Two-Sample Hypothesis Testing Practice Solutions

1. Let p_1 represent the population proportion of U.S. Senate and Congress (House of Representatives) democrats who are in favor of a new modest tax on "junk food". Let p_2 represent the population proportion of U.S. Senate and Congress (House of Representative) republicans who are in favor of a new modest tax on "junk food". Out of the 265 democratic senators and congressman 106 of them are in favor of a "junk food" tax. Out of the 285 republican senators and congressman only 57 of them are in favor a "junk food" tax.

   a) Find a 95 percent confidence interval for the difference between proportions 1 and 2.

   Soln: $\hat{p}_1 = \frac{106}{265} = 0.4,\ \ \hat{p}_2 = \frac{57}{285} = 0.2.$

   $$se_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{0.4(0.6)}{265} + \frac{0.2(0.8)}{285}} = 0.0383.$$

   $95\%CI: (0.4 - 0.2) \pm 1.96(0.0383) \to [0.125, \quad 0.275].$

   b) At α = 0.01, can we conclude that the proportion of democrats who favor "junk food" tax is more than 5% higher than proportion of republicans who favor the new tax?

   Soln: $H_0: p_1 - p_2 \leq 0.05$ vs $H_a: p_1 - p_2 > 0.05$. $Z = \frac{(0.4-0.2)-0.05}{0.0388} = 3.866.$ Since 3.866 > 2.33, we *do* have enough evidence to suggest the democratic percentage who favor the tax is more than 5% higher than for the republicans.

2. The mid-distance running coach, Zdravko Popovich, for the Olympic team of an eastern European country claims that his six-month training program significantly reduces the average time to complete a 1500-meter run. Five mid-distance runners were randomly selected before they were trained with coach Popovich's six-month training program and their completion time of 1500-meter run was recorded (in minutes). After six months of training under coach Popovich, the same five runners' 1500 meter run time was recorded again the results are given below.

Runner	1	2	3	4	5
Completion time before training	5.9	7.5	6.1	6.8	8.1
Completion time after training	5.4	7.1	6.2	6.5	7.8
Difference	0.5	0.4	–0.1	0.3	0.3

At α = 0.05, can we conclude there has been a significant decrease in the mean time per mile?

Soln: $H_0: \mu_{before} - \mu_{after} \leq 0$ vs $H_a: \mu_{before} - \mu_{after} > 0$. $\bar{d} = 0.28$ and $s_{\mathrm{d}=0.228}$.

Then $t = \frac{0.28}{0.228/\sqrt{5}} = 2.746$. Since the cutoff for *df* = 4 and α = 0.05 is 2.132, there IS enough evidence to reject the null and assume the coach is able to decrease the runners' times.

3. A fast food company uses two management-training methods. Method 1 is a traditional method of training and Method 2 is a new and innovative method. The company has just hired 31 new management trainees. 15 of the trainees are randomly selected and assigned to the first method, and the remaining 16 trainees are assigned to the second training method. After three months of training, the management trainees took a standardized test. The test was designed to evaluate their performance and learning from training and the sample mean and standard deviation of the two methods are given below. The population standard deviations are assumed to be equal. The management wants to determine if the company should implement the new training method. *Note: assume here that they will only implement the new method if the mean has *increased* (measuring productivity).

	n	Mean	Standard deviation
Method 1	15	69	3.4
Method 2	16	72	3.8

a) Write the null hypothesis and alternative hypotheses.
Soln: $H_0: \mu_1 - \mu_2 \geq 0$ vs $H_a: \mu_1 - \mu_2 < 0$.

b) What is the rejection point at α = 0.05? At α = 0.01?
Soln: *df* here is $n_1 + n_2 - 2 = 15 + 16 - 2 = 29$. Since this is a one-sided *less than* test, the α = 0.05 cutoff is -1.699 and the α = 0.01 cutoff is –2.462.

c) What is the value of the test statistic?
Soln: $se_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{11.56}{15} + \frac{14.44}{16}} = 1.294$. Then the t-statistic is $t = \frac{(\bar{x}_1-\bar{x}_2)}{se_{\bar{x}_1-\bar{x}_2}} = \frac{(69-72)}{1.294} = -2.318$.

d) Is there sufficient evidence to conclude that the new training method is more effective than the traditional method? What is the decision at α = 0.05? At α = 0.01?
Soln: Based on the cutoffs, for α = 0.05 I would reject the null and conclude there *is* sufficient evidence that the second method is better. For α = 0.01, though, I do not have enough evidence to conclude this.

4. A marketing research company surveyed grocery shoppers in the east and west coasts to see the percentage of the customers who prefer chicken to other meat. The data are given below.

	Sample size	Number of customers in the sample that prefer chicken
East Coast	492	156
West Coast	386	172

a) At α = 0.10, is the proportion of customers who prefer chicken the same for the 2 regions?
Soln: $H_0: p_{east} - p_{west} = 0$ vs $H_a: p_{east} - p_{west} \neq 0$.

$\hat{p}_{east} = \frac{156}{492} = 0.317$, $\hat{p}_{west} = \frac{172}{386} = 0.446$. $se_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{0.317(0.683)}{492} + \frac{0.446(0.554)}{386}} = 0.0329.$ $z = \frac{0.317-0.446}{0.0329} = -3.925.$ Since $|z|$ = 3.925 > 1.645, we should *reject* H_0 and conclude there IS a difference between the 2 regions.

b) Determine the 95% confidence interval for the difference between the proportion of customers who prefer chicken at the West Coast and the proportion of customers who prefer chicken at the East Coast.
Soln: $\hat{p}_{east} = \frac{156}{492} = 0.317$, $\hat{p}_{west} = \frac{172}{386} = 0.446$, and $se_{\hat{p}_1 - \hat{p}_2} = 0.0329$.
95% CI: $-0.129 \pm 1.96(0.0329) \rightarrow [-0.1935, -0.0645]$.

5. Two hospital emergency rooms use different procedures for triage of their patients. We want to test the claim that the mean waiting time of patients is the same for both hospitals. The 40 randomly selected subjects from one hospital produce a mean of 18.3 minutes. The 50 randomly selected patients from the other hospital produce a mean of 25.31 minutes. Assume a σ_a = 2.1 minutes and σ_b=2.92 minutes.
   a) Setup the null and alternative hypothesis to determine if there is a difference in the mean waiting time between the two hospitals.
   Soln: $H_0: \mu_a - \mu_b = 0$ vs $H_a: \mu_a - \mu_b \neq 0$.

   b) Calculate the test statistic for testing these hypotheses and test the hypothesis at α = 0.05.
   Soln: $se_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{4.41}{40} + \frac{8.526}{50}} = 0.5299.$ The t-statistic is $t = \frac{(18.3-25.31)}{0.5299} = -13.229$, which is *significantly* beyond the cutoff of $t_{88,0.05} = -1.987$.

   c) What do you conclude about the waiting time for patients in the two hospitals at α = 0.05?
   Soln: Based on the cutoff for α = 0.05, I would reject the null and conclude there *is* sufficient evidence of a significant difference in the mean waiting time between the two hospitals.

6. An e-commerce research company claims that 60% or more graduate students have bought merchandise on-line. A consumer group is suspicious of the claim and thinks that the proportion is lower than 60%. A random sample of 80 graduate students show that only 22 students have ever done so. Is there enough evidence to show that the true proportion is lower than 60%? Conduct the test at a significance level of 10%.

   Soln: $H_0: p = 0.60\ vs\ H_a: p < 0.60$. Our sample proportion is $\hat{p} = \frac{22}{80} = 0.275.$ For a one-sided, less-than test at 10%, the cutoff is Z = -1.28. Our sample gives $Z = \frac{0.275-0.60}{\sqrt{\frac{0.60(0.40)}{80}}} = -5.934.$ Obviously there is *plenty* of evidence to suggest the consumer group is correct and the proportion is significantly less than 60%.

7. A local high school polled a random sample of 400 students about whether to put on a school play. The music director wondered if there was a difference between boys' and girls' opinions. From the sample, 45 of the 150 boys wanted a school play and 113 of the 250 girls sampled wanted a school play. Test this at a 1% level of significance.

   Soln: $H_0: p_{boys} = p_{girls}\ vs\ H_a: p_{boys} \neq p_{girls}$ or $H_0: p_{boys} - p_{girls} = 0\ vs\ H_a: p_{boys} - p_{girls} \neq 0$. Our sample proportions are $\hat{p}_{boys} = \frac{45}{150} = 0.30$ and $\hat{p}_{girls} = \frac{113}{250} = 0.452$. For a two-sided test at 1%, the cutoffs are $Z = \pm 2.575$. Our sample gives $Z = \frac{0.30-0.452}{\sqrt{\frac{0.30(0.70)}{150}+\frac{0.452(0.548)}{250}}} = \frac{-0.152}{0.049} - 3.109.$

   Therefore, there is enough evidence to suggest there is a significant difference in the choices between the boys and girls.

8. A newspaper journalist wants to compare the proportion of readers under 35 years old that read his column to the proportion of readers 35 or older that read his column. He believes that the proportion of younger people (Y) is less than the proportion of older people (O). What is the appropriate alternative hypotheses to test his claim? The newspaper journalist takes a survey of residents in his distribution area. Out of 150 randomly selected individuals under the age of 35, 65 state that they read his column. Out of the 250 randomly selected individuals over 35, 110 state that they read his column. What is the conclusion at $\alpha = 0.05$?

   Soln: $H_0: p_y = p_o\ vs\ H_a: p_y < p_o$ or $H_0: p_y - p_o = 0\ vs\ H_a: p_y - p_o < 0$. Our sample proportions are $\hat{p}_y = \frac{65}{150} = 0.433$ and $\hat{p}_o = \frac{110}{250} = 0.44$. For a one-sided, less-than test at 5%, the cutoff is $Z = -1.645$. Our sample gives $Z = \frac{0.433-0.44}{\sqrt{\frac{0.433(0.567)}{150}+\frac{0.44(0.56)}{250}}} = -0.137$. Therefore, there is *not* enough evidence to suggest the younger readers are less likely to read the column than the older readers.

9. A research firm wants to know if there is a difference in miles per gallon between US and Japanese cars. To test this, the following MPG summary statistics were obtained from two samples, the first from US cars and the second from Japanese cars. Test the hypotheses at a 5% significance level.

US Car MPG	Japanese Car MPG
$n_1 = 249$	$n_2 = 79$
$\bar{x}_1 = 20.145$	$\bar{x}_2 = 30.481$
$s_1 = 6.415$	$s_2 = 6.108$

   Soln: $H_0: \mu_{US} = \mu_{Jap}\ vs\ H_a: \mu_{US} \neq \mu_{Jap}$ or $H_0: \mu_{US} - \mu_{Jap} = 0\ vs\ H_a: \mu_{US} - \mu_{Jap} \neq 0$. For a two-sided test at 5% with *df* = 259+79-2 = 336, the cutoffs are $t = \pm 1.96$.
   Our sample gives $t = \frac{20.145-30.481}{\sqrt{\frac{6.415^2}{249}+\frac{6.108^2}{79}}} = -12.945.$
   Therefore, there is *plenty* of enough evidence to suggest there is a significant difference in MPG between US and Japanese cars.