_r)
                    val_r = rnext(it_r)

                # Write out their product.
                for l,r in product(val_ls,val_rs):
                    out.write(combine(l,r))

            # Push the non-equal records to the antijoins
            while extractl(val_l) < extractr(val_r):
                anti_l.write(val_r)
                val_r = rnext(it_r)
            while extractl(val_r) < extractr(val_l):
                anti_r.write(val_l)
                val_l = lnext(it_l)
    except RightFinished:
        # if we ran out of records from the right operand
        # Push the remaining if the left operand to left antijoin
        for l in it_l:
            left_anti.write(r)
    except LeftFinished:
        # if we ran out of records from the left operand
        # Push the remaining if the right operand to right antijoin
        for r in it_r:
            right_anti.write(r)
```

Listing 6.14.: The equi-join algorithm first sorts in place the inputs w.r.t. equal subtuples and then merges them.

The type of the join operator is defined in terms of a template such that it can be specialized at compile time (see listing 6.15). The template arguments are the following:

- Three file paths are provided, `OutFile`, `LeftAnti` and `RightAnti` are each either of type `Nothing` indicating to the compiler not to generate any code that relates to the particular output or of type `Just<std::string>` indicating to the compiler the opposite.

- The subtuple extraction functions `LeftExtract` and `RightExtract`. These are record transformation functions that extract the subtuple that needs to

```
template <typename LeftExtract, typename RightExtract, typename Combine,
          typename OutFile,    // Maybe(std::string)
          typename LeftAnti,   // Maybe(std::string)
          typename RightAnti>  // Maybe(std::string)
class Join;
```

Listing 6.15.: Class declaration of the join operator

The reverse join operator is composed of two pipelined steps:

- A projection and deduplication based on the unique subtuple of the input on the join output to get a left or right semijoin.
- A union of the semijoin with the antijoin to get the input table.

As with the selection operator, the correctness of this reverse operation is predicated on the fact that FluiDB has no notion of `NULL` values.

Aggregation and sort algorithms

The main challenge to implementing aggregations in the context of FluiDB is maintaining constant space during its evaluation, as the planner tries to use as much of the memory budget as possible to maintain intermediate results. Therefore, we opted against using an auxiliary hash map to group records. Like we did to implement joins we perform aggregation in two steps: first we sort the records in place based on grouping columns and then aggregate in a single pass.

6.5. Conclusion

In this chapter we discussed some physical attributes of how FluiDB evaluates the final plans it constructs via code generation. We discussed the storage layout and the fundamental ideas the code generator was built around to generate highly specialized code. Finally, we discussed some specific algorithms used by the FCL library that is called by the generated code to implement the actual operators.

Among this we discussed some caveats that relate to the FluiDB's approach to code generation, namely, that highly specific code generated can be expensive to compile from scratch and that the shmem in-memory filesystem provided by Linux is a cheap solution in terms of engineering effort but would be better replaced with a solution that does not involve system calls.

Evaluation

Tis time to save the few remains of war.
But let some prophet, or some sacred sage,
Explore the cause of great Apollo's rage;
Or learn the wasteful vengeance to remove

(The Iliad of Homer, Book I)

Chapter summary

- FluiDB is well suited for join-heavy star schemata so we evaluated using the SSB-TPC-H benchmark.
- Our evaluation shows that FluiDB is able to plan around the space constraints and come up with plans that materialize intermediate results that are useful for future queries.
- FluiDB is generally faster than the baseline due to caching but at times may be slower in individual queries when it receives "unexpected" queries.
- FluiDB generally performs better when allowed larger memory budgets but this speedup is based on heuristic assumptions that sometimes break in interesting ways.

We based our evaluation of FluiDB on the Star Schema Benchmark (SSB) [73] which is a variation of TPC-H. Like TPC-H it models the data warehouse of a wholesale supplier.

As the name suggests, these queries query a star schema. The star schema is derived from the TPC-H schema (figure 7.2) by merging the `linenumber`, `orders` and `partsupp` and completely dropped tables `nation` and `region` tables as shown in figure 7.1.

The particular queries contained in the workload are presented in listing A.1 in the appendix (A).

We generated a workload where queries were compiled and planned one by one in a loop and were run over a database of data generated with a modified version of the TPC-H dbgen [114] with scaling of size 1, meaning that the total primary data totals around 1GB in CSV format. The exact size of the data generated after formatting them to the format required by FluiDB (described in chapter 6) are

```
$ du -sh *.dat
4.1M    customer.dat
256K    date.dat
757M    lineorder.dat
31M     part.dat
268K    supplier.dat
```

Which makes a total of approximately 200k pages of size 4KB.

In our experiment, we use page IO as a proxy for performance, despite the fact that FluiDB is an in-memory database. We think that this is a reasonable experimental approach because, as FluiDB leans heavily on code generation, it is unlikely that the actual instruction retiring will have a major impact on the performance. Instead, the performance cost is dominated by page IO that will certainly cause cache misses at the LLC. Another thing to note is that FluiDB generates code that focuses on performance and not on compilation time. Making heavy use of metaprogramming like `constexpr` makes compilation fairly slow. There are ways to speed up compilation time like using pre-compiling header files [115] and fine-tuning the compiler optimization passes. These techniques are beyond the scope of this work, FluiDB is focused on analytics workloads that do not include sub-second queries.

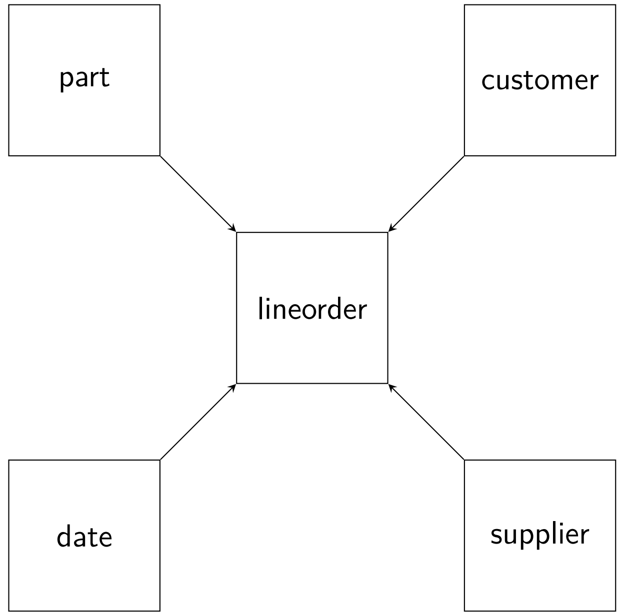


Figure 7.1.: The foreign key links in a SSB-TPC-H

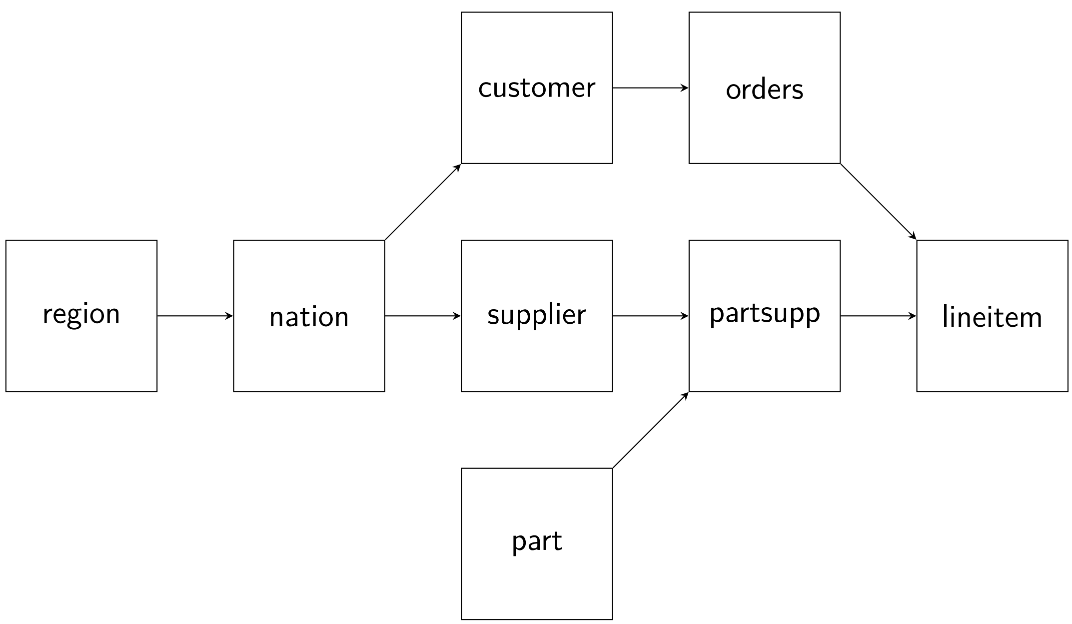


Figure 7.2.: The foreign key links in a traditional TPC-H schema

An important factor to the performance of FluiDB is the available memory budget. Most traditional systems plan independently of their memory constraints will die will an OOM exception if they are unable to run the plan they constructed. FluiDB takes budget into account while planning and is able to produce better plans under laxer restrictions. We evaluate the performance of the system under different budget restrictions.

The lowest budget within which FluiDB was able to plan all 13 queries of SSB-TPC-H was 2300k pages and therefore that was the lowest budget we used in our benchmarks. The highest budget we used was 6500K pages which is a memory budget that allows the execution of all queries without the GC needing to be triggered. Therefore FluiDB will have the same performance running our benchmark workload for any memory budget over 6500K pages.

7.1. Analysis

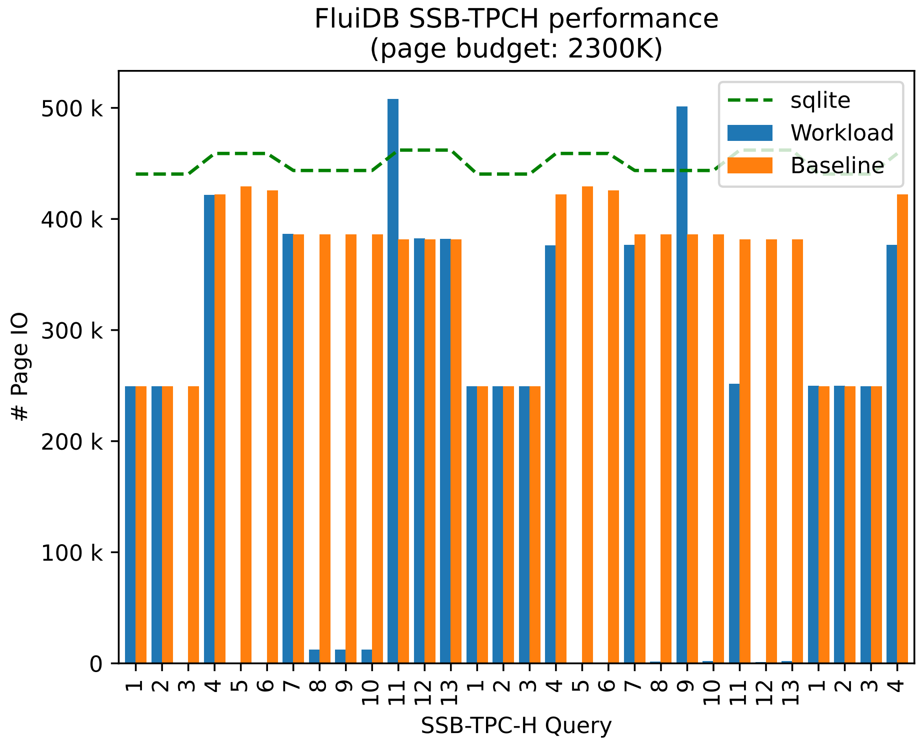


Figure 7.3.: The total page read/writes for each query of SSB TPC-H. The baseline is the query being run by FluiDB directly without any materialized n-nodes. The workload bars represent the cost of each query in a workload being accumulated into the same QDAG. The dashed line represents the performance of sqlite3 with all indexes stripped.The budget allowed for this is the minimum budget within which FluiDB can run each individual query (2300K pages).

Figure 7.3 demonstrates that FluiDB running a workload versus running each query individually causes speedups even in constrained budgets. This particular run is run on the minimum space budget for which the planner is able to create plans for every individual query. In some cases, however, the garbage collector is forced to delete tables that need to be recreated later in the workload causing FluiDB to be sporadically less performant than the base case.

For a larger budget, the FluiDB is able to store more useful intermediate results as demonstrated in figure 7.4.

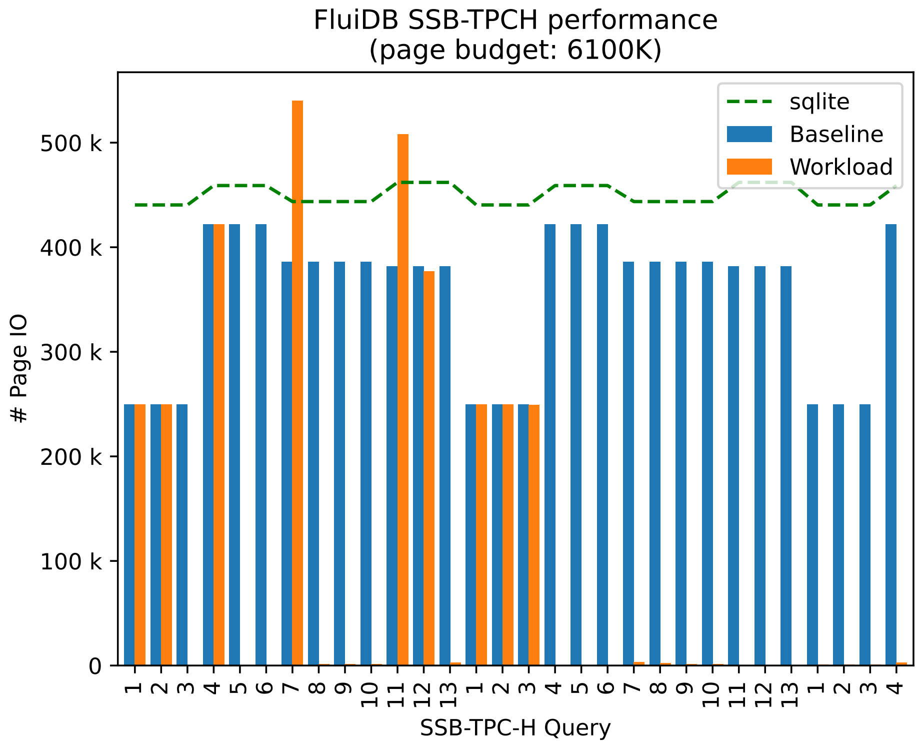


Figure 7.4.: The total page read/writes for each query of SSB TPC-H. The baseline is the query being run by FluiDB directly without any materialized n-nodes. The workload bars represent the cost of each query in a workload being accumulated into the same QDAG. The dashed line represents the performance of sqlite3 with all indexes stripped.The budget allowed for this is about triple the minimum budget within which FluiDB can run each individual query (6100K pages). A badly timed GC run causes some individual queries to be slower than their counterparts from a workload being run in a tight budget (figure 7.3).

An interesting point here is the plan for evaluating query 7 is more expensive in the workload run under laxer budgetary constraints. This may seem strange but it is an example demonstrative of the fundamental operation of FluiDB. The high level explanation for this is that the high-budget plan needs to evaluate $lineorder$ via the reverse trigger of a join as it was deleted during the evaluation of query 6. Mysteriously, during the strict-budget planning the $lineorder$ relation is readily available during the planning of query 7! The key lies slightly earlier in the workload (see listing 7.1).

FluiDB materializes in query 4 the join

$$Q_{36} := supplier \bowtie_{lo\_suppkey=s\_suppkey} lineorder$$

and the corresponding antijoins Q_{35} and Q_{37}, making the node $lineorder$ deletable. However, when running the workload under strict budgetary constraints, it is forced to garbage collect shortly after materializing said join at a moment while both Q_{36} and $lineorder$ are protected (see section 4.2.1 on garbage collection). Therefore, FluiDB is forced to delete both Q_{35} and Q_{37}, making $lineorder$ non-deletable when planning for query 4 finishes.

On the other hand, with laxer budgetary constraints, no garbage collection is triggered during query 4. The next garbage collection is triggered during query 6, at a moment when $lineorder$ is deletable, unprotected, and a prime candidate for deletion based on the GC heuristics. Alas, when query 7 requires $lineorder$ for its plan, FluiDB needs to reconstruct it in the case of lax budgetary constraints but not in the case of strict constraints.

This example of FluiDB being forced to locally produce more expensive plans is an effect of FluiDB being more opportunistic, the lower the available budget is, and more adventurous when operating with high budgets. When FluiDB is frugal, it is generally prone to miss opportunities to share computation between queries. There are times however that this frugality saves it from bad heuristic-based decisions that it is allowed to make otherwise.

```
# Query 4
Query $s\gamma\pi\sigma(supplier \bowtie lineorder \bowtie date \bowtie part)$ {
  # There is enough space to keep both and the complements
  $Q_{36}, Q_{35}, Q_{37}$ :=
  ↪ Materialize[$supplier \bowtie lineorder, supplier \ltimes\!\!\!\!/\, lineorder, supplier \rtimes\!\!\!\!/\, lineorder$]
  $Q_{41}, Q_{40}, Q_{42}$ := Materialize[$Q_{36} \bowtie date, Q_{36} \ltimes\!\!\!\!/\, date, Q_{36} \rtimes\!\!\!\!/\, date$]
  $Q_{46}, Q_{45}, Q_{47}$ := Materialize[$Q_{41} \bowtie part, Q_{41} \ltimes\!\!\!\!/\, part, Q_{41} \rtimes\!\!\!\!/\, part$]
  $Q_{50}$ := Materialize[$\sigma Q_{46}$]
  $Q_{90}$ := Materialize[$\gamma\pi Q_{50}$]
  $Q_{91}$ := Materialize[$s Q_{90}$]
}
```

```
# Query 5
Query $s\gamma\pi\sigma(supplier \bowtie lineorder \bowtie date \bowtie part)$ {
  $Q_{92}$ := Materialize[$\sigma Q_{46}$]
  $Q_{118}$ := Materialize[$\gamma\pi Q_{92}$]
  $Q_{119}$ := Materialize[$sQ_{118}$]
}

# Query 6
Query $s\gamma\pi\sigma(supplier \bowtie lineorder \bowtie date \bowtie part))$ {
  # FluiDB decides delete lineorder since it has the complements
  GC { Delete[$\ldots, lineorder, \ldots$] }
  $Q_{120}$ := Materialize[$\sigma Q_{46}$]
  $Q_{146}$ := Materialize[$\gamma\pi Q_{120}$]
  $Q_{147}$ := Materialize[$sQ_{146}$]
}

# Query 7
Query $s\gamma\pi\sigma(customer \bowtie date \bowtie lineorder \bowtie supplier)$ {
  # Ooops... this would be avoided if we hadn't deleted lineorder.
  $lineorder$ := Materialize[$\bar{\pi} Q_{36} \cup Q_{37}$]
  $date$ := Materialize[$\bar{\pi} Q_{41} \cup Q_{42}$]
  GC { Delete[$\ldots$] }
  $Q_{149}, Q_{148}, Q_{150}$ := Materialize[$date \bowtie lineorder, date \not\ltimes lineorder, date \not\rtimes lineorder$]
  GC { Delete[$\ldots$] }
  $Q_{154}, Q_{153}, Q_{155}$ := Materialize[$customer \bowtie Q_{149}, customer \not\ltimes Q_{149}, customer \not\rtimes Q_{149}$]
  GC { Delete[$\ldots$] }
  $Q_{165}$ := Materialize[$\sigma Q_{154}$]
  $Q_{182}, Q_{181}, Q_{183}$ := Materialize[$Q_{165} \bowtie supplier, Q_{165} \not\ltimes supplier, Q_{165} \not\rtimes supplier$]
  GC { Delete[$\ldots$] }
  $Q_{163}$ := Materialize[$\sigma Q_{182}$]
  $Q_{203}$ := Materialize[$\gamma\pi Q_{163}$]
  $Q_{204}$ := Materialize[$sQ_{203}$]
}
```

Listing 7.1.: Abbreviated version of the plans of queries 4 to 7 of SSB TPC-H. This demonstrates how an unfortunately timed GC can cause cause FluiDB to make some bad decisions

FluiDB aspires to deal with the entire workload as if it were planning a single query. While any decision during the planning of a single query can be scrapped in the backtracking process, FluiDB is tragically forced to commit to whatever adventurous or conservative decisions it makes at the end of every planning iteration, doomed to pay dearly for every misstep but to reap the rewards of every insightful choice.

Interestingly, this problem goes away when we run with a budget of 3500k pages (figure 7.5) as the GC run is delayed to a time when other relations are better candidates for deletion than $lineorder$. A carefully designed set of heuristics for the GC should avoid this problem in most cases.

It becomes clear from the benchmark results so far that while the GC often makes the difference between FluiDB being able to successfully run a query and throwing OOM, and while we have put a lot of care in tuning it to make good decisions, it can have detrimental effects to the performance of the system as a whole. We can't get away from the fact that the less the GC is triggered, the less likely it is to delete a query useful in the future.

For that reason, as we discussed in chapter 4 in detail, FluiDB will only trigger the GC when it runs out of memory, and the GC itself goes to great lengths to delete as few tables as possible. As demonstrated in figure 7.6, however, the harsh the memory constraints will cause the GC not only to run more often it will also be forced to evict relations that are likely to be necessary for future queries.

While the GC is a central component of FluiDB, the latter is more than a simple cache system as it involves an advanced planner that can utilize reversible operations. This means that the search space for plans is, in principle, larger than the search space that would be available to a recycling RDBMS that does not take advantage of reversible operations. As mentioned the performance of FluiDB without triggering the GC is demonstrated in figure 7.5.

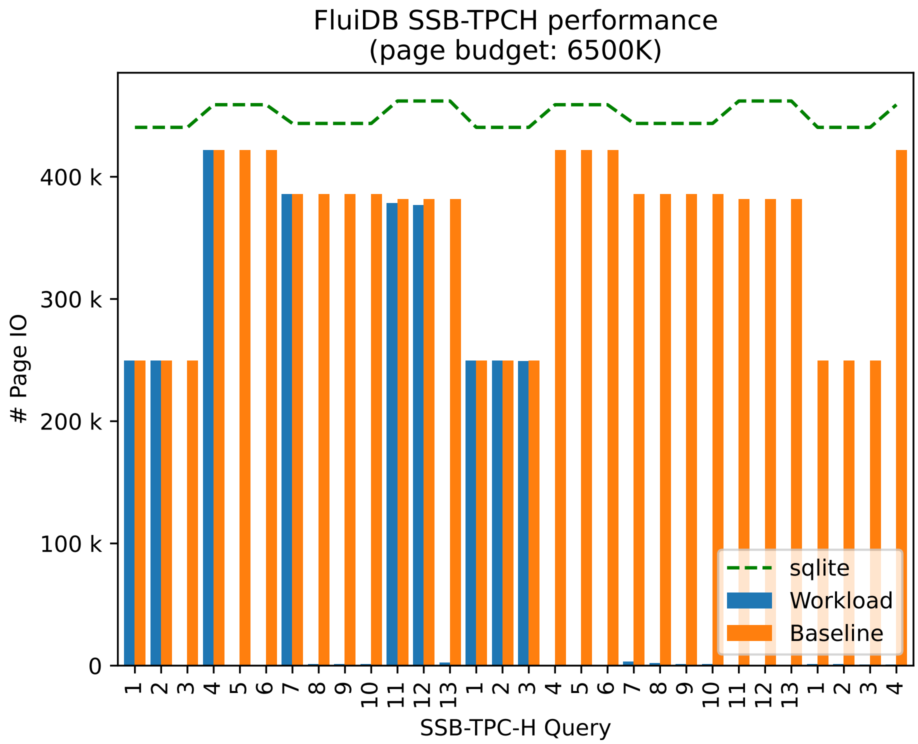


Figure 7.5.: The total page read/writes for each query of SSB TPC-H. The baseline is the query being run by FluiDB directly without any materialized n-nodes. The workload bars represent the cost of each query in a workload being accumulated into the same QDAG. The dashed line represents the performance of sqlite3 with all indexes stripped.. The budget allowed for this is about triple the minimum budget within which FluiDB can run each individual query (6500K pages).

7.2. Best/worst case scenarios for FluiDB performance

The actual best case scenario for FluiDB is a workload consisting of a single query being executed repeatedly. FluiDB will just repeatedly return a reference to the table created at the first execution. A realistic scenario that is quite good for FluiDB is the presented star schema which is the SSB-TPC-H where the number of the possible expensive queries (joins) is fairly limited and FluiDB is able to eventually figure out which views to materialize and stick with them.

The worst case scenario in the absence of memory constraints is a workload that has minimal reuse, the queries that reuse are very far apart and therefore no sharing can be exploited. This extreme case can be simulated by repeating the schema such that a workload of N queries we would have a repeated N times TPC-H-SSB schema amounting to $5 \times N$ tables:

$$\begin{aligned}
customer_1, date_1 lineorder_1, part_1, supplier_1 \\
customer_2, date_2 lineorder_2, part_2, supplier_2 \\
\ldots \\
customer_N, date_N lineorder_N, part_N, supplier_N
\end{aligned}$$

Each query Q_i will reference tables $< table >_i$ instead of $< table >$. In terms of the query runtime this is equivalent to running each query from scratch each time, i.e. the /"baseline"/ in each figure.

In the presence of constrained memory the more nuanced worst case is when the the GC always deletes primary nodes that are needed by the next query so each query needs to recreate the primary tables. A simple example is:

$$
\begin{aligned}
Q_1 &:= lineorder \bowtie date \bowtie customer \\
Q_2 &:= lineorder \bowtie part \bowtie supplier \\
Q_3 &:= lineorder \bowtie date \bowtie customer \\
Q_4 &:= lineorder \bowtie part \bowtie supplier \\
&\dots
\end{aligned}
$$

In a memory constrained situation the garbage collector needs to delete $lineorder$ in order to make space for the final join of each Q_i, meaning that it needs to recreate $lineorder$ at each step by projecting on $lineorder \bowtie part$ for Q_{2i+1} and in $lineorder \bowtie data$ for each Q_{2i}. In slightly more general terms this kind of form oscillation that is the worst case workload for FluiDB means that every operator in the plan is, not only used only once, but needs to be undone. This renders the worst case cost of each operator to be its cost $+$ the cost of it's reverse.

It is hard to compare this case with other databases or even with non-GC FluiDB operation because most database systems will fail with OOM if their budget can't accommodate all the primary, and every stage of intermediate and final relations for the particular workload. While this is not a great situation for FluiDB, then, it is in fact much better than a failure to resolve the query.

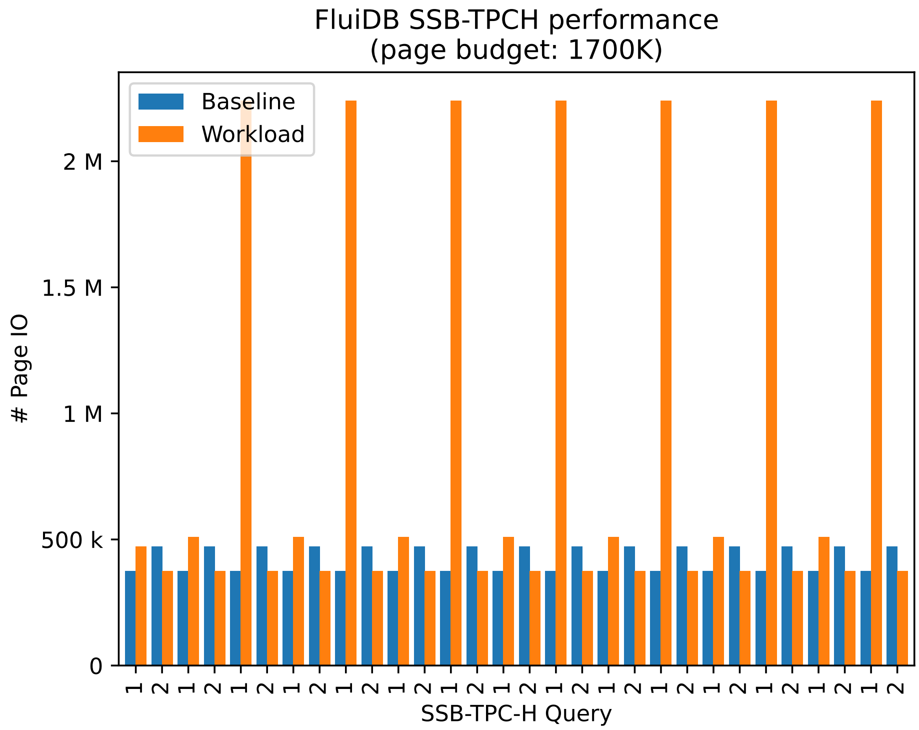


Figure 7.6.: This plot shows a workload where the GC opts to remove primary tables in order to make space for intermediate results. The primary tables are then required so they need to be reconstructed. The general shape of the queries is $A \bowtie B_0 \bowtie C_0$ and $A \bowtie B_1 \bowtie C_1$ and the budget is small enough that it can't fit the final result, the intermediate result and the primary tables simultaneously. The GC is therefore required to delete the primary tables forcing their reconstruction the next time it is required.

The compilation overheads range between a staggering 9 and 11 secs per query. While it is not ideal, there are things that can be done to mitigate it like prerecompiling the headers (age with the -fmodule-header flag) or changing the granularity of the code generation to generate individual operators, which would allow both caching of compiled units as well as parallel compilation. Optimizing the compilation process itself, however, is beyond the scope of this thesis as the overhead is constant (or more precisely, a function of the query complexity) and this work focuses on optimizing long running queries.

7.3. Conclusion

FluiDB is a complex database system built from scratch focused on a specific idea: making optimal use of the storage budget by fully adapting the data layout to the workload. While FluiDB is an experimental system that is probably not stable enough system to be used in a production setting, it is a complete end-to-end system and the results presented in this chapter demonstrate that it is based on ideas worth considering in the design of a commercial database system.

CHAPTER 8

Conclusions and future perspectives

What do we do now, now that we are happy?

(S. Beckett – Waiting for Godot (Estragon))

Chapter summary

- Summary of FluiDB concepts.
- Future work.

In this thesis we covered the important and interesting aspects of FluiDB. Implementing the system itself involved much work that is not covered in this thesis to avoid flooding the reader with details that might remove focus from the important aspects of the system. To name a few we implemented a pretty printing library, a query decorrelation algorithm, a set of tools for debugging and tracing backtracking algorithms, etc.

In the introduction we discussed some examples where data layout can radically adapt to the workload by making use of reversible operations and integrating a garbage collector into the planner. In the background chapter, we further introduced the relational database model and provided an overview of the various stages of query processing. We focus on two subfields of database research that are of particular relevance to FluiDB: intermediate result recycling, and code generation for in-memory databases.

The first non-introductory chapter (chapter 3) goes in depth about the semantics of reversible operations and how they, along with possible intermediate results for each query form the QDAG. It covers topics that relate to the construction of the QDAG, the use of the QDAG for enumerating plans, higher levels of organization of the QDAG n-nodes into clusters and how those clusters are used by FluiDB to infer properties of intermediate query results like cardinality and relation extent using propagators.

Chapter 4 on physical planning starts off by introducing a novel framework for implementing search algorithms, the `HCntT` monad transformer. Computation expressed in terms of `HCntT` can dynamically switch between branches of the backtracking algorithm essentially implementing a weighted search. The main novelty lies in the implementation of `once` and `fallback` operations that, to our knowledge, are not supported by any similar weighted search framework.

Based on `HCntT` we describe the implementation of the physical planner that lies at the core of FluiDB. The physical planner uses an $A^{\star}$-like algorithm to traverse the QDAG in order to enumerate query plans. Query plan fragments are interleaved with GC runs that produce sequences of table deletions when the planner detects that the storage budget would be exceeded.

The decisions made by the planner are based on computations that depend on the inventory of materialized relations, namely variations of cost estimation of intermediate result and intermediate result materializability. It makes sense to implement computations in terms of an incremental computation framework as there are major opportunities for sharing between evaluations. Furthermore, the performance of these computations is sensitive to the order of evaluation due to absorbing elements and $\min$ operations. To solve these problems we developed *Antisthenis* which was described in detail in chapter 5.

We wrap up the technical description of FluiDB with chapter 6 that goes in depth about how FluiDB transpiles physical plans into C++ and what algorithms are used to implement each operation and their reverse.

Finally, we present some experimental results to demonstrate the performance characteristics of FluiDB and any system that might implement similar concepts to it. Particularly, we run the SSB TPC-H benchmark and show that FluiDB can successfully outperform a similar system that only does per-query optimization in amortized cost and even in most per-query cases. We also demonstrate the expected result that FluiDB's repertoire of heuristics may cause individual queries planned within the context of a workload to be be less performant than the same query planned in isolation. We further demonstrate how FluiDB usually succeeds at making better use of larger storage budgets but occasionally, like every heuristic based system, will make decisions that hurt performance.

8.1. Future perspectives

FluiDB steps into a new path towards what we believe to be a *truly* adaptive storage, and indicates many interesting research trajectories. There are many interesting tangents to follow and many places where FluiDB does not take full advantage of the state of the art in database research.

Starting with the latter, the most important place where FluiDB can do better is cardinality estimation. As mentioned FluiDB is very naive w.r.t. to how it estimates sizes. That said the propagator network structure used to estimate cardinalities can be augmented to propagate information about selectivity and statistics about each n-node. These statistics could be collected for every materialized intermediate relation to enrich the much needed information available during planning.

Another low hanging fruit that can be exploited is parallelism. All plans that FluiDB produces are single threaded but there is no fundamental reason for that to be so. A simple data dependency analysis could reveal opportunities for parallelism and operators themselves could be easily replaced with parallel versions. A more complex but very interesting problem would be how to teach the planner to take this into account in order to produce plans that aren't just accidentally parallel, but are designed taking such speedups into account.

Another important shortcoming of FluiDB as it was implemented is its complete lack of support for updates. There are heaps of literature to draw from and many excellent approaches to the problem. It would be interesting to find out which of these and how they could be applied to the peculiar case of FluiDB's data model where primary tables are likely not materialized in memory.

Furthermore, a system based on FluiDB could take advantage of non-relational operations like table compression, materialization of indexes, movement of data in a memory hierarchy in order to produce plans that can better take advantage of the various possibilities provided by the state of the art in database research.

Finally, a major bottleneck especially for cheap queries, is the latency of the C++ compiler that can take up to 10 sec to build a query plan into an executable. The FluiDB code generation make absolutely no attempt to make life easier for the C++ compiler, making heavy use of metaprogramming facilities like templates, traits, and `constexpr` expressions. There has been a lot of work to mitigate this problem, in the literature, some good examples of which are outlined in the background section 2.3.1

Appendix I: SSB TPC-H Queries

```sql
-- Flight 1
-- Query 1
select sum(lo_extendedprice*lo_discount) as revenue
from lineorder, date
where lo_orderdate = d_datekey
and d_yearmonthnum = 199401
and lo_discount between 4 and 6
and lo_quantity between 26 and 35

-- Query 2
select sum(lo_extendedprice*lo_discount) as revenue
from lineorder, date
where lo_orderdate = d_datekey
and d_year = 1993
and lo_discount between 1 and 3
and lo_quantity < 25

-- Query 3
select sum(lo_extendedprice*lo_discount) as revenue
from lineorder, date
where lo_orderdate = d_datekey
and d_yearmonthnum = 199401
```

```sql
and lo_discount between 4 and 6
and lo_quantity between 26 and 35

-- Flight 2
-- Query 4
select sum(lo_revenue), d_year, p_brand1
from lineorder, date, part, supplier
where lo_orderdate = d_datekey
and lo_partkey = p_partkey
and lo_suppkey = s_suppkey
and p_category = 'MFGR#12'
and s_region = 'AMERICA'
group by d_year, p_brand1
order by d_year, p_brand1

-- Query 5
select sum(lo_revenue), d_year, p_brand1
from lineorder, date, part, supplier
where lo_orderdate = d_datekey
and lo_partkey = p_partkey
and lo_suppkey = s_suppkey
and p_brand1 between 'MFGR#2221' and 'MFGR#2228'
and s_region = 'ASIA'
group by d_year, p_brand1
order by d_year, p_brand1

-- Query 6
select sum(lo_revenue), d_year, p_brand1
from lineorder, date, part, supplier
where lo_orderdate = d_datekey
and lo_partkey = p_partkey
and lo_suppkey = s_suppkey
and p_brand1 = 'MFGR#2221'
and s_region = 'EUROPE'
```

```sql
group by d_year, p_brand1
order by d_year, p_brand1

-- Flight 3
-- Query 7
select c_nation, s_nation, d_year, sum(lo_revenue) as revenue
from customer, lineorder, supplier, date
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_orderdate = d_datekey
and c_region = 'ASIA' and s_region = 'ASIA'
and d_year >= 1992 and d_year <= 1997
group by c_nation, s_nation, d_year
order by d_year, revenue desc

-- Query 8
select c_city, s_city, d_year, sum(lo_revenue) as revenue
from customer, lineorder, supplier, date
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_orderdate = d_datekey
and c_nation = 'UNITED STATES'
and s_nation = 'UNITED STATES'
and d_year >= 1992 and d_year <= 1997
group by c_city, s_city, d_year
order by d_year, revenue desc

-- Query 9
select c_city, s_city, d_year, sum(lo_revenue) as revenue
from customer, lineorder, supplier, date
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_orderdate = d_datekey
and (c_city='UNITED KI1' or c_city='UNITED KI5')
```

```sql
and (s_city='UNITED KI1' or s_city='UNITED KI5')
and d_year ⩾ 1992 and d_year ⩽ 1997
group by c_city, s_city, d_year
order by d_year, revenue desc

-- Query 10
select c_city, s_city, d_year, sum(lo_revenue) as revenue
from customer, lineorder, supplier, date
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_orderdate = d_datekey
and (c_city='UNITED KI1' or c_city='UNITED KI5')
and (s_city='UNITED KI1' or s_city='UNITED KI5')
and d_yearmonth = 'Dec1997'
group by c_city, s_city, d_year
order by d_year, revenue desc

-- Flight 4
-- Query 11
select d_year, c_nation, sum(lo_revenue - lo_supplycost) as profit
from date, customer, supplier, part, lineorder
where lo_custkey = c_custkey
 and lo_suppkey = s_suppkey
 and lo_partkey = p_partkey
 and lo_orderdate = d_datekey
 and c_region = 'AMERICA'
 and s_region = 'AMERICA'
 and (p_mfgr = 'MFGR#1' or p_mfgr = 'MFGR#2')
group by d_year, c_nation
order by d_year, c_nation

-- Query 12
select d_year, s_nation, p_category, sum(lo_revenue - lo_supplycost) as profit
from date, customer, supplier, part, lineorder
```

```sql
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_partkey = p_partkey
and lo_orderdate = d_datekey
and c_region = 'AMERICA'
and s_region = 'AMERICA'
and (d_year = 1997 or d_year = 1998)
and (p_mfgr = 'MFGR#1'
or p_mfgr = 'MFGR#2')
group by d_year, s_nation, p_category order by d_year, s_nation, p_category

-- Query 13
select d_year, s_city, p_brand1, sum(lo_revenue - lo_supplycost) as profit
from date, customer, supplier, part, lineorder
where lo_custkey = c_custkey
and lo_suppkey = s_suppkey
and lo_partkey = p_partkey
and lo_orderdate = d_datekey
and c_region = 'AMERICA'
and s_nation = 'UNITED STATES'
and (d_year = 1997 or d_year = 1998)
and p_category = 'MFGR#14'
group by d_year, s_city, p_brand1 order by d_year, s_city, p_brand1
```

Listing A.1.: The SQL code for the SSB queries

Appendix II: Computation representation in Haskell

Here we will describe the the basic model of computation as understood by ML-like languages, and specifically Haskell. These languages define computations as first class values that have parametric types. A computation of type `f a` can be understood as a computation that when executed would generate a value of type `a`. In that sense `f` is a generic type with one type parameter `a`, thereby being of kind $* \rightarrow *$, whereas `a` and `f a` are both of kind $*$. For example, in Haskell, we represent as `IO Int` the type of computation that interacts with the outside world in order to compute an integer, e.g. by reading user input, by writing files, or, as the meme in the community goes, by launching missiles.

From the perspective of the programming language, in order to manipulate computation it is useful to be able to *manipulate* computations, i.e., to be able to create morphisms `f a → f b` from simpler primitives in the language. The notation of the arrow `→` used here is a function within the basic framework of the language, a simple mapping between a value of type `f a` to a value of type `f b`. This is indeed already a computation carried out by the runtime. In that sense one might consider it *implicit* computation: We have little control over the conditions under which it is carried out, especially in a lazy language like Haskell, and we consider it to be pure, with no effects.

Returning to our higher kinded (of kind `Type → Type`), explicit computation types, commonly in Haskell implement a hierarchy of 3 typeclasses (interfaces), each of which provides a different way of producing morphisms of the computation itself, and therefore each of which allowing different computations to be expressed by type `f`. While these typeclasses are more general than referring only to computations, we will try to provide an intuition of each in reference to computations, to avoid overwhelming the reader with the full generality of these constructs. The reader is encouraged to notice how the weaker, more general, typeclasses presented first, more intuitively describe containers of values and as we strengthen the constraints and require more operations of the computation objects, it becomes harder to think of containers that fit the constraints and notions of the values as computations become more natural.

B.0.1. Functor

For computations implementing the `Functor` typeclass we can create a morphism of the computation from a normal Haskell function: for every functor `f` then, there must be a function `fmap :: (a → b) → f a → f b` that abides by the functor (see listing B.1) [34]. In plain English, given a computation and a function that can transform the result of the computation we can get a new computation that is equivalent to the original computation but yielding the transformed result. The functor laws (see listing B.2), which all valid implementations of a functor must follow, assert that applying the identity function to the result of a computation does not change the computation itself and that `fmap`ing has no other effect on the computation other than changing its result.

```
class Functor f where
  fmap :: (a → b) → f a → f b
```

Listing B.1.: The functor interface in Haskell.

It is common to understand functors in the context of haskell as mappable containers that can hold any type value. One could think of a computation that supports the functor interface as a computation that results in a functorial container. An example of a computation that is *not* a functor is that produces a set of values, because a set of values requires that the values are comparable, and not all per element value transformations of a set are valid. For example a set of integers `Set Int` can be unambiguous for the language but a set of functions `Set (Int → String)` is not unless we define a notion of equality for functions `Int → String`.

```
-- Identity
fmap id == id
```

```haskell
-- Composition
fmap (f . g) == fmap f . fmap g
```

Listing B.2.: Laws that any value implementing the functor interface must obey.

B.0.2. Applicative

Computations that are applicative functors [34] are computations that can be combined in "no particular order". In particular an applicative functor must implement `ap :: f (a -> b) -> f a -> f b` (more commonly written as `<*>` which is a bit harder to pronounce) and `pure :: a -> f a` (see listing B.3). Applicatives are more often described in terms of computation than functors.

Implementing an applicative interface means first and foremost that we must be able to construct a trivial computation that just returns a given value, meaning that there must be no restriction on the kinds of values an applicative computation can yield. For example a computation that yields only integers is not an applicative because then there would no way to universally quantify the argument of `pure`. Incidentally it is not a functor either for the same reason, namely that we would not be able to universally quantify the output of the input function of `fmap`.

```haskell
class Functor f => Applicative f  where
  (<*>) :: f (a -> b) -> f a -> f b
  pure :: a -> f a
```

Listing B.3.: The interface of a haskell applicative functor.

Furthermore, from the definition of `<*>` (pronounced `ap`) we understand that two computations that are applicative functors can be combined into one computation with no restriction on the order that they are evaluated. There are a few examples of applicatives being used to represent parallelisable computation, the most prominent of which being Facebook's Haxl [63]

Like with `Functor` the interface of applicative must be subject to the applicative laws presented in B.4. Essentially, these formalize the triviality of `pure`.

```haskell
-- Identity
pure id <*> v = v
```

```
-- Composition
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
-- Homomorphism
pure f <*> pure x = pure (f x)
-- Interchange
u <*> pure y = pure ($ y) <*> u
```

Listing B.4.: Laws that any valid applicative interface must obey. They mean what one might intuitively understand as "`pure` must be trivial".

B.0.3. Monad

A meme in the functional community defines monads as "monoids in the category of endofunctors". To the unfamiliar reader this may take a second to parse, but it is really just a fancy way of saying something fairly simple: we can transform any nested monadic functor `f (f a)` into a flat one `f a`, and for a nested monad `f (f ( ... f (f a)))` the order in which the `f`s are collapsed does not matter.

In the context of computations, this implies that nested computations run from inside out, the inner computation is run first and then the outer one is run. The "and then" part is what a monad is meant to bring to the table. Viewed as computations, monads must have all the properties of functors and applicatives, but they must also implement the operation `(>>=) :: f a -> (a -> f b) -> f b` where `>>=` is pronounced `bind`. The computation described by the first argument of `>>=` must be executed **first** in order to generate the argument for the function in the second argument which must **then** produce the result. We commonly represent a monadic functor by the character `m` rather than the character `f` that we used for functors and applicatives. Monads are the most commonly used abstraction to describe computations comprised of interdependent steps. The Haskell interface for monads is succinctly presented in listing B.5

```
class Applicative m => Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b
```

Listing B.5.: Definition of the interface of a haskell monad.

Monad implementations need to adhere to the laws laid out in listing B.6. These laws are similar to the applicative laws in that they formalize the triviality of `pure`, now called `return`, in relation to the `bind` operator this time. However, unlike the applicative laws, monadic laws introduce the law of associativity that may seem strange to a reader unfamiliar with Kleisli arrows [32]. The essence of the monad, when focusing on the second argument of bind (the one typed `a → m b`), and which is codified by the monad laws is that a monad `m` must give rise to a category, referred to in the literature as the Kl category.

A category needs a set of objects and a domain of arrows that can be composed. Furthermore, we need the arrow composition to be associative and an identity arrow that maps objects to themselves. An obvious category is formed by all objects in Haskell, Haskell functions as arrows and `id :: a → a` as the identity arrow. This is commonly referred to as the *Hask category*. As alluded to earlier, this category does describe computation, albeit in a more implicit manner. The monad laws assert that each monad gives rise to a Kleisly category where objects are the objects of Hask, arrows `a ⇝ b ≡ a → m b` and the identity arrow is `return`. Moving forward we will see how, capitalizing on this conception we can come up with a more flexible

```
-- Left identity
return a >>= k = k a
-- Right identity
m >>= return = m
-- Associativity
m >>= (\x → k x >>= h) = (m >>= k) >>= h
```

Listing B.6.: Laws that any valid monad implementation must abide [42].

Monad examples

To make the concept tangible, we provide a few examples of how monads encapsulate computational effects by looking at four monads: `Maybe`, `State` , `IO`, and `Free`.

Maybe The `Maybe` functor (defined in listing B.7) implements function partiality. In other languages, the same concept may be called `opt` or `Option`. A partial function is denoted `a → Maybe b` because it may or not return a value.

```
data Maybe a = Just a | Nothing
```

Listing B.7.: Definition of the `Maybe` monad.

Composability of the Kleisli arrow allows us to compose multiple partial functions into new ones, thus creating computations that can trivially fail. `Maybe` provides a good opportunity to distinguish between `Applicative` and `Monad`. Considering the two programs in listing B.8 where we only care about `Maybe` as an `Applicative`. The semantics of the program do not specify whether calling `invAdd` will cause `inverse a` or `inverse b` to be computed first or if they will be computed in parallel. No order is imposed between them. However, in the program presented in B.9 when evaluating the function `f` the expression `inverse a` *must* be evaluated before the `toInt` so that `toInt` has an input to operate on.

```haskell
inverse :: Double -> Maybe Double
inverse a = if a == 0 then Nothing else Just (1 / a)

invAdd :: Double -> Double -> Maybe Double
invAdd a b = (+) <$> inverse a <*> inverse b
```

Listing B.8.: Example usage of the `Maybe` applicative functor.

```haskell
inverse :: Double -> Maybe Double
inverse a = if a == 0 then Nothing else Just (1 / a)

toInt :: Double -> Maybe Int
toInt a =
  if a == fromInteger (round a)
    then Just (round a)
    else Nothing

f :: Double -> Maybe Int
f a = inverse a >>= toInt
```

Listing B.9.: Example usage of the `Maybe` monad functor.

State A `State` monad (listing B.10) encapsulates a computation that depends on mutable state of type `s`.

```haskell
newtype State s a = State (s -> (a,s))
```

Listing B.10.: The state monad describes mutable state.

`State` is essentially a function that accepts some state, modifies it and returns it along with a value. In C-like languages, the operator to compose different components that operate on the and local state is `;`. `f() ; g() ;` means that `f()` can have arbitrary side effects on the global state and must be computed entirely before `g()`. In our slightly more precise flavor of effectful computations `f >> g` as a `State` composes a state functor that expects some state value, passes it to `f` which modifies it and passes it to `g`.

IO The `IO a` monad is "magical" in the sense that it is completely opaque to the constructs of the language. It represents the interaction of our Haskell program with the outside world, and since all Haskell functions are pure, Haskell can be thought of as metalanguage that defines programs composed of different `IO a` components called the `main :: IO ()`. The components of an `IO` computation are guaranteed to be evaluated one after the other.

This is demonstrated in B.11. Because monads are so ubiquitous Haskell provides some syntactic sugar called the `do` notation. The same program is presented in a much more readable for in B.12.

The functions `putStrLn :: String -> IO ()` takes a string and returns a computation that would show the string to stdout. `getLine :: IO String` is a computation that reads a line from stdin and retains its value. The `main :: IO ()` is a special variable that a Haskell program needs to define. All the Haskell runtime essentially does is run the IO computation that `main` refers to.

```haskell
main :: IO ()
main = putStrLn "What's your name?"
       >> getLine
       >>= (\name -> putStrLn ("Hello " ++ name))
```

Listing B.11.: Sequencing IO interactions using the `IO` monad.

```haskell
main :: IO ()
main = do
  putStrLn "What's your name?"
```

```haskell
  name <- getLine
  putStrLn ("Hello " ++ name)
```

Listing B.12.: Sequencing IO interactions using the `IO` monad also using the `do` notation.

Free monad The *free monad* is slightly more convoluted than the ones we talked about so far. Without getting to deep into what free structures are in general ([101] is a recommended source that goes into the subject), a free monad is a structure that barely supports the monad interface in a law-abiding way. The only difference is that instead of focusing on the `>>=` operator that we saw so far, it instead provides a constructor corresponding to the equivalent operator `join` presented in listing B.13.

```haskell
join :: Monad m => m (m a) -> m a
join m = m >>= id

-- and also
(>>=) :: Monad m => m a -> (a -> m b) -> m b
m >>= f = join (fmap f m)
```

Listing B.13.: The bind (`>>=`) and join operations on a monad are equivalent given that monads are also functors.

A free monad `Free f m` then is equivalent to the *functor* `f` that is stacked on top of itself zero or more times `f (f ( ... f (f a)))`. It essentially turns a functor into a monad for "free" – as long as one does not mind that it is actually a stack of nested functor rather than a single layer like monads are. Listing B.14 presents a simple implementation, although the implementation in B.15 is more common. Free monads are particularly important for our implementation of Antisthenis (chapter 5).

```haskell
data Free f a
  = Pure a
  | Free (f (Free f a))
```

Listing B.14.: A simple implementation of the free monad type.

```haskell
newtype FreeF f a x = Pure a | Free (f x)
data FreeT f m a = m (FreeF f a (FreeT f m a))
```

Listing B.15.: A simple implementation of the free monad type.

Bibliography

[1] Yehoshua Sagiv and Mihalis Yannakakis. “Equivalences Among Relational Expressions with the Union and Difference Operators”. In: *Journal of the ACM* 27.4 (Oct. 1980), pages 633–655.

[2] Donald D. Chamberlin et al. “A History and Evaluation of System R”. In: *Communications of the ACM* 24.10 (Oct. 1, 1981), pages 632–646.

[3] Christopher T. Haynes. “Logic Continuations”. In: *The Journal of Logic Programming* 4.2 (June 1, 1987), pages 157–176.

[4] T. K. Sellis. “Multiple-Query Optimization.” In: (1987).

[5] G. Graefe and K. Ward. “Dynamic Query Evaluation Plans”. In: *ACM SIGMOD Record* 18.2 (June 1, 1989), pages 358–366.

[6] W. Pugh and T. Teitelbaum. “Incremental Computation via Function Caching”. In: *Proceedings of the 16th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages - POPL ’89*. The 16th ACM SIGPLAN-SIGACT Symposium. Austin, Texas, United States: ACM Press, 1989, pages 315–328.

[7] G. Graefe and W.J. McKenna. “The Volcano Optimizer Generator: Extensibility and Efficient Search”. In: *Proceedings of IEEE 9th International Conference on Data Engineering*. Proceedings of IEEE 9th International Conference on Data Engineering. Apr. 1993, pages 209–218.

[8] Goetz Graefe. “The Cascades Framework for Query Optimization”. In: *IEEE Data Eng. Bull.* 18.3 (1995), pages 19–29.

[9] Sheng Liang, Paul Hudak, and Mark Jones. “Monad Transformers and Modular Interpreters”. In: *Proceedings of the 22nd ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. POPL ’95. New York, NY, USA: Association for Computing Machinery, Jan. 25, 1995, pages 333–343.

[10] Ralf Hinze. *Monadic Style Backtracking*. Citeseer, 1996.

[11] Peter Scheuermann, Junho Shim, and Radek Vingralek. “Watchman: A Data Warehouse Intelligent Cache Manager”. In: *Proceedings of the VLDB Conference*. 1996.

[12] Himanshu Gupta. "Selection of Views to Materialize in a Data Warehouse". In: *Database Theory — ICDT '97*. Edited by Foto Afrati and Phokion Kolaitis. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 1997, pages 98–112.

[13] Gérard Huet. "The Zipper". In: *Journal of Functional Programming* 7.5 (Sept. 1997), pages 549–554.

[14] Dimitrios Theodoratos and Timos Sellis. "Data warehouse configuration". In: *Proceedings of the 23rd International Conference on Very Large Databases, VLDB 1997*. 23rd International Conference on Very Large Databases, VLDB 1997. Morgan Kaufmann, Jan. 1, 1997, pages 126–135.

[15] Jeffrey D. Ullman. "Information Integration Using Logical Views". In: *International Conference on Database Theory*. Springer, 1997, pages 19–40.

[16] Yannis Kotidis and Nick Roussopoulos. "Dynamat: A Dynamic View Management System for Data Warehouses". In: *ACM Sigmod record* 28.2 (1999), pages 371–382.

[17] Chuan Zhang and Jian Yang. "Genetic Algorithm for Materialized View Selection in Data Warehouse Environments". In: *International Conference on Data Warehousing and Knowledge Discovery*. Springer, 1999, pages 116–125.

[18] Ralf Hinze. "Deriving Backtracking Monad Transformers". In: *ACM SIGPLAN Notices* 35 (Aug. 15, 2000).

[19] Prasan Roy et al. "Efficient and Extensible Algorithms for Multi Query Optimization". In: *Proceedings of the 2000 ACM SIGMOD International Conference on Management of Data*. 2000, pages 249–260.

[20] Hoshi Mistry et al. "Materialized View Selection and Maintenance Using Multi-Query Optimization". In: *Proceedings of the 2001 ACM SIGMOD International Conference on Management of Data*. SIGMOD '01. New York, NY, USA: Association for Computing Machinery, May 1, 2001, pages 307–318.

[21] Umut A. Acar et al. "Dynamizing Static Algorithms, with Applications to Dynamic Trees and History Independence". In: (2004).

[22] Umut A. Acar. *Self-Adjusting Computation*. Carnegie Mellon University, 2005.

[23] Stavros Harizopoulos and Anastassia Ailamaki. "StagedDB: Designing Database Servers for Modern Hardware." In: *IEEE Data Eng. Bull.* 28.2 (2005), pages 11–16.

[24] Stavros Harizopoulos, Vladislav Shkapenyuk, and Anastassia Ailamaki. "Qpipe: A Simultaneously Pipelined Relational Query Engine". In: *Proceedings of the 2005 ACM SIGMOD International Conference on Management of Data*. 2005, pages 383–394.

[25] John Hughes. "Programming with Arrows". In: *Advanced Functional Programming*. Edited by Varmo Vene and Tarmo Uustalu. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 2005, pages 73–129.

[26] Oleg Kiselyov et al. "Backtracking, Interleaving, and Terminating Monad Transformers: (Functional Pearl)". In: *ACM SIGPLAN Notices* 40.9 (2005), pages 192–203.

[27] Tarmo Uustalu and Varmo Vene. *Comonadic Functional Attribute Evaluation*. Volume 6. Jan. 1, 2005, page 162. 145 pages.

[28] Umut A. Acar et al. "Kinetic Algorithms via Self-Adjusting Computation". In: *European Symposium on Algorithms*. Springer, 2006, pages 636–647.

[29] Gang Gou, J. X. Yu, and Hongjun Lu. "A/Sup */ Search: An Efficient and Flexible Approach to Materialized View Selection". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)* 3.36 (2006), pages 411–425.

[30] Umut A. Acar et al. "Adaptive Bayesian Inference". In: *Neural Information Processing Systems (NIPS)*. Volume 10. 2981562.2981743. Citeseer, 2007.

[31] M. Bayir, I. H. Toroslu, and A. Cosar. "Genetic Algorithm for the Multiple-Query Optimization Problem". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)* (2007).

[32] Jeremy E Dawson. "Compound Monads and the Kleisli Category". In: *Unpublished note. Available online at http://users. cecs. anu. edu. au/~ jeremy/pubs/cmkc* (2007).

[33] Surajit Chaudhuri, Vivek Narasayya, and Ravi Ramamurthy. "A Pay-as-You-Go Framework for Query Execution Feedback". In: *Proceedings of the VLDB Endowment* 1.1 (2008), pages 1141–1152.

[34] Conor McBride and Ross Paterson. "Applicative Programming with Effects". In: *Journal of functional programming* 18.1 (2008), pages 1–13.

[35] Thomas Phan and Wen-Syan Li. "Dynamic Materialization of Query Views for Data Warehouse Workloads". In: *2008 IEEE 24th International Conference on Data Engineering*. 2008 IEEE 24th International Conference on Data Engineering. Apr. 2008, pages 436–445.

[36] Janis Voigtländer. "Asymptotic Improvement of Computations over Free Monads". In: *Mathematics of Program Construction*. Edited by Philippe Audebaud and Christine Paulin-Mohring. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 2008, pages 388–403.

[37] Umut A. Acar et al. "An Experimental Analysis of Self-Adjusting Computation". In: *ACM Transactions on Programming Languages and Systems (TOPLAS)* 32.1 (2009), pages 1–53.

[38] George Candea, Neoklis Polyzotis, and Radek Vingralek. "A Scalable, Predictable Join Operator for Highly Concurrent Data Warehouses". In: *Proceedings of the VLDB Endowment* 2.1 (Aug. 1, 2009), pages 277–288.

[39] Sebastian Fischer. *Reinventing Haskell Backtracking*. Gesellschaft für Informatik e. V., 2009.

[40] Alexey Andreyevich Radul. "Propagation Networks: A Flexible and Expressive Substrate for Computation". In: (2009).

[41] Gerald Jay Sussman and Alexey Radul. "The Art of the Propagator". In: (Jan. 26, 2009).

[42] Brent Yorgey. "The Typeclassopedia". In: (2009).

[43] Umut A. Acar et al. "Traceable Data Types for Self-Adjusting Computation". In: *ACM Sigplan Notices* 45.6 (2010), pages 483–496.

[44] Subi Arumugam et al. "The DataPath System: A Data-Centric Analytic Processing Engine for Large Data Warehouses". In: *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data*. 2010, pages 519–530.

[45] Georgios Giannikis et al. "Crescando". In: *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data*. 2010, pages 1227–1230.

[46] Pradeep Kumar Gunda et al. "Nectar: Automatic Management of Data and Computation in Datacenters." In: *OSDI*. Volume 10. 2010, pages 1–8.

[47] Herodotos Herodotou and Shivnath Babu. "Xplus: A Sql-Tuning-Aware Query Optimizer". In: *Proceedings of the VLDB Endowment* 3.1-2 (2010), pages 1149–1160.

[48] Milena G. Ivanova et al. "An Architecture for Recycling Intermediates in a Column-Store". In: *ACM Transactions on Database Systems (TODS)* 35.4 (2010), pages 1–43.

[49] Konstantinos Krikellas, Stratis D. Viglas, and Marcelo Cintra. "Generating Code for Holistic Query Evaluation". In: *2010 IEEE 26th International Conference on Data Engineering (ICDE 2010)*. 2010 IEEE 26th International Conference on Data Engineering (ICDE 2010). Mar. 2010, pages 613–624.

[50] Simon Marlow. "Haskell 2010 Language Report". In: *Available on: https://www. haskell. org/onlinereport/haskell2010* (2010).

[51] Tomasz Nykiel et al. "MRShare: Sharing across Multiple Queries in MapReduce". In: *Proceedings of the VLDB Endowment* 3.1-2 (Sept. 1, 2010), pages 494–505.

[52] Pramod Bhatotia et al. "Incoop: MapReduce for Incremental Computations". In: *Proceedings of the 2nd ACM Symposium on Cloud Computing*. SOCC '11. New York, NY, USA: Association for Computing Machinery, Oct. 26, 2011, pages 1–14.

[53] Swati V. Chande and Madhavi Sinha. "Genetic Optimization for the Join Ordering Problem of Database Queries". In: *2011 Annual IEEE India Conference*. IEEE, 2011, pages 1–5.

[54] Alfons Kemper and Thomas Neumann. "HyPer: A Hybrid OLTP&OLAP Main Memory Database System Based on Virtual Memory Snapshots". In: *2011 IEEE 27th International Conference on Data Engineering*. IEEE, 2011, pages 195–206.

[55] Xiaodan Wang et al. "Coscan: Cooperative Scan Sharing in the Cloud". In: *Proceedings of the 2nd ACM Symposium on Cloud Computing*. 2011, pages 1–12.

[56] Iman Elghandour and Ashraf Aboulnaga. "ReStore: Reusing Results of MapReduce Jobs". Comment: VLDB2012. Feb. 29, 2012. arXiv: 1203.0061 [cs].

[57] Georgios Giannikis, Gustavo Alonso, and Donald Kossmann. "SharedDB: Killing One Thousand Queries with One Stone". 2012. arXiv: 1203.0056.

[58] S. Idreos et al. "Monetdb: Two Decades of Research in Column-Oriented Database". In: *IEEE Data Engineering Bulletin* 35.1 (2012), pages 40–45.

[59] Imene Mami and Zohra Bellahsene. "A Survey of View Selection Methods". In: *Acm Sigmod Record* 41.1 (2012), pages 20–29.

[60] Srinath Shankar et al. "Query Optimization in Microsoft SQL Server PDW". In: *Proceedings of the 2012 International Conference on Management of Data - SIGMOD '12*. The 2012 International Conference. Scottsdale, Arizona, USA: ACM Press, 2012, page 767.

[61] T. V. Vijay Kumar and Santosh Kumar. "Materialized View Selection Using Genetic Algorithm". In: *International Conference on Contemporary Computing*. Springer, 2012, pages 225–237.

[62] Lindsey Kuper and Ryan R. Newton. "LVars: Lattice-Based Data Structures for Deterministic Parallelism". In: *Proceedings of the 2nd ACM SIGPLAN Workshop on Functional High-Performance Computing*. FHPC '13. New York, NY, USA: Association for Computing Machinery, Sept. 23, 2013, pages 71–84.

[63] Simon Marlow et al. "The Haxl Project at Facebook". In: *Proceedings of the Code Mesh London* (2013).

[64] Fabian Nagel, Peter Boncz, and Stratis D. Viglas. "Recycling in Pipelined Query Evaluation". In: *2013 IEEE 29th International Conference on Data Engineering (ICDE)*. IEEE, 2013, pages 338–349.

[65] Guoping Wang and Chee-Yong Chan. "Multi-Query Optimization in Mapreduce Framework". In: *Proceedings of the VLDB Endowment* 7.3 (2013), pages 145–156.

[66] Matthew A. Hammer et al. "Adapton: Composable, Demand-Driven Incremental Computation". In: *Proceedings of the 35th ACM SIGPLAN Conference on Programming Language Design and Implementation*. PLDI '14: ACM SIGPLAN Conference on Programming Language Design and Implementation. Edinburgh United Kingdom: ACM, June 9, 2014, pages 156–166.

[67] Jules Hedges. "Monad Transformers for Backtracking Search". 2014. arXiv: 1406.2058.

[68] Luis L. Perez and Christopher M. Jermaine. "History-Aware Query Optimization with Materialized Intermediate Views". In: *2014 IEEE 30th International Conference on Data Engineering*. 2014 IEEE 30th International Conference on Data Engineering. Mar. 2014, pages 520–531.

[69] Luis L. Perez and Christopher M. Jermaine. "History-Aware Query Optimization with Materialized Intermediate Views". In: *2014 IEEE 30th International Conference on Data Engineering*. IEEE, 2014, pages 520–531.

[70] Mohamed A. Soliman et al. "Orca: A Modular Query Optimizer Architecture for Big Data". In: *Proceedings of the 2014 ACM SIGMOD International Conference on Management of Data*. 2014, pages 337–348.

[71] Mohamed A. Soliman et al. "Orca: A Modular Query Optimizer Architecture for Big Data". In: *Proceedings of the 2014 ACM SIGMOD International Conference on Management of Data*. SIGMOD '14. New York, NY, USA: Association for Computing Machinery, June 18, 2014, pages 337–348.

[72] Michael Armbrust et al. "Spark SQL: Relational Data Processing in Spark". In: *Proceedings of the 2015 ACM SIGMOD International Conference on Management of Data*. SIGMOD '15. New York, NY, USA: Association for Computing Machinery, May 27, 2015, pages 1383–1394.

[73] Melyssa Barata, Jorge Bernardino, and Pedro Furtado. "An Overview of Decision Support Benchmarks: TPC-DS, TPC-H and SSB". In: *New Contributions in Information Systems and Technologies*. Edited by Alvaro Rocha et al. Advances in Intelligent Systems and Computing. Cham: Springer International Publishing, 2015, pages 619–628.

[74] Viktor Leis et al. "How Good Are Query Optimizers, Really?" In: *Proceedings of the VLDB Endowment* 9.3 (Nov. 2015), pages 204–215.

[75] Jack Chen et al. "The MemSQL Query Optimizer: A Modern Optimizer for Real-Time Analytics in a Distributed Database". In: *Proceedings of the VLDB Endowment* 9.13 (Sept. 1, 2016), pages 1401–1412.

[76] Alvin Cheung, Samuel Madden, and Armando Solar-Lezama. "Sloth: Being Lazy Is a Virtue (When Issuing Database Queries)". In: *ACM Transactions on Database Systems* 41.2 (June 6, 2016), 8:1–8:42.

[77] Darko Makreshanski et al. *Mqjoin: Efficient Shared Execution of Main-Memory Joins*. ETH Zurich, 2016.

[78] Wentao Wu, Jeffrey F. Naughton, and Harneet Singh. "Sampling-Based Query Re-Optimization". In: *Proceedings of the 2016 International Conference on Management of Data*. 2016, pages 1721–1736.

[79] Tarun Kathuria and S. Sudarshan. "Efficient and Provable Multi-Query Optimization". In: *Proceedings of the 36th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems*. 2017, pages 53–67.

[80] Darko Makreshanski et al. "BatchDB: Efficient Isolated Execution of Hybrid OLTP+ OLAP Workloads for Interactive Applications". In: *Proceedings of the 2017 ACM International Conference on Management of Data*. 2017, pages 37–50.

[81] Prashanth Menon, Todd C. Mowry, and Andrew Pavlo. "Relaxed Operator Fusion for In-Memory Databases: Making Compilation, Vectorization, and Prefetching Work Together at Last". In: *Proceedings of the VLDB Endowment* 11.1 (Sept. 1, 2017), pages 1–13.

[82] William Arthur Rogers et al. "Multi-Query Optimization". Patent. US Patent App. 15/222,229. Mar. 23, 2017.

[83] E. Yu. Sharygin et al. "Query Compilation in PostgreSQL by Specialization of the DBMS Source Code". In: *Programming and Computer Software* 43.6 (Nov. 1, 2017), pages 353–365.

[84] Edmon Begoli et al. "Apache Calcite: A Foundational Framework for Optimized Query Processing over Heterogeneous Data Sources". In: *Proceedings of the 2018 International Conference on Management of Data*. 2018, pages 221–230.

[85] Bailu Ding et al. "Plan Stitch: Harnessing the Best of Many Plans". In: *Proceedings of the VLDB Endowment* 11.10 (June 2018), pages 1123–1136.

[86] Stavros Harizopoulos et al. "OLTP through the Looking Glass, and What We Found There". In: *Making Databases Work: The Pragmatic Wisdom of Michael Stonebraker*. 2018, pages 409–439.

[87] Rudi Horn, Roly Perera, and James Cheney. "Incremental Relational Lenses". In: *Proceedings of the Acm on Programming Languages-Pacmpl* 2 (Sept. 2018), page 74.

[88] Sven Keidel and Sebastian Erdweg. "Sound and Reusable Components for Abstract Interpretation". In: *Proceedings of the ACM on Programming Languages* 3 (OOPSLA Oct. 10, 2019), 176:1–176:28.

[89] Ryan Marcus et al. "Neo: A Learned Query Optimizer". In: *Proceedings of the VLDB Endowment* 12.11 (July 2019), pages 1705–1718.

[90] Mark Raasveldt and Hannes Mühleisen. "DuckDB: An Embeddable Analytical Database". In: *Proceedings of the 2019 International Conference on Management of Data*. SIGMOD/PODS '19: International Conference on Management of Data. Amsterdam Netherlands: ACM, June 25, 2019, pages 1981–1984.

[91] Ana Nunes Alonso et al. "Building a Polyglot Data Access Layer for a Low-Code Application Development Platform: (Experience Report)". In: *Distributed Applications and Interoperable Systems*. Edited by Anne Remke and Valerio Schiavoni. Volume 12135. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2020, pages 95–103.

[92] Daniel Selsam, Simon Hudon, and Leonardo deMoura. "Sealing Pointer-Based Optimizations behind Pure Functions". In: *Proceedings of the ACM on Programming Languages* 4 (ICFP Aug. 2, 2020), 115:1–115:20.

[93] Zechao Shang et al. "CrocodileDB: Efficient Database Execution through Intelligent Deferment." In: *CIDR*. 2020.

[94] Haitao Yuan et al. "Automatic View Generation with Deep Learning and Reinforcement Learning". In: *2020 IEEE 36th International Conference on Data Engineering (ICDE)*. IEEE, 2020, pages 1501–1512.

[95] Daniel Anderson et al. "Efficient Parallel Self-Adjusting Computation". 2021. arXiv: `2105.06712`.

[96] Chris Hanson and Gerald Jay Sussman. *Software Design for Flexibility: How to Avoid Programming Yourself into a Corner*. MIT Press, Mar. 9, 2021. 449 pages. Google Books: `iM_tDwAAQBAJ`.

[97] Donnacha Oisín Kidney and Nicolas Wu. "Algebras for Weighted Search". In: *Proceedings of the ACM on Programming Languages* 5 (ICFP Aug. 18, 2021), 72:1–72:30.

[98] Guoliang Li, Xuanhe Zhou, and Lei Cao. "Machine Learning for Databases". In: *The First International Conference on AI-ML-Systems*. AIMLSystems 2021. New York, NY, USA: Association for Computing Machinery, Oct. 21, 2021, pages 1–2.

[99] Guoliang Li, Xuanhe Zhou, and Lei Cao. "Machine Learning for Databases". In: *The First International Conference on AI-ML-Systems*. 2021, pages 1–2.

[100] Pietro Michiardi, Damiano Carra, and Sara Migliorini. "Cache-Based Multi-Query Optimization for Data-Intensive Scalable Computing Frameworks". In: *Information Systems Frontiers* 23.1 (2021), pages 35–51.

[101] Bartosz Milewski. *The Dao of Functional Programming*.

[102] 14:00-17:00. *ISO/IEC 14882:2020*. ISO. URL: `https://www.iso.org/cms/render/live/en/sites/isoorg/contents/data/standard/07/93/79358.html` (visited on 10/28/2021).

[103] *Associated Type Synonyms | ACM SIGPLAN Notices*. URL: https://dl.acm.org/doi/abs/10.1145/1090189.1086397?casa_token=HLcvyM59laEAAAAA:3yH9-bJAMmzqia0ewv_MReCDVnbfydL5nsIokMy4g8KWDC12huZV0_X7kMSVaE257aTivPkkuZLO0A (visited on 10/10/2021).

[104] "Computing Queries from Derived Relations". In: (), page 11.

[105] Ralf Hinze. "Efficient Monadic-Style Backtracking". In: ().

[106] *Implementing Data Cubes Efficiently | Proceedings of the 1996 ACM SIGMOD International Conference on Management of Data*. URL: https://dlnext.acm.org/doi/10.1145/233269.233333 (visited on 11/26/2021).

[107] Oleg Kiselyov et al. "Backtracking, Interleaving, and Terminating Monad Transformers". In: (), page 12.

[108] Edward Kmett. *Propagators*.

[109] Tim Kraska et al. "SageDB: A Learned Database System". In: (), page 10.

[110] Daan Leijen. "Parsec: Direct Style Monadic Parser Combinators For The Real World". In: (), page 22.

[111] Thomas Neumann. *Evolution of a CompilingQuery Engine*. URL: https://www.semanticscholar.org/paper/Evolution-of-a-CompilingQuery-Engine-Neumann/2cd16bb2d3680d04948f4536270c36d98fd4e768 (visited on 11/28/2021).

[112] Luis L. Perez and Christopher M. Jermaine. *History-Aware Query Optimization with Materialized Intermediate Views*.

[113] Chris Perivolaropoulos. *Fakedrake/Ssb-Dbgen*.

[114] Chris Perivolaropoulos. *Fakedrake/Ssb-Dbgen*.

[115] *Precompiled Headers (PCH)*. URL: https://releases.llvm.org/3.1/tools/clang/docs/PCHInternals.html (visited on 12/03/2021).