USC School of Business

# Data Driven Decision Making: Correlations & Portfolio Analysis

*GSBA 545, Fall 2021*

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# Correlations

- Relationship between Two Variables
- Covariance & Correlation
- Portfolio Basics
- Reducing Risk
- Independent & Dependent Calculations

# Covariance

The **covariance** is a measure of the linear association between two variables.

- Positive values indicate a positive relationship.
- Negative values indicate a negative, or inverse, relationship.

Calculation of the covariance for a **population**:

$$\sigma_{xy} = \frac{\sum(x_i - \mu_x)(y_i - \mu_y)}{N}$$

And for a **sample**:

$$s_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

# Correlation

**Correlation** measures linear association, not necessarily causation.

– If two variables are correlated, it may not mean that one causes the other.

Correlation calculation for a **population**: $\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$

And the calculation for a **sample**: $r_{xy} = \frac{s_{xy}}{s_x s_y}$

The coefficient can take on values between -1 and +1.

– Values near -1 indicate a strong negative linear relationship.
– Values near +1 indicate a strong positive linear relationship.
– The closer the correlation is to zero, the weaker the relationship.

## Golfing Study

A golfer is interested in investigating the relationship, if any, between driving distance and 18-hole score.

| Average Driving Distance (yds) | Average 18-hole Score |
|---|---|
| 277.6 | 69 |
| 259.5 | 71 |
| 269.1 | 70 |
| 267.0 | 70 |
| 255.6 | 71 |
| 272.9 | 69 |

# Golfing Study

## Sample Covariance:

$$s_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{-35.40}{6-1} = -7.08$$

## Sample Correlation:

$$r_{xy} = \frac{-7.08}{(8.2192)(0.8944)} = -0.9631$$

There does, indeed, appear to be a fairly strong negative correlation between driving distance and 18-hole score.

# Portfolio Analysis

**Basic definition**: A combination of stocks, bonds, etc. that form one investment.

Why invest this way?

- Generally, the benefit is a reduction of risk.

How is risk measured?

- Typically, the variance or standard deviation is used.
- More fluctuation implies higher risk, so lower standard deviations indicate more stability and lower risk. Examples of high and low risk funds?

How can we control the risk?

- Aggregate funds from different sources.

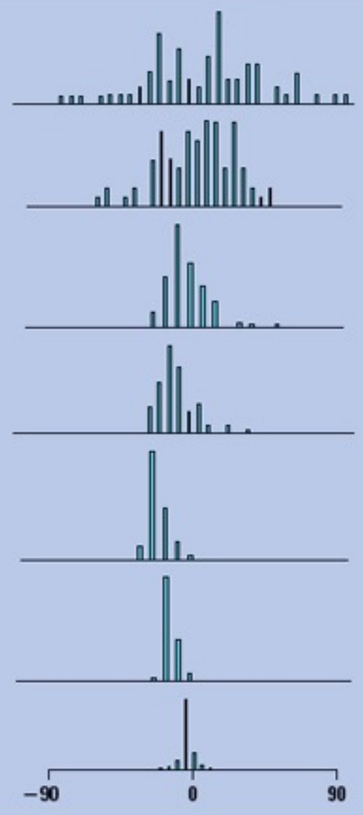
# Historical Returns

The table shows the average stock return, bond return, T-bill return, and inflation rate (1926 – 2012).

We can derive average *excess returns*. The average *excess return* from large-company common stocks relative to T-bills for the entire period was 8.2% (11.8% – 3.6%).

The average *excess return* on common stocks is called the **historical equity risk premium** because it is the additional return from bearing risk.

| Series | Arithmetic Mean (%) | Standard Deviation (%) | Distribution (%) |
|---|---|---|---|
| Small-company stocks* | 16.5 | 32.3 | |
| Large-company stocks | 11.8 | 20.2 | |
| Long-term corporate bonds | 6.4 | 8.3 | |
| Long-term government bonds | 6.1 | 9.7 | |
| Intermediate-term government bonds | 5.5 | 5.6 | |
| U.S. Treasury bills | 3.6 | 3.1 | |
| Inflation | 3.1 | 4.1 | |




* Modified from Ibbotson® SBBI® 2013 Classic Yearbook™ (Chicago: Morningstar).

# Definitions

**Notation**:

$a$ and $b$ represent amounts invested in two funds $X$ and $Y$

Let $W$ represent the portfolio combination

**Expected Value**, or average: $E(W) = E(aX + bY) = aE(X) + bE(Y)$

**Variance**:

1. Independent: $Var(W) = Var(aX + bY) = a^2Var(X) + b^2Var(Y)$
2. Dependent: $Var(W) = Var(aX + bY) = a^2Var(X) + b^2Var(Y) + 2ab\ Cov(X,Y)$

The covariance term is needed to evaluate portfolio risk.

*Most funds are assumed to be normally distributed, letting us make easy calculations. ☺

# What is Risk?

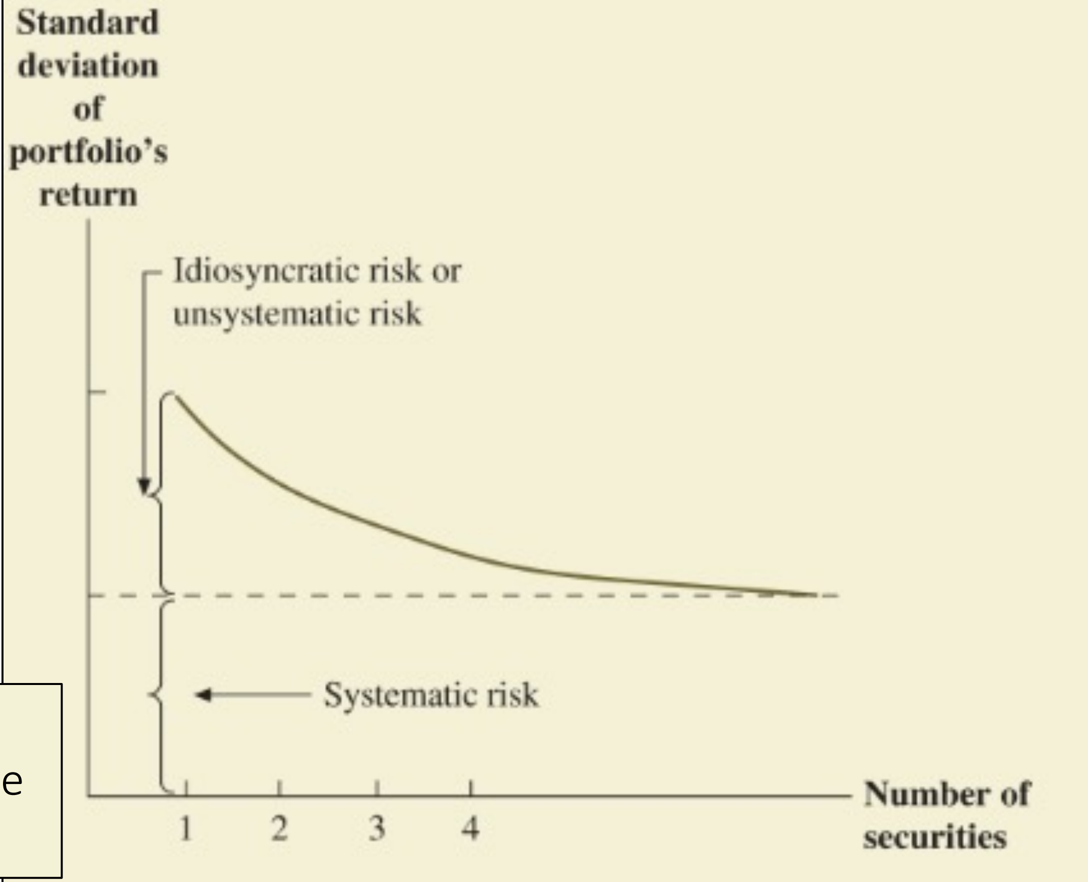
- *Total risk = systematic risk + unsystematic risk*
- The standard deviation of returns is a measure of total risk.
- For well-diversified portfolios, unsystematic risk is very small. Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk.
- A systematic risk is any risk that affects a large number of assets, each to a greater or lesser degree.
  - Uncertainty about general economic conditions like GNP, interest rates or inflation.
- An unsystematic risk is a risk that specifically affects a single asset or small group of assets and can be diversified away.
  - Announcements specific to a single company are examples of unsystematic risk.

# How Many Stocks?

Notice the standard deviation declines as the number of securities is increased.

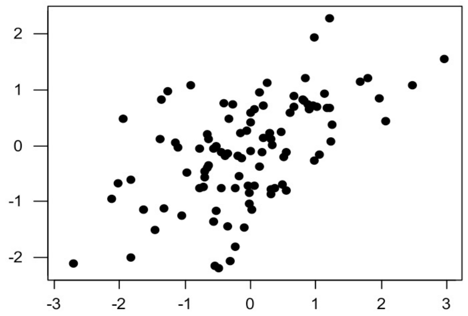
Diversification does not allow the *total risk* to go to zero because only *unsystematic risk* is controlled here.

Relationship between the standard deviation of a portfolio's return and the number of securities in the portfolio.

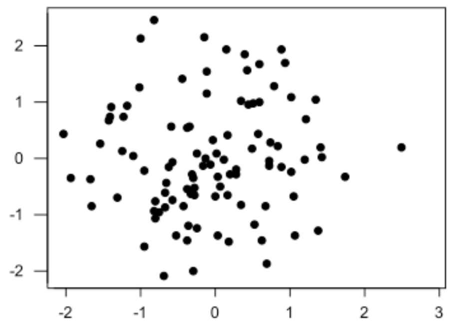



The standard deviation of a portfolio drops as more securities are added to the portfolio. However, it does not drop to zero. Rather, while unsystematic risk can be eliminated through diversification, systematic risk cannot be.

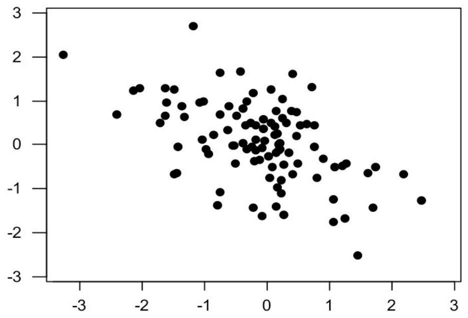
# Various Correlations



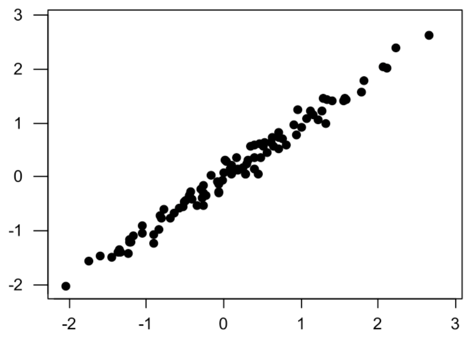

*r* = 0.55



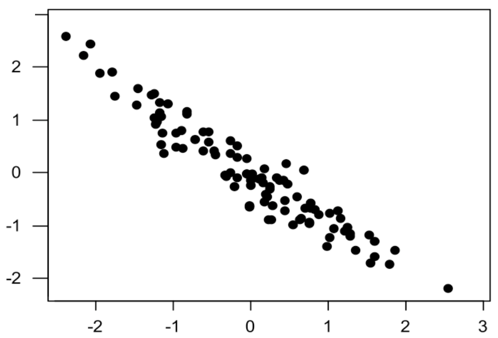

*r* = 0.06




*r* = -0.52




*r* = 0.99




*r* = -0.96

# Independent Portfolio

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 1 | **Investment in a portfolio of stocks** | | | | | |
| 2 | | | | | | |
| 3 | | Stock A | Stock B | Stock C | Stock D | Total |
| 4 | Weights % | 25.00% | 25.00% | 25.00% | 25.00% | 100.00% |
| 5 | Weights $ | $10,000 | $10,000 | $10,000 | $10,000 | $40,000 |
| 6 | | | | | | |
| 7 | | Stock A | Stock B | Stock C | Stock D | |
| 8 | Means | 0.13 | 0.16 | 0.12 | 0.15 | |
| 9 | Stdevs | 0.04 | 0.06 | 0.02 | 0.05 | |
| 10 | | | | | | |
| 11 | Correlations between stock returns: | | | | | |
| 12 | | Stock A | Stock B | Stock C | Stock D | |
| 13 | Stock A | 1.00 | 0.00 | 0.00 | 0.00 | |
| 14 | Stock B | 0.00 | 1.00 | 0.00 | 0.00 | |
| 15 | Stock C | 0.00 | 0.00 | 1.00 | 0.00 | |
| 16 | Stock D | 0.00 | 0.00 | 0.00 | 1.00 | |
| 17 | | | | | | |
| 32 | | | | | | |
| 33 | | Exp Value | $5,600.00 | | | |
| 34 | | Variance | $810,000.00 | | | |
| 35 | | St Dev | $900.00 | | | |
| 36 | | | | | | |

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| 5 | Weights $ | $10,000 | $10,000 | $10,000 | $10,000 | $40,000 |
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| 11 | Correlations between stock returns: | | | | | |
| 12 | | Stock A | Stock B | Stock C | Stock D | |
| 13 | Stock A | 1.00 | 0.45 | 0.75 | -0.60 | |
| 14 | Stock B | 0.45 | 1.00 | 0.55 | -0.35 | |
| 15 | Stock C | 0.75 | 0.55 | 1.00 | -0.80 | |
| 16 | Stock D | -0.60 | -0.35 | -0.80 | 1.00 | |
| 32 | | | | | | |
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| 35 | | St Dev | $817.31 | | | |

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