Week 8: Concrete Formulation I

Session 15: Appropriately Phrasing the English Description

Recap of the Four Steps of Optimization Modeling

1. English description: write a succinct verbal description of the decision, objective and constraints.
2. Concrete formulation: translate the above into a linear optimization formulation, illustrating with made-up numbers from a toy example.
3. Abstract formulation
4. Reusable software

Example from Week 7: Assortment Planning

Amazon.com is expanding its business by launching a physical store in West LA. As the manager, you need to select which bestsellers to carry at the store's grand opening. The following table provides the list of Top 10 Bestsellers in Literature & Fiction, along with their genres. Note that some bestsellers belong to more than one genre.

Rank \ Genre	Literary	Sci-Fi	Romance	Thriller
1	√			
2		√		√
3			√	√
4	√		√	
5	√			
6			√	
7		√		
8				√
9	√	√		
10			√	

You wish to carry the minimum number of bestsellers, while ensuring that there are at least two bestsellers in each genre. Formulate this as a linear optimization problem.

Sample English Description

Decision: Which bestsellers to carry.
Objective: Minimize the total number of bestsellers carried.
Constraints: For each of the four genres, we need to carry at least two books of that genre.

Sample Concrete Formulation

Decision variables: Let x_i denote whether to carry book i. (Binary)
Objective:

$$\text{Minimize:} \quad x_1 + x_2 + \cdots + x_{10}$$

Constraints:

$$\begin{aligned} \text{(Literary)} \quad & x_1 + x_4 + x_5 + x_9 \geq 2 \\ \text{(Sci-Fi)} \quad & x_2 + x_7 + x_9 \geq 2 \\ \text{(Romance)} \quad & x_3 + x_4 + x_6 + x_{10} \geq 2 \\ \text{(Thriller)} \quad & x_2 + x_3 + x_8 \geq 2 \end{aligned}$$

Note on Latex: to type x_{10}, you need `$x_{10}$`, otherwise it will look like x_10.

Sample Problem: Project Sub-Contracting

Tom Burke, owner of Burke Construction, has promised to complete five projects this winter. Burke Construction has 10 workers that will work 40 hours a week for 12 weeks this winter. Since this is a limited workforce, Tom knows that he will not be able to complete all of his construction projects without subcontracting some of them. In the table below, he has estimated the amount of labor hours required by each project, and the profit to his company. (All profits are in thousands of dollars.)

Project	1	2	3	4	5
Labor hours required	1300	950	1000	1400	1600
Profit (if done by own company)	30	10	26	18	20
Profit (if subcontracted)	6	2	8	9	4

To maximize profit, which jobs should Tom schedule for his company, and which should be subcontracted? (Assume that projects cannot be partially subcontracted; that is, a project will be completed entirely by either Burke Construction or the subcontractor.)

In-class Exercise: English Description

Describe in English the decision, objective and constraints in the above problem. Try your best to use **helpful keywords** and ensure that the descriptions are **precise**, **complete** and **succinct**.

Decision:

Which jobs should be scheduled for own company & which one should be subcontracted?

Objective:

Maximize profit. $24x_1 + 8x_2 + 18x_3 + 9x_4 + 16x_5$

Constraints:

Total hours available = 4800 hours.

$1300x_1 + 950x_2 + 1000x_3 + 1400x_4 + 1600x_5 \leq 4800$

$x_1, x_2, x_3, x_4, x_5 = \{0, 1\}$

Every project needs to either be done by self or needs to be subcontracted

Exercise 8.1: Concrete Formulation for Project Sub-Contracting

Download the Jupyter notebook attached to the link on Blackboard for this exercise and submit it there. The notebook asks you to write a linear optimization formulation of the above problem and typeset it in Latex.

Concrete Formulation

Decision variables:

Objective:

In notebook

Constraints:

Sample Problem: Supply Chain Planning

The following table provides the shipping cost for a certain item, from three of Amazon's fulfillment centers (FC) to four regions (A, B, C and D).

Region FC	1	2	3
A. Kings County, NY	20	8	25
B. Los Angeles County, CA	18	23	8
C. King County, WA	21	24	8
D. Harris County, TX	8	8	19

The following table summarizes the weekly demand for the item from each region.

Region A	Region B	Region C	Region D
30	50	10	20

Suppose that each FC is able to ship up to 40 units each week in total. Formulate a linear program to determine the minimum transportation cost needed to satisfy all demand while respecting FC capacities, as well as the optimal shipment plan.

In-class Exercise: English Description

Describe in English the decision, objective and constraints in the above problem. Try your best to use **helpful keywords** and ensure that the descriptions are **precise**, **complete** and **succinct**.

Decision:

Determine how many units which FC should deliver to which regions

Objective:

Minimize transportation cost while satisfying all demand.

Constraints:

The amount shipped out of each FC is at most 40

The amount shipped into each region is equal to its demand.

Exercise 8.2: Concrete Formulation for Supply Chain Planning

Download the Jupyter notebook attached to the link on Blackboard for this exercise and submit it there. The notebook asks you to write a linear optimization formulation of the above and typeset it using Latex.

Concrete Formulation

Decision variables:

X_{ij} => Amount of units fulfillment centre i should deliver to region j.

Objective:

minimize: $20x_{11} + 18x_{12} + 21x_{13} + 8x_{14} + 8x_{21} + 23x_{22} + 24x_{23} + 8x_{24} + 25x_{31} + 8x_{32} + 8x_{33} + 19x_{34}$

Constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 40$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 40$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 40$$

$$x_{11} + x_{21} + x_{31} = 30$$
$$x_{12} + x_{22} + x_{32} = 50$$
$$x_{13} + x_{23} + x_{33} = 10$$
$$x_{14} + x_{24} + x_{34} = 20$$

Writing good English Description:

1. Use helpful keywords

Decision

Continuous { - how much, - amount of }

Integer { - how many, - number of }

Binary { - whether, - which }

Objective

maximize

minimize

Constraints

$\leq$ { - at most, - no more than, - less than equal to }

$\geq$ { - at least, - no less than, - greater than or equal to }

$=$ equal to

2. Precise
3. Complete (Encoding all you control, all the constraints)
4. Succinct.