機率與統計期中考

[1]

.1 $$f_x(X) = \binom{10}{X} p^X q^{10-X}$$

f_x (X)=

3.48678440e-01, 3.87420489e-01, 1.93710245e-01, 5.73956280e-02, 1.11602610e-02, 1.48803480e-03, 1.37781000e-04, 8.74800000e-06, 3.64500000e-07, 9.00000000e-09, 1.00000000e-10 (0<=X<=10)

.2 E[X]= 1.0000000000000004

.3 STD[X]= 1.3784048752090223

.4 $$f_y(Y) = \frac{\binom{Y}{10}\binom{10-Y}{90}}{\binom{10}{100}}$$

f_y (Y)=

3.30476211e-01, 4.07995322e-01, 2.01509885e-01, 5.17937053e-02, 7.55324869e-03, 6.39804595e-04, 3.09982847e-05, 8.14404851e-07, 1.04114257e-08, 5.19921381e-11, 5.77690423e-14 (0<=Y<=10)

.5 E[Y]+Std[Y]= 2.348399724926484

.6 $$f_z(Z) = \binom{Z-1}{5-1} p^5 q^{Z-5}$$

f_z (Z)=

1.00000000e-05, 4.50000000e-05, 1.21500000e-04, 2.55150000e-04, 4.59270000e-04, 7.44017400e-04, 1.11602610e-03, 1.57837977e-03, 2.13081269e-03, 2.77005650e-03, 3.49027119e-03, 4.28351464e-03, 5.14021756e-03, 6.04964067e-03, 7.00029849e-03, 7.98034028e-03, 8.97788281e-03, 9.98129325e-03, 1.09794226e-02, 1.19617920e-02, 1.29187353e-02, 1.38415021e-02, 1.47223250e-02, 1.55544564e-02, 1.63321792e-02, 1.70507951e-02, 1.77065949e-02, 1.82968147e-02, 1.88195809e-02, 1.92738466e-02, 1.96593236e-02, 1.99764094e-02, 2.02261145e-02, 2.04099883e-02, 2.05300471e-02, 2.05887043e-02, 2.05887043e-02, 2.05330592e-02, 2.04249905e-02, 2.02678752e-02, 2.00651964e-02, 1.98204989e-02, 1.95373489e-02, 1.92192990e-02, 1.88698572e-02, 1.84924601e-02, 1.80904501e-02, 1.76670566e-02, 1.72253802e-02, 1.67683803e-02, 1.62988656e-02, 1.58194872e-02, 1.53327338e-02, 1.48409291e-02, 1.43462315e-02, 1.38506344e-02, 1.33559689e-02, 1.28639069e-02, 1.23759656e-02, 1.18935127e-02,

1.14177722e-02, 1.09498307e-02, 1.04906442e-02, 1.00410452e-02,
9.60174947e-03, 9.17336372e-03, 8.75639264e-03, 8.35124612e-03,
7.95824630e-03, 7.57763452e-03, 7.20957799e-03, 6.85417626e-03,
6.51146744e-03, 6.18143416e-03, 5.86400916e-03, 5.55908069e-03,
5.26649749e-03, 4.98607360e-03, 4.71759271e-03, 4.46081235e-03,
4.21546767e-03, 3.98127502e-03, 3.75793521e-03, 3.54513646e-03,
3.34255724e-03, 3.14986864e-03, 2.96673675e-03, 2.79282459e-03,
2.62779405e-03, 2.47130744e-03, 2.32302899e-03, 2.18262614e-03,
2.04977064e-03, 1.92413953e-03, 1.80541603e-03, 1.69329019e-03,
1.58745955e-03, 1.48762962e-03, 1.39351428e-03, 1.30483610e-03

[2]

.1 $$f_w(W) = \frac{e^{-100}(100)^W}{W!}$$

f_w (W)=

3.72007598e-44, 3.72007598e-42, 1.86003799e-40, 6.20012663e-39,
1.55003166e-37, 3.10006331e-36, 5.16677219e-35, 7.38110313e-34,
9.22637891e-33, 1.02515321e-31, 1.02515321e-30, 9.31957466e-30,
7.76631221e-29, 5.97408632e-28, 4.26720451e-27, 2.84480301e-26,
1.77800188e-25, 1.04588346e-24, 5.81046366e-24, 3.05813877e-23,
1.52906938e-22, 7.28128278e-22, 3.30967399e-21, 1.43898869e-20,
5.99578622e-20, 2.39831449e-19, 9.22428649e-19, 3.41640240e-18,
1.22014372e-17, 4.20739212e-17, 1.40246404e-16, 4.52407755e-16,
1.41377423e-15, 4.28416435e-15, 1.26004834e-14, 3.60013811e-14,
1.00003836e-13, 2.70280639e-13, 7.11264839e-13, 1.82375600e-12,
4.55938999e-12, 1.11204634e-11, 2.64772938e-11, 6.15751018e-11,
1.39943413e-10, 3.10985363e-10, 6.76055137e-10, 1.43841518e-09,
2.99669830e-09, 6.11571082e-09, 1.22314216e-08, 2.39831797e-08,
4.61214994e-08, 8.70216969e-08, 1.61151291e-07, 2.93002347e-07,
5.23218476e-07, 9.17927151e-07, 1.58263302e-06, 2.68242885e-06,
4.47071474e-06, 7.32904056e-06, 1.18210332e-05, 1.87635447e-05,
2.93180386e-05, 4.51046748e-05, 6.83404163e-05, 1.02000621e-04,
1.50000914e-04, 2.17392629e-04, 3.10560898e-04, 4.37409716e-04,
6.07513494e-04, 8.32210266e-04, 1.12460847e-03, 1.49947796e-03,
1.97299731e-03, 2.56233417e-03, 3.28504381e-03, 4.15828330e-03,
5.19785413e-03, 6.41710386e-03, 7.82573641e-03, 9.42859809e-03,
1.12245215e-02, 1.32053195e-02, 1.53550226e-02, 1.76494513e-02,
2.00561946e-02, 2.25350502e-02, 2.50389446e-02, 2.75153238e-02,

2.99079606e-02, 3.21590974e-02, 3.42118058e-02, 3.60124271e-02,
3.75129449e-02, 3.86731391e-02, 3.94623868e-02, 3.98609968e-02,
3.98609968e-02, 3.94663335e-02, 3.86924838e-02, 3.75655183e-02,
3.61206906e-02, 3.44006577e-02, 3.24534507e-02, 3.03303278e-02,
2.80836368e-02, 2.57648044e-02, 2.34225495e-02, 2.11013959e-02,
1.88405321e-02, 1.66730372e-02, 1.46254713e-02, 1.27178011e-02,
1.09636216e-02, 9.37061678e-03, 7.94120066e-03, 6.67327786e-03

.2 E[Z]+Std[Z]= 110.0

.3

根據標準差的定理，再平均數正負差距兩個標準差內的數據約佔總體數據 95%
故機率應為 0.95

.4 P(W>120)= 0.028230393964806644

.5 0.028230393964806644 大致為 0.03，也就是 3%，大樂透中隨便一個獎的機率也大概是 3%。

當人中獎時，都會說是偶然運氣好，但是關於生命安全的事情，不能用一時運氣不好就這麼帶過去，我個人理想的狀況是「完全不可能發生火災」，所以對我而言，只要有那麼一點點機率，我都覺得不大好，要改進。

[3]

.1 發現“有 10 個或更多項目存在缺陷”的機率為
0.031828057306205304

.2 我不能接受，如果我今天身為消費者，購買產品時買到那跟樂透中獎一樣稀少的瑕疵品，我會覺得很嘔，何況如果今天那名消費者，有些特殊情況呢？
有一次，曾經看過一個阿嬤帶著孫子，他們坐在一旁，孫子一直在哭，我過去關心，結果原來是小孩新買的玩具壞掉了，由於我有些相關知識，我想說要幫忙修修看，結果發現是原本就有點問題，我沒有零件，所以不能修。
因為一個「有瑕疵」，讓一個阿嬤買東西送小孩的心意就這麼被糟蹋，這怎麼能接受。

[3]

$$B(p,n)=P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

$$\lambda=np$$

$$\Rightarrow p=\frac{\lambda}{n}$$

$$\lim_{n\to\infty}P(X=k)=\lim_{n\to\infty}\frac{n!}{k!(n-k)!}\left(\frac{\lambda}{n}\right)^k\left(1-\frac{\lambda}{n}\right)^{n-k}$$

λ^k

$\frac{1}{k!}$

$$\left(\frac{\lambda^k}{k!}\right)\lim_{n\to\infty}\frac{n!}{(n-k)!}\left(\frac{1}{n^k}\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-k}$$

$$\lim_{n\to\infty}\frac{n!}{(n-k)!}\left(\frac{1}{n^k}\right)$$

$$\lim_{n\to\infty}\frac{n(n-1)(n-2)...(n-k)(n-k-1)...(1)}{(n-k)(n-k-1)...(1)}\left(\frac{1}{n^k}\right)$$

$$\lim_{n\to\infty}\frac{n(n-1)(n-2)...(n-k+1)}{n^k}$$

$$\lim_{n\to\infty}\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)...\left(\frac{n-k+1}{n}\right)$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n$$

$$e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$x=-\tfrac{n}{\lambda}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{1}{x}\right)^{x(-\lambda)}=e^{-\lambda}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-k}$$

$$\left(\frac{\lambda^k}{k!}\right)\lim_{n\to\infty}\frac{n!}{(n-k)!}\left(\frac{1}{n^k}\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-k}=\left(\frac{\lambda^k}{k!}\right)(1)\left(e^{-\lambda}\right)(1)$$

$$P(\lambda,k)=\left(\frac{\lambda^k e^{-\lambda}}{k!}\right)$$