Planet in a Binary-Star System

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Abstract

I studied a restricted three body problem: planetary orbits around a binary star on 2D plane. Simulations of orbits of three astrophysical objects using Velocity Verlet algorithm are presented, along with comparison of energy conservation with other algorithms like fourth order Runge-Kutta. Through choices of initial velocities and positions, orbital stability is tested for a range of P-type and S-type orbits. For S-type orbits, there seems to be a relationship between orbital stability and initial displacement between planet and the primary star.

References

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Introduction

1. Physical Motivations: Planets in binary star systems may be candidates for supporting extraterrestrial life. Studying the conditions for orbital stability will give more insights to habitable zones. Assuming the system as a classic Newtonian, isolated, three-body problem, no general analytical solution exists, also in the special case where the mass of the planet is negligible for the restricted three body problem, so we are motivated to study the numerical simulations of the planet in binary star system.

2. Equations of Motion: Let $G = 1$, $\ddot{\mathbf{r}}_i = \sum_{j\neq i} -m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{r_{ij}^3}$.

3. Velocities of planet for Circular Orbits:

$$v_c = \sqrt{\frac{M}{r}}.$$

   For inner orbits, the velocity v_c is computed taking $M = M_A$, since the primary star is the only body enclosed by the orbit of the planet, and $r = R_{AP}$. For outer orbit, the velocity v_c is computed taking $M = M_A + M_B$, since now the planet's orbit encloses both stars and use r with distance of planet to c.o.m of two stars.

4. Kepler's Two Body Problem: analytical ellipse orbit solution and use the Vis-viva equation, the relative speed at apoapsis

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \quad \text{and} \quad v = \sqrt{\frac{m_1+m_2}{|\mathbf{x}_2 - \mathbf{x}_1|}(1-e)}$$

   and velocities of stars are

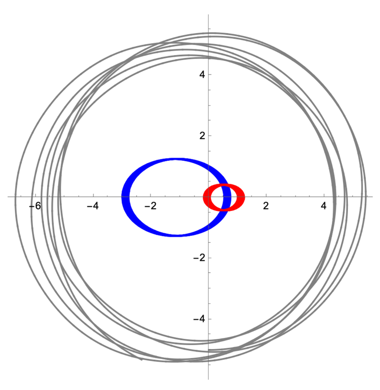
$$v_1 = \frac{m_2}{m_1+m_2}\mathbf{v},\ v_2 = -\frac{m_1}{m_1+m_2}\mathbf{v}.$$

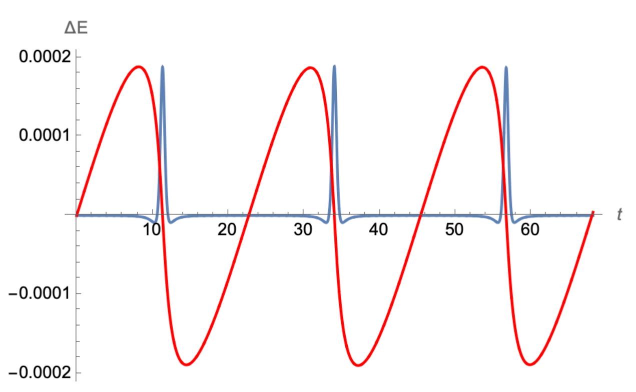
5. Criterion of Stability: Stable (S) $\langle\Delta r\rangle \leq 5\%$, marginally stable (MS) $5\% < \langle\Delta r\rangle < 35\%$, and unstable (U) $\langle\Delta r\rangle \geq 35\%$, where $\langle\Delta r\rangle$ refers to the orbital variability with respect to the initial distance between the primary star and the giant planet

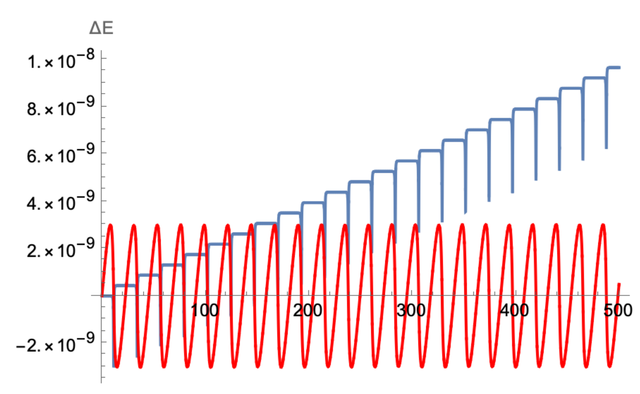
$$\langle\Delta r\rangle = \frac{\langle r(t) - r_i\rangle}{r_i}$$

Method & Results

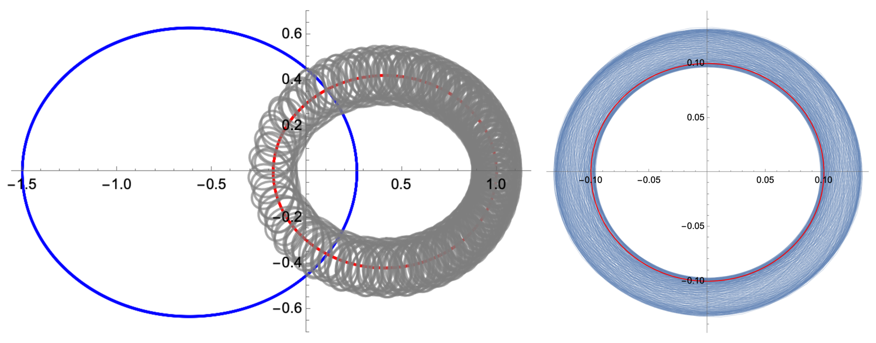
1. Integration methods: Runge-Kutta methods have smaller local error of order $\mathcal{O}(\Delta t^5)$ than the Velocity Verlet method $\mathcal{O}(\Delta t^2)$, but it is also more computationally expensive. On the other hand, the Verlet method is a sympletic type, so it conserves energy.

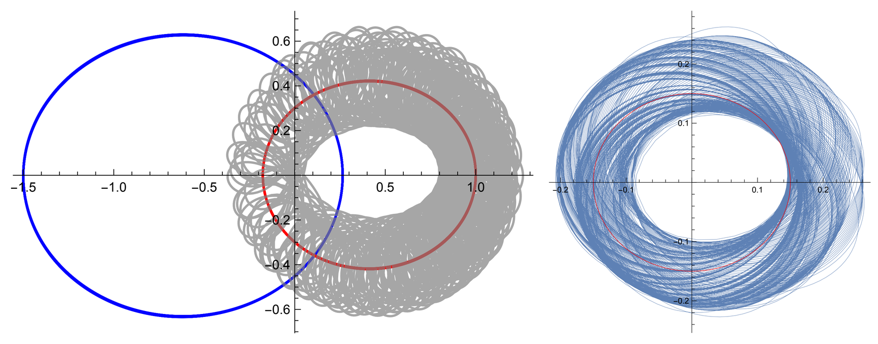




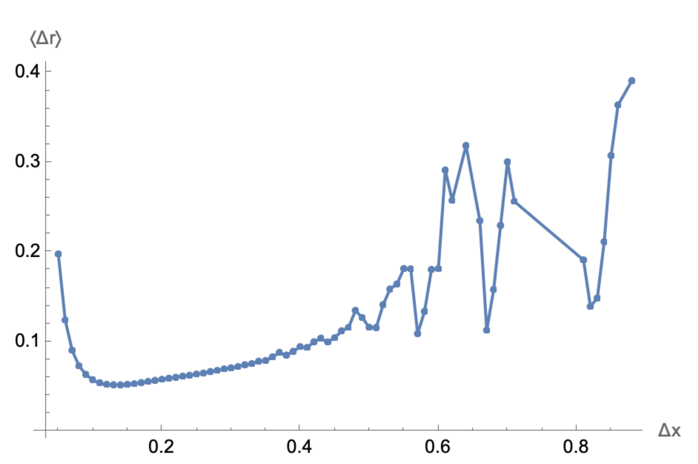


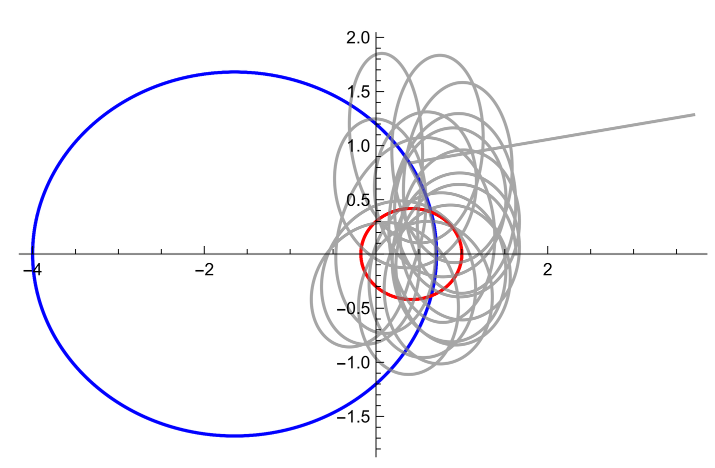
2. Stability of orbits: The stability of an orbit could be visualize by the "thickness" of the orbits. For S-type orbit, using the co-rotating system of the primary star is more intuitive.





3. Stability v.s. initial position of planet: Fix primary star mass $M_A = 0.8$, secondary star mass $M_B = 0.2$, and planet $M_P = 10^{-6}$, eccentrity of binary star orbits $e = 0.7$, and initial position of primary star $x_A = \{1, 0\}$, change the initial distance of planet to the primary star.





4. Limitations: simulation time might not be long enough, choice of step size could lead to significant errors in the calculation of stability, random choice of parameters of the system.

Conclusion

1. The periodicity of error of energy is related to the periodicity of the orbits of the binary stars.

2. For S-type orbit, a clear relationship between stability and initial position is presented. There seems to be a local minimum of orbital stability of the planet as we changes its initial position. Generally, as the distance between planet and its primary star increases, the orbits of planet become less stable and are more likely to be ejected away from the system.

3. For P-type orbit, the stability and initial position should also be associated. As the distance of the planet from the binary increases, the orbit should be more stable.

4. Some future directions: study the relationship between orbital stability with other parameters of the system, increase time length, introduce more bodies to the system, and use data from existing exoplanet missions to identify and study planets in the real physical world.

The Fun-Zone

For a taste of the blend of this physical problem with philosophical speculation, politics and history, conspiracy theory, and alien epistemology crisis, please refer to the science fiction *Three Body Problem*.