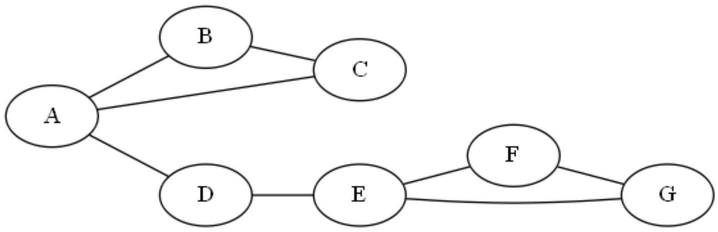
Week 8: Concrete Formulation I

Session 16: Encoding Logical Constraints using Linear Expressions

Sample Problem: Project Selection

Ebony is an ambitious master's student who would like to maximize the number of extra-curricular business analytics projects she takes part of this year. However, projects may conflict with one another. The following graph summarizes the conflicts. (For example, project A conflicts with B, C and D, but projects B and D can be done together.)



Beside the conflict above,

- Project A is a prerequisite to project F (meaning that pursuing F requires also pursuing A.)
- Project B is a prerequisite to project G.

Formulate a linear optimization problem to help her decide which projects to pursue.

In-Class Exercise

Write the English Description and Concrete Formulation for this Problem. You don't have to submit anything on Blackboard.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision Variables:

x_i : whether project i is pursued (Binary)

Objective and Constraints:

$$
\left.\begin{aligned}
x_A + x_B &\le 1\\
x_A + x_C &\le 1\\
x_A + x_B &\le 1\\
x_D + x_E &\le 1\\
x_E + x_F &\le 1\\
x_E + x_G &\le 1\\
x_F + x_G &\le 1
\end{aligned}\right\}
$$

A is prereq of F :

A	F
0	0
~~0~~	~~1~~
1	0
1	1

$$\Rightarrow \boxed{\begin{aligned} x_A &\ge x_F \\ x_B &\ge x_G \end{aligned}}$$

Sample Problem: Warehouse Planning

The following table provides the shipping cost for one-pound packages, from 7 of Amazon's fulfillment centers (FC) to 4 regions.

Region	1	2	3	4	5	6	7
A. Kings County, NY	20.25	7.70	24.59	23.26	7.69	7.70	7.69
B. Los Angeles County, CA	18.43	23.30	7.69	7.69	24.16	22.12	24.91
C. King County, WA	21.28	24.18	7.70	17.67	23.91	22.98	24.57
D. Harris County, TX	7.69	7.70	18.73	7.71	18.79	7.70	19.47

A shipping cost of $10 or less indicates that the package will be transported via ground shipping; otherwise, it will be transported via air shipping.

For a certain item that weights a pound, Amazon would like to stock it in as few FCs as possible, while guaranteeing that it can fulfill demand in all 4 regions via ground shipping. Moreover,

- the item must be stocked in at least one of FCs 5 or 7;
- the item cannot be stocked in FC 4 unless it is also stocked in FC 1;
- if the item is stocked in FC 2, then it cannot also be stocked in FC 3.

Formulate a linear optimization problem to find the minimum number of FCs needed.

In-Class Exercise: English Description

Decision:

Whether to stock item in FC_i or not

Objective:

Minimize number of FCs to stock product in

Constraints: Every region's location must be met.

Stock should be present in FC_5 and/or FC_7

Cost of transportation $\leq \$10$

If in FC_2 then not in FC_3

If in FC_4 then also in FC_1

Exercise 8.3 Concrete Formulation for Warehouse Planning

Download the Jupyer notebook attached to this exercise on Blackboard and submit it there. The notebook asks you to write the Concrete Formulation for the above problem.

Sample Concrete Formulation

Decision Variables:

x_i: stock or not in FC_i (binary)

Objective and constraints:

$$x_2 + x_5 + x_6 + x_7 \geq 1$$

$$x_3 + x_4 \geq 1$$

$$x_3 \geq 1$$

$$x_1 + x_2 + x_4 + x_6 \geq 1$$

$$x_1 \geq x_4$$

$$x_2 + x_3 \leq 1$$

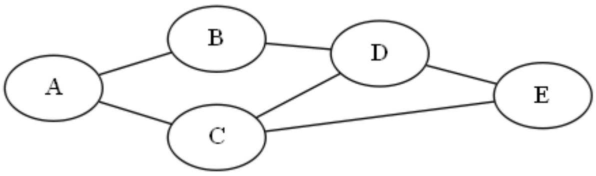
$$x_5 + x_7 \geq 1$$

$$\min \quad x_1 + x_2 + \ldots\ldots + x_7$$

Exercise 8.4: Optimal Sensor Placement

Download the Jupyer notebook attached to this exercise on Blackboard and submit it there. The notebook asks you to write the English Decription and Concrete Formulation for the following problem.

To reduce traffic congestion around USC, the university plans to install sensors at various streets in order to monitor every segment of every street around the university. To be precise, a segment is defined to be the line between two adjacent intersections. In order to monitor a segment, it suffices to have a sensor installed at one of the two ends. For example, consider the following map, which has 5 intersections (A, B, C, D, E) and 6 segment (A-B, A-C, C-D, B-D, C-E, and D-E). Installing a sensor at intersection A would monitor segments A-B and A-C, but no other segments.



The following table summarizes the cost of installing a sensor at each intersection

	A	B	C	D	E
Cost ($)	100	150	180	160	130

Moreoer, in order to install sensors at intersections B or C, the university would have to apply for a special permit from the city, which costs $50, and this cost is in addition to the installation costs above. Once the university has this permit, then it can install sensors at both B and C under the same permit.

Formulate a linear optimiztaion problem to decide which intersections to install sensors in order to minimize cost, subject to monitoring every segment.

English Description

Decision:

Determine which intersections to install sensors on.

Objective:

Minimize installation cost of sensors while covering every section. $100x_A + 150x_B + 180x_C + 160x_D + 130x_E + 50x_p$

Constraints:

$x_A + x_B \geq 1$

$x_A + x_C \geq 1$

$x_C + x_D \geq 1$

$x_D + x_E \geq 1$

$x_C + x_E \geq 1$

$x_B + x_D \geq 1$

$x_B \leq x_p$

$x_C \leq x_p$

$\Rightarrow x_B + x_C \leq 2x_p$

Concrete Formulation

Decision Variables:

Objective:

Constraints: