

(b)

Figure 5.9 Continued

frequency is at the band edge, 280 Hz. Using $C = 0.3 \mu\text{F}$, we find $R = 1.9 \text{ k}\Omega$, which we may set equal to R/k ($k = 1$) if we pick $d = 1.381$; also $QR = 1.52 \text{ k}\Omega$. Finally, we need to set b ; from $[a - b(kQ)] = 0$ we find

$$b = a/(kQ) = 0.0316/0.8 = 0.0395$$

The lowpass notch circuit and its performance are shown in Fig. 5.9. The transmission zero is at 1.9 kHz ($= f_0\sqrt{\sqrt{2}/a}$) as expected. The summer bandwidth limitations reduce the high-frequency gain by 3 dB (to -33 dB) at approximately 650 kHz, in agreement with Eq. (5.4): $1.5 \text{ MHz}/(1 + a + b + d) = 1.5 \text{ MHz}/2.45 \approx 610 \text{ kHz}$.

We now see the versatility of the four-opamp biquad circuit. Starting with the most general structure, Fig. 5.1 described by Eq. (5.5), simply disconnecting certain resistors from the circuit and reevaluating others, we can design entirely arbitrary transfer functions. In this sense the circuit in Fig. 5.1 can be considered a universal filter, easily and economically implemented using packages with four opamps (so-called *quads*) on one integrated circuit. The circuit can be tuned in a noninteractive manner for precise filter parameters. The pole frequency ω_0 is tuned by varying the resistor R at the output of the first opamp (R_4 in Fig. 4.10), Q is set by the resistor QR , and for bandpass and lowpass filters the gain is controlled by R/k . In the full biquadratic function of Eq. (5.5), the parameter a controls the high-frequency gain and c or d are chosen to set the low-frequency gain. At the same time c and/or d , of course, also determine the zero (notch) frequency, and b is adjusted to control the notch depth. We present in Table 5.1 a summary of the transfer functions that we found to be realizable with the four-opamp circuit. Next we shall discuss briefly the phase response of the circuits.

5.1.1 Phase Response of the General Biquadratic Circuit

Apart from the allpass, biquadratic circuits are normally designed for their magnitude behavior. Nevertheless, the engineer should have an understanding of the phase response of biquads. The circuits are described by Eq. (5.1). On the $j\omega$ -axis we have

$$\begin{aligned} T(j\omega) &= \frac{N(j\omega)}{D(j\omega)} = \frac{N(j\omega)}{-\omega^2 + j\omega\omega_0/Q + \omega_0^2} \quad (\text{comp) } 220 \text{ for } S = j\omega \\ &= \frac{|N(j\omega)| e^{j\theta_1}}{|D(j\omega)| e^{j\theta_2}} = \left| \frac{N(j\omega)}{D(j\omega)} \right| e^{j(\theta_1 - \theta_2)} \end{aligned} \quad (5.22)$$

TABLE 5.1 Standard Forms of Second-Order Responses

	Frequency Response	Poles/Zeros	Name
$T_{LP} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$			Lowpass
$T_{BP} = \frac{\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$			Bandpass
$T_{BE} = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$			Bandstop "notch"
$T_{HP} = \frac{s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$			Highpass
$T_{AP} = \frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$			Allpass
$T_{LPN} = \frac{s^2 + k^2 \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $\omega_z = k \cdot \omega_0, k > 1$			Lowpass notch
$T_{HPN} = \frac{s^2 + h^2 \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $\omega_z = k \cdot \omega_0, k < 1$			Highpass notch

el denominador
D(s) es el
mismo en todos
los casos.

nuevas
configuraciones.

De esta manera, le puede agregar una ganancia
"K" a la banda de pasobaja correspondiente.
 Si es notch → pasa altos en NHP tengo ganancia.
 Si es notch → pasa bajos en NHP tengo derivación.

We discussed in Chapters 1 and 4 that the phase of $T(j\omega)$ is obtained by subtracting the phase of the denominator from that of the numerator:

$$\phi(\omega) = \theta_1(\omega) - \theta_2(\omega) = \theta_1 - \tan^{-1} \frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2} = \theta_1 - \tan^{-1} \frac{(\omega/\omega_0)(1/Q)}{1 - \omega^2/\omega_0^2} \quad (5.23)$$

We recognize

$$\theta_{LP}(\omega) = -\tan^{-1} \frac{\omega\omega_0/Q}{\omega_0^2 - \omega^2} \quad (5.24)$$

as the phase of a (noninverting) lowpass function, Eq. (4.31). It was plotted in Fig. 4.13b for various values of Q and normalized frequencies ω/ω_0 . The phase of the numerator has in general the same form as $\theta_2(\omega)$ but depends, of course, on the specific transfer function. We show in Table 5.2 the most common forms of $N(s)$ and the phases of $N(j\omega)$. According to Eq. (5.23) we only have to add the phases in Table 5.2 to $\theta_2(\omega)$ to obtain the total phase of the biquad. We note that in a lowpass, $N(j\omega) = \omega_0^2$ so it adds zero to $\theta_2(\omega)$; in a bandpass function, we have $N(j\omega) = j(\omega\omega_0/Q)$, which adds $+90^\circ$ to $\theta_2(\omega)$, and a highpass has $N(j\omega) = -\omega^2$, which means the total phase equals $\phi(\omega) = 180^\circ - \theta_2(\omega)$. Since the difference between the phases contributed by the different filters is only a constant, we show in Fig. 5.10 the resulting phase shifts for various values of Q plotted versus different axes. Finally we note that the phase of an allpass filter, Eq. (5.18), is simply twice that of a lowpass filter, and that the phase of the numerator of a notch filter changes abruptly at $\omega = \omega_0$ from zero to 180° . Figure 5.10 shows the result. Our discussion assumed that all filters are noninverting; if the filter is inverting, the factor (-1) simply adds 180° to the results in Fig. 5.10.

fase del numerador $N(s)$
constante

TABLE 5.2 Common Forms of $N(s)$ and Phases of $N(j\omega)$

Name	$N(s)$	$N(j\omega)$	Plot of $\theta_1(\omega)$
Lowpass	ω_0^2	ω_0^2	
Bandpass	$\frac{\omega_0}{Q}s$	$j\frac{\omega\omega_0}{Q}$	
Bandstop	$s^2 + \omega_0^2$	$-\omega^2 + \omega_0^2$	
Highpass	s^2	$-\omega^2$	

Piso de fase
constante que
aporta el
numerador en
estas
transferencias.

salto
de
fase en
en
eliminación
banda.

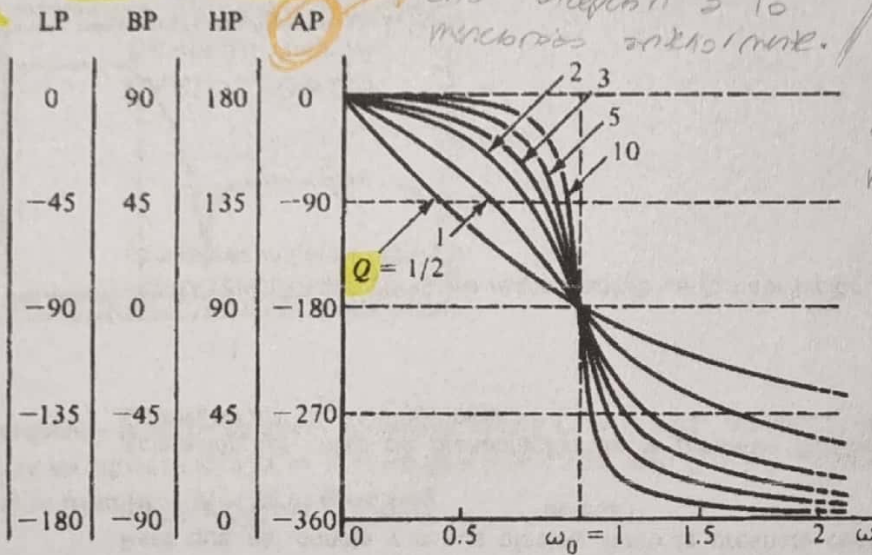
Importante: la fase del numerador $N(s)$ es constante para todos los valores de ω (salvo en el filtro notch), la fase que va variando es la del denominador $D(s)$, por eso varía la fase total de la transferencia.

Cada numerador aporta un "Piso" de fase distinto.

Para los 225 } El numerador SIEMPRE tiene un término ω_0^2 , mientras que la fase
 Para los 225 } del denominador varía entre 0° y 180° (π rad)
 Para los 225 } el ejemplo más claro es en el LPF.
 Para los 225 }

Tipo de
 filtro

ϕ , degrees for case:



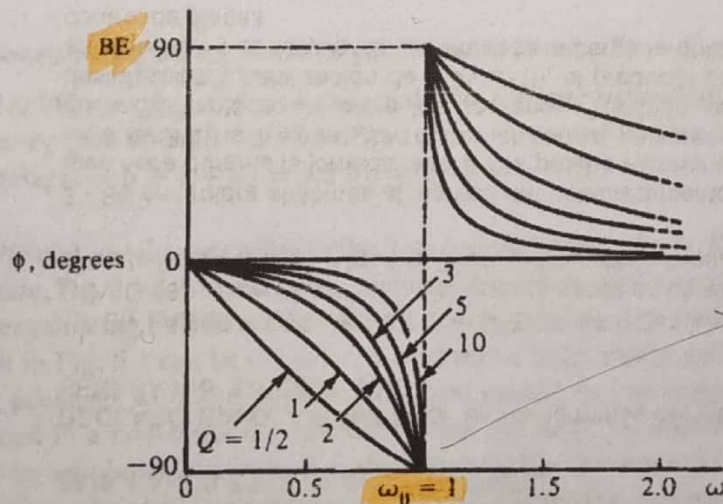
La forma de la
 fase es siempre
 igual, cambia
 solamente los puntos
 en donde
 empieza y termina,
 los grados (º)
 que recibe.

Siempre la fase
 con pendiente
 negativa, pero
 obtener con
 sistemas casuales.

Ver como afecta
 la variación de Q
 a la fase.

BE \rightarrow Band
 elimination filter

Filtro notch



el filtro notch no
 comparte el mismo
 gráfico de fase que
 los filtros anteriores,
 por eso está a parte.

Se puede apreciar el
 salto de fase en
 $\omega_0 = 1$, característica del
 filtro notch.

Figure 5.10 The phases of the most common second-order filters.

We next discuss the second method for the creation of transmission zeros, feeding the input signal into different nodes in the circuit.

5.2 BY VOLTAGE FEEDFORWARD

Poles are the *natural frequencies* of a system and we recall from mathematics or elementary circuits courses that the natural frequencies of a system are determined when the excitation is zero. In our situation, circuit analysis, this means all inputs are set to zero. This insight provides us with the clue on how to generate the needed inputs. For achieving some desired polynomial $N(s)$ in Eq. (5.1) without disturbing the roots of $D(s)$, the poles, we must feed the input voltages to the circuit in such a fashion that the core pole-generating circuit is restored when the inputs are set to zero. Figure 5.11 illustrates how this can be done. The figure shows a part of a circuit, with two admittances displayed explicitly. We may now feed an input into