Week 9: Concrete Formulation II

Session 17: Effectively Utilizing Auxiliary Decision Variables

Sample Problem: Food Production

A food factory requires 2000 tons of canola oil every month as a raw ingredient. The price of canola oil fluctuates from month to month due to market conditions. The predicted prices for the next six months are as follows

Month	1	2	3	4	5	6
Price ($) per ton	150	160	180	170	180	160

The factory's supplier for canola oil delivers it on the first day of every month, and charges the prices above. The factory can decide how much oil to buy each month from the supplier. At the end of each month, the factory can also store unused oil for future use, but the inventory of oil (the total amount stored) cannot exceed 1000 tons at any given time. (The current inventory of oil before the shipment in Month 1 is zero.) Formulate a linear optimization problem to decide how much canola oil to buy for each of the six months in order to minimize the total purchase cost over these six months.

In-Class Exercise

Write the English description and the corresponding concrete formulation for the above problem.

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision Variables:

Objective:

Constraints:

In-Class Exercise

Suppose that in each month in which any amount of oil is bought, there is a fixed cost of $50,000. Furthermore, if the amount of oil bought in a month is not zero, then it must be at least 200 tons. Modify the above formulation to incorporate these considerations.

Modified Concrete Formulation

Decision Variables:

Objective:

Constraints:

Exercise 9.1: Optimal Box Selection

Download the Jupyter notebook attached to the Blackboard link for this exercise and submit it there. The notebook asks you to write a English description and concrete formulation for the following problem, a variant of which appeared in a previous final exam.

A company sells items of various sizes and ships them to customers using special boxes. While the sizes of the items are fixed, the company can decide what sized boxes to use for shipping. The following table lists the types of items, along with the minimum box size needed for each item, as well as the number of each item that needs to be shipped.

Item type	1	2	3
Minimum box size (in cubit feet)	1.5	1.7	2.0
Demand	400	500	200

For simplicity, the company limits the set of possible box sizes to be exactly the sizes listed in the table above. In order to satisfy demand, the company can always use a larger box to ship a smaller item. For example, a type-1 item can be shipped with a box of size 1.5, but can also be shipped using boxes of sizes 1.7 or 2.0.

While larger boxes are more flexible, they are also more expensive to make: **the variable cost** (in dollars) of making each box is exactly equal to the box size. However, the higher variable cost might be worth it since to produce a box of a certain size, the company needs to pay a **fixed cost** of 1000 to create the mold. So using larger boxes might allow the company to make do with fewer box types, which would lower the total fixed cost. For example, using boxes of all three types would incur a fixed cost of 3000, while using only boxes of size 2.0 would incur a fixed cost of 1000.

Write a linear optimization formulation to help the company determine which box types to produce, as well as how many boxes of each size to produce, in order to minimize the total cost while satisfying all demand. (The total cost is the sum of the total variable cost and the total fixed cost.)

English Description

Decision:

Objective:

Constraints:

Concrete Formulation

Decision Variables:

Objective:

Constraints: