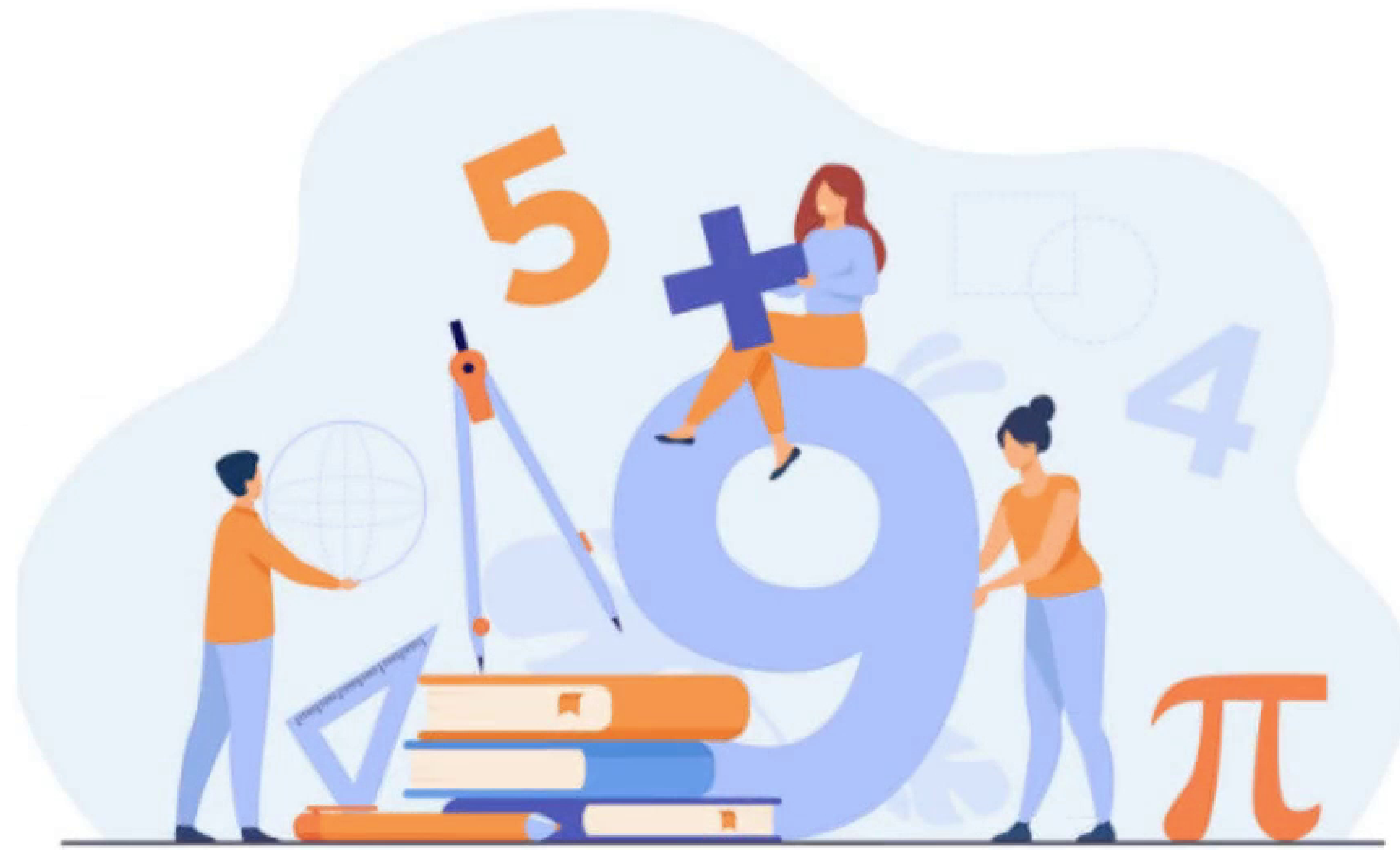
Greatest common divisor
Euclidean Algorithm

Arabic Animated Intuition



Greatest Common Divisor

The greatest common divisor (GCD), also called the greatest common factor, of two numbers is the largest number that divides them both.

find the gcd
for the given two
numbers

gcd(20,15) =5

gcd(5,5)=5

gcd(6,3)=3

gcd(7,5)=1

gcd(5,0)=5

a simple for loop for i=1 to min(a,b) will do the job
time complexity o(min(a,b))

Greatest Common Divisor

can we do better ?

let's notice

30=2*3*5

45=3*3*5

gcd(30,45)=3*5=15

so we can do this using the prime factorization

time complexity sqrt(max(a,b))

Euclidean Algorithm

let's find the gcd(20,15)

we will subtract the smaller number from the larger number

i encourge you to code this

20 15

5 15

5 10

5 5

0 5

so the gcd is 5

Euclidean Algorithm

why subtracting the smaller number from the larger number works ?

intuitive way to understanding it:

30=2*3*5

45=3*3*5

gcd(30,45)=3*5=15

this works because after the subtraction all what's left is always the wanted prime factors only

Euclidean Algorithm

from the modular arithmetic

video : A=B*Q+m

m=A-B*Q

so we can get that our subtraction was just getting the mod

so we can say that

gcd(A,B)=gcd(B,A%B)

time complexity

is o(log(max(a,b)))

we will stop when B=0

we can easily code that using recursion

but as we didn't discuss recursion yet

we will code it by a for loop + the recursion takes more memory