KALMAN FILTER BASED GPS TRACKING

Prepared by: Falak Shah (0BEC082)
Semester VIII (EC), Nirma University
Guided by: Mr. Ankesh Garg (DCTG-SAC/ISRO)

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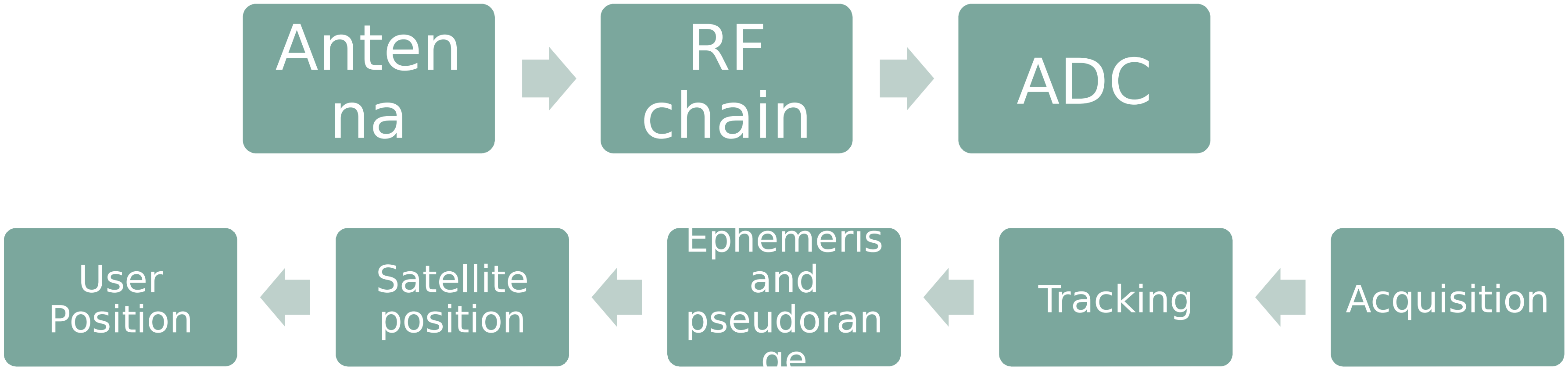
Global positioning system: overview

- Constellation of 24 satellites
- Performance requirements of GPS systems
- 10-30 m accuracy
- User dynamics
- Worldwide coverage
- Minimum number of required references for determining position in 3-D is 4
- 5 used to calculate user position out of 7 visible

Global positioning system: overview



Block diagram of GPS receiver

Signal structure of GPS

- Coarse/ Acquisition (C/A code) and P codes
- 2 frequencies L1 centered at1575.42 MHz and L2 at 1227.6 MHz. (multiple of 10.23 MHz clock)

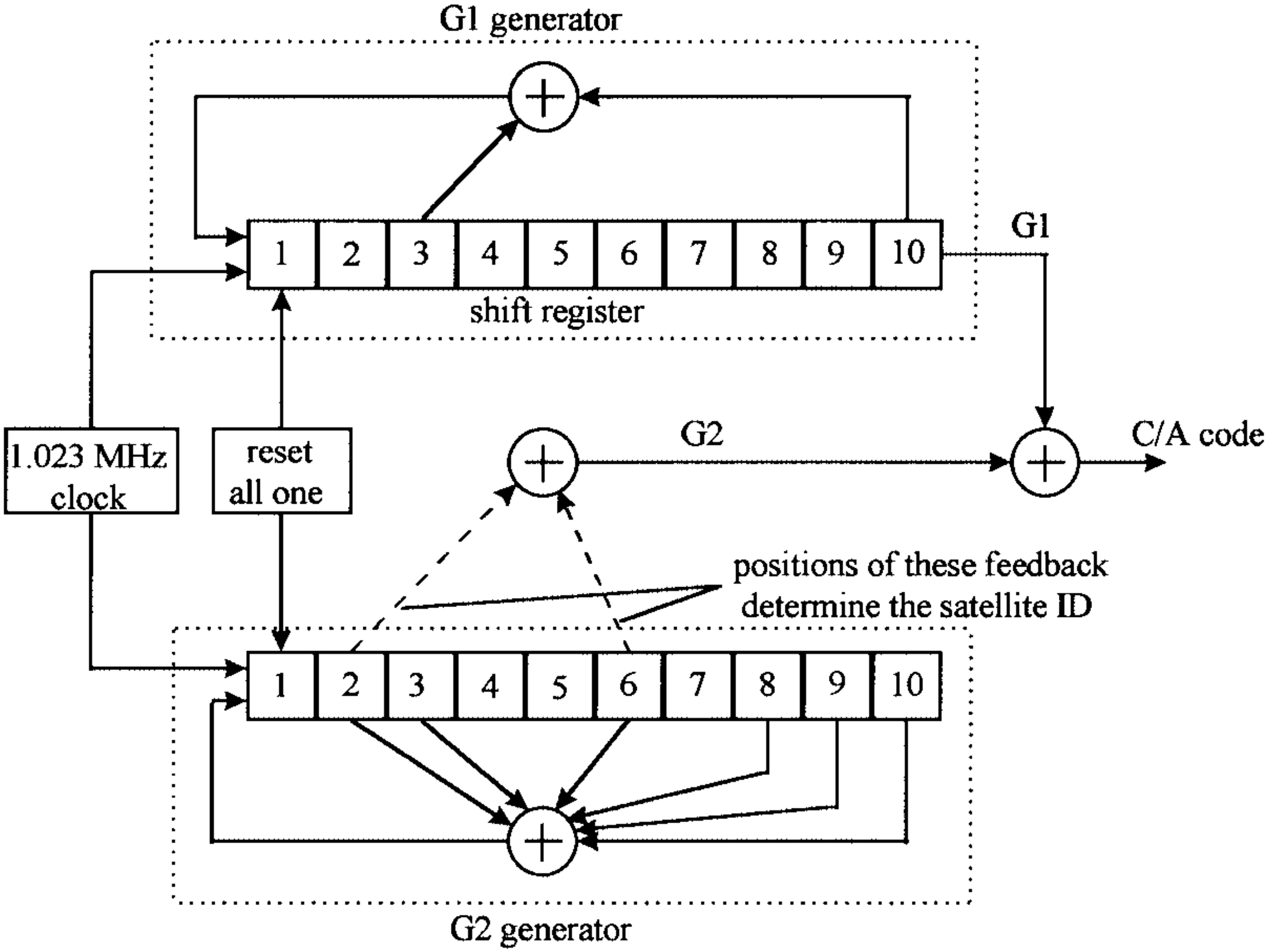
$$S_{L1} = A_p P(t)D(t)\cos(2\pi f_1 t + \phi) + A_c C(t)D(t)\sin(2\pi f_1 t + \phi)$$

- where S_{L1} *is the signal at L1 frequency,* A_p *is the amplitude of the P code, P(t) and D(t)* represent the phase of the *P code and* data code. f_1 *is the L1 frequency,* *is the initial phase.*
- GPS signal ϕ phase modulated BPSK

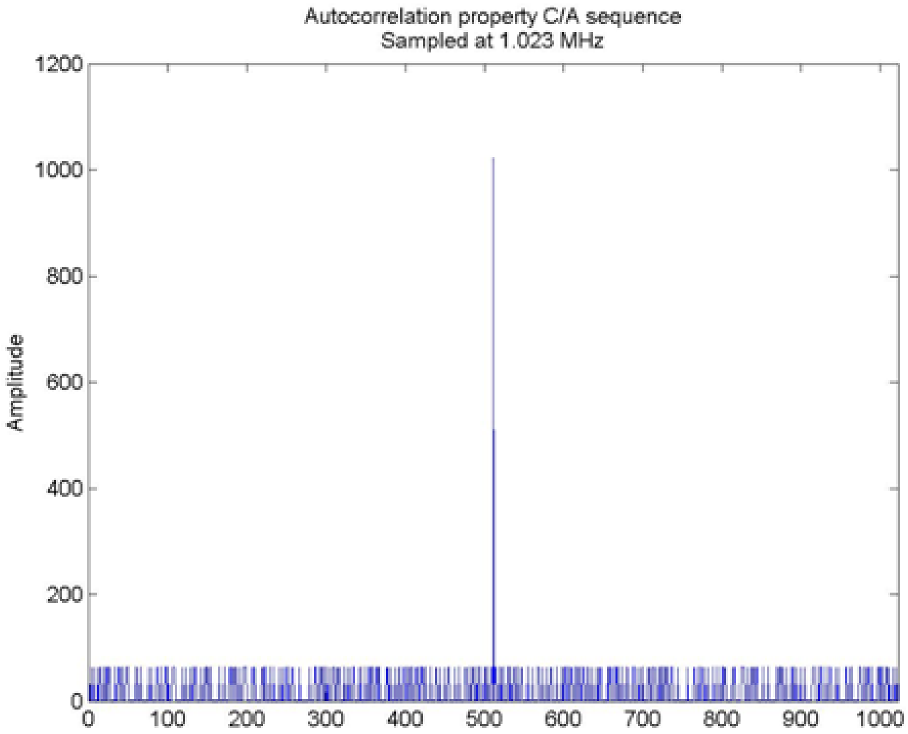
Signal structure of GPS

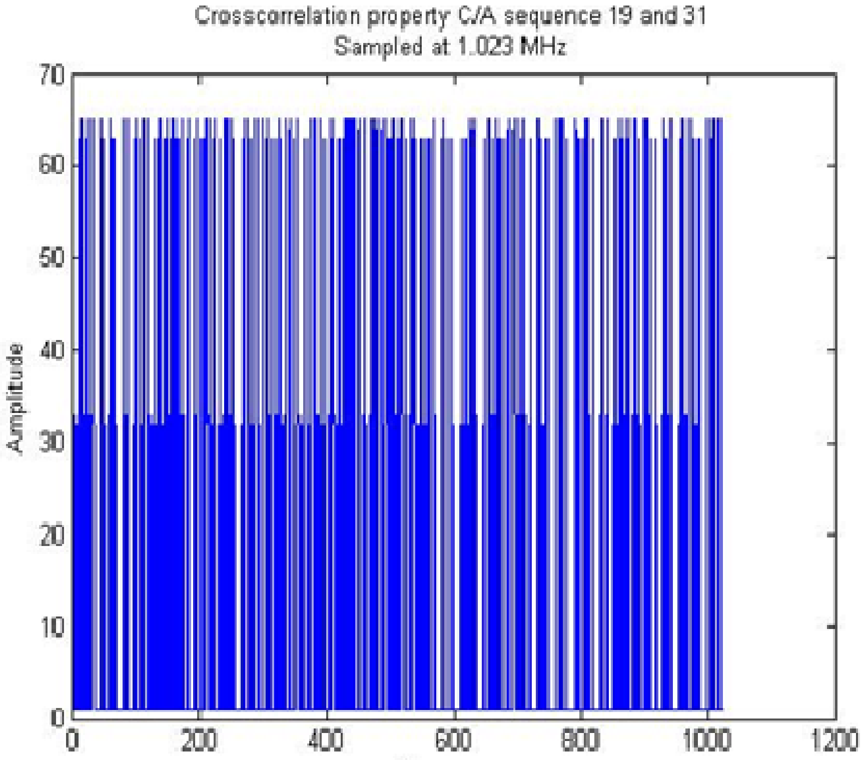
- P code and C/A code are bi-phase modulated at 10.23 MHz and 1.023 MHz Gold codes
- C/A code generated with use of a LSR of length 10 and is 1023 bits long
- 1ms data enough to find its beginning
- C/A codes generated and autocorrelation and cross correlation properties verified
- Peak provides starting point of the code
- Navigation data is 20 ms long- contains 20 C/A codes

C/A code generator



Autocorrelation and cross-correlation properties of C/A code





Acquisition

- Search for frequency over is +/- 10 kHz which is maximum Doppler range
- 2-D search for carrier frequency and starting point of C/A code
- IF is at 21.25 MHz, sampling frequency is 5 MHz. So, center of the signal is at 1.25 MHz.
- Maximum data record used for acquisition is 10 ms since data is 20 ms long

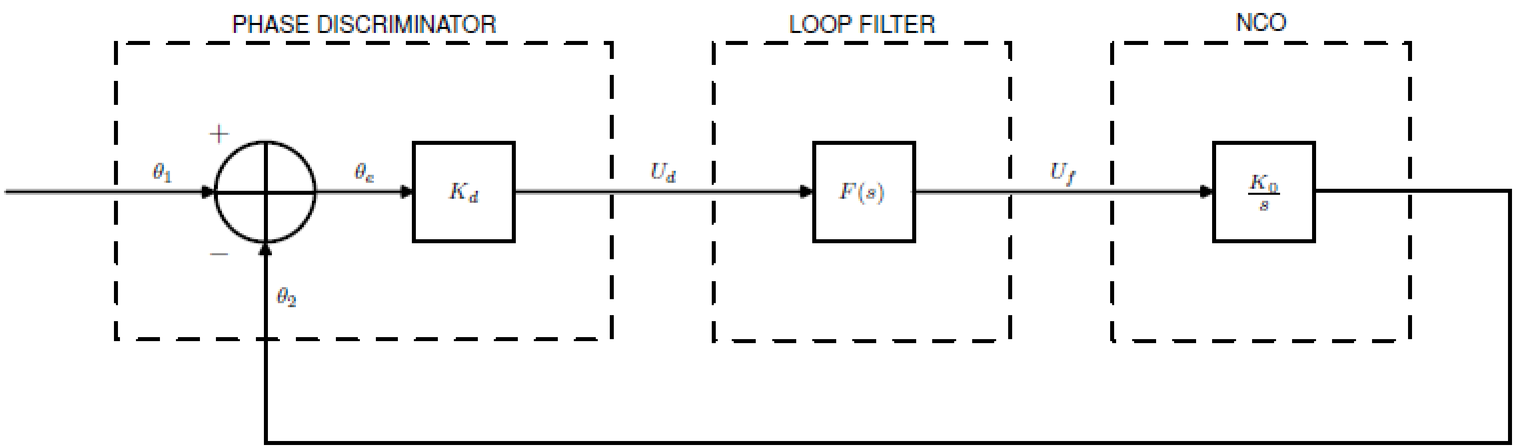
$$\left|Z(k)\right| = \left|X(k)H^*(k)\right|$$

- Acquisition uses circular correlation for

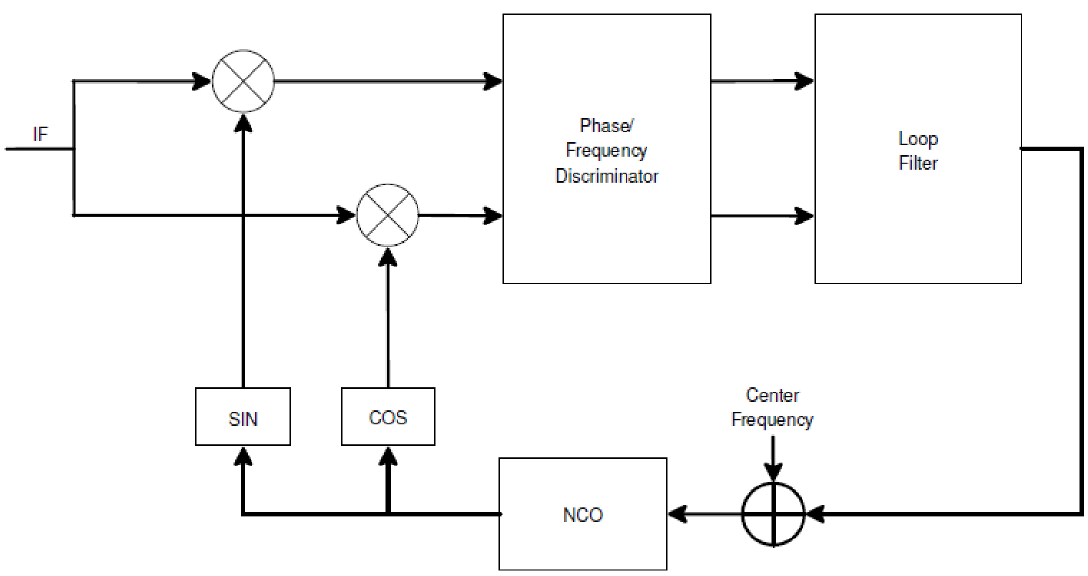
Tracking loop

- After acquisition- approximate carrier frequency and starting point of C/A code known
- More accurate approximation by tracking
- Continuously following the incoming carrier & code
- Acquisition performed at regular intervals or on unlocking of loop
- Maximum noise bandwidth, sampling time and damping factor are known
- Loop gain parameters, filter coefficients calculated

Tracking loop



Conventional tracking loop



Costas phase lock loop

PLL equations

$$\textit{Overall transfer function } \mathrm{H(s)} = \mathrm{k}_0 k_d F(s)/s + \mathrm{k}_0 k_d F(s)$$

Where k0 and kd are gain of discriminator and NCO respectively. F(s) being the loop filter transfer function given by

$$F(s) = 1 \text{ for first order PLL}$$

$$F(s) = \tau_2 s + 1/\tau_1 s \text{ for 2nd order PLL}$$

τ_2 *and* τ_1 are calculated based on given noise bandwidth and sampling frequency.

Noise bandwidth is given by

$$B_n = \int_0^{\infty} \left|H(j\omega)\right|^2 df$$

Which comes out to be

$$\mathrm{B_n} = \omega_n / 2\,(\delta + 1/4\delta)$$

Continuous to discrete domain transforms

- Bilinear transform and infinite impulse transform most commonly used
- Convert individual blocks from continuous to discrete domain rather than complete digital design

Equation in discrete domain becomes

$$\frac{k_0 k_d F(z) N(z)}{1 + k_0 k_d F(z) N(z)}$$

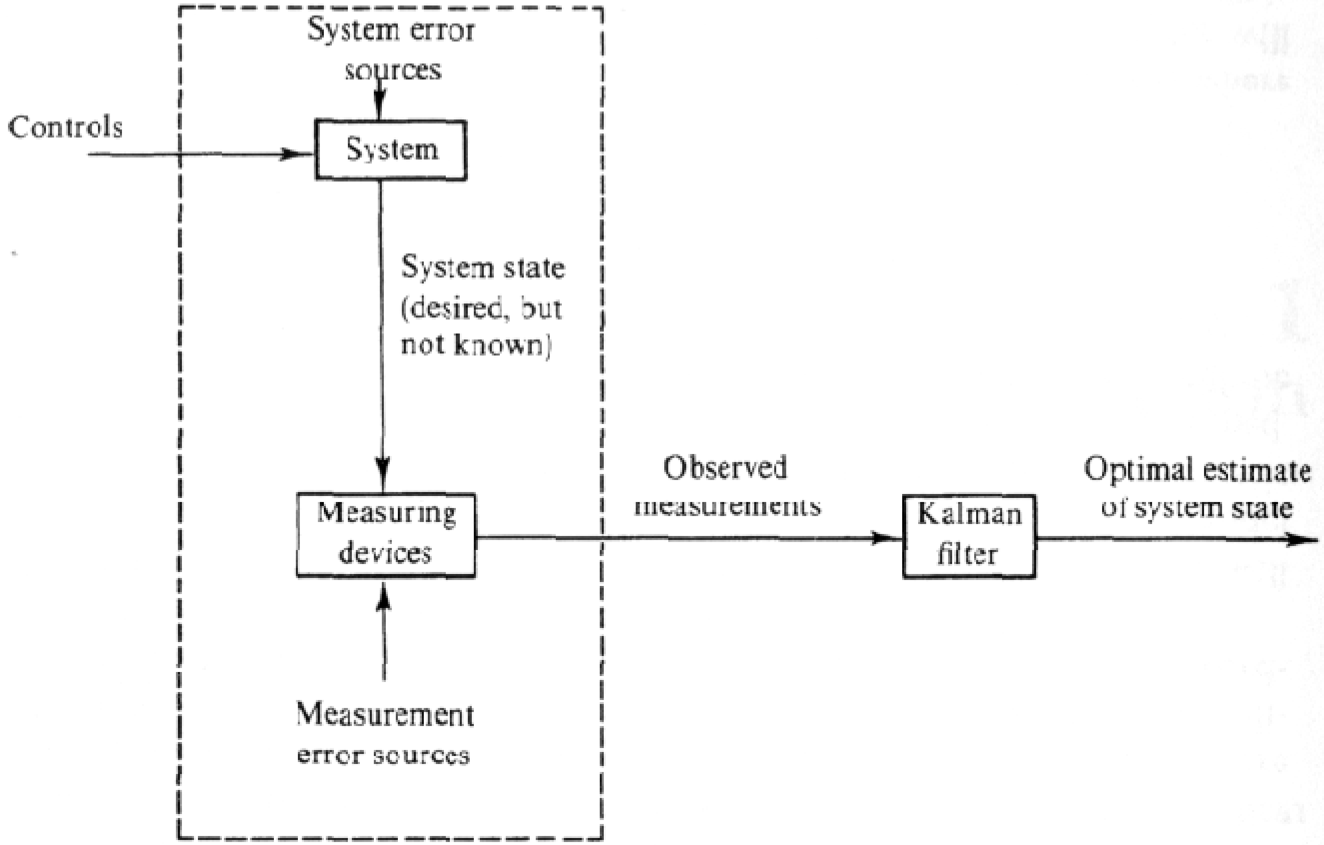
Bilinear transform performed as

$$s = 2/t_s \left(\frac{1 + z^{-1}}{1 - z^{-1}} \right)$$

and impulse invariant transform as

$$\left(\frac{a_k}{s - s_k} \right) \Rightarrow \left(\frac{a_k T_s}{z - e^{s_k} T_d} \right)$$

Kalman Filter

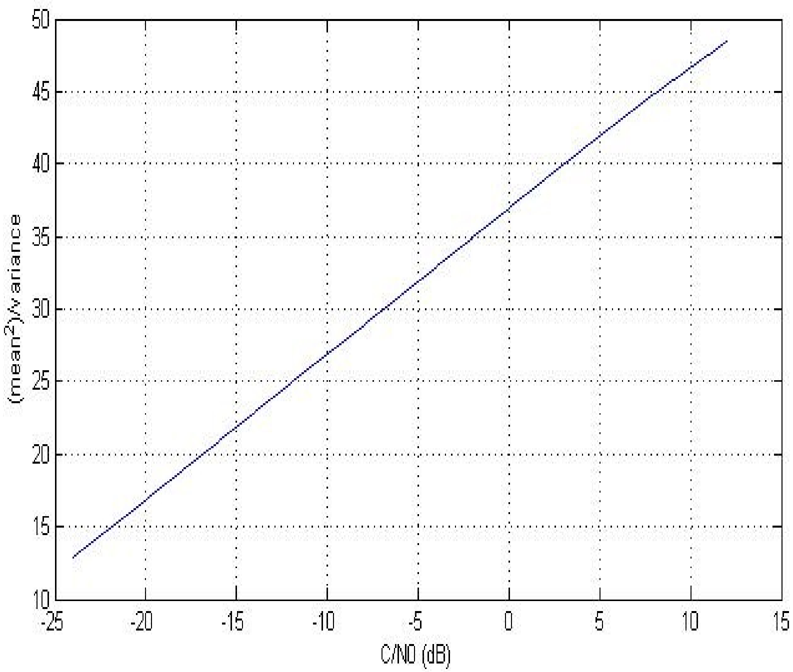


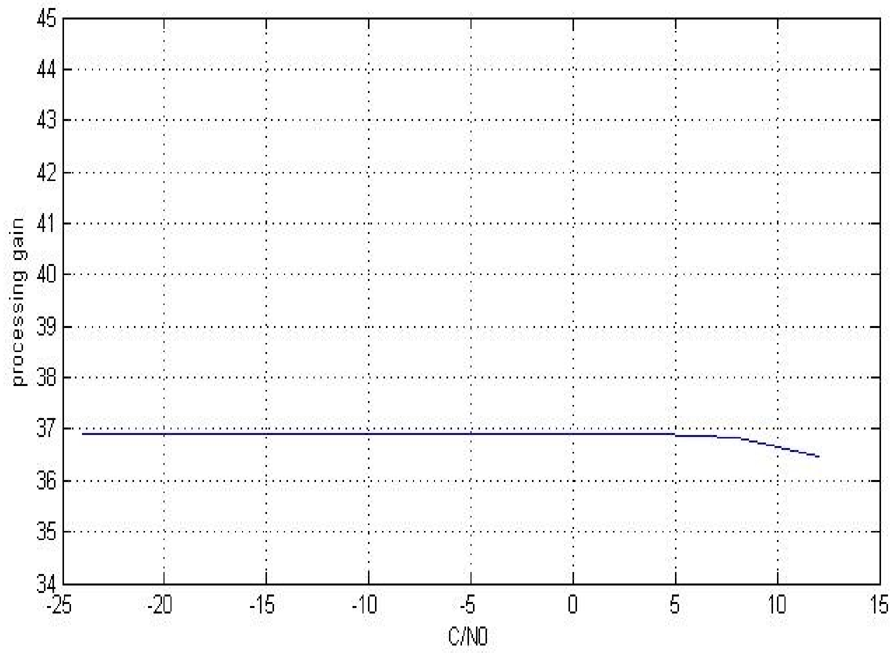
Kalman filter

- Optimal recursive data processing algorithm
- Optimal in all aspects- uses all measurements available regardless of their precision
- Minimises statistical error by doing this
- Recursive-does not need to keep all previous measurements in memory
- Assumptions- linear system model with white and Gaussian noise
- Whiteness- noise value not correlated in time & equal power at all frequencies
- Gaussian- number of noise sources in system and measurement device so sum Gaussian

Review II

Gain due to Integrate and dump





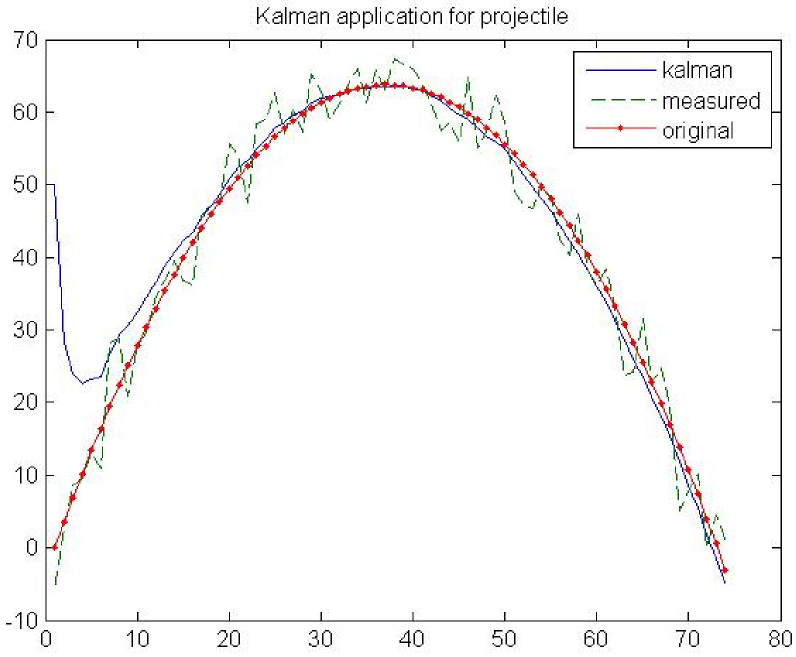
Values of SNR (dB) after integrate and dump block and its constant processing gain value.

Kalman filter Equations

BLUE = inputs ORANGE = outputs BLACK = constants GRAY = intermediary variables

State Prediction (Predict where we're gonna be)	$\mathbf{x}_{predicted} = \mathbf{A}\mathbf{x}_{n-1} + \mathbf{B}\mathbf{u}_n$
Covariance Prediction (Predict how much error)	$\mathbf{P}_{predicted} = \mathbf{A}\mathbf{P}_{n-1}\mathbf{A}^{\mathbf{T}} + \mathbf{Q}$
Innovation (Compare reality against prediction)	$\tilde{\mathbf{y}} = \mathbf{z}_n - \mathbf{H}\mathbf{x}_{predicted}$
Innovation Covariance (Compare real error against prediction)	$\mathbf{S} = \mathbf{H}\mathbf{P}_{predicted}\mathbf{H}^{\mathbf{T}} + \mathbf{R}$
Kalman Gain (Moderate the prediction)	$\mathbf{K} = \mathbf{P}_{predicted}\mathbf{H}^{\mathbf{T}}\mathbf{S}^{-1}$
State Update (New estimate of where we are)	$\mathbf{x}_n = \mathbf{x}_{predicted} + \mathbf{K}\tilde{\mathbf{y}}$
Covariance Update (New estimate of error)	$\mathbf{P}_n = (I - \mathbf{K}\mathbf{H})\mathbf{P}_{predicted}$

An example tested for Kalman filter



- Test run for projectile motion
- Measurements corrupted with noise provided with incorrect initial values yet tracking done
- Role of parameters present in the equation understood

Formation of filter equations for PLL

- Phase difference and frequency difference were selected to be the parameters for state equations
- Requirements: Design of FLL, error variance in measurement of phase and frequency
- Deciding the control inputs for the state equations
- Finding variance for ϕ different values of C/N_0 which are 40 dB-Hz to 80 dB-Hz for GPS signals

FLL Design

- Provides frequency difference between incoming and locally generated signals
- Equation as follows:

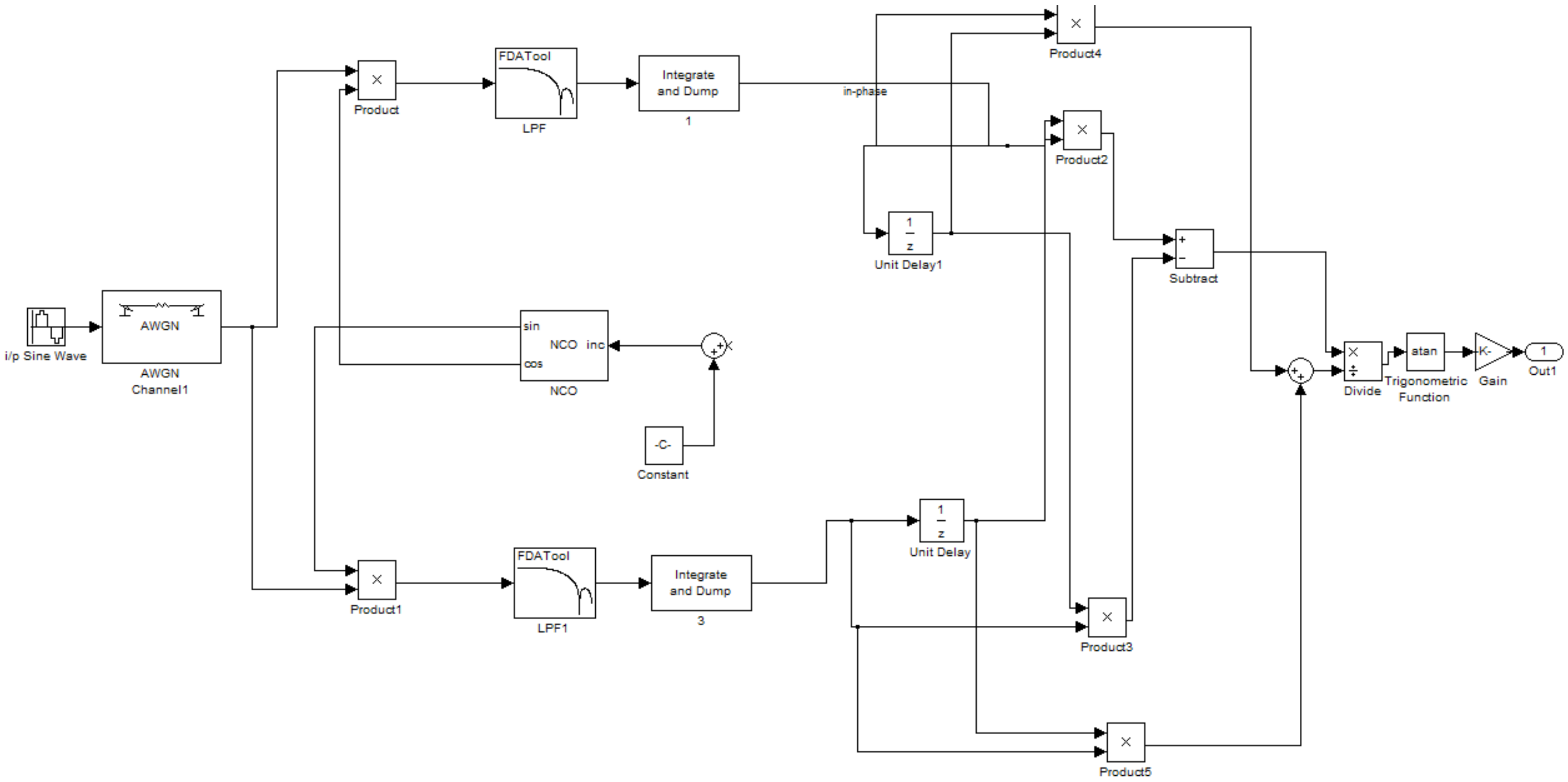
$$\text{atan2(dot,cross)/(t2-t1)}$$

- Where $dot = IP_1\ IP_2 + QP_1\ QP_2$

$$cross = IP_1\ QP_2 - IP_2 \times QP_1$$

- This discriminator is optimal at low to high signal to noise ratios

FLL Loop



i/p Sine Wave
AWGN
AWGN Channel1
Product
FDATool
LPF
Integrate and Dump
1
in-phase
Product4
Product2
1
z
Unit Delay1
Subtract
sin
NCO inc
cos
NCO
-C-
Constant
Divide
atan
Trigonometric Function
-K-
Gain
1
Out1
Product1
FDATool
LPF1
Integrate and Dump
3
1
z
Unit Delay
Product3
Product5

Benefits of kalman filtering and costas loop

- Loop is unaffected by sign change due to data bits due to use of division of in-phase and quadrature components
- Limitation of low bandwidth of loop filter is overcome by using kalman filter.
- Able to track even in case of high velocity users having high doppler frequency
- Optimal estimate using all available measurements

Different Filter implementations

- Smoother-Based GPS Signal Tracking in a Software Receiver, Mark L. Psiaki, *Cornell University*
- Kalman Filter Based Tracking Algorithms For Software GPS Receivers, Mathew Lashley, Thesis, Auburn University
- Above two implemented and innovation of change in input values done- desired output values obtained

Challenges in design

- Non functioning of PLL due to divide by zero at

 t=T as sine value dropped to zero

- Kalman filter not showing any output values beyond first sample as first output came NaN (not a number)
- Selection of values of error variance in measurement (open loop or closed loop)
- State equation not producing desired output
- Very high variance in FLL output

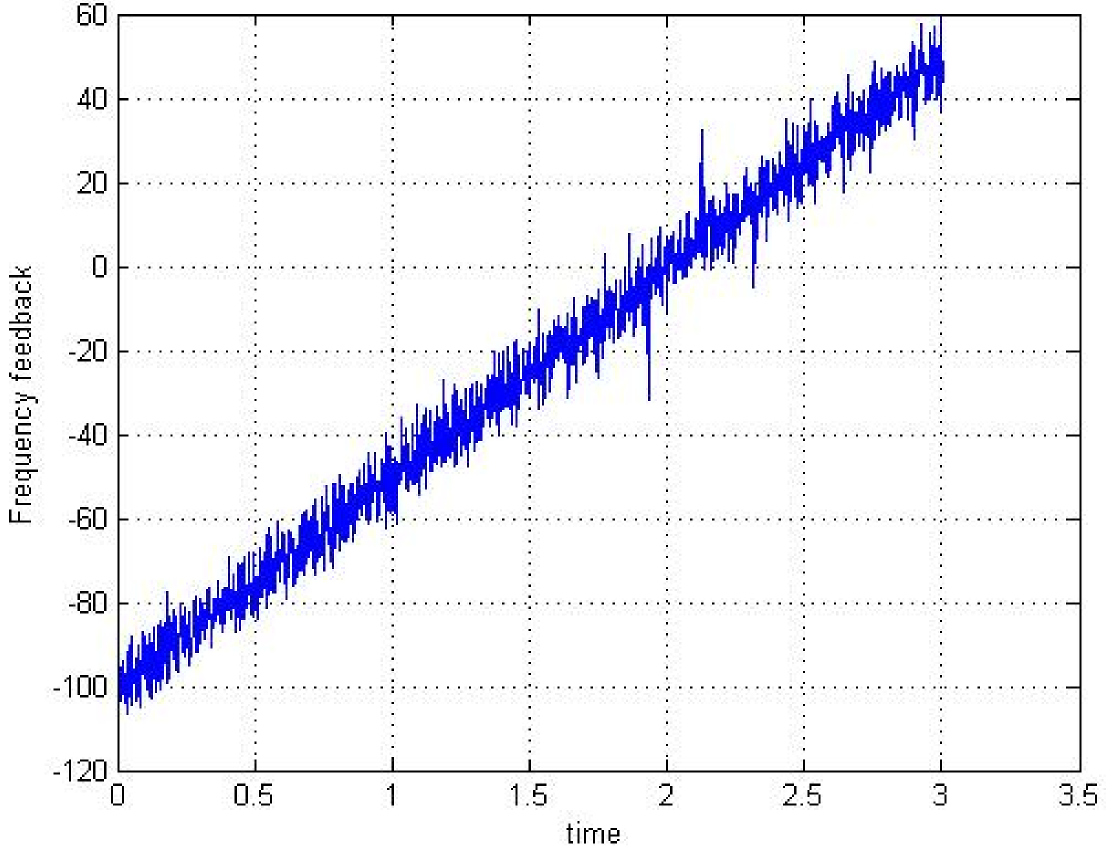
Solutions to design challenges

- Addition of very small value (10e-6) to Quadrature phase component to prevent divide by zero error
- Debugging to find reason for non functioning Kalman filter after first sample
- Source found to be initial values of FLL set as zero causing NaN in output and the same loop iterations again and again
- Initial values altered and output obtained
- Error variance measured for closed loops

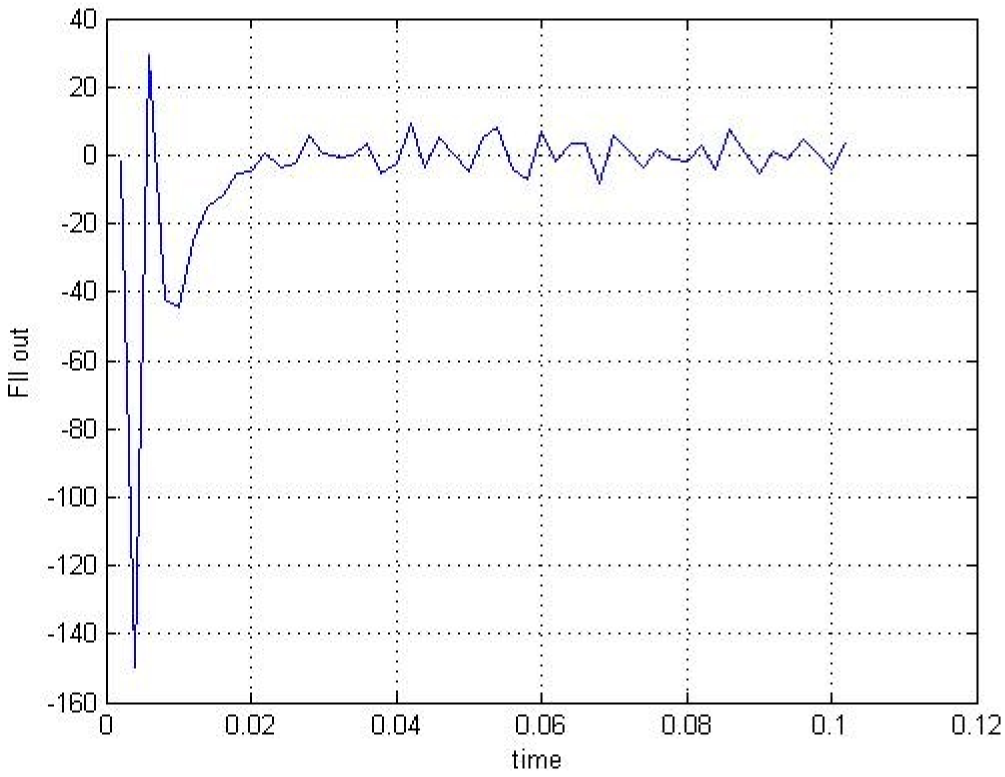
Solutions to design challenges

- Process error variance and measurement error variance measured setting up high values for other
- State equation used input as frequency difference output from FLL which failed to get desired output
- So, difference of previous state of frequency and FLL output taken
- FLL outputs averaged for decreasing variance

Results: Lock even in high dynamics



Results: Higher loop bandwidth and locking

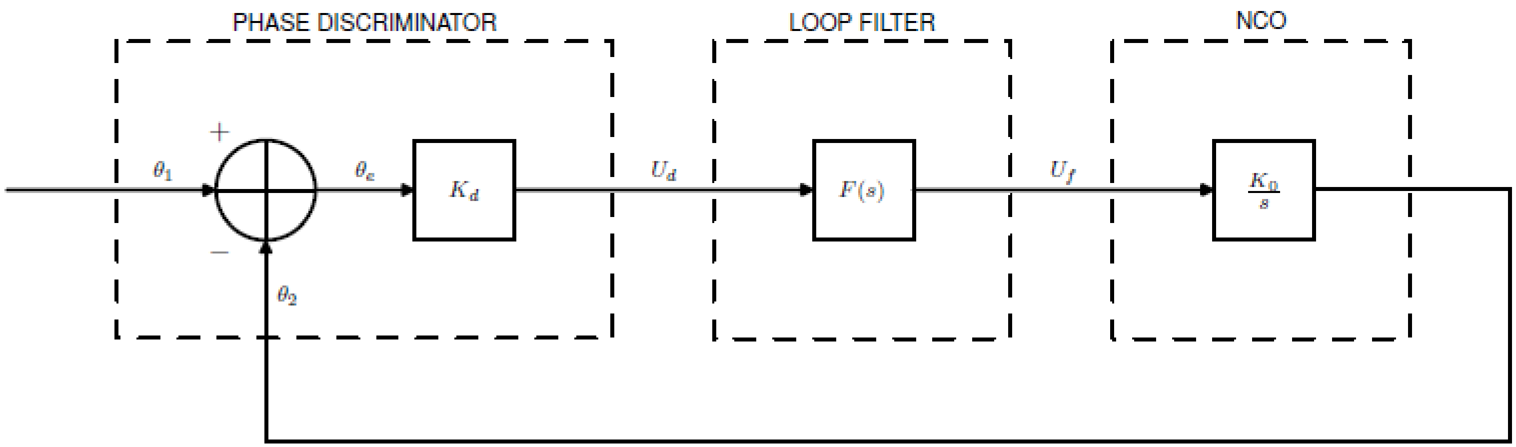


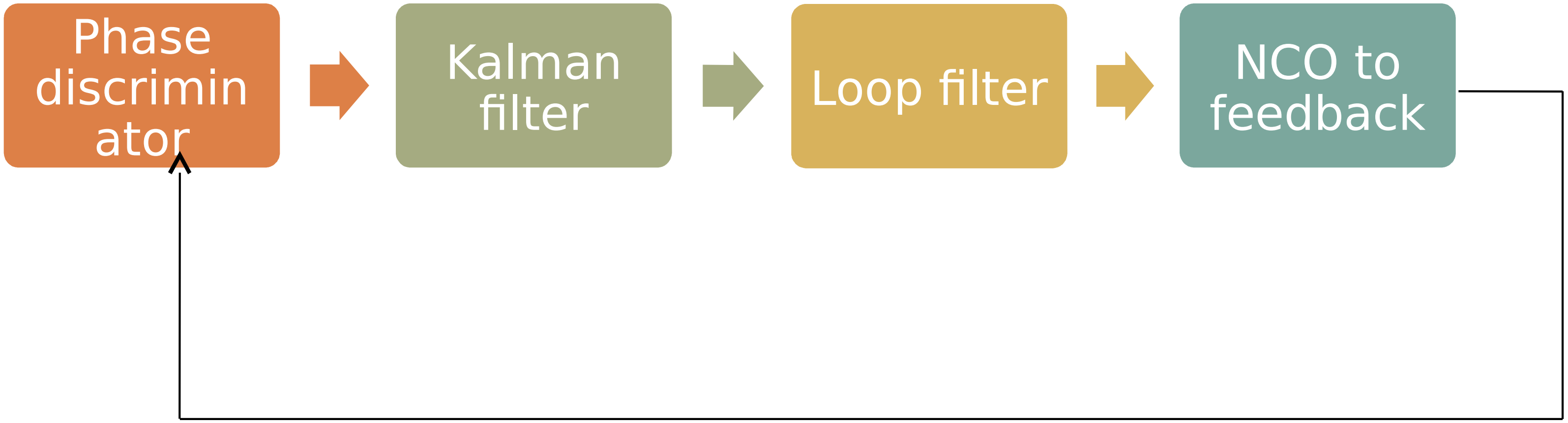
Review III-Kalman filter based GPS tracking

List of Simulations carried out

- PLL and FLL based on thesis by Matthew Lashley
- PLL based on 2 papers by M L Psiaki - GPS Signal Tracking in high dynamics and low C/N_0
- One innovation in design of FLL using different equations from those given in the papers
- Cascading of Kalman filter
- All simulations performed at 40 dB-Hz-at lower value of SNR than the average 45

Kalman filter incorporation





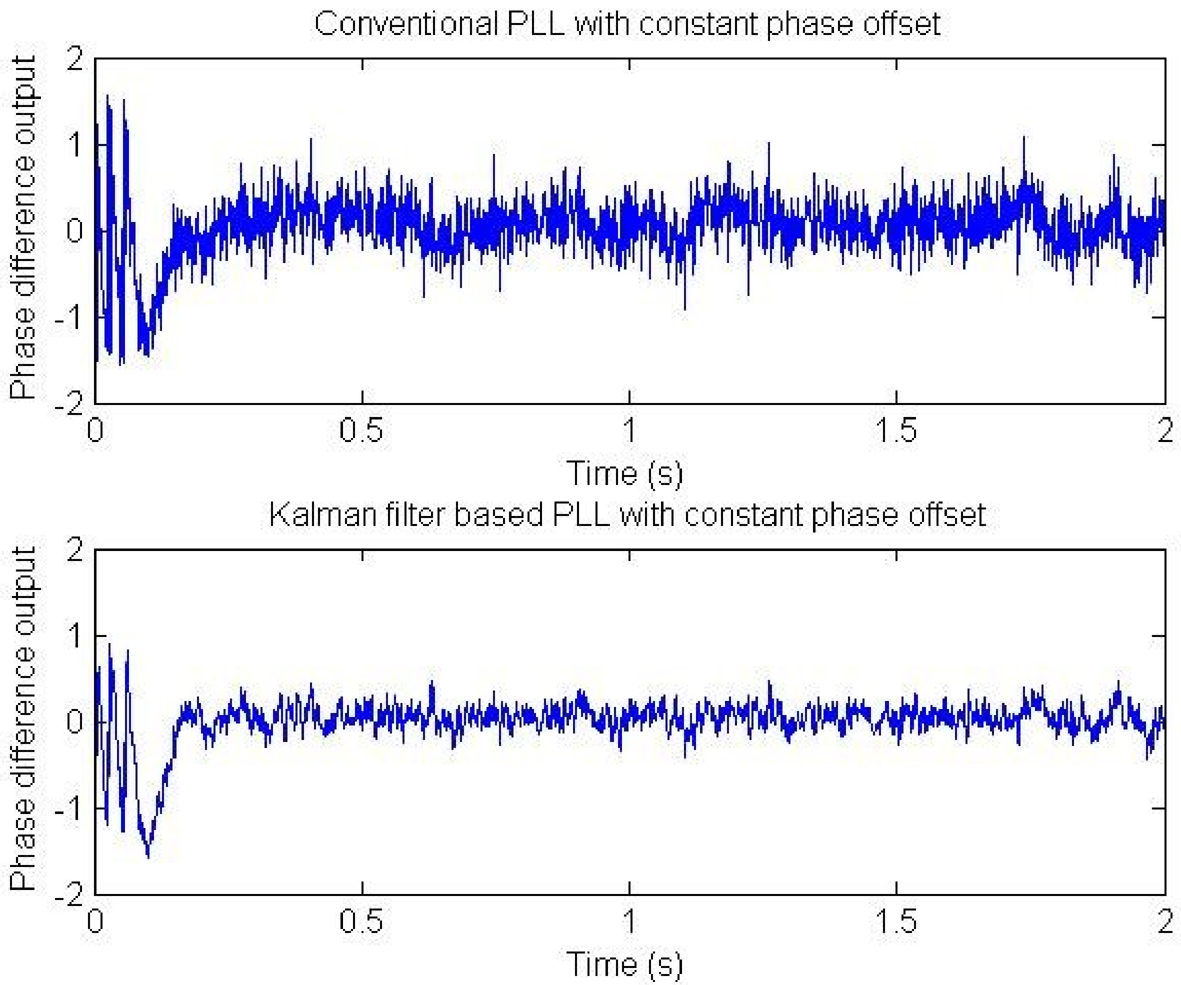
Matthew Lashley thesis

- Equations

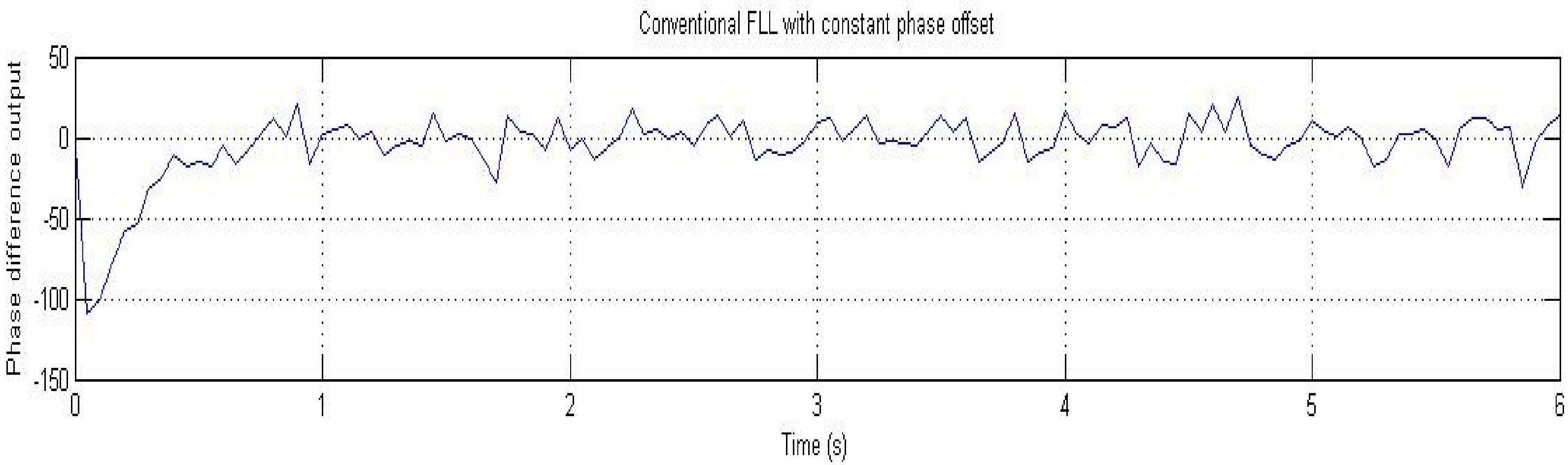
$$\begin{bmatrix} \theta_{c,k+1} \\ f_{c,k+1} \\ \frac{df}{dt}_{c,k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \theta_{c,k} \\ f_{c,k} \\ \frac{df_{c,k}}{dt} \end{bmatrix}$$

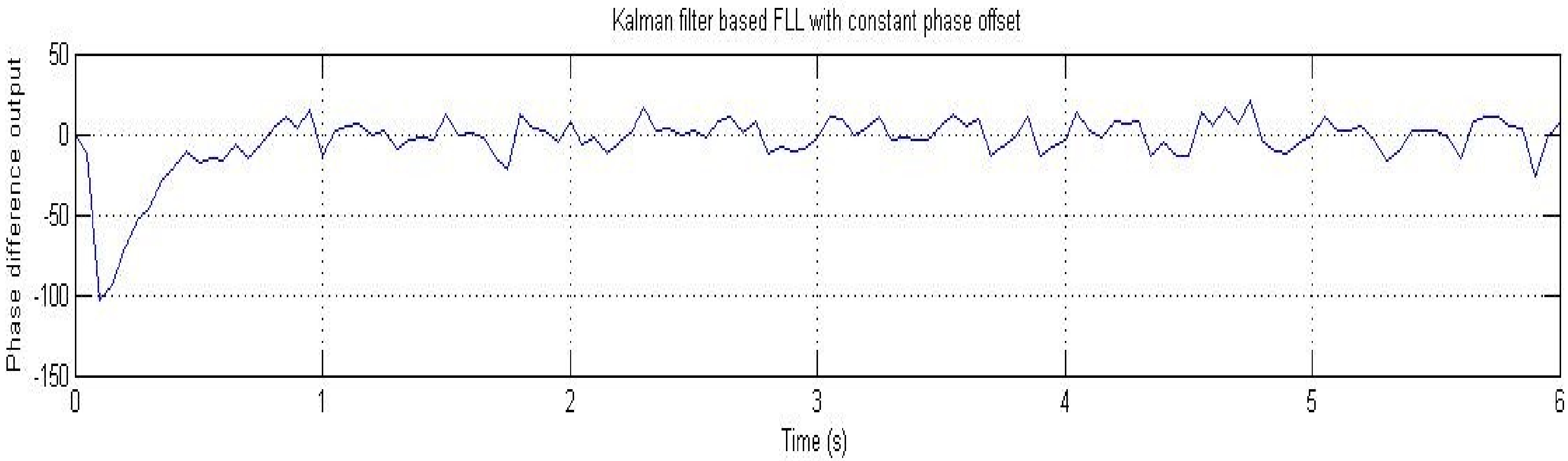
- Just depends on previous state, no input values.
- Test performed show it is able to maintain phase lock even after outage situation.

Comparison: discriminator output and kalman filtered output



Another variant: Using frequency difference for feedback

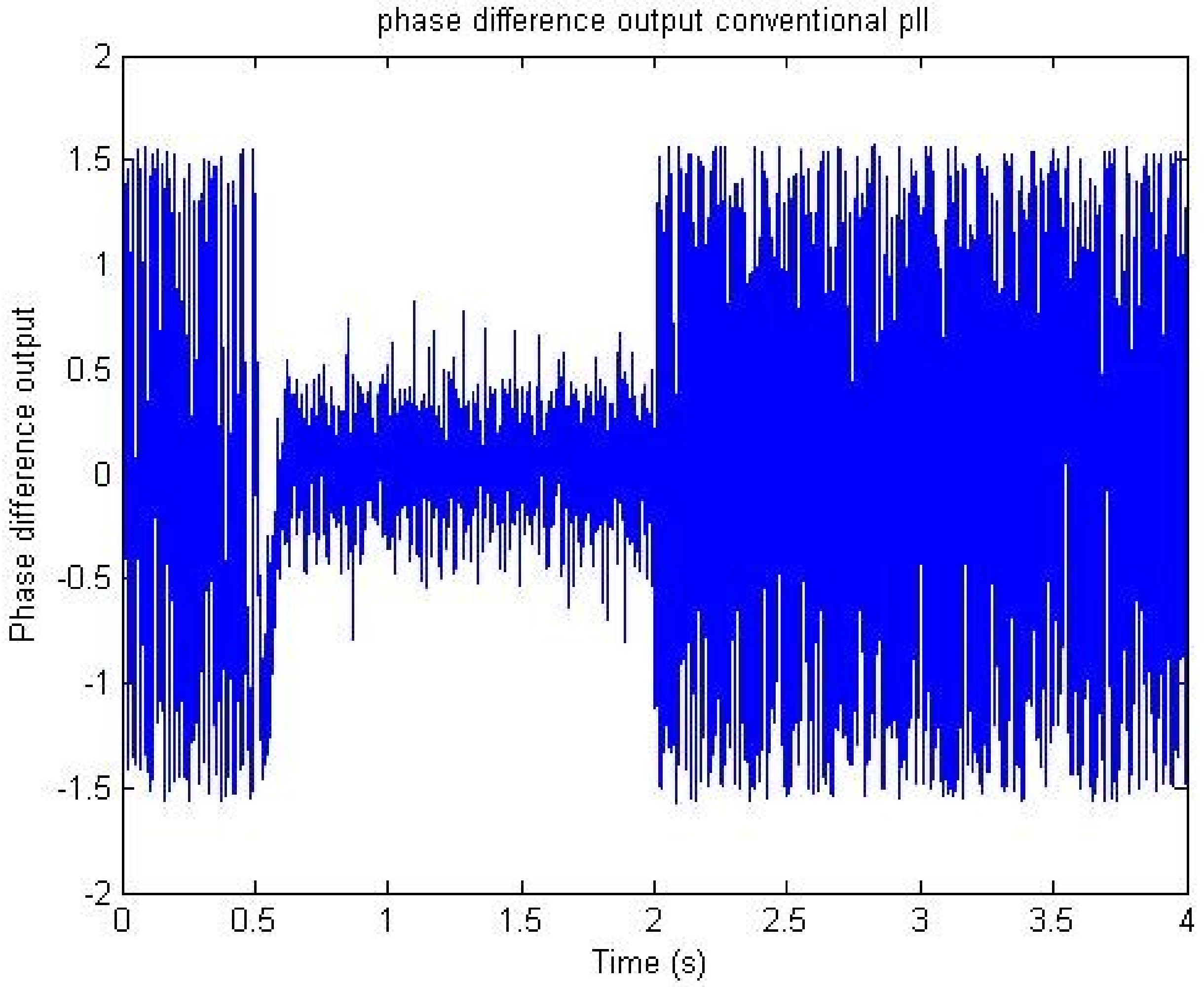




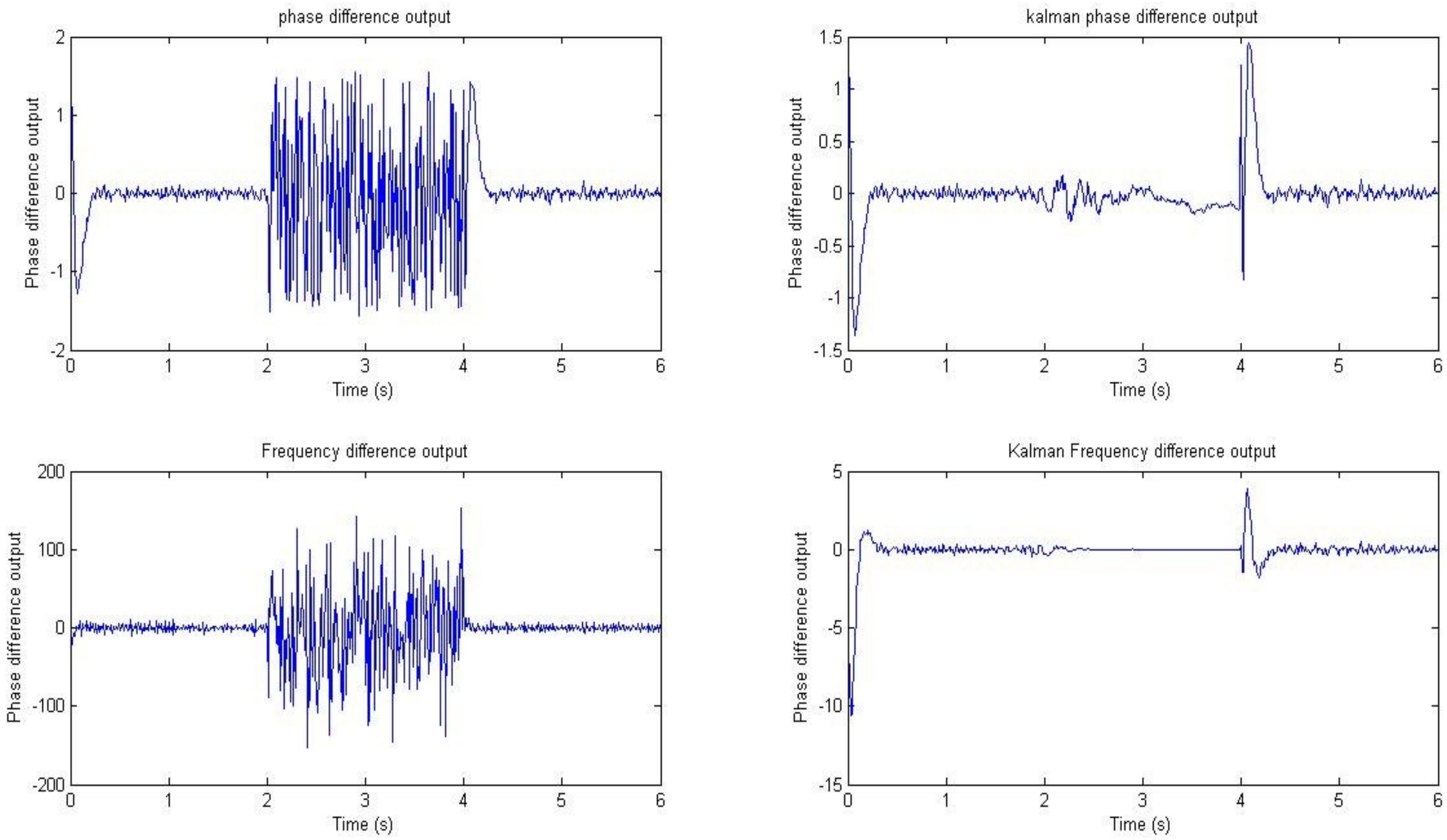
PLL with 10 ms integration time in outage

- Increased integration time improves gain of the integrate and dump block
- Upper limit is 10ms due to data bit transition boundary
- Filter coefficients modified as sampling time changed
- Initially it was taken as 1 ms, now increased to 10 ms which increases gain to 47 dB from 37 dB.
- Entire loop along with Kalman filter now works on 10 ms sampling time after integrate & dump
- Improvement in PLL performance even in outage case as shown in results

Coventional PLL: failure during outage



Outage case in PLL with kalman filter



How this works

- Conventional PLL is not weighted and thus due to f_d being > f_{pullin} it loses lock during outage
- Kalman filter being weighted filter it effectively neglects measurements during outage
- It predicts the present state values based on the values just before signal outage occurs and thus maintains phase lock
- Measurement error variance values increased during outage to make it neglect measurements
- Process error variance nearly zero

Model 2: based on papers by M L Psiaki

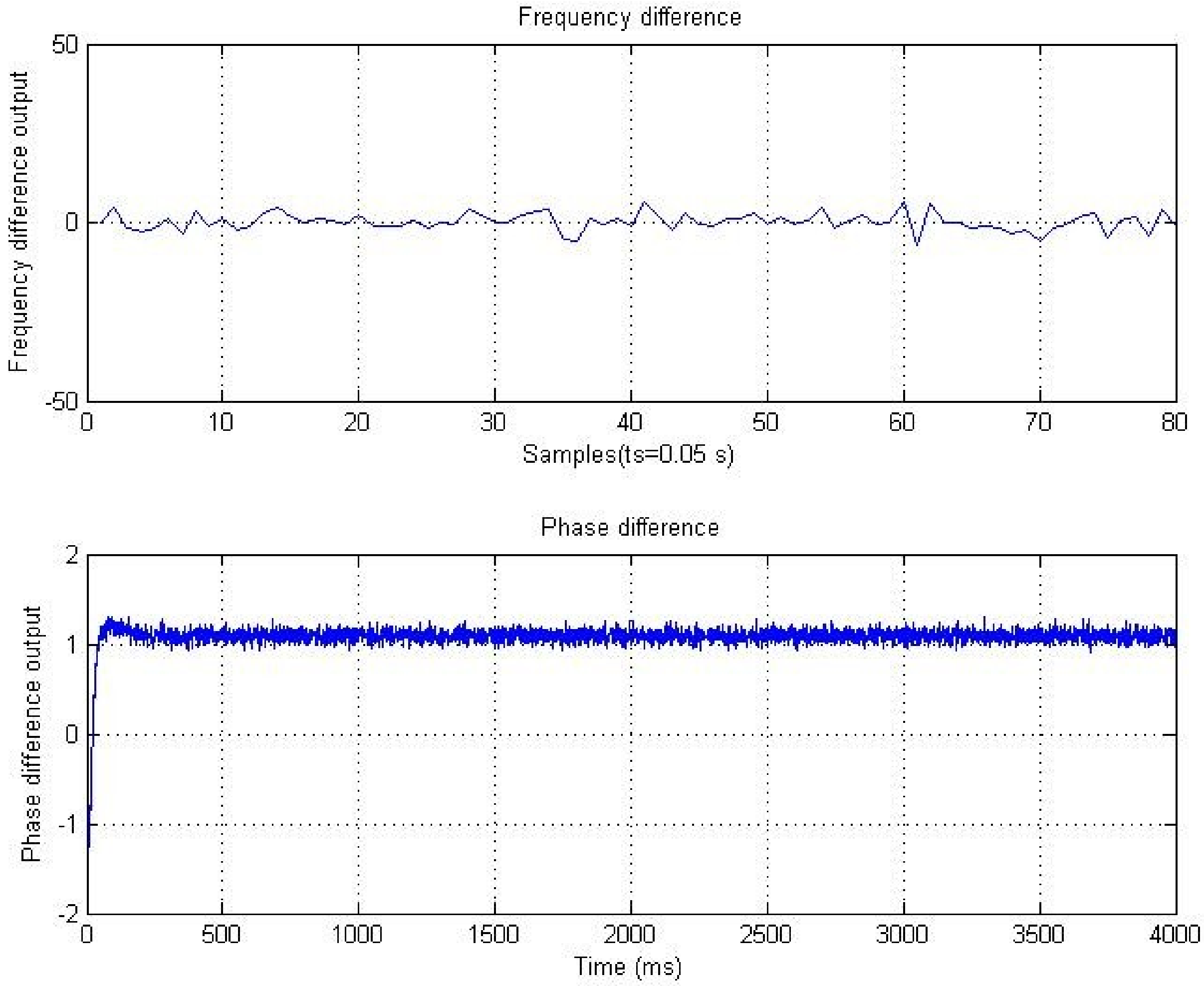
- Equations:

$$x_{k+1} = \begin{bmatrix} \theta_{c,k+1} \\ f_{c,k+1} \\ \frac{df}{dt}_{c,k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \theta_{c,k} \\ f_{c,k} \\ \frac{df_{c,k}}{dt} \end{bmatrix} - 2 * pi * \Delta t * \begin{bmatrix} f_{re} \\ 0 \\ 0 \end{bmatrix}$$

$$f_{re(k+1)} = \left[1 \quad 2 * \pi * \Delta t \quad 4 * \pi * \Delta t\right] * x_k - f_{re(k)}$$

- Locks frequency but cannot lock phase offset hence constant phase offset remains

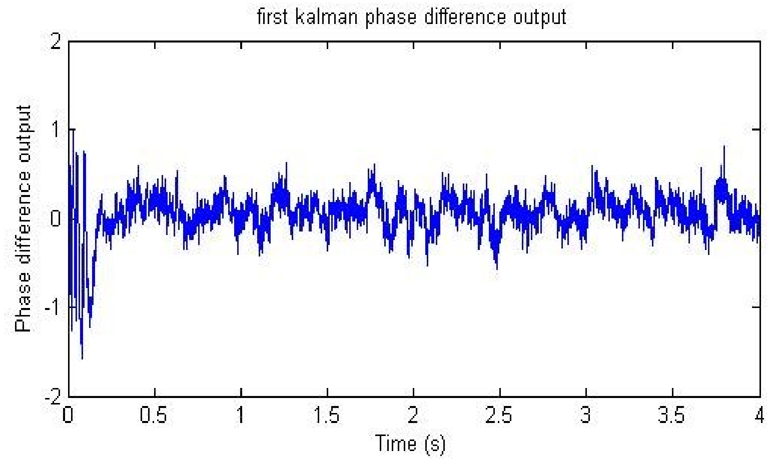
Results by use of the model

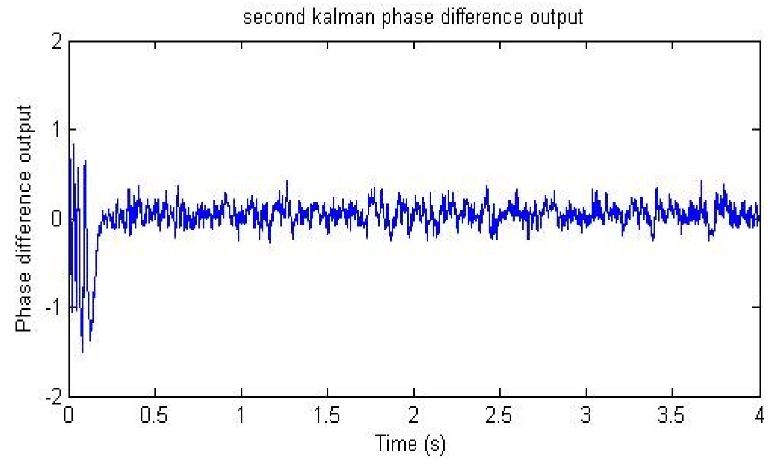


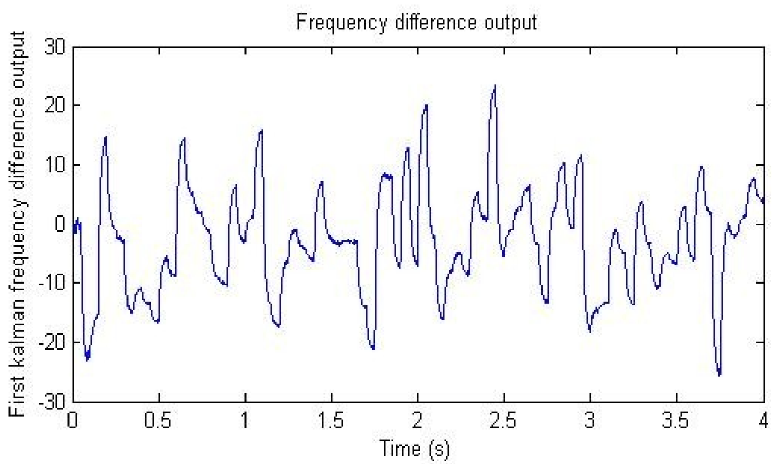
Cascade of kalman filters

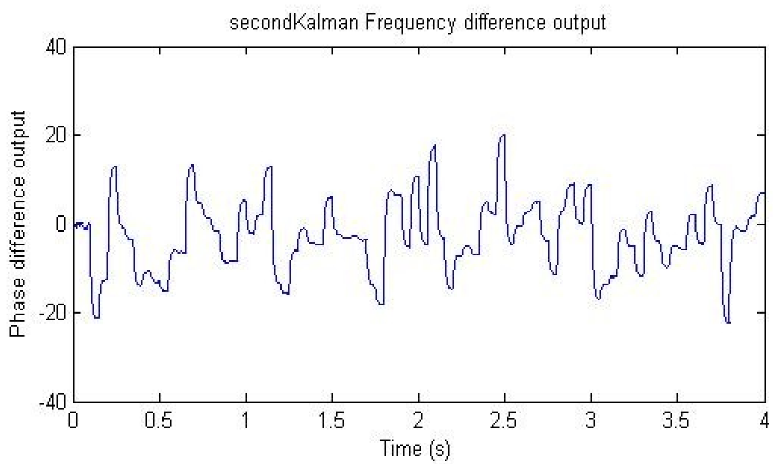
- Cascade tried for further reduction in noise error variance
- Too much increase in computational complexity
- One kalman filter block followed by other with reduced measurement error variance

Results comparison









Conclusion

Criteria	Conventional PLL	K F based PLL
Constant phase offset	Tracks for any value	Tracks for any value
Constant frequency offset	Can track upto certain range (pull-in)	Can track upto much higher range than conventional
Frequency modulated signal	Can handle for very low dynamics	Can track high dynamics
Signal outage	Completely unlocks and cannot attain lock again	Attains lock immediately on re appearance of signal
Post signal outage	Re-acquisition required	No requirement of carrying out acquisition again
Measurement noise variance	Same amount as found in actual measurements	Reduction in variance by use of state equations

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Thank You

On a lighter note…