Week 9: Concrete Formulation II

Session 18: Creative Problem Solving

Tips for Solving Difficult Problems

1. Be in a relaxed state of mind. If needed, take a break and go to the bathroom, or work on something else and come back later.

2. Make sure you understand the problem. It might be helpful to sketch a drawing to visualize what is going on at a high level, or talk yourself through a numerical example to make sure you internalize the key tradeoffs. → don't start w/ 1st idea right away.

3. Brainstorm a few ideas before committing. The first idea you think of is often not the best one. Spend a few minutes to jot down 3-5 different ideas of how to approach the problem before exploring any idea in depth. For tricky constraints, think of several ways of phrasing the constraint in English, before trying to formulate anything using variables.

4. Clarify the formulation in English first. Before writing any mathematical expressions, ensure that the English description of your formulation is precise, complete and succinct, and try to use helpful keywords as much as possible.

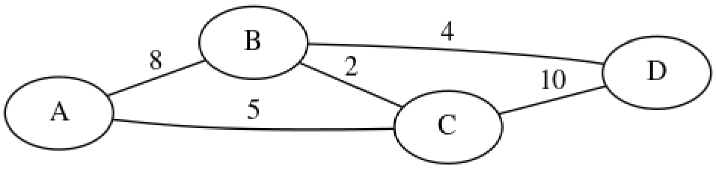
5. Use auxiliary decision variables correctly. Auxiliary decision variables can be helpful for clarifying complex logic or for expressing non-linear logic in a linear way. When you use these variables, ensure that you connect them to the main decision variables using appropriate linear constraints.

6. Plug in concrete numbers as a sanity check. After writing down a mathematical formulation, make up some values for the decision variables and plug them in, to see whether the objective and constraints are what they should be. In particular, check whether the constraints always rule out what should be ruled out and always allow what should be allowed. Plug in several sets of numbers until you are convinced that the logic of your formulation is correct.

Exercise 9.2: Maximizing Supply Chain Throughput

Download the Jupyter notebook attached to the Blackboard link for this exercise and submit it there after completing it. The notebook contains the following problem.

A company manufactures a type of heavy machinery in city A and would like to determine the fastest rate at which it can deliver machines to customers in city D. (Rate, or throughput, is measured in the average number of machines delivered per day.) The bottleneck is that the company must use a special type of truck to ship the machine, and a limited number of these trucks travel between two adjacent cities each day. Each truck can carry only one machine at a time, and each truck only makes trips between two specified cities and will not go anywhere else. The following figure shows which cities are adjacent and how many trucks travel between each pair of adjacent cities in either directions each day.



For example, 8 trucks travel from A to B per day, and all 8 return from B to A on the same day. Since a truck can bear load when travelling in either directions, the rate at which machines travel between A and B is at most 8 per day in either directions. Machines that arrive at city B must be immediately unloaded from the truck it came from (as the truck is going back to city A); later on, that machine can be loaded unto other trucks that travel for example to city C or D. Because all the demand are in city D, the rate at which machines arrive into city B must equal the rate at which machines leave city B, and similarly for city C.

i) Formulate a linear optimization model to determine the fastest rate at which the company can satisfy demand in city D. The formulation may assume the network structure above, but it must continue to work if any combination of numbers are changed.

Decision Variables:

Find the fastest rate of delivery or maximize number of machines delivered

Objective and Constraints:

Max: $x_{CD} + x_{BD}$

$x_{AB} \le 8$ $\quad x_{BD} \le 4$ $\quad x_{CD} \le 10$

$x_{CD} \le 10$ $\quad x_{BC} \le 2$ $\quad x_{CB} \le 2$

$AB + BC = BC + BD$

$AC + BC = CB + CD$

$X_{ij} \ge 0$

ii) Suppose that there is an additional constraint: if the company uses any trucks that travel directly between A and C, then it cannot use trucks that travel directly between B and D. **Define additional decision variables and linear constraints to implement this.**

$$y_{AC} + y_{BD} \leq 1$$

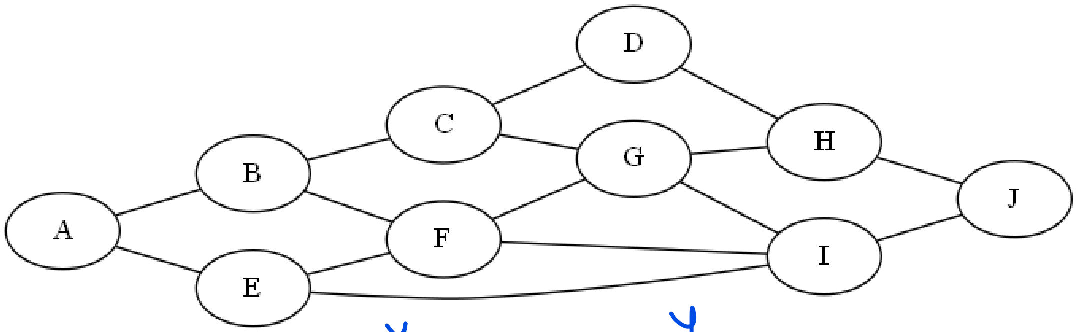
$$x_{AC} \leq 5y_{AC}$$

$$x_{BD} \leq 4y_{BD}$$

Exercise 9.3: Campus Security

Download the Jupyter notebook attached to the Blackboard link for this exercise and submit it there after completing it. The notebook contains the following problem.

USC would like to protect certain streets around USC by stationing security staff at intersections. A street is defined to be the segment connecting two neighboring intersections. For example, the following map has 10 intersections and 15 streets. The street connecting A-B can be protected by staffing either at intersection A or intersection B (these are the two ends of the street), and staffing at one of these would suffice, although it's okay to staff at both ends as well. For example, staffing someone at intersection A would protect the streets A-B and A-E simultaneously; staffing someone at intersection B would protect the streets A-B, B-C, and B-F simultaneously.



min $Y_A + \dots + Y_J$

For the above map, formulate a linear optimization problem to minimize the number of staff needed, subject to the following constraints:

$Y_A+Y_B+Y_C+Y_D \geq 3$

$X_{AB} + \dots \geq 12$

$L_1 + L_2 \geq 1$

- At least 12 of the 15 streets must be protected.
- The segments connecting intersections A, B, C and D are part of a large avenue, and the segments connecting E, F, G and H is another large avenue. At least one of these two large avenues must have at least 3 out of 4 intersections stationed.
- If someone is stationed at intersection I, then someone must be also be stationed at J. However, it is okay to station only J but not I. $Y_I \leq Y_J$
- Intersections A and F must either both be stationed or both be unstationed. $Z_{AF} \leq 1$
- The streets C-G, G-H and B-F are especially dangeous. At least 2 out of 3 of these especially dangeous streets must have staff stationed at both ends. $Z_{CG} + Z_{GH} + Z_{BF} \geq 2$

I	J
0	0
0	1
~~1~~	~~0~~
1	1

Write a concrete formulation of a linear optimization problem, specifying the decision variables, the objective function and all constraints.

Decision Variables:

Objective and Constraints: