4.3分部积分法

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§3. 分部积分法

$$(uv)' = u'v + uv'$$

$$uv' = (uv)' - u'v$$

定理：$$\int uv'\,dx = \int \left((uv)' - u'v\right)dx$$

$$\int u\,dv = uv - \int v\,du$$

例1. $$\int \underset{u}{x}\underset{v'}{e^x}\,dx$$

$$= \int \underset{u}{x}\,d\underset{v}{e^x} = xe^x - \int e^x\,dx = xe^x - e^x + C$$

$$\int \underset{v'}{x}\underset{u}{e^x}\,dx$$

$$= \int \underset{u}{e^x}\,d\underset{v}{\left(\tfrac{1}{2}x^2\right)} = \tfrac{1}{2}x^2e^x - \int \tfrac{1}{2}x^2 e^x\,dx$$

例 $$\int x\cos x\,dx$$

$$= \int \underset{u}{x}\,d\sin x$$

$$= \int \underset{u}{\cos x}\,d\left(\tfrac{1}{2}x^2\right)$$

注：u 的优先顺序：

反三角函数 → 对数 → 幂函数 → 指数 → 三角

例3 $\int x^2 e^x \, dx$

$= \int x^2 \, d e^x$

$= x^2 e^x - \int 2x e^x \, dx = x^2 e^x - 2 \int x \, de^x$

$= x^2 e^x - 2 \left[x e^x - \int e^x \, dx \right] = (x^2 - 2x + 2) e^x + C$

4. $\int x \arctan x \, dx$

$= \frac{1}{2} \int \arctan x \, dx^2$

$= \frac{1}{2} \left(x^2 \arctan x - \int \frac{x^2+1-1}{1+x^2} dx \right)$

$= \frac{1}{2} x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx$

$= \frac{1}{2} x^2 \arctan x - \frac{1}{2} x + \frac{1}{2} \arctan x + C$

5 $\int \frac{\arcsin x}{u} \frac{dx}{v}$

$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \arcsin x + \int \frac{d(1-x^2)}{2\sqrt{1-x^2}} = x \arcsin x + \sqrt{1-x^2} + C$

6. $I = \int e^x \cos x \, dx$

$= \int e^x \, d \sin x$

$= e^x \sin x - \int e^x \sin x \, dx$

$= e^x \sin x + \int e^x \, d \cos x$

$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$$= e^x \sin x + e^x \cos x - \int \cos x\, e^x \, dx$$

$$I = e^x (\sin x + \cos x) - I$$

$$\Rightarrow I = \frac{1}{2} e^x (\sin x + \cos x) + C$$