Exponential & Poisson Distribution Practice Problems Solutions

1. Assume that you usually get 2 phone calls per hour. Calculate the probability that at least one phone call will come within the next hour.
   Solution: Let X be the number of calls occurring within one hour and the average is 2 phone calls per hour. Then $X \sim \text{Poisson}(\lambda = 2)$. We want $P(X \geq 1) = 1 - P(X < 1)$.
   Excel: 1 – POISSON.DIST(0, 2, 1) = 0.8647.

2. Suppose that historically 10 customers arrive at the checkout lines each hour. What is the probability of 15 people arriving at the checkout counter in the next hour?
   Solution: $x = 15$ and $\lambda = 10$. Excel: POISSON.DIST(15, 10, 0) = 0.0347.

3. Suppose that an average of 30 customers per hour arrive at a store and the time between arrivals is exponentially distributed.
   a) On average, how many minutes elapse between two successive arrivals?
      Solution: Since we expect 30 customers to arrive per hour (60 minutes), we expect on average one customer to arrive every two minutes on average.

   b) When the store first opens, how long on average does it take for three customers to arrive?
      Solution: Since one customer arrives every two minutes on average, it will take six minutes on average for three customers to arrive.

   c) After a customer arrives, find the probability that it takes less than one minute for the next customer to arrive.
      Solution: Let X = the time between arrivals, in minutes. By part a, $\mu = 2$, so $\lambda = 1/2 = 0.5$. Therefore, $X \sim \text{Exp}(0.5)$. Excel: EXPON.DIST(1, 0.5, 1) = 0.3935.

   d) After a customer arrives, find the probability that it takes more than five minutes for the next customer to arrive.
      Solution: Let X = the time between arrivals, in minutes. By part a, $\mu = 2$, so $\lambda = 1/2 = 0.5$. Therefore, $X \sim \text{Exp}(0.5)$. Excel: 1–EXPON.DIST(5, 0.5, 1) = 0.0821.

4. At a police station in a large city, calls come in at an average rate of four calls per minute. Assume that the time that elapses from one call to the next follows an exponential distribution. We are concerned only with the rate at which calls come in, and we are ignoring the time spent on the phone, and we can also assume that the times spent between calls are independent. This means that a particularly long delay between two calls does not mean that there will be a shorter waiting period for the next call.
   a) What distribution will the total number of calls received during a time period follow?
      Solution: $X \sim \text{Poisson}(\lambda = 4)$.

   b) What is the average time between two successive calls?

Solution: On average there are four calls occur per minute = 15 secs → μ = 0.25 mins between calls.

c) Find the probability that after a call is received, the next call occurs in less than ten seconds.
Solution: Let T = time between calls. From part b, $\mu = 0.25$, so $\lambda = 1/0.25 = 4$ and T ~ Exp(λ = 4).
The probability that the next call occurs in less than ten seconds (ten seconds = 1/6 minute).
Excel: EXPON.DIST(1/6, 4, 1) = 0.4866.

d) Find the probability that exactly five calls occur within a minute.
Solution: Let X = the number of calls per minute and X ~ Poisson(λ = 4). We want $P(X = 5)$.
Excel: POISSON.DIST(5, 4, 0) = 0.1563.

e) Find the probability that less than five calls occur within a minute.
Solution: Let X = the number of calls per minute and X ~ Poisson(λ = 4). We want $P(X < 5)$.
Excel: POISSON.DIST(4, 4, 1) = 0.6288.

f) Find the probability that more than 40 calls occur in an eight-minute period.

Solution: Let Y = the number of calls that occur during an 8-minute period. Since there is an average of four calls per minute, there is an average of (8)(4) = 32 calls during each 8-minute period and then Y ~ Poisson(λ = 32). Therefore, $P(Y > 40) = 1 – P(Y \leq 40)$.
Excel: 1 – POISSON.DIST(40, 32, 1) = 0.0707. 0.1563.

5. A bus comes every 15 minutes on average and follows an exponential distribution. You just missed the bus! The moment you arrived, the driver closed the door and left. If the next bus doesn't arrive within the next ten minutes, you have to call an Uber or you'll be late to your Statistics class.
   a) What's the probability that it takes less than ten minutes for the next bus to arrive?
   Solution: Let T be the amount of time until the next bus. 1 bus every 15 min → $\lambda = 1/15$ for time unit of 1 min and T ~ Exponential(λ = 1/15). We want $P(T < 10)$. Excel: EXPON.DIST(10, 1/15, 1) = 0.4866.
   b) How long on average does it take for two buses to arrive?
   Solution: If we assume it takes, on average, 15 mins for one bus to arrive, then it should take 2*15 = 30 mins for two busses to arrive.