Week 7: Introduction to Linear Optimization

Session 14: Typesetting and Solving a Linear Optimization Model

Example from Last Session

A small factory can make two products, X and Y. The following table summarizes the required inputs to produce each product and the profit of each.

	Product X	Product Y
Steel	4 kg	1 kg
Plastic	0 kg	2 kg
Labor	1 hour	1 hour

Suppose that each unit of X makes a profit of 100 dollars and each unit of Y a profit of 200 dollars. Moreover, the daily supply of steel is 60kg, of plastic is 48 kg and of labor is 30 hours. The following linear program (LP) helps the factory to decide how much of each product to produce so as to maximize profit.

Decision Variables:

- X: the amount of product X to produce per day. (Continuous)
- Y: the amount of product Y to produce per day. (Continuous)

Objective:

$$\text{Maximize:} \quad 100X + 200Y$$

Constraints:

$$\begin{aligned} \text{(Steel)} \quad & 4X + Y \leq 60 \\ \text{(Plastic)} \quad & 2Y \leq 48 \\ \text{(Labor)} \quad & X + Y \leq 30 \\ \text{(Non-negativity)} \quad & X, Y \geq 0 \end{aligned}$$

```
[1]: # Solving numerically using Gurobi
     from gurobipy import Model, GRB
     mod=Model()
     X=mod.addVar()
     Y=mod.addVar()
     mod.setObjective(100*X+200*Y,sense=GRB.MAXIMIZE)
     mod.addConstr(4*X+Y<=60)
     mod.addConstr(2*Y<=48)
     mod.addConstr(X+Y<=30)
     mod.optimize()
     print(f'Optimal daily profit:',mod.objVal)
     print(f'Optimal daily production: X={X.x} Y={Y.x}')

... (Gurobi's intermediate outputs are omitted for the handout)
Optimal daily profit: 5400.0
Optimal daily production: X=6.0 Y=24.0
```

In-Class Exercise: GTC Production Planning

Formulate a linear program to solve the following problem: The Gemstone Tool Company (GTC) produces wrenches and pliers. Each product is made of steel, and requires using a Molding Machine and an Assembly Machine. The daily availability of each resource, as well as the resources required to produce one units of each tool, are shown below.

	Wrench (1 unit)	Plier (1 unit)	Daily Availability
Steel	1.5 lbs	1.0 lbs	27,000 lbs
Molding Machine	1.0 hours	1.0 hours	21,000 hours
Assembly Machine	0.3 hours	0.5 hours	9,000 hours

There is demand for 16,000 wrenches and 15,000 pliers per day, and the amount produced cannot exceed the demand. Each wrench earns a profit of .10 dollars and each plier earns a profit of .13 dollars. GTC would likes to decide the amount of wrenches and pliers to produce in order to maximize its profit. For simplicity, assume that the amount of each product produced in a day can be fractional.

Decision Variables

Objective:

Constraints:

Exercise 7.1 Numerically Solving the GTC Production Planning LP

In a blank Jupyter notebook, numerically solve the linear program from the in-class exercise using Gurobi, and upload this notebook on Blackboard. You may follow the "template" code for LP from last session, which is given immediately before the in-class exercise.

Typesetting a Linear Optimization Model using Latex

The following example illustrates how to nicely display a linear optimization formulation using Latex, which is the most widely used method of typesetting mathematics among technical people.

```
**Decision Variables:**

- $X$: the amount of product X to produce per day. (Continuous)
- $Y$: the amount of product Y to produce per day. (Continuous)

**Objective:**
$$\text{Maximize:} \qquad 100X+200Y$$

**Constraints:**
$$\begin{aligned}
\text{(Steel)} && 4X+Y & \le 60 \\
\text{(Plastic)} && 2Y & \le 48 \\
\text{(Labor)} && X+Y & \le 30 \\
\text{(Non-negativity)}&& X,Y & \ge 0 \\
\end{aligned}$$
```

Decision Variables:

- X: the amount of product X to produce per day. (Continuous)
- Y: the amount of product Y to produce per day. (Continuous)

Objective:

$$\text{Maximize:} \qquad 100X + 200Y$$

Constraints:

$$\begin{aligned}
\text{(Steel)} \quad & 4X + Y \le 60 \\
\text{(Plastic)} \quad & 2Y \le 48 \\
\text{(Labor)} \quad & X + Y \le 30 \\
\text{(Non-negativity)} \quad & X, Y \ge 0
\end{aligned}$$

Notice that the variables X and Y are in a special font denoting mathematical variables. Moreover, notice that the linear program above is centered and aligned, both at the signs as well as at the constraint labels.

Explanation of above

To render an expression using LaTex, the expression must be enclosed with dollar signs. For example, the expression `$X>0$` is rendered $X > 0$. A single dollar sign is for mathematical expressions within the same line, and double dollar signs are for a standalone mathematical expressions in its own line. Hence, `$$X > 0.$$` is rendered as

$$X > 0.$$

To make the linear program aligned, we not only use the double dollar signs, but also use the `\begin{aligned} \end{aligned}` commands. (Double click the linear program above to see the code.) Within this block of LaTex script,

- `\text{ }` is for displaying the enclosed string as plain text, without the mathematical rendering.

- `\quad` is for creating a horizontal space. `\qquad` is the same as two `\quad`'s.
- `&` is for alignment. The convention is right align before the first `&`, then left align between the first and second `&` of each line, then right align again between the second and third `&` and so on. Hence, to make it right aligned both before and after the alignment character, we use a double `&&` after the `\text{}`. If this is confusing, you can simply copy the above convention (`&&` after the explanation of constraint, and `&` before the sign).
- `\\` is for creating a new line. Notice that we end the line early using `\\` for "subject to" and "maximize".
- `\le` (less than or equal to) is for $\leq$, and `\ge` (greater than or equal to) is for $\geq$. This looks better than $<=$ and $>=$.
- `_` can be used for subscripts, such as a_1. If multiple characters need to be subscripted, you need `_{...}`, such as `a_{12}`: a_{12}.

Unfortunately, it is difficult to debug LaTex within Jupyter notebook. Hence, if there is any error at all, then the LaTex will not render and you will see your original script. When this happens, try to render one line at a time and see which line is causing the error. Common errors include:

- Not having matching braces, { }. Similarly, not having matching dollar signs to begin and end the block of math, or not matching begin{aligned} with end{aligned}.
- Having blank lines in the aligned environment. (Latex in Markdown cells is very fragile)
- Too few or two many alignment characters &, or not ending the line with \\. The last line in the aligned environment does not need a \\.
- Using & outside of the aligned environment.

Exercise 7.2: Typesetting the GTC Production Planning LP

In a blank Jupyter notebook, typeset the GTC Production Planning LP from the In-Class Exercise in a Markdown cell using Latex. You can use the example in the previous section as a template and follow the given instructions.