Sampling & Confidence Interval Practice Problems Solutions (1)

1. At a local manufacturing plant, bolts being produced should be no more than 25 millimeters in diameter. Historically, the bolts were supposed to be produced to follow a normal distribution with a mean of 22 mms and a standard deviation of four mms.
   a) What percent of bolts produced are more than 30 mms in diameter?
   Soln: $P(X > 25) = P\left(Z > \frac{25-22}{4}\right) = P(Z > 0.75) = 1 - 0.7734 = 0.2266.$

   b) For quality control purposes, a random sample of 36 bolts was taken and measured to assess the proportion that actually is larger than 25 mms. What is the probability of getting a *sample average* greater than 25 mms?
   Soln: $P(\bar{X} > 25) = P\left(Z > \frac{25-22}{4/\sqrt{36}}\right) = P(Z > 4.5) \approx 0.$

2. During the recent Summer Olympic Games, you found out that the length an athlete throws a hammer is normally distributed with a mean of 50 feet and a standard deviation 5 feet.
   a) What is the probability an athlete throws the hammer between 55 feet and 60 feet?
   Soln: $P(55 < X < 60) = P\left(\frac{55-50}{5} < Z < \frac{60-50}{5}\right) = P(1 < Z < 2) = 0.9772 - 0.8413 = 0.1359.$

   b) Instead of scoring the athlete based on just one throw, the average of 5 throws is used. What is the probability the *average* of 5 throws is between 50 and 60 feet?
   Soln: $P(55 < \bar{X} < 60) = P\left(\frac{55-50}{5/\sqrt{5}} < Z < \frac{60-50}{5/\sqrt{5}}\right) = P(2.24 < Z < 4.47) = 1 - 0.9875 = 0.0125.$

3. A chain of "quick lube" shops has a standard service for performing oil changes and basic checkups on automobiles. The chain has a standard that says that the average time per car for this service should be 12.5 minutes. The manager picked 48 random times and found the mean to be 13.104 minutes with a standard deviation of 2.4 minutes. What is a 95% confidence interval for the mean? Does this interval indicate that the mean for this shop differs from the 12.5 minute standard?
   Soln: $\bar{x} \pm z\frac{s}{\sqrt{n}} \rightarrow 13.104 \pm 1.96\frac{2.4}{\sqrt{48}} \rightarrow 13.104 \pm 0.679 \rightarrow [12.425,\ 13.783].$ Since the claim of 12.5 minutes falls within the confidence interval, we have no evidence that the shop's mean differs from that.

4. The amount of fill in a half-liter bottle of Diet Coke is normally distributed. From past experience, the process standard deviation is about 1.2 ml. The mean amount of fill can be adjusted. A sample of 50 bottles gives a sample mean of 503.4 ml. Create a 99% confidence interval for what you think the true population average is based on the sample results.
   Soln: $\bar{x} \pm z\frac{s}{\sqrt{n}} \rightarrow 503.4 \pm 2.575\frac{1.2}{\sqrt{50}} \rightarrow 503.4 \pm 0.437 \rightarrow [502.963,\ 503.837].$

5. A business magazine samples 90 individuals responsible for economic forecasting for regional banks. Suppose that the sample of 90 forecasts yields an average prediction of a 2.7% growth in

real disposable income with a standard deviation of 0.4%. Calculate a 99% confidence interval for the population mean forecast.

Soln: $\bar{x} \pm z\frac{s}{\sqrt{n}} \rightarrow 2.7 \pm 2.575\frac{0.4}{\sqrt{90}} \rightarrow 2.7 \pm 0.109 \rightarrow [2.591,\ 2.809].$

6. A questionnaire of spending habits was given to a random sample of college students. Each student was asked to report the amount of money they spent on textbooks in a semester. The sample of 130 students resulted in an average of $422 with standard deviation of $57. Give a 90% confidence interval for the mean amount of money spent by college students on textbooks.

   Soln: $\bar{x} \pm Z\frac{s}{\sqrt{n}} \rightarrow 422 \pm 1.645\frac{57}{\sqrt{130}} \rightarrow [\$413.78,\ \$430.22].$