GTDMO - 1st assignment
Group 2

Bertoletti, Luca
Degl'innocenti, Giorgio
Funicello, Alfredo
Heinrich Bruckner, Xaver Matthias
Ly Nguyen, Thi Phuong
Luraschi, Matias Santiago

A farm is producing two different types of beans, type A and type B. For each Kg of production of beans A, the farm is spending 2€ workers salary, 1€ in gasoline, and 3€ in fertilizers. For each Kg of production of beans B, the farm is spending 5€ in workers salary, 2€ gasoline, and 1€ fertilizers.

In one season of production of beans A and B, in total the company would like to spend exactly 310€ in workers salary, 140€ in gasoline and 270€ in fertilizers. Is it possible? And if yes how many Kg of beans A and beans B should they produce for each season of production?

Formulate the problem in mathematical terms and explain how do you find the solution.

Solution

We first stablished the problem knowing that the price vector for workers salary, gasoline and fertilizers is given in the following way for each type.

$$Beans\ A = \begin{matrix} Workers\ Salary & €2 \\ Gasoline & €1 \\ Fertilizer & €3 \end{matrix}$$

$$Beans\ B = \begin{matrix} Workers\ Salary & €5 \\ Gasoline & €2 \\ Fertilizer & €1 \end{matrix}$$

Knowing this, we stablished the problem as a system of equations in the following way. Where X_A and X_B correspond to the production of beans type A and beans type B respectively.
We took into account the fact that the farm has a budget constrain for each type of input. Then, by creating the equation system stated below we could find out how much to produce of each type of bean.

In this sense the system we stablished is the following:

$$\begin{aligned} 2X_A + 5X_B &= 310 \\ 1X_A + 2X_B &= 140 \\ 3X_A + 1X_B &= 270 \end{aligned}$$

Moreover, after stating this system of equations we generated the augmented matrix, which is made by adding the right column into the coefficient matrix. Afterwards we did fundamental operations to solve it. The steps are shown below.

Steps:

Using the components we made the augmented matrix and we started to work on it in order to get the reduced echelon form.

The initial matrix is

$$\begin{bmatrix} 2 & 5 & 310 \\ 1 & 2 & 140 \\ 3 & 1 & 270 \end{bmatrix}$$

We then exchanged rows 1 for 3

$$\begin{bmatrix} 3 & 1 & 270 \\ 1 & 2 & 140 \\ 2 & 5 & 310 \end{bmatrix}$$

Then we multiplied the row 1 by 1/3

$$\begin{bmatrix} 1 & 1/3 & 90 \\ 1 & 2 & 140 \\ 2 & 5 & 310 \end{bmatrix}$$

Afterwards we subtracted row 1 from row 2

$$\begin{bmatrix} 1 & 1/3 & 90 \\ 0 & 5/3 & 50 \\ 2 & 5 & 310 \end{bmatrix}$$

Then we multiplied row 1 times 2 and subtracted it from row 3

$$\begin{bmatrix} 1 & 1/3 & 90 \\ 0 & 5/3 & 50 \\ 0 & 13/3 & 130 \end{bmatrix}$$

Being done with the first column, we then proceeded with the second column in order to get the echelon form. In that sense, we exchanged row 2 and 3

$$\begin{bmatrix} 1 & 1/3 & 90 \\ 0 & 13/3 & 130 \\ 0 & 5/3 & 50 \end{bmatrix}$$

Afterwards we multiplied row 2 by 3/13 to get a pivot element.

$$\begin{bmatrix} 1 & 1/3 & 90 \\ 0 & 1 & 30 \\ 0 & 5/3 & 50 \end{bmatrix}$$

Moreover, we multiplied row 2 times 1/3 and then subtracted it from row 1

$$\begin{bmatrix} 1 & 0 & 80 \\ 0 & 1 & 30 \\ 0 & 5/3 & 50 \end{bmatrix}$$

Then we did the same by multiplying row 2 times 5/3 and subtracting it from row 3

$$\begin{bmatrix} 1 & 0 & 80 \\ 0 & 1 & 30 \\ 0 & 0 & 0 \end{bmatrix}$$

This last matrix gives us the solution, which is $X_A = 80$ and $X_B = 30$. In this sense, they should produce 80kgs of beans type A and 30kgs of beans type B. This is the only solution for the system since there are no free variables.

Which problem should you solve instead to know how many Kg of beans A and B should they produce, if the farm wants to spend in total 720€ per season, whatever the splitting into costs for workers salary, gasoline and fertilizers? Do you think that the problem has a unique solution in this case?

To solve problem number 2 we figured what was the cost provided by each production function. In this sense we summed up the cost of the inputs for producing each type of bean. We arrived to the following price vector.

$$Cost\ Beans\ Type\ A = 6€$$
$$Cost\ Beans\ Type\ B = 8€$$

Knowing that the needed inputs for the production function remain constant as well as the prices, we then just stated a linear equation with the following form:

$$6\ Q_A + 8\ Q_B = 720$$

Where Q_A and Q_B correspond to the amounts of beans type A and B respectively.

We can notice right away that we have two unknowns but only one equation. Therefore, the system has infinite solutions. Moreover, there would be infinite combinations of the quantities for beans type A and B that could be produced (assuming that $Q_A \wedge Q_B \in \mathbb{R}$).

Appendix. R Code Reduced Echelon Form

```
> A <- matrix(c(2, 5,
+             1, 2,
+             3, 1), 3, 2, byrow=TRUE)
> b <- c(310, 140, 270)
>
> echelon(A, b, reduced=TRUE, verbose=TRUE, fractions=TRUE) # row-echelon form
```