Channel Models for FSO

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Wireless model for FSO

- The wireless channel model is given by [1],

$$y = sx + n = \eta I x + n$$

- Where s = ηI denotes the instantaneous intensity gain.
- x ∈ {0, 1} the OOK modulated signal.
- n ∼ N (0,N0/2) the white Gaussian noise with mean 0 and variance N0/2 because of random nature of electrons at receiver electronic circuitry.
- η the effective photo-current conversion ratio of the receiver. Where η is defined by,

$$\eta = \gamma \frac{e\lambda}{h_p c}$$

  Where γ is the quantum efficiency of the photo receiver, e the electron charge, λ the signal wavelength, hp Plank's constant and c is the speed of light.
- I is the irradiance And definition of I will change according to models.

Kalmogrov theory

- Eddies are vortex due to flow of wind at any geographical location and are main cause of turbulence.
- Kolmogorov’s theory describes how energy is transferred from larger to smaller eddies and how much energy is dissipated by eddies of each size. Atmospheric turbulence is derived using Kalmogorov theory as,

$$\text{Ø}_n(\kappa) = 0.033 C_n^2 \kappa^{-\frac{11}{3}}, \qquad \text{where } \frac{1}{L_0} \ll \kappa \ll \frac{1}{l_0}$$

- Where L0 and l0 are large and small eddy size of 10-100 m and 1 cm, respectively, $C_n{}^2$ is the refractive index parameter giving the spatial frequency based on geographical location, altitude and time of day. Values of $C_n{}^2$ [2] for different turbulence levels like weak turbulence, moderate turbulence and strong turbulence:

C_n^2= 10-17 m-2/3 for weak turbulence

= 10-15 m-2/3 for moderate turbulence

= 10-13 m-2/3 for strong turbulence

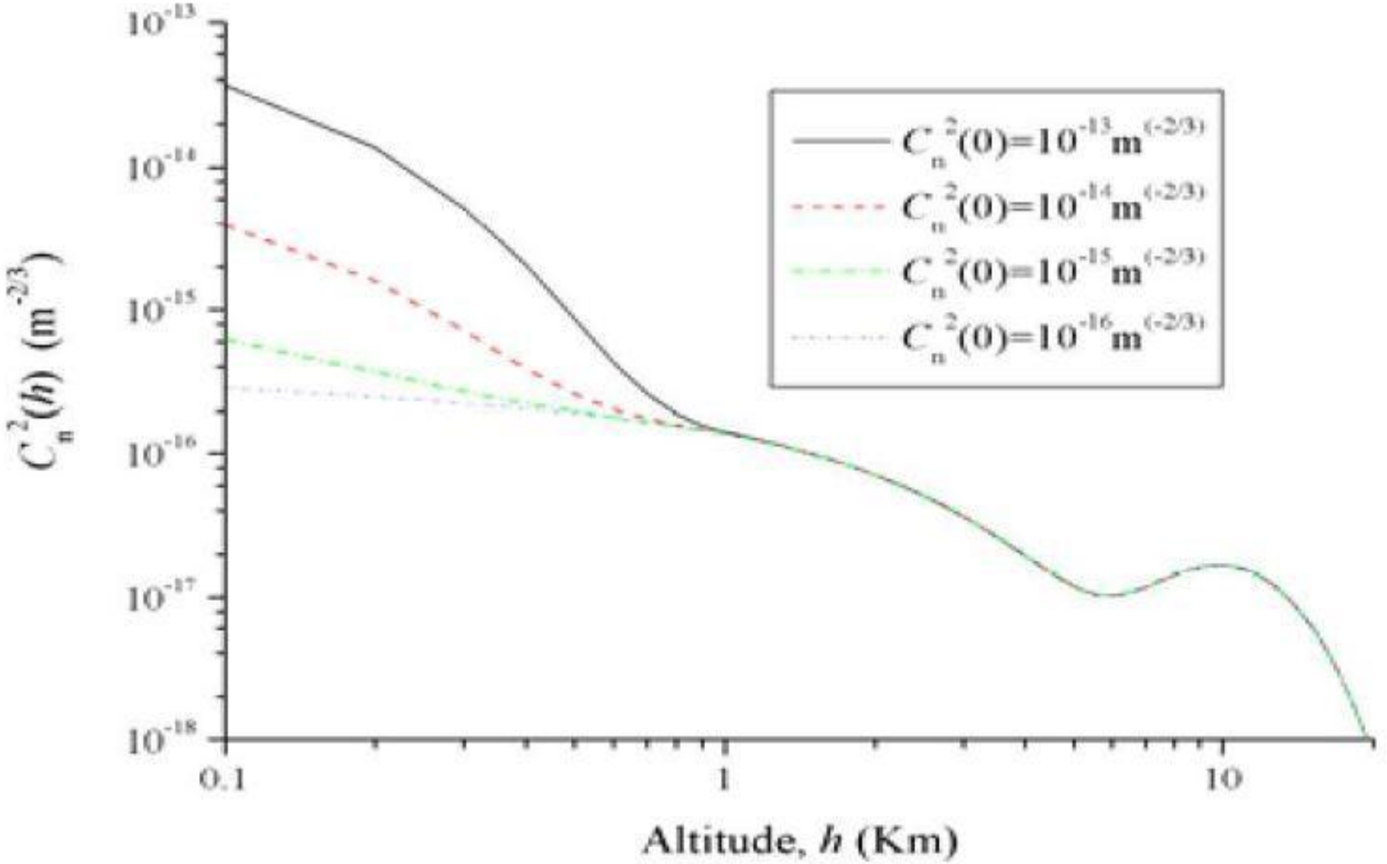
Hufnagel valley boundary theorem

- Refractive index structure parameter is almost constant for horizontal path propagation. But in vertical path propagation, temperature gradient is different at different altitude so it varies with altitude.. The mathematical model of HVB is as shown below,

$$C_n^2(h) = a_1 \left(\frac{V}{27}\right)^2 \left(\frac{h}{s_1}\right)^{10} \exp\left(-\frac{h}{s_1}\right) + a_2 \exp\left(-\frac{h}{s_2}\right) + C_n^2(0) \exp\left(-\frac{h}{s_3}\right)$$

- Where *a1= 5.94 x 10-23, a2= 2.7 x 10-16,*
- *s1=1000 m, s2= 1500 m, s3= 100 m*
- *h is altitude (m),*
- V is the rms wind speed in m/s. The refractive index structure parameter versus the altitude, h has been shown in figure for HVB-21 model with V= 21 m/s.
- For different values of ${}_n{}^2(0)$, $C_n{}^2$ decreases with increasing height and is nearly independent of $C_n{}^2(0)$ for altitude greater than 1 km.

HVB-21 model C_n^2 variation



Scintillation Index

- Scintillation is caused by small temperature variations in the atmosphere, which results in index of refraction fluctuations.
- Scintillation is defined by, S

$$S = \frac{\sqrt{\left\langle \left(I - \overline{\langle I \rangle}\right)^2 \right\rangle}}{\overline{\langle I \rangle}} = \frac{\textit{standard deviation}}{\textit{mean}}$$

- where *I* denote irradiance that is the received intensity of the optical field after passing it through turbulent medium.
- Based on the value of S, turbulence can be identified as strong or weak.

Lognormal Channel

- In lognormal channel model, irradiance *I is*

$$I = e^{2z}$$

- Where, Z is the Gaussian distribution with Mean 0 and variance σ^2.
- *So, I* will follow log-normal distribution with

  mean $e^{2\sigma^2}$

  and variance $e^{4\sigma^2}(e^{4\sigma^2} - 1)$ [3] .
- Based on the relation, $I = \frac{\gamma}{\xi}$
- we obtain $P_\gamma(\gamma)$ and for summing this, we use power series to obtain BER equation as,

$$Pe, L(\Upsilon g, \sigma x) = \frac{1}{2} - \frac{1}{\sqrt{\pi}} e^{\frac{\sigma x}{2}} \sum_{k=0}^{\infty} \frac{(-1)^k \Upsilon g^{(2k+1)/2}}{2^{(2k+1)/2}(2k+1)k!} \exp\left(\frac{(4k+1)\sigma x}{\sqrt{2}}\right)^2$$

Lognormal Channel

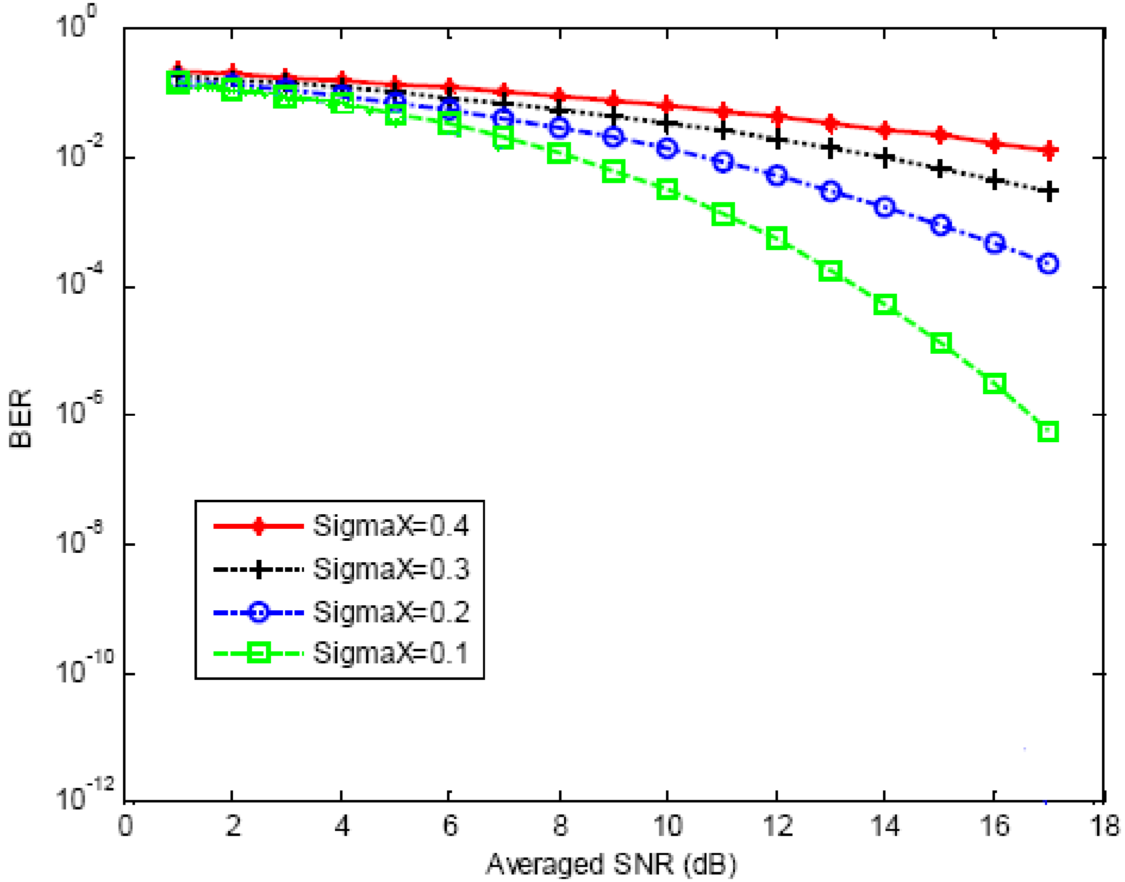
- Where $P_{e,L}$ is bit error rate probability which is a function of Υ_g (signal to noise ratio) and σ_x (fading intensity).SNR can be calculated by,

$$SNR = \frac{4R^2P^2}{(\sigma 1 + \sigma 0)^2}$$

- Where, R is responsivity of receiver,
- P is transmitted power.
- σ1 and σ0 are standard deviation of noise currents for symbols '1' and '0'.
- The above equation of SNR can be expressed in terms of h for fading channels as,

$$SNR = \frac{4h^2R^2P^2}{(\sigma 1 + \sigma 0)^2}$$

Simulation BER v/s SNR lognormal



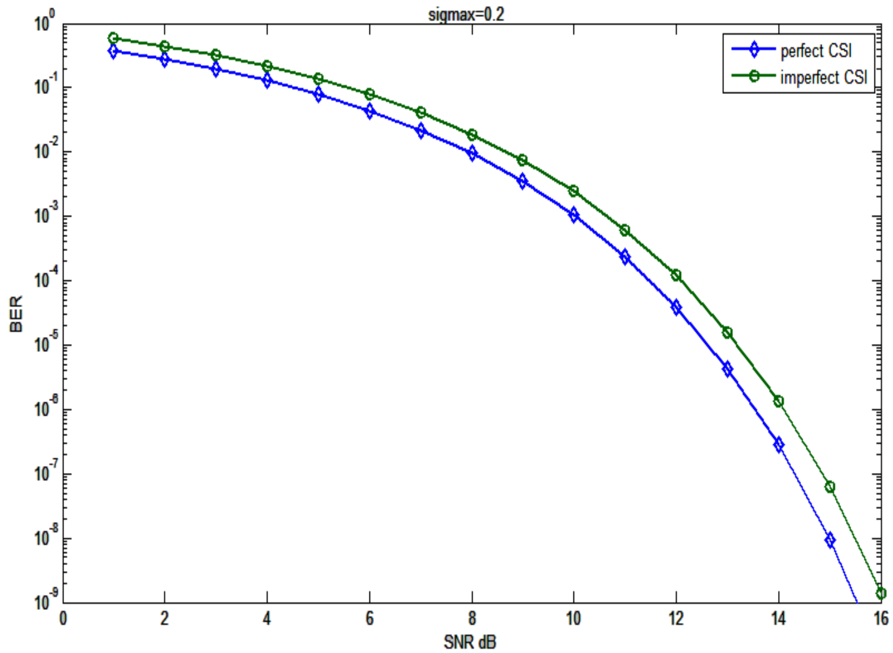
Performance of perfect CSI at receiver for log-normal channel model

Lognormal with imperfect CSI

- Gauss-Markov Model is described as,

$$h1 = \delta h + \sqrt{1-\delta^2}\, w$$

- Using h1 instead of h in BER equation, we have got comparison of with CSI and without CSI as below,



Modified Rytov Theory

- Andrews proposed the *modified Rytov theory [4,5], which defines the optical field as,*

$$U(r,L) = U_0(r,L)\exp(\Psi_x(r,L) + \Psi_y(r,L))$$

- Where $\Psi_x(r,L) \,\&\, \Psi_y(r,L)$ are statistically independent complex perturbations which are due only to large-scale and small-scale atmospheric effects, respectively.
- The perturbation ψ 1(*r, L)= χ1(r, L)+ jξ1(r, L) is* a complex Gaussian random process
- *U_0(r, L) is the optical field in the absence* of turbulence
- where *r is the observation point in transverse plane at propagation* distance *L,*

Gamma Gamma Channel

- Specifically, in [6], gamma pdf is used to model both small-scale and large scale fluctuations, leading to the so-called *gamma-gamma pdf,*

$$P(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right); \qquad I > 0$$

- Where $K_a(.)$ is the modified Bessel function of second kind of order a. α and β are the effective number of small scale and large scale eddies of the scattering environment.

$$\beta = \left[\exp\left(\frac{0.51\chi^2(1+0.69\chi^{12/5})^{-5/6}}{(1+0.9d^2+0.62d^2\chi^{12/5})^{5/6}}\right)-1\right]^{-1}$$

$$\alpha = \left[\exp\left(\frac{0.49\chi^2}{(1+0.18d^2+0.56\chi^{12/5})^{7/6}}\right)-1\right]^{-1}$$

Gamma gamma channel

- Now from this result, we can find the BER performance of scheme as,

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} \frac{D^6(\theta)}{(1-2D^2(\theta))^2}\,d\theta$$

- Where D(θ) is given by,

$$D(\theta) = 2^{\frac{\alpha-\beta+4}{4}} c1\left(\frac{c2}{\alpha}\right)^{\frac{\alpha-\beta}{2}} \left(\frac{sin\theta}{\sqrt{\tau}}\right)^{\frac{\alpha+\beta}{2}} K_{\alpha-\beta}\left(2^{5/4}\sqrt{\frac{c2\,\alpha\, sin\theta}{\sqrt{\tau}}}\right)$$

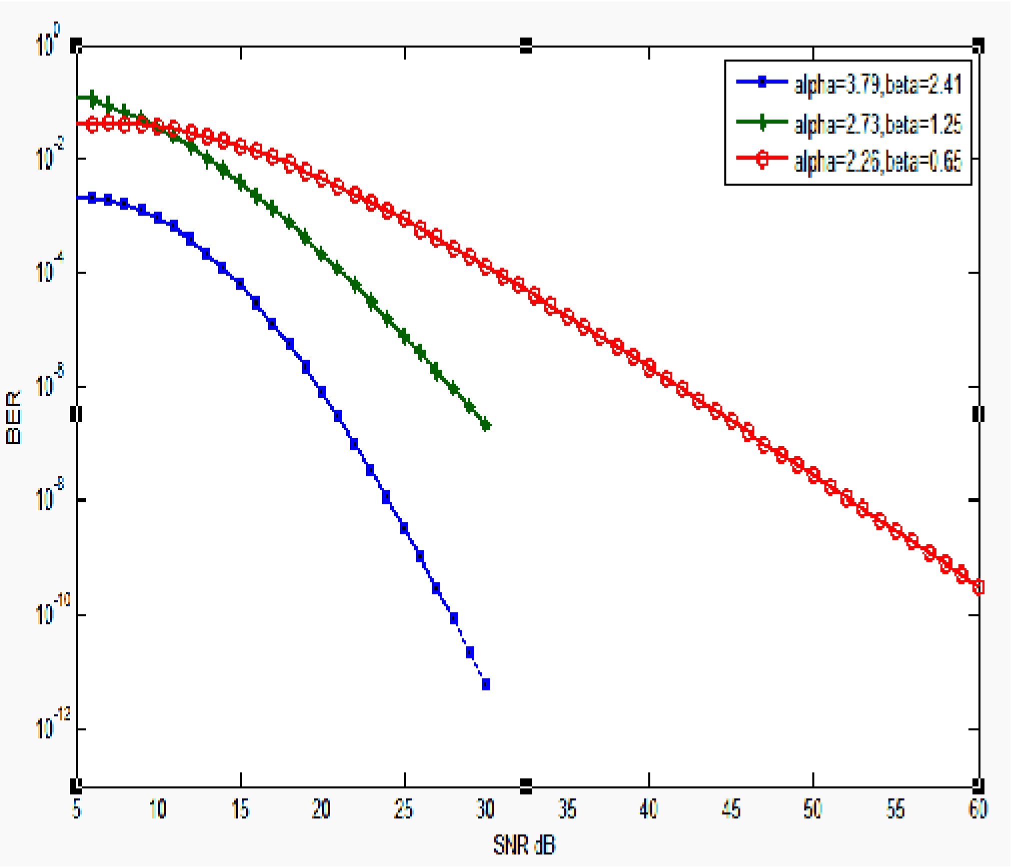
- Where c1 is given by,

$$c1 = \frac{\sqrt{\pi}\alpha^{\alpha}\beta^{\beta}}{\Gamma(\alpha)\Gamma(\frac{\beta+1}{2})}$$

- τ = Es/N0
- And c2 is given by,

$$c2 = \beta\left(\sqrt{\beta-\frac{1}{2}} + \frac{1}{16}\left(\beta-\frac{1}{2}\right)\right)^{-\frac{3}{2}}$$

BER v/s SNR for gamma gamma



Negative exponential model

- In strong turbulence, more irradiance fluctuations.
- Where link length spans several kilometers, number of independent scatter become large.
- Signal amplitude - Rayleigh fading distribution,

  Signal intensity - negative exponential statistics.
- Where I_0 is mean radiance of channel.

$$P(I) = \frac{1}{I_0} \exp\left(-\frac{I}{I_0}\right); \qquad where\, I > 0$$

K Channel model

- For strong turbulence channels.
- Excellent similarity between theoretical and experimental values.
- pdf of irradiance - governed by the negative exponential distribution [8].

$$f_{\frac{I}{\mu}}\left(\frac{I}{\mu}\right)=\frac{1}{\mu}\exp\left(-\frac{1}{\mu}\right); where I > 0$$

- Where μ is mean irradiance of channel.

K Channel model

Converting the above equation,

$$f_{\mu}\left(\mu\right)=\frac{\alpha^{\alpha}\mu^{\alpha-1}}{\Gamma(\alpha)}\exp\left(-\alpha\mu\right); where \mu>0$$

$$\Gamma\left(\alpha\right)=\int_{0}^{\infty}e^{-t}t^{\alpha-1}dt$$

We can find pdf of I as follows,

$$P_{I}\left(I\right)=\int_{0}^{\infty}f_{\frac{I}{\mu}}\left(\frac{I}{\mu}\right)*f_{\mu}\left(\mu\right)dI$$

This integration results as,

$$P_{I}\left(I\right)=2\frac{\alpha^{\frac{\alpha+1}{2}}}{\Gamma(\alpha)}\frac{I^{\frac{\alpha+1}{2}}}{\xi^{\frac{\alpha+1}{4}}}K_{\alpha-1}\left(2\sqrt{\alpha I}\right)$$

K Channel model

Using a simple transformation, SNR is obtained as,

$$P_\gamma(\gamma) = \frac{\alpha^{\frac{\alpha+1}{2}}}{\Gamma(\alpha)} \frac{\gamma^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} K_{\alpha-1}\left(2\sqrt{\alpha\sqrt{\frac{\gamma}{\xi}}}\right) \qquad \xi = {(\eta E[I])^2}/{N_0}$$

Using the equation of Chanel capacity,

$$C = B \times \log_2(1 + SNR)$$

Pdf of C is,

$$P_C(C) = \frac{\ln(2)\alpha^{\frac{\alpha+1}{2}}}{\Gamma(\alpha)2^{-C/B}} \frac{(2^{C/B}-1)^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} K_{\alpha-1}\left(2\sqrt{\alpha\sqrt{\frac{(2^{C/B}-1)}{\xi}}}\right)$$

$$r = \int_0^{Cout} P_C(C)\,dC$$

I-K Channel model

- In both scenarios- weak turbulence and strong turbulence.
- Less computation complexity than gamma-gamma channel model.
- This channel model is preferred over others [8].
- The PDF of normalized signal irradiance is given as,

$$P_I(I) = \begin{cases} 2\alpha(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \times K_{\alpha-1}\left(2\sqrt{\alpha\rho}\right) \times \\ I_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)I}\right);\ for I < \dfrac{\rho}{1+\rho} \\ 2\alpha(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \times I_{\alpha-1}\left(2\sqrt{\alpha\rho}\right) \times \\ K_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)I}\right);\ for I > \dfrac{\rho}{1+\rho} \end{cases}$$

I-K Channel model

- Again using a simple transformation, SNR is obtained as,

$$P_\gamma(\gamma) = \begin{cases} 2\alpha(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \dfrac{\gamma^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} K_{\alpha-1}\left(2\sqrt{\alpha\rho}\right)\times \\ I_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{\gamma}{\xi}}}\right); for\gamma < \dfrac{\rho^2\xi}{(1+\rho)^2} \\ 2\alpha(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \dfrac{\gamma^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} I_{\alpha-1}\left(2\sqrt{\alpha\rho}\right)\times \\ K_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{\gamma}{\xi}}}\right); for\gamma > \dfrac{\rho^2\xi}{(1+\rho)^2} \end{cases}$$

I-K channel model

Using equation of channel capacity,

$$C = B \times \log_2(1 + SNR)$$

The pdf of capacity can be given as,

$$P_c(c) = \begin{cases} \dfrac{2^{C/B+1} ln(2)}{B\alpha^{-1}}(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \dfrac{(2^{C/B}-1)^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} K_{\alpha-1}\left(2\sqrt{\alpha\rho}\right) \times \\ I_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{2^{C/B}-1}{\xi}}}\right) \ \mathit{for}\ C < B\log_2 \dfrac{1+\rho^2\xi}{(1+\rho)^2} \\ \dfrac{2^{C/B+1} ln(2)}{B\alpha^{-1}}(1+\rho)\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \dfrac{(2^{C/B}-1)^{\frac{\alpha-3}{4}}}{\xi^{\frac{\alpha+1}{4}}} I_{\alpha-1}\left(2\sqrt{\alpha\rho}\right) \times \\ K_{\alpha-1}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{2^{C/B}-1}{\xi}}}\right) \ \mathit{for}\ C > B\log_2 \dfrac{1+\rho^2\xi}{(1+\rho)^2} \end{cases}$$

I-K channel model

Outage probability can be found using channel capacity as,

$$r = \int_0^{Cout} P_C(C)\,dC$$

Value of outage probability can be shown as,

$$r = \begin{cases} 2\sqrt{\alpha\rho}\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha}{2}} \dfrac{(2^{Cout/B}-1)^{\frac{\alpha}{4}}}{\xi}\; K_{\alpha-1}\left(2\sqrt{\alpha\rho}\right)\times \\ I_{\alpha}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{2^{Cout/B}-1}{\xi}}}\right) \; ; Cout < B\log_2 \dfrac{1+\rho^2\xi}{(1+\rho)^2} \\ 1-2\sqrt{\alpha(1+\rho)}\left(\dfrac{1+\rho}{\rho}\right)^{\frac{\alpha-1}{2}} \dfrac{(2^{Cout/B}-1)^{\frac{\alpha}{4}}}{\xi}\; I_{\alpha-1}\left(2\sqrt{\alpha\rho}\right)\times \\ K_{-\alpha}\left(2\sqrt{\alpha(1+\rho)\sqrt{\dfrac{2^{Cout/B}-1}{\xi}}}\right) \; ; Cout > B\log_2 \dfrac{1+\rho^2\xi}{(1+\rho)^2} \end{cases}$$

Summary

- Lognormal channel model is used in weak turbulence scenario and key factor is σ .
- Gamma-Gamma channel model is used in weak to strong turbulence scenario and key factors are α, β .
- K channel model is used in strong turbulence scenario and key factor is β
- I-K channel model is used in strong turbulence scenario and key factor is ρ .

Reference

1. Murat Uysal, Jing (Tiffany) Li, Error Rate Performance of Coded Free-Space Optical Links over Gamma-Gamma Turbulence Channels, Communications, 2004 IEEE International Conference, 20-24 June 2004, 3331 - 3335 Vol.6.
2. ARNON S.: ‘Optical wireless communications’, chapter in the Encyclopedia of Optical Engineering (EOE), R.G. DRIGGERS (ED.) (Marcel Dekker, 2003), pp. 1866–1886
3. Hassan Moradi, Maryam Falahpour, Hazem H. Refai, Peter G. LoPresti, Mohammed Atiquzzaman, BER Analysis of Optical Wireless Signals through Lognormal Fading Channels with Perfect CSI, Telecommunications (ICT), 2010 IEEE 17th International Conference Publications, 493 – 497..
4. L. C. Andrews, R. L. Phillips, C. Y. Hopen and M. A. Al-Habash, “Theory of optical scintillation”, *Journal of Optical Society America A,* vol. 16, no.6, p. 1417-1429, June 1999.
5. L. C. Andrews, R. L. Phillips and C. Y. Hopen, “Aperture averaging of optical scintillations: Power fluctuations and the temporal spectrum”, *Waves Random Media, vol. 10, p. 53-70, 2000.*

References

6. M. A. Al-Habash, L. C. Andrews and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media", *Optical Engineering, vol. 40, no.* 8, p. 1554-1562, August 2001
7. LETZEPIS, N., FABREGAS, A. G. Outage probability of the free space optical channel with doubly stochastic scintillation. IEEE Transactions on Communications, 2009, vol. 57, no. 10.
8. Hector E. NISTAZAKIS, Andreas D. TSIGOPOULOS, Michalis P. HANIAS, Christos. D. PSYCHOGIOS, Dimitris MARINOS, Costas AIDINIS, George S. TOMBRAS, Estimation of Outage Capacity for Free Space Optical Links over I-K and K Turbulent Channels, RADIOENGINEERING, VOL. 20, NO. 2, JUNE 2011.