MiT Single Variable Calculus, Fall 2010 Pending Questions

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https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/

Questions that I had in my mind while doing this course:

1 Differentiation

1.1 Session 1, 2

- 1. How do we cut a cone using a plane so that it gives us the hyperbola 1/x (because this particular curve has a weird skewed shape. Pretty hard to imagine the resulting cross-section unless the cones are themselves skewed somehow.)
- 2. The "difference quotient" formula for differentiation has delta x or dx in the denominator where $dx \to 0$. We can't just put dx to 0 "INITIALLY" because that will give us 0/0. But after simplifying the formula (and some cancellations), as a very last step we plug 0 to dx and everything works out magically! So What is happening here? Is the simplification process changing anything? (even though it shouldn't)

1.2 Session 3

- 1. The geometric interpretation of derivative is that its the "slope of a line". Are there any other interpretations? Yes:
 - (a) Rate of change (average change)
 - (b) ...

1.3 Session 5

- 1. Stupid question: Why do we study limits in "calculus"? Shouldn't this be a part of say functions or real analysis or something?
- 2. Is the "limit" a "linear operator"? i.e:

$$\lim_{x \to \infty} (f(x) + g(x)) = \lim_{x \to \infty} f(x) + \lim_{x \to \infty} g(x)$$

- 3. My Comment: Most of the proofs we do in math appear to be "trial and error". We keep on discarding things that do not work and accept things that do work (at least appear to). But once we reach at the end, the result may give an insight to something totally new, something that we were not even looking for initially, a novel insight into the reality (its like traveling to the unknown place through a worm hole or suddenly being illuminated by the divine).
- 4. I didn't quite understand what he means by "one sided limit" in the MiT solutions file. In some places (Q4 and Q5) he has used "no need to use one sided limit here because..", not too sure what he is saying. I thought one sided limit means the special case when either $\lim_{x\to k^+}$ exists or $\lim_{x\to k^-}$ but not both. Am I missing something here?
- 5. Show that \sqrt{x} is a continuous function specially at x = 0 (beware: it has one sided limit).
- 6. Show that $|\sin x|$ is a continuous function specially at x=0