Session 9

Fakhir

November 15, 2017

Solutions

1. (a) For continuity $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 6$.

   (b) First derivative of the first piece must be equal to the first derivative of the second piece at 0. This means:

   $$f'(0^-) = f'(0^+)$$

   $$2ax + b = 10x^4 + 12x^3 + 8x + 5$$

   Putting $x = 0$ we get:

   $$b = 5$$

   (c) *(According to MiT solutions, no need to check this)* Second derivative of the first piece must be equal to the second derivative of the second piece at 0. This means:

   $$f''(0^-) = f''(0^+)$$

   $$2a = 40x^3 + 36x^2 + 8$$

   Putting $x = 0$ we get:

   $$a = 4$$

   MiT Solution says: The first derivative has to be equal on both sides. We do not need to check the second derivative. So $b = 5$ and a can be any real number.

2. (a) For continuity $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$. So plugging in $x = 1$ in both pieces we get:

   $$a + b + 6 = 20$$

   $$a + b = 14$$

(b) First derivative of the first piece must be equal to the first derivative of the second piece at 1. This means:

$$f'(1^-) = f'(1^+)$$

$$2a + b = 10 + 12 + 8 + 5$$

$$2a + b = 35$$

Solving these two simultaneous equations we get:

$$2a + (14 - a) = 35$$

$$a + 14 = 35$$

$$a = 21$$

$$b = -7$$

Hence $a = 21$ and $b = -7$.