Discrete Distributions Practice Problems Solutions

1. Utilizing the data below concerning ratings of a new intake system at a local hospital's emergency room, calculate the mean and standard deviation of the system's ratings.

X, Rating	P(X)	$X \cdot P(X)$	$(X-\mu)^2 \cdot P(X)$
1	0.07	0.07	0.573
2	0.07	0.14	0.242
3	0.22	0.66	0.163
4	0.21	0.84	0.004
5	0.43	2.15	0.559
		μ =3.86	σ^2 = 1.540
			σ = 1.2411

Soln: The mean rating is 3.86 and the standard deviation is 1.2411.

2. The following table summarizes investment outcomes and corresponding probabilities for a particular oil well: (calculate the mean and standard deviation of the profits)

X, Outcome in $	P(X)	$X \cdot P(X)$	$(X-\mu)^2 \cdot P(X)$
-40,000 (no oil)	0.25	-10000.000	410062500.00
10,000 (some oil)	0.70	7000.000	63175000.00
70,000 (much oil)	0.05	3500.000	241512500.00
		μ =500.00	σ^2 =714750000
			σ =$26734.81

Soln: The expected profit is $500 and the standard deviation is $26,734.81.

3. An insurance company sells a $20,000 whole life insurance policy for an annual premium of $300. Actuarial tables show that a person who would be sold such a policy with this premium has a 0.001 probability of death during a year. Let X be a random variable representing the insurance company's profit made on one of these policies during a year. Find the expected profit and standard deviation for the insurance company. The probability distribution of X is:

X, Profit	P(X)	$X \cdot P(X)$	$(X-\mu)^2 \cdot P(X)$
$300 (if policyholder lives)	0.999	299.70	399.60
$300-$20,000 = -$19,700 (if policyholder dies)	0.001	-19.70	399200.40
		μ =280.00	σ^2=399600.00
			σ = $632.14

Soln: The expected profit is $280 and the standard deviation is $632.14.

4. The Bay Street Inn is a seven-room bed-and-breakfast in the California city of Santa Theresa. Demand for rooms generally is strong during February, a prime month for tourists. However, experience shows that demand is quite variable. The probability distribution of room rentals during February (from historical data) is shown below, where X = number of rooms rented.

X, Rooms Rented	P(*X*)	$X \cdot P(X)$	$(X-\mu)^2 \cdot P(X)$
0	0.05	0.00	1.109
1	0.05	0.05	0.688
2	0.06	0.12	0.441
3	0.10	0.30	0.292
4	0.13	0.52	0.066
5	0.20	1.00	0.017
6	0.15	0.90	0.250
7	0.26	1.82	1.363
		$\mu = 4.71$	$\sigma^2 = 4.226$

a) What is the mean, or expected value of the number of rooms rented?
Soln: The mean number of rooms rented is 4.71.

b) What is the standard deviation of the rooms rented?
Soln: The standard deviation is the square root of 4.226, which is 2.056.

c) What is the probability that fewer than 4 rooms are rented?
Soln: There is a 0.05 + 0.05 + 0.06 + 0.10 = 0.26 = 26% chance that fewer than 4 are rented.