Uniform & Normal Distribution Practice Problems Solutions

1. You arrive at a building and are about to take an elevator to your floor. Once you call the elevator, it will take between 0 and 40 seconds to arrive to you. We will assume that the elevator arrives uniformly between 0 and 40 seconds after you press the button. In this case a = 0 and b = 40.
   a) Calculate the probability that the elevator takes less than 15 seconds to arrive.
      Soln: The correct probability is P(X < 15) = $P(X < 15) = \frac{15-0}{40-0} = 0.375.$
   b) What is the *expected* amount of time you will need to wait for the elevator?
      Soln: The expected value of a uniform distribution is: $E(X) = \frac{a+b}{2} = \frac{0+40}{2} = 20 \text{ secs.}$
   c) What is the standard deviation of your waiting time?
      Soln: The variance of a uniform distribution is: $Var(X) = \frac{(b-a)^2}{12} = \frac{(40-0)^2}{12} = 133.33 \text{ secs.}$
      Therefore, the standard deviation will be: $sd = \sqrt{133.33} = 11.55 \text{ secs.}$

2. Suppose the time it takes a nine-year old to eat a donut is uniformly distributed between 0.5 and 4 minutes. Let *X* = the time, in minutes, it takes a nine-year old child to eat a donut. Then X~ U (0.5, 4).
   a) What is the probability it takes a randomly selected 9-year old child at least 2 mins to eat a donut?
      Soln: $P(X > 2) = \frac{(4-2)}{4-0.5} = 0.5714.$
   b) What is the probability a different 9-year old eats a donut in more than 2 mins, given the child has already been eating the donut for more than 1.5 mins?
      Soln: This requires some *conditional probability.* Since we already know it will take at least 1.5 mins to finish the donut, the space is now defined as X~U(1.5, 4). Now $P(X > 2) = \frac{(4-2)}{4-1.5} = 0.8.$

3. The average amount of salt consumption per day by an American is 15 grams (or 15,000 milligrams), although the actual physiological minimum daily requirement for salt is only 220 mgs. Suppose the amount of salt intake per day is approximately normally distributed with a std deviation of 5 grams.
   a) What proportion of all Americans consume between 14 and 22 grams of salt per day?
      Soln: P(14 < *X* < 22) = P(-0.2 < *Z* < 1.4) = 0.9192 – 0.4207 = 0.4985

   b) Physicians recommend that those Americans who want to reach a level of salt intake at which hypertension is less likely to occur should consume less than 1 gram of salt per day. What is the probability that a randomly selected American consumes less than 1 gram of salt per day?
      Soln: P(*X* < 1) = P(*Z* < -2.8) = 0.0026

4. The annual rate of return on an equity fund is normally distributed with mean 12% and standard deviation 6%. The annual rate of return on an income fund is normally distributed with mean 10% and standard deviation 2%.
   a) What is the probability that the rate of return of the equity fund will exceed 15%?
      Soln: P(Equity > 15) = P(*Z* > 0.5) = 1.0 – 0.6915 = 0.3085

   b) What is the probability that the equity fund will lose money (the annual rate of return will be less than 0)?
      Soln: P(Equity < 0) = P(*Z* < -2) = 0.0228

c) Find the 90th percentile of the equity fund's rate of return and interpret.
Soln: I'm looking for the *z*-value that gives the 90th percentile. This happens at 1.28, so the answer is:
mean + 1.28 SD = 12 + 1.28 (6) = 19.68

This can be interpreted as: There is a 90% chance that the equity fund will have a return of 19.68 or below. Also, 90% of the time the fund will have a return of 19.68 or below.

d) What is the probability that the rate of return on the income fund will be more than 8%?
Soln: P(Income > 8) = P(Z > -1) = 1 – 0.1587 = 0.8413

e) What is the probability that the rate of return on the equity fund will be more than 8%?
Soln: P(Equity > 8) = P(Z > -0.67) = 1 – 0.2514 = 0.7486

f) Explain why the answer to (*d*) is greater than the answer to (*e*), even though the average rate of return of the income fund is less than the average rate of return on the equity fund.
Soln: Since the standard deviation is smaller for the Income fund, we will cover more of the distribution in a smaller range than for the Equity fund.

5. A certain amount of material is wasted in cutting patterns for garments. A producer of army uniforms has found that the lot-to-lot waste is normally distributed with mean 4.1% and std deviation 0.6%.
   a) In a particular lo*t, what is the probability that the wastage exceeds 5%?*
   Soln: P(X > 5) = *P(*Z > 1.5) = 1 – 0.9332 = 0.0668

   b) If the actual amount of material required for a lot is 4700 yards, and 5000 yards of material are available, what is the probability that the supply of material is adequate?
   Soln: With 5000 yds to start, we could waste up to 300 yds to meet the 4700 yd requirement. This is equivalent to wasting 6%. Therefore, we want: P(X < 6) = P(Z < 3.17), which is almost 1.00. So we are almost *certain* to have enough material.

6. The time required to verify and fill a common prescription at a neighborhood pharmacy is normally distributed with a mean of 10 minutes and a standard deviation of 3 minutes.
   a) What proportion of the time should a customer expect to wait at least 15 minutes?
   Soln: P(X > 15) = P(Z > 1.67) = 1 – 0.9525 = 0.0475, or 4.75% of the time.

   b) What is the 99th percentile of wait times?
   Soln: I'm looking for the point on the *z*-table that gives an area of 0.99 (so look INSIDE the table first). This is closest to z = 2.33, so I'll use that approximation. Therefore, the wait time that is 2.33 standard deviations above the mean will mark the 99th percentile: 10 + (2.33)(3) = 16.99 minutes.

7. The population mean heart rate is about 74.0 bpm with a standard deviation of 7.5 bpm and is assumed to be a normally distributed random variable.
   a) What is the *Z*-score of a person with a heart rate of 80 bpm?

Soln: $Z = \frac{80-74}{7.5} = \mathbf{0.8}$. So that person's heart rate is 0.8 standard deviations above average.

b) What is the *percent* of people with a heart rate of 80 bpm or more?
Soln: $P(X \geq 80) = P(Z \geq 0.8) = 1 - 0.7881 = \mathbf{0.2119}$. About 21% of people have a heart rate of at least 80 bpm.

c) What is a *reasonable range* of heart rates (assume this to be 95%)?
Soln: An approximate 95% range should fall between the mean and 2 standard deviations.

$$\mu \pm 2\sigma \rightarrow 74.0 \pm (2)(7.5) \rightarrow [59.0, 89.0]$$