Session 5

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Solutions

1. Plot $\sqrt{x}$. Observing it gives us a one sided limit i.e limit from the right. It is a continuous function (how?) but it is not defined for $x < 0$:

$$\lim_{x \to 0^+} \sqrt{x} = 0$$

$$\lim_{x \to 0^-} \sqrt{x} = invalid$$

2. Visualizing the graph gives us two sided limit. This function is not defined at $x = -1$:

$$\lim_{x \to -1^+} \frac{1}{x+1} = +\infty$$

$$\lim_{x \to -1^-} \frac{1}{x+1} = -\infty$$

3. First by visualizing the graph (and then bit cheating by plotting it in maxima) gives us two sided limit. This function is not defined at $x = 1$:

$$\lim_{x \to 1^+} \frac{1}{(x-1)^4} = +\infty$$

$$\lim_{x \to 1^-} \frac{1}{(x-1)^4} = +\infty$$

Since in this case $\lim_{x \to 1^-} \frac{1}{(x-1)^4} = \lim_{x \to 1^+} \frac{1}{(x-1)^4}$

We can simply say:

$$\lim_{x \to 1} \frac{1}{(x-1)^4} = +\infty$$

4. First by visualizing the graph (couldn't draw on maxima) gives us two sided limit. This function appears to be continuous at all points:

$$\lim_{x \to 0^+} |\sin x| = 0$$

$$\lim_{x \to 0^-} |\sin x| = 0$$

   Again we can say:

$$\lim_{x \to 0} |\sin x| = 0$$

5. Couldn't plot it in maxima. In my opinion it is a piece wise graph given by:

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \\ undefined, & \text{if } x = 0 \end{cases}$$

   So this implies that it has a two sided limit as follows:

$$\lim_{x \to 0^+} \frac{|x|}{x} = +1$$

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

Extra questions that need to be answered:

1. I didn't quite understand what he means by "one sided limit" in the MiT solutions file. In some places (Q4 and Q5) he has used "no need to use one sided limit here because..", not too sure what he is saying. I thought one sided limit means the special case when either $\lim_{x \to k^+}$ exists or $\lim_{x \to k^-}$ but not both. Am I missing something here?

2. Show that $\sqrt{x}$ is a continuous function specially at $x = 0$ (beware: it has one sided limit).

3. Show that $|\sin x|$ is a continuous function specially at $x = 0$