

GSBA 545 Formula Sheet 2021

Descriptive Statistics

Population: $\mu = \frac{1}{N} \sum X_i$ $\sigma^2 = \frac{1}{N} \sum (X_i - \mu)^2$ $\sigma = \sqrt{\sigma^2}$

Sample: $\bar{x} = \frac{1}{N} \sum X_i$ $s^2 = \frac{1}{n-1} \sum (X_i - \bar{x})^2$ $s = \sqrt{s^2}$

Coefficient of Variation: $c_v = \frac{\sigma}{\mu}$ or $\frac{s}{\bar{x}}$

Discrete Distributions

$\mu = E(X) = \sum X \cdot P(X)$

$\sigma^2 = Var(X) = \sum (X - \mu)^2 \cdot P(X)$

$\sigma = SD(X) = \sqrt{Var(X)}$

Normal Distribution/Sampling

If X is normal with mean μ and std dev σ , then $Z = \frac{X - \mu}{\sigma}$.

If $n \geq 30$, then $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

Other Distributions

	Mean	Variance
Uniform:	$\mu = \frac{a+b}{2}$	$\sigma^2 = \frac{(b-a)^2}{12}$
Exponential:	$\mu = \frac{1}{\lambda}$	$\sigma^2 = \frac{1}{\lambda^2}$
Poisson:	$\mu = \lambda$	$\sigma^2 = \lambda$

Hypothesis Testing

$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ or $Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$

$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$ with $df = n - 1$

	Z-values	
α	One-sided	Two-sided
0.10	1.28	1.645
0.05	1.645	1.96
0.01	2.33	2.575

Confidence Intervals/Sample Size Determination

Means: $n \geq 30$: $\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $\bar{X} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$

$n < 30$: $\bar{X} \pm t_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ or $\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$, $df = n - 1$

$n = \left(\frac{Z_{\alpha/2} \times \sigma}{E} \right)^2$

Proportions: $\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$n = p(1-p) \left(\frac{Z_{\alpha/2}}{E} \right)^2$

Confidence Interval

Z-values:

90% $Z = 1.645$

95% $Z = 1.96$

99% $Z = 2.575$

Two-Sample Hypothesis Testing

Independent: $df = n_1 + n_2 - 2$

$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{se(\bar{x}_1 - \bar{x}_2)}$, where

$se(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Proportions:

$Z = \frac{\hat{p}_1 - \hat{p}_2}{se(\hat{p}_1 - \hat{p}_2)}$, where

$se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Paired: $df = n - 1$, $t = \frac{\bar{d} - D_0}{s_d/\sqrt{n}}$

Linear Regression

$\hat{Y} = b_0 + b_1X$ Error: $e_i = Y_i - \hat{Y}_i$

PI (estimate): $\hat{Y} \pm t_{0.025, df} RMSE$

CI (estimate): $\hat{Y} \pm t_{0.025, df} \frac{RMSE}{\sqrt{n}}$

Confidence Interval (β_j): $b_j \pm t_{0.025, dfError} S_{b_j}$