Decentralized Dispatch of Distributed Multi-Energy Systems with Comprehensive Regulation of Heat Transport in District Heating Networks

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***Abstract*—Distributed Energy Systems(DES) interconnected with Electric Power Network(EPN) and District Heating Network(DHN) have drawn great attention recently as they promote user-side coordination of multi-energy flows. However, the difference of physical nature between electric power transmission and heat transport has brought difficulties to the modelling and decentralized optimization. In this work, a new DHN model considering delay and storage features of pipeline heat migration and heat transfer between fluids is proposed through trigonometric expansion of the decision series and the heat current method. The model comprehensively characterizes the heat transport in the system and a dispatch problem considering hybrid regulation of fluid flow rates and temperatures in DHN is then established. A primal-decomposition-based decentralized gradient descent method in accompany with Alternating Direction Method of Multipliers(ADMM) is proposed to optimize the DESs in a fully decentralized manner. Case study validates the effectiveness of the proposed model and method to further harness the potential of DHN, which reduces renewable energy curtailment from 17.3% to 0.**

***Index Terms*—Distributed Energy Systems, District Heating Networks, Decentralized Optimization, Renewable Energy Sources, Alternating Direction Method of Multipliers**

I. Introduction

DISTRIBUTED Energy System(DES) is a promising solution to accomplish carbon neutrality, which consists of multi-energy sources, e.g., distributed renewables, fossil-fuel-based cogenerators and energy conversion and storage devices to meet end-users' diverse demands [1]. Meanwhile, with the Electric Power Network(EPN), the interconnected DESs are more resilient to demand and renewable energy fluctuations and thus have the potential to behave collaboratively. Besides, the District Heating Network(DHN) with delay and storage characteristics temporally decouples the heat generations and loads and thus can provide more flexibility to the renewable sources. Structurally, the DESs, usually belonging to different agents, also reveal a decentralized nature. Therefore, a decentralized coordination scheme may be preferred compared to the centralized dispatch through a system operator. The decentralized scheme encourages each DES agent to share information with neighbours and participate in the dispatch directly, which protects privacy, enables parallel calculation and improves solution robustness in case of single-point failure [2].

Nevertheless, the difference of physical nature between multi-energy flows has brought difficulties to the modelling and optimization of networked DESs. Specifically, heat transport in DHN consists of both heat migration accompanying with fluid flow and heat transfer between fluids with different temperatures, which are obviously different from EPN. From the modelling perspective, extensive efforts have been made to concise mathematical descriptions of DHN. As for an individual DHN, a common approach is to solve the partial differential equations of pipeline flow through numerical methods [3]. However, the highly accurate differential equations can hardly be embedded into a programming model. Therefore, early attempts [4], [5] to optimize integrated power and heating systems usually consider energy conservation only and dismiss the pipeline delay and losses. Presuming that the fluid flow rates in DHNs are known constants, a node method [6] is proposed to capture the quasi-dynamics of temperatures and widely applied to multi-energy analysis [7]–[9]. The node method represents pipeline outlet temperatures as the weighted average temperature of several historical time sections at the inlet, which is not accurate when the pipeline delay time is not an integer multiple of the dispatch interval. In [10], Chen et al. propose a general phasor method, which uses Fourier transform to reformulate mass-constant heat flow constraints into phasor forms and reduces calculation errors. The node method and the general phasor method mainly concern the heat migration through pipeline flows, and the heat transfer constraints of heat exchange stations are ignored casually. Consequently, the decisions of flow temperatures may be indeterminate. In [11], a heat current method is employed to model the nonlinear heat transfer processes in DHN. This method embeds nonlinearity into nonlinear circuit elements and linear circuits, enabling a timing simulation of integrated power and heating systems. Despite the current lack of practice, through a combination of the reviewed research the heat transport in DHN with various DESs can be modelled comprehensively and properly.

However, most researches leave over one problem in common in terms of regulation techniques. Typically, the DHN is assumed to operate at quality-regulation mode where flow temperatures are adjustable variants while fluid flow rates are

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constants. This is because frequent adjustment of fluid flow rates, i.e., quantity regulation, may induce hydraulic imbalances. Under this circumstance, the methods reviewed above give a linear DHN model which can be optimized through various centralized or decentralized algorithms. However, quantity regulation can be executed every few days and the fluid flow rates should be carefully selected as their effects permeate the entire system with respect to delay time, heat capacities and thermal resistances at the heat exchange stations. Therefore, the quality regulation alone cannot fully reflect and exploit the storage potential of heat transport in DHN. When the decisions of those fluid flow rates are involved, the DHN modelling methods can exhibit strong nonlinear properties. For example, in the node method, the weights are non-smooth functions of fluid flow rates and thus gradient-based numerical optimization is not applicable. Possible solutions considering quantity regulation include sequential programming which is a variant of the fixed-point method [12], [13] and heuristic techniques such as the genetic method [14]. Notwithstanding, the discontinuous mapping relationship of the fixed-point method and the randomness consequent upon the heuristic algorithms create an obstacle for the design of an effective decentralized optimization process.

So far, few researchers have yet modelled the heat migration and transfer in DHN thoroughly considering complementary heating techniques among DESs and analyzed how can the overall heat transport be rationally regulated through a co-optimization of fluid flow rates and temperatures in DHN under decentralized schemes. To fill the gaps, this work aims to present a synthetic model of the DHN and propose a fully decentralized optimization method for connected multi-energy DESs where hybrid regulation of both fluid flow rates and temperatures in DHN, i.e., hybrid quantity and quality regulation is possible. The main contributions are as follows:

1) A new DHN model following the basic idea of phasor methods, i.e., applying Fourier Transform or trigonometric expansion to nodal temperatures, is established to model pipeline delay and losses considering the variation of fluid flow rates. Besides, the heat transfer constraints between fluids of different temperatures are also considered through the heat current method. On this basis, an optimization model is established where the heat transport between interconnected DESs is comprehensively characterized.
2) To deal with the nonconvexity of the model, a primal-decomposition-based gradient descent method in company with the Alternating Direction Method of Multipliers(ADMM) is proposed to optimize distributed multi-energy systems considering hybrid regulation of fluid flow rates and temperatures in DHN in a fully decentralized manner.
3) Case study on an integrated system validates the effectiveness of the proposed method. A direct application of ADMM with various relaxation methods does not converge, while the proposed method reduces the curtailment rate of renewable energy from 17.3% to 0 compared to the case without fluid flow rate regulation.

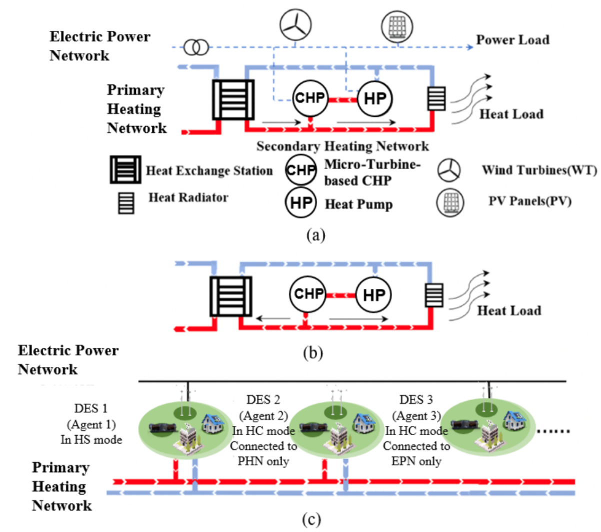

Fig. 1. The general structure of networked DESs: (a) DES in the HC mode. (b) DES in the HS mode, the flow directions are different from (a). (c) Multiple DESs connected with the PHN and EPN.

This article is an extension of our previous conference paper [15]. The initial conference paper combines the node method and heat current method to model the heat transport in DHN and uses ADMM to do agent-based fully decentralized dispatch, while fluid flow rates in the DHN are preset parameters. This manuscript addresses this problem and provides more insight into the significance of comprehensive modelling and regulation of heat transport. The remainder of this article is organized as follows: Section II describes the distributed multi-energy systems with EPN and DHN. Section III proposes the fully decentralized solution. Section IV presents the case study.

II. Mathematical Model of Networked DESs

A. System Structure and Regulation Framework

Fig. 1(a) reveals the general structure of a DES to be studied, which is implemented with micro-turbine-based combined heat and power(CHP) systems, heat pumps(HP) and renewable sources such as photovoltaic(PV) panels and wind turbines(WT). These multi-energy sources and energy converters work together to meet the demands of shopping malls, hospitals or industrial facilities. The DES can exchange power with the EPN to maintain power balance. Besides, the heating devices in the system are connected through a secondary heating network(SHN), which exchange heat with the primary heating network(PHN) through a heat exchange station(HES). The SHN and PHN constitute the self-sufficient DHN. The DESs can either work in heat source(HS) or heat customer(HC) modes. In the HC mode(Fig. 1(a)), local devices and the PHN work together to meet the heat demand of end-users. In the HS mode(Fig. 1(b)), the fluid flow direction in SHN is different and the DES will work reversely as a heat source of the PHN. Fig. 1.(c) shows the structure of networked DESs connected through the EPN and PHN. The DESs can connect to one or both of the networks.

As can be seen from Fig. 1, the PHN and SHN may have quite different spatial sizes. The SHNs reveal the topology of local energy sources and belong to specific DESs. Contrariwise, the PHN connects multiple SHNs, i.e., multiple DESs, on a larger scale. Due to safety reasons, we consider an interday quantity regulation and intraday quality regulation scheme of the PHN. The quantity regulation of flow rates is executed at the beginning of every day, while quality regulation of flow temperatures is performed hourly or every 15 minutes. As for the SHN, the flow rates of fluids that enter the HES and the fluid temperatures can be flexibly adjusted at all times under operational constraints. A day-ahead dispatch based on both historical DHN operation status and demand and renewable energy predictions is considered.

B. Modelling of PHN

The PHN includes symmetric supply and return pipelines and HESs to exchange heat with the DESs. We consider a radial PHN.

1) Pipelines: We first consider a constant fluid flow rate $\dot{m}$ in the PHN pipelines and neglect the heat conduction between fluid microelements, the inlet and outlet temperatures have the following relationship [16].

$$\theta_{out}(t) = \theta_{in}(t - t_d)e^{-\frac{UL}{\dot{m}c_p}} \tag{1}$$

where $\theta_{out}(t) = T_{out}(t) - T_a$ and $\theta_{in}(t) = T_{in}(t) - T_a$ are the excess temperatures at the pipeline inlet and the outlet. T_a is the ambient temperature. U is the heat transfer coefficient between the fluid and the surrounding environment. L denotes the pipeline length. ρ and c_p are the density and the heat capacity of the fluid. $t_d = \frac{\rho AL}{\dot{m}}$ is the delay time, where A is the pipeline cross section area.

Since optimization can only be performed on discrete time slots, (1) should be discretized based on the dispatch time intervals, Δt. Noting that the time delay t_d may not be an integer multiple of Δt, interpolation of temperature decisions adjacent in time may lead to a non-smooth structure in terms of $\dot{m}$. To deal with this problem, we follow the idea of the general phasor method [10] to apply Discrete Fourier Transform(DFT) to discrete temperatures to obtain phasor forms. For a better illustration, we rewrite the constant-fluid-flow phasor model in the form of trigonometric series on the real number field:

$$\begin{aligned}
\boldsymbol{\theta}_{out}[i] &= \sum_{k=0}^{\frac{M}{2}} \widetilde{\boldsymbol{\theta}}_{out}[k]\cos k\omega i + \sum_{k=1}^{\frac{M}{2}-1} \widetilde{\boldsymbol{\theta}}_{out}[k+\frac{M}{2}]\sin k\omega i \\
&= \theta_{out}(i\Delta t) = \theta_{in}(i\Delta t - t_d)e^{-\frac{UL}{\dot{m}c_p}} \\
&= \left(\sum_{k=0}^{\frac{M}{2}} \widetilde{\boldsymbol{\theta}}_{in}[k]\cos k\omega\left(i - \frac{t_d}{\Delta t}\right)\right. \\
&\quad \left. + \sum_{k=1}^{\frac{M}{2}-1} \widetilde{\boldsymbol{\theta}}_{in}[k+\frac{M}{2}]\sin k\omega\left(i - \frac{t_d}{\Delta t}\right)\right) e^{-\frac{UL}{\dot{m}c_p}}
\end{aligned} \tag{2}$$

where $\boldsymbol{\theta}[i] = \theta(i\Delta t)$ defines the discrete decision series. M denotes the length of $\boldsymbol{\theta}$ and is assumed to be even-numbered. $\tilde{\boldsymbol{\theta}}$ is the coefficient of the trigonometric series, and can also be treated as decisions in the frequency domain. $\omega = 2\pi/M$ is the base frequency. The matrix form of (2) is presented as:

$$\boldsymbol{\theta}_{out} = \mathbf{A}\widetilde{\boldsymbol{\theta}}_{out} = \mathbf{AB}\widetilde{\boldsymbol{\theta}}_{in} = \mathbf{ABA}^{-1}\boldsymbol{\theta}_{in} \tag{3}$$

where $\mathbf{A}$ is a constant matrix representing trigonometric interpolation and $\mathbf{B}$ is a $\dot{m}$-dependent matrix indicating delay and loss characteristics.

$$\mathbf{A}_{ij} = \begin{cases} \cos\omega ij & j = 0, ..., \frac{M}{2} \\ \sin\omega i\left(j - \frac{M}{2}\right) & j = \frac{M}{2}+1, ..., M-1 \end{cases} \quad i = 0, ..., M-1 \tag{4}$$

$$\begin{aligned}
\mathbf{B} &= e^{-\frac{UL}{\dot{m}c_p}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathbf{B}_{\cos} & 0 & -\mathbf{B}_{\sin} \\ 0 & 0 & \cos\frac{Mt_d\omega}{2\Delta t} & 0 \\ 0 & \mathbf{B}_{\sin} & 0 & \mathbf{B}_{\cos} \end{bmatrix} \\
\mathbf{B}_{\cos} &= diag\left(\cos\frac{t_d\omega}{\Delta t}, \cos\frac{2t_d\omega}{\Delta t}, ..., \cos\frac{\left(\frac{M}{2}-1\right)t_d\omega}{\Delta t}\right) \\
\mathbf{B}_{\sin} &= diag\left(\sin\frac{t_d\omega}{\Delta t}, \sin\frac{2t_d\omega}{\Delta t}, ..., \sin\frac{\left(\frac{M}{2}-1\right)t_d\omega}{\Delta t}\right)
\end{aligned} \tag{5}$$

The above deduction is based on the premise that fluid flow rates remain constants and that the temperature decisions are periodically executed. However, this is impractical and the history temperature series with possible different fluid flow rates should be modelled. Since fluid flow rates are regulated every day, we can safely assume that the flow microelements entering every pipeline will encounter velocity variation once at most before the outlet. In other words, there are no "super-long" pipelines. In this case, the fluid microelement that encounter flow rate variations can be considered as successively passing two pipelines at different velocities. Bearing this in mind, we further assume that the flow direction remains unchanged. Without loss of generality, we stipulate that the fluid flow rates are regulated at $t = 0$ and dispatch decisions are made for $t \geq 0$. Then the analytic solution (1) has the following form:

$$\begin{aligned}
\theta_{out}(t) &= \begin{cases} \theta_{in}(t - t_d)\,e^{-\frac{UL}{\dot{m}c_p}} & t \geq t_d \\ \theta_{in}^{his}\left(\frac{\dot{m}}{\dot{m}^{his}}(t - t_d)\right)e^{-\frac{UL}{\dot{m}c_p}+a(t-t_d)} & t \leq t_d \end{cases} \\
a &= \frac{-U}{\rho A c_p}\left(1 - \frac{\dot{m}}{\dot{m}^{his}}\right)
\end{aligned} \tag{6}$$

where θ_{in}^{his} and $\dot{m}^{his}$ represent historical inlet temperature and fluid flow rates for $t \leq 0$.

Let $\boldsymbol{\theta}_{in}^{his}[i] = \theta_{in}^{his}(i\Delta t)$ with $i \in \{-M_{his}, ..., -1\}$ be the discrete sampling of historical data. Repeat the process of trigonometric interpolation in (2), we can obtain a matrix form similar to (3):

$$\begin{aligned}
\boldsymbol{\theta}_{out} &= \begin{bmatrix} \mathbf{0}_{M_{his}\times M} \\ \mathbf{I}_M \end{bmatrix}^T \mathbf{A}'\mathbf{B}'\mathbf{A}'^{-1} \begin{bmatrix} \mathbf{L}_s\mathbf{L}_d\widetilde{\boldsymbol{\theta}}_{in}^{his} \\ \boldsymbol{\theta}_{in} \end{bmatrix} \\
\mathbf{L}_s &= diag\left(e^{(-M_{his})a\Delta t}, e^{(-M_{his}+1)a\Delta t}, ..., e^{(-1)a\Delta t}\right) \\
\mathbf{L}_{d,ij} &= \begin{cases} \cos\frac{\dot{m}}{\dot{m}^{his}}\omega\left(i - M_{his}\right)j & j = 0, ..., \frac{M_{his}}{2} \\ \sin\frac{\dot{m}}{\dot{m}^{his}}\omega\left(i - M_{his}\right)\left(j - \frac{M_{his}}{2}\right) & j = \frac{M_{his}}{2}+1, ..., M_{his}-1 \end{cases} \\
& i = 0, ..., M_{his}-1
\end{aligned} \tag{7}$$

The definition of $A', B' \in \mathbb{R}^{(M+M_{his})\times(M+M_{his})}$ above is similar to those in (4) and (5) except that they are defined for different discrete series and thus have different dimensions. From (7), we can obtain an intuitive understanding of the pipeline dynamics considering quantity regulation. The historical flow temperature curve is first reshaped through the scaling matrix $\mathbf{L}_s$ and the dilation matrix $\mathbf{L}_d$ to include the impact of flow rate variation. The transformed historical curve and the pending decisions for the current period $\boldsymbol{\theta}_{in}$ are then spliced and delay and losses are considered through the matrix $\mathbf{B}'$. It is worth noting that the converting matrix $\mathbf{A}'\mathbf{B}'\mathbf{A}'^{-1}$ is pre-multiplied by an extra $[\mathbf{0}_{M\times M_{his}} \quad \mathbf{I}_M]$ to take the last M rows. This is because we only concern the decisions for $t \geq 0$. The reduced form of (7) is presented as:

$$\boldsymbol{\theta}_{ij,out} = \mathbf{C}_{ij,his}\left(\dot{m}_{ij}\right)\boldsymbol{\theta}_{ij,in}^{his} + \mathbf{C}_{ij,cur}\left(\dot{m}_{ij}\right)\boldsymbol{\theta}_{ij,in} \tag{8}$$

Herein, the subscript ij is involved to label the pipeline and define the flow direction from node i to j. $\mathbf{C}_{ij,his}(\cdot)$ and $\mathbf{C}_{ij,cur}(\cdot)$ are differentiable matrix functions of flow rate $\dot{m}_{ij}$, which enable the gradient-based optimization approach.

Noting that the pipelines also exhibit storage capabilities, a periodic boundary condition should be imposed:

$$\begin{aligned}\int_{t=0}^{t_{end}} \frac{\mathrm{d}H_{sto}}{\mathrm{d}t}dt &= \int_{t=0}^{t_{end}} (\Phi_{ij,in} - \Phi_{ij,out} - \Phi_{ij,loss})dt \\ &\approx \int_{t=0}^{t_{end}} \dot{m}c_p\left(\theta_{ij,in}e^{-\frac{U_{ij}L_{ij}}{\dot{m}_{ij}c_p}} - \theta_{ij,out}\right)dt = 0\end{aligned} \tag{9}$$

The discrete form of constraint (9) is given as:

$$e^{-\frac{U_{ij}L_{ij}}{\dot{m}_{ij}c_p}}\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}_{in} = \mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}_{out} \tag{10}$$

The detailed proof for (6)-(9) are provided in the supplementary materials [17].

2) Pipeline junctions: At the junction i of the pipelines, the conservation of fluid flow rates gives the following constraints:

$$\sum_{j\to i}\dot{m}_{ji}^{s} = \sum_{i\to k}\dot{m}_{ik}^{s} + \dot{m}_i \tag{11}$$

$$\sum_{i\to j}\dot{m}_{ij}^{r} = \sum_{k\to i}\dot{m}_{ki}^{r} + \dot{m}_i \tag{12}$$

where s and r discriminate between the symmetric supply and return pipelines and it is obvious that $\dot{m}_{ji}^{s} = \dot{m}_{ij}^{r}$. $\dot{m}_i$ denotes the fluid flow rate that enters the HES at node i, which is positive for HES in HC modes and negative for HES in HS modes.

Besides, energy conservation at the junctions gives:

HES in HC modes:

$$\sum_{j\to i}\dot{m}_{ji}^{s}\mathbf{T}_{ji,out} = \mathbf{T}_i^{s,\mathrm{MIX}}\left(\sum_{i\to k}\dot{m}_{ik}^{s} + \dot{m}_i\right) \tag{13}$$

$$\sum_{k\to i}\dot{m}_{ki}^{r}\mathbf{T}_{ki,out} + \dot{m}_i\mathbf{T}_i^{r,\mathrm{PHN}} = \mathbf{T}_i^{r,\mathrm{MIX}}\sum_{i\to j}\dot{m}_{ij}^{r} \tag{14}$$

HES in HS modes:

$$\sum_{j\to i}\dot{m}_{ji}^{s}\mathbf{T}_{ji,out} - \dot{m}_i\mathbf{T}_i^{s,\mathrm{PHN}} = \mathbf{T}_i^{s,\mathrm{MIX}}\sum_{i\to k}\dot{m}_{ik}^{s} \tag{15}$$

$$\sum_{k\to i}\dot{m}_{ki}^{r}\mathbf{T}_{ki,out} = \mathbf{T}_i^{r,\mathrm{MIX}}\left(\sum_{i\to j}\dot{m}_{ij}^{r} - \dot{m}_i\right) \tag{16}$$

where $\mathbf{T}_i^{s,\mathrm{MIX}}$ and $\mathbf{T}_i^{r,\mathrm{MIX}}$ are temperatures of the mixed flow at the junctions. $\mathbf{T}_i^{s,\mathrm{PHN}}$ and $\mathbf{T}_i^{r,\mathrm{PHN}}$ are the supply and return flow temperature at the primary side of HES at node i. $\mathbf{T}_i^{s,\mathrm{MIX}} = \mathbf{T}_i^{s,\mathrm{PHN}}$ for HC nodes and $\mathbf{T}_i^{r,\mathrm{MIX}} = \mathbf{T}_i^{r,\mathrm{PHN}}$ for HS nodes.

Besides, the safety constraints include:

$$\underline{\dot{m}}_{ij} \leq \dot{m}_{ij} \leq \overline{\dot{m}}_{ij} \tag{17}$$

$$\underline{\dot{m}}_{i} \leq \dot{m}_{i} \leq \overline{\dot{m}}_{i} \tag{18}$$

$$\mathbf{T}_i^{s,\mathrm{PHN}} \leq \overline{T}_i \tag{19}$$

The pumping cost is considered as:

$$C_i^{\mathrm{PHN}} = \sum_{j\to i} f_{ji}^{\mathrm{PIPE}}(\dot{m}_{ij}^{s}) + \sum_{k\to i} f_{ki}^{\mathrm{PIPE}}(\dot{m}_{ki}^{r}) \tag{20}$$

where C_i^{PHN} covers the pumping cost of fluid flow towards node i. $f_{ji}^{\mathrm{PIPE}}(\dot{m}_{ij}^{s})$ and $f_{ki}^{\mathrm{PIPE}}(\dot{m}_{ki}^{r})$ are arbitrary convex functions.

The heat exchange between PHN and SHN at the HES is:

$$\mathbf{H}_i = \dot{m}_i c_p\left(\mathbf{T}_i^{s,\mathrm{PHN}} - \mathbf{T}_i^{r,\mathrm{PHN}}\right) \tag{21}$$

Apart from the energy conservation equation above, the heat transfer processes between fluids with different temperatures at the HES and the user-side radiator should be further modelled because they are important to the decisions of flow temperatures and make the optimization problem determinate [15]. The heat transfer constraints will be explained comprehensively in section II-C

C. Modelling of DESs

The DESs are prosumers of power and heat. Considering their relatively small spatial scale, we neglect the power flow constraints and delay and losses of SHN in DESs. Compacting the time-dependent variables into vector forms, the power and heat balance equations are listed as follows:

$$\mathbf{P}_i = \mathbf{P}_i^{l} + \mathbf{P}_i^{\mathrm{HP}} - \mathbf{P}_i^{\mathrm{CHP}} - \mathbf{P}_i^{\mathrm{WT}} - \mathbf{P}_i^{\mathrm{PV}} \tag{22}$$

$$\mathbf{Q}_i = \mathbf{Q}_i^{l} - \mathbf{Q}_i^{\mathrm{CHP}} - \mathbf{Q}_i^{\mathrm{WT}} - \mathbf{Q}_i^{\mathrm{PV}} \tag{23}$$

$$w_i\mathbf{H}_i = \mathbf{H}_i^{l} - \mathbf{H}_i^{\mathrm{CHP}} - \mathbf{H}_i^{\mathrm{HP}} \tag{24}$$

where the superscript l denotes loads. $\mathbf{P}_i$, $\mathbf{Q}_i$ denote the active and reactive power extracted from the EPN. w_i discriminates between HS and HC nodes by taking -1 or 1. The other terms with superscripts such as CHP represent the generation of corresponding devices. The operational constraints are:

$$\underline{P}_i^{\mathrm{dev}} \leq \mathbf{P}_i^{\mathrm{dev}} \leq \overline{P}_i^{\mathrm{dev}}, \mathrm{dev} = \text{CHP, HP, WT or PV} \tag{25}$$

$$\mathbf{Q}_i^{\mathrm{dev}} = \mathbf{P}_i^{\mathrm{dev}}\tan\varphi_i^{\mathrm{dev}}, \mathrm{dev} = \text{CHP, WT or PV} \tag{26}$$

$$\mathbf{H}_i^{\mathrm{CHP}} = k_i^{\mathrm{CHP}}\mathbf{P}_i^{\mathrm{CHP}} \tag{27}$$

$$\mathbf{H}_i^{\mathrm{HP}} = \mathrm{COP}_i^{\mathrm{HP}}\mathbf{P}_i^{\mathrm{HP}} \tag{28}$$

where φ represent the power angles. k_i^{CHP} is the heat-to-power ratio of CHP i and $\mathrm{COP}_i^{\mathrm{HP}}$ denotes the coefficient of performance of heat pump i. k_i^{CHP} and $\mathrm{COP}_i^{\mathrm{HP}}$ are considered constant.

As for the generation cost, we only consider those of the CHPs and the renewables:

$$\mathbf{C}_i^{\mathrm{DES}} = f_i^{\mathrm{CHP}}\left(\mathbf{P}_i^{\mathrm{CHP}}\right) + \gamma_i^{\mathrm{WT}}\mathbf{P}_i^{\mathrm{WT}} + \gamma_i^{\mathrm{PV}}\mathbf{P}_i^{\mathrm{PV}} \tag{29}$$

where $f_i^{\mathrm{CHP}}(\cdot)$ is a quadratic function. Since $\mathbf{H}_i^{\mathrm{CHP}}$ is coupled with $\mathbf{P}_i^{\mathrm{CHP}}$ through the heat-to-power ratio, $f_i^{\mathrm{CHP}}(\cdot)$ is treated as a univariate function of $\mathbf{P}_i^{\mathrm{CHP}}$. γ is the cost coefficient of the renewable sources and has a small value.

As for the heat transfer process between the PHN and end-users, the nearly proposed heat current method which provides an analogy to linear electrical circuits can be used to characterize them [18]. For completeness, the established model in [15] is reintroduced as follows.

1) Heat current model of HC nodes: Fig. 2 presents the heat current model of the SHN in Fig. 1(a) which functions as a PHN heat customer. The constraints are:

$$\mathbf{T}_i^{s,\text{PHN}} - \mathbf{T}_i^{r,\text{SHN}} = \mathbf{R}_i^{\text{HES}} \circ \mathbf{H}_i \tag{30}$$

$$\mathbf{T}_i^{s,\text{SHN}} - \mathbf{T}^{\text{room}} \geq \mathbf{R}_i^{\text{RAD}} \circ \mathbf{H}_i^l \tag{31}$$

$$c_p \dot{\mathbf{m}}_i^{\text{RAD}} \circ \left(\boldsymbol{\epsilon}_i^{\text{HES}} + \boldsymbol{\epsilon}_i^{\text{CHP-HP}}\right) = \mathbf{H}_i + \mathbf{H}_i^{\text{CHP}} + \mathbf{H}_i^{\text{HP}} \tag{32}$$

$$\mathbf{T}_i^{r,\text{SHN}} + \boldsymbol{\epsilon}_i^{\text{HES}} + \boldsymbol{\epsilon}_i^{\text{CHP-HP}} = \mathbf{T}_i^{s,\text{SHN}} \tag{33}$$

where $\boldsymbol{\epsilon}_i^{\text{HES}}$ and $\boldsymbol{\epsilon}_i^{\text{CHP-HP}}$ are thermal potentials reflecting temperature increase of fluids in the SHN. $\circ$ represent the element-wise product of vecotrs. $\mathbf{T}_i^{s,\text{SHN}}$ and $\mathbf{T}_i^{r,\text{SHN}}$ denote the supply and return temperature of the heat radiator. In (31), we have relaxed the original equality constraint to allow waste of heat.

$\mathbf{R}_i^{\text{HES}}$ and $\mathbf{R}_i^{\text{RAD}}$ are the thermal resistance of the HES and the radiator, and can be represented as a function of the fluid flow rates that enter them, $\dot{\mathbf{m}}_i^{\text{SHN,HES}}$ and $\dot{\mathbf{m}}_i^{\text{RAD}}$:

$$\mathbf{R}_i^{\text{HES}} = R_{\text{HES}}\left(\dot{m}_i, \dot{\mathbf{m}}_i^{\text{SHN,HES}}\right) \tag{34}$$

$$\mathbf{R}_i^{\text{RAD}} = R_{\text{RAD}}\left(\dot{\mathbf{m}}_i^{\text{RAD}}\right) \tag{35}$$

The detailed expression of nonconvex $R_{\text{HES}}(\cdot)$ and $R_{\text{RAD}}(\cdot)$ can be found in [17] and [18].

2) Heat current model of HS nodes: Fig. 2(b) displays the heat current model of SHN in HS modes. The constraints are:

$$\mathbf{T}_i^{s,\text{SHN}} - \mathbf{T}_i^{r,\text{PHN}} = \mathbf{R}_i^{\text{HES}} \circ \mathbf{H}_i \tag{36}$$

$$\mathbf{T}_i^{s,\text{SHN}} - \mathbf{T}^{\text{room}} \geq \mathbf{R}_i^{\text{RAD}} \circ \mathbf{H}_i^l \tag{37}$$

For simplicity, we assume that the rate of fluid flow that directly enters the user-side radiator, i.e., $\dot{\mathbf{m}}_i^{\text{RAD}}$, cannot be freely adjusted, while the flow rate that enters the HES at the SHN side, i.e., $\dot{\mathbf{m}}_i^{\text{SHN,HES}}$, can be regulated within a given range $\left[\underline{\dot{m}_i^{\text{SHN,HES}}}, \overline{\dot{m}_i^{\text{SHN,HES}}}\right]$. Therefore, (30) and (36) can be equivalently rewritten for HC and HS nodes respectively as follows:

$$\underline{R_i^{\text{HES}}}(\dot{m}_i)\mathbf{H}_i \leq \mathbf{T}_i^{s,\text{PHN}} - \mathbf{T}_i^{r,\text{SHN}} \leq \overline{R_i^{\text{HES}}}(\dot{m}_i)\mathbf{H}_i \tag{38}$$

$$\underline{R_i^{\text{HES}}}(\dot{m}_i)\mathbf{H}_i \leq \mathbf{T}_i^{s,\text{SHN}} - \mathbf{T}_i^{r,\text{PHN}} \leq \overline{R_i^{\text{HES}}}(\dot{m}_i)\mathbf{H}_i \tag{39}$$

where $\underline{R_i^{\text{HES}}}(\dot{m}_i)$ and $\overline{R_i^{\text{HES}}}(\dot{m}_i)$ are calculated by substituting $\overline{\dot{m}_i^{\text{SHN,HES}}}$ and $\underline{\dot{m}_i^{\text{SHN,HES}}}$ into (34) respectively.

D. Modelling of Electric Power System

The power grid that connects the DESs is assumed to be a radial distribution network. According to the second-order-cone relaxed distflow model [19], [20], the constraints are:

$$\sum_{j:j\to i} \mathbf{P}_{ji} = \sum_{i:i\to k} \left(\mathbf{P}_{ik} + \mathbf{l}_{ik} R_{ik}\right) + \mathbf{P}_i \tag{40}$$

$$\sum_{j:j\to i} \mathbf{Q}_{ji} = \sum_{i:i\to k} \left(\mathbf{Q}_{ik} + \mathbf{l}_{ik} X_{ik}\right) + \mathbf{Q}_i \tag{41}$$

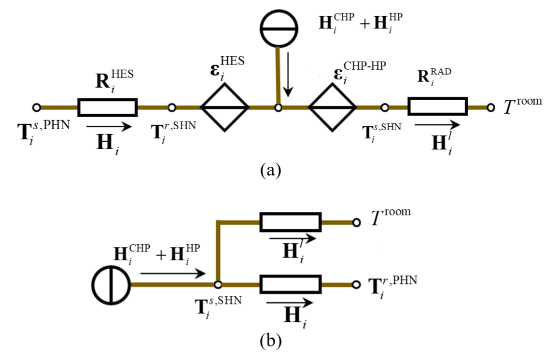

Fig. 2. Heat current model of the secondary heating network: (a) SHN in HC modes. PHN and local devices provide heat to the users. (b) SHN in HS modes. Local devices provide heat to the PHN and users.

$$\mathbf{v}_i = \mathbf{v}_j - 2\left(R_{ji}\mathbf{P}_{ji} + X_{ji}\mathbf{Q}_{ji}\right) + \left(R_{ji}^2 + X_{ji}^2\right)\mathbf{l}_{ji} \tag{42}$$

$$\mathbf{l}_{ji} \circ \mathbf{v}_i \geqslant \mathbf{P}_{ji}^2 + \mathbf{Q}_{ji}^2 \tag{43}$$

The direction $j \to i$ indicates that node j is the parent of i in the tree-like grid. $\mathbf{P}_{ji}$ and $\mathbf{Q}_{ji}$ denote power flow, while $\mathbf{P}_i$ and $\mathbf{Q}_i$ represent the net active and reactive power loads of node i. $\mathbf{v}$ and $\mathbf{l}$ are the squared magnitudes of nodal voltage and transmission line current. The safety constraints are:

$$\underline{v_i} \leq \mathbf{v}_i \leq \overline{v_i} \tag{44}$$

$$\mathbf{l}_{ij} \leq \overline{l_{ij}} \tag{45}$$

To sum up, the dispatch problem for the DESs minimizes the following objective:

$$\min \sum_i C_i = \sum_i \left(\mathbf{1}^{\text{T}}\mathbf{C}_i^{\text{DES}} + C_i^{\text{PHN}}\right) \tag{46}$$

The constraints include: 1) PHN constraints: (8), (10)-(19); 2) DES and SHN constraints: (22)-(24), (31)-(34), (37)-(39); 3) EPN constraints: (40)-(45).

III. Fully decentralized solution method

Considering the multi-agent structure of networked DESs, decentralized optimization methods are considered. Typical decentralized algorithms include dual decomposition method [21], ADMM [22], and optimality condition decomposition methods [23], which split the optimization problem into several subproblems and solve them iteratively. Generally, the methods are designed for convex problems. However, the model introduced in section II reveals strong nonlinear properties. Specifically, (8), (21), (38) and (39) are complicated nonconvex constraints. Therefore, a direct application of popular decentralized methods may face great difficulties.

Despite the obstacles, it can be observed that all the constraints are convex as long as the fluid flow rates are fixed. Besides, the constraints related only to fluid flow rates, i.e., (11), (12), (17) and (18), are very simple. These properties inspire a primal-decomposition-based optimization method. In primal decomposition, the complicated decision variables are fixed so that we can easily solve the rest problem and provide guidance for the update of complicated variables. In this work, we denote the complicated decisions of PHN fluid flow rates as $\mathbf{Y}$ and the rest decision variables as $\mathbf{X}$. For easy reference, the optimization model is compacted to the following form:

$$\min \sum_{i=1}^{N} C_i\left(\mathbf{X}_i, \mathbf{Y}_i\right) \tag{47a}$$

$$\text{s.t. } \mathbf{X}_i \in \mathcal{G}_{\mathbf{Y}_i, i} \tag{47b}$$

$$\mathbf{Y} \in \mathcal{H} \tag{47c}$$

$$\mathbf{C}_{ij}\mathbf{X}_i + \mathbf{C}_{ji}\mathbf{X}_j = \mathbf{0} \tag{47d}$$

$$\mathcal{G}_{\mathbf{Y}_i, i} = \{\mathbf{X}_i | \mathbf{G}_i^1(\mathbf{X}_i, \mathbf{Y}_i) = 0,\ \mathbf{G}_i^2(\mathbf{X}_i, \mathbf{Y}_i) \leq 0\} \tag{47e}$$

$$\mathcal{H} = \{\mathbf{Y} | \mathbf{H}^1\mathbf{Y} = 0, \mathbf{H}^2\mathbf{Y} \leq 0\} \tag{47f}$$

$$(i,j) \in \mathcal{E}\ ,\ i = 0, 1, ..., N-1$$

where $\mathbf{X}_i$ and $\mathbf{Y}_i$ collects specific decision variables concerning DES node i. The variables with subscript ij in the model are duplicated and involved in both $\mathbf{X}_i(\mathbf{Y}_i)$ and $\mathbf{X}_j(\mathbf{Y}_j)$. Therefore, the coupling constraint (47d) is introduced to ensure that the duplicated variables are consistent. $(i,j) \in \mathcal{E}$ indicates that DES i and j are connected with a transmission line or PHN pipelines. Besides, (47c) indicates both the consistency constraints and safety and conservation constraints of fluid flow rates, where $\mathcal{H}$ is a convex set. $\mathcal{G}_{\mathbf{Y}_i, i}$ imposes DES local constraints on $\mathbf{X}_i$ and is convex for arbitrary fixed $\mathbf{Y}_i$.

Relaxing coupling constraints (47d) and fixing $\mathbf{Y}$, the partial augmented lagrangian is given as:

$$\begin{aligned} L\left(\mathbf{X}, \mathbf{Y}, \boldsymbol{\lambda}\right) &= \sum_{i=1}^{N} C_i\left(\mathbf{X}_i, \mathbf{Y}_i\right) \\ &+ \sum_{\substack{\forall i, \forall j \\ (i,j) \in \mathcal{E}}} \left[\boldsymbol{\lambda}_{ij}^T\left(\mathbf{C}_{ij}\mathbf{X}_i + \mathbf{C}_{ji}\mathbf{X}_j\right) + \frac{\rho}{2}\|\mathbf{C}_{ij}\mathbf{X}_i + \mathbf{C}_{ji}\mathbf{X}_j\|^2\right] \end{aligned} \tag{48}$$

By minimizing over $\mathbf{X}$ and maximizing over $\boldsymbol{\lambda}$, the optimal dual objective is:

$$\xi\left(\mathbf{Y}\right) = \max_{\boldsymbol{\lambda}} \psi\left(\mathbf{Y}, \boldsymbol{\lambda}\right) = \max_{\boldsymbol{\lambda}} \min_{\mathbf{X}_i \in \mathcal{G}_{\mathbf{Y}_i, i}} L\left(\mathbf{X}, \mathbf{Y}, \boldsymbol{\lambda}\right) \tag{49}$$

which is also the optimal value of problem (47) for a given $\mathbf{Y}$. Herein, the basic idea is to employ ADMM to obtain an approximate value of $\xi\left(\mathbf{Y}\right)$ and then apply decentralized gradient descent to update $\mathbf{Y}$. In ADMM, the subproblem for each DES agent i is given by decomposing the partial augmented lagrangian (48):

$$\begin{aligned} \min_{\mathbf{X}_i \in \mathcal{G}_{\mathbf{Y}'_i, i}} & L_i\left(\mathbf{X}_i, \mathbf{Y}'_i, \boldsymbol{\lambda}'_i\right) = C_i\left(\mathbf{X}_i, \mathbf{Y}'_i\right) \\ &+ \frac{1}{2} \sum_{\substack{\forall j \\ (i,j) \in \mathcal{E}}} \left[\boldsymbol{\lambda}_{ij}'^T\left(\mathbf{C}_{ij}\mathbf{X}_i + \mathbf{C}_{ji}\mathbf{X}_j\right) + \frac{\rho}{2}\|\mathbf{C}_{ij}\mathbf{X}_i + \mathbf{C}_{ji}\mathbf{X}_j\|^2\right] \end{aligned} \tag{50}$$

where $\boldsymbol{\lambda}'_i$ collects the multipliers $\boldsymbol{\lambda}'_{ij}$ for $(i,j) \in \mathcal{E}$. The $\frac{1}{2}$ indicates that the lagrangian terms are split equally between L_i and L_j

The decentralized optimization framework is given in **Algorithm 1**, where each DES agent collects information from neighbours, calculates local subproblems and updates the multipliers. The framework is similar to the parallel ADMM proposed in [24], except that $\mathbf{Y}_i$ is not to be updated directly in the subproblem. Instead, a relatively loose threshold ϵ_1^{tol} is used to estimate the calculation accuracy of $\xi\left(\mathbf{Y}\right)$ and **Algorithm 2** is employed to update $\mathbf{Y}_i$ for PHN nodes.

Algorithm 1 Decentralized Optimization Framework

Initialize
  Set $k := 0$, residual tolerances $\epsilon_1^{\text{tol}} > \epsilon_2^{\text{tol}}$
  Each agent i sets arbitrary initial $\mathbf{X}_i^{(0)}$ and multipliers $\lambda_{ij}^{(0)}$ for $\forall j \in \mathcal{N}_i$ and a feasible initial $\mathbf{Y}_i'^{(0)} = \mathbf{Y}_i^{(0)}$
repeat
  for agent i=0,1,...,$N-1$ parallelly **do**
    Collect $\mathbf{X}_j^{(k)}$ and $\boldsymbol{\lambda}_{ij}^{(k)}$ from each node $j \in \mathcal{N}_i$
    $\boldsymbol{\lambda}_{ij}'^{(k+1)} := \left(\boldsymbol{\lambda}_{ij}^{(k)} - \boldsymbol{\lambda}_{ji}^{(k)}\right)/2$
    Solve the subproblem (50) to obtain $\mathbf{X}_i^{(k+1)}$
    Update $\boldsymbol{\lambda}_{ij}^{(k+1)} := \boldsymbol{\lambda}_{ij}^{(k)} + \rho\left(\mathbf{C}_{ij}\mathbf{X}_i^{(k+1)} + \mathbf{C}_{ji}\mathbf{X}_j^{(k)}\right)$ for $\forall j \in \mathcal{N}_i$
    Calculate the primal and dual residuals of $\mathbf{X}_i$:
    $R_{p,i}^{(k+1)} := \|\mathbf{C}_{ij}\mathbf{X}_i^{(k+1)} + \mathbf{C}_{ji}\mathbf{X}_j^{(k)}\|_\infty$
    $R_{d,i}^{(k+1)} := \|\mathbf{X}_i^{(k+1)} - \mathbf{X}_i^{(k)}\|_\infty$
    if not converge with the looser tolerance ϵ_1^{tol} **then**
      Broadcast partial update signals in the system.
    else
      Broadcast full update signals in the system.
    end if
    if partial update signals received **then**
      $\mathbf{Y}_i^{(k+1)} := \mathbf{Y}_i^{(k)}$, $\mathbf{Y}_i'^{(k+1)} := \mathbf{Y}_i^{(k+1)}$
    else
      Execute **Algorithm 2** for PHN nodes to obtain $\mathbf{Y}_i^{(k+1)}$, $\mathbf{Y}_i'^{(k+1)}$ and the residual $R_{\mathbf{Y}_i}^{(k+1)}$ of $\mathbf{Y}_i$.
    end if
    if $\mathbf{X}_i$ and $\mathbf{Y}_i$ not converge with threshold ϵ_2^{tol} **then**
      Broadcast signals of continuous iteration.
    end if
  end for
until No iteration signals are broadcast and received

Algorithm 2 can be divided into three parts: the decentralized gradient calculation from step 4 to 13, the decentralized step size calculation from step 14 to 25, and the gradient descent part from step 26 to 29.

In the gradient calculation part, since the residuals of $\mathbf{X}_i^{(k+1)}$ and $\boldsymbol{\lambda}_i^{(k+1)}$ have been small enough, we can safely estimate that:

$$\begin{aligned} \xi^{(k)} &\triangleq \xi\left(\mathbf{Y}'^{(k)}\right) \approx L\left(\mathbf{X}^{(k+1)}, \mathbf{Y}'^{(k)}, \boldsymbol{\lambda}'^{(k+1)}\right) \\ &= \sum_{i=1}^{N} L_i^{(k)} = \sum_{i=1}^{N} L_i\left(\mathbf{X}_i^{(k+1)}, \mathbf{Y}_i'^{(k)}, \boldsymbol{\lambda}_i'^{(k+1)}\right) \end{aligned} \tag{51}$$

It should be noted that $\mathbf{Y} \in \mathcal{H}$ indicates that the terms in $\mathbf{Y}$ are not independent. Therefore, we select $\dot{m}_i, i \neq l_0$ as independent decision variables and estimates their impacts on $\xi^{(k)}$ by calculating the gradients c_i through the chain rule,

$$\begin{aligned} c_i &\triangleq \frac{\partial \xi^{(k)}}{\partial \dot{m}_i'^{(k)}} \\ &= \sum_{j=l_0}^{l_{N_h-1}} \left(\frac{\partial L_j^{(k)}}{\partial \mathbf{Y}_j'^{(k)}} + \boldsymbol{\mu}_{\mathbf{A}_j}^{(k)T} \frac{\partial \mathbf{A}_j^{(k)}}{\partial \mathbf{Y}_j'^{(k)}}\right) \frac{\partial \mathbf{Y}_j'^{(k)}}{\partial \dot{m}_i'^{(k)}} \end{aligned} \tag{52}$$

where $\mathbf{A}_j'^{(k)}$ collects all active constraints in $\mathcal{G}_{\mathbf{Y}_j, j}$ and $\boldsymbol{\mu}_{\mathbf{A}_j}^{(k)}$ denotes the corresponding multipliers. l_0, ..., l_{N_h-1} are indices

Algorithm 2 Decentralized Gradient Descent of $\mathbf{Y}$

1: **Assumptions**
The PHN is a radial connected network. One arbitrary PHN node is taken as a slack node(l_0).
2: **Initialize**
Let $b_{ij}=1$ if water flows from node i to j in the supply PHN and $b_{ij}=-1$ for the opposite case. Let $t_i=1$ if DES node i is a HS node and $t_i=-1$ if i is a HC node.
Set auxiliary $a_i=a_{ij}=0$. Set gradients $c_i=0$.
Set auxiliary $s_i^{\max}=s_{ij}^{\max}=d_{ij}=0$.
Set the reference step size s^{ref} and actual step size of gradient descent $s=0$
3: **for** each PHN agent i=l_0, l_1, ..., l_{N_h-1} parallelly **do**
4: **(Decentralized calculation of gradients)**
5: Calculate $\frac{\partial \xi_i^{(k)}}{\partial \mathbf{Y}_i'^{(k)}}$, i.e., calculate $\frac{\partial \xi_i^{(k)}}{\partial \dot{m}_i'^{(k)}}$ and $\frac{\partial \xi_i^{(k)}}{\partial \dot{m}_{ij}'^{(k)}}$.
6: **if** i is the slack node l_0 **then**
7: $a_i := t_i \frac{\partial \xi_i^{(k)}}{\partial \dot{m}_i'^{(k)}}$,
8: **else**
9: Wait for $a_{u_i i}$ from some neighbour node u_i.
10: $a_i := a_{u_i i} + b_{u_i i}\frac{\partial \xi_i^{(k)}}{\partial \dot{m}_{u_i i}'^{(k)}}$, $c_i := -t_i a_i + \frac{\partial \xi_i^{(k)}}{\partial \dot{m}_i'^{(k)}}$.
11: **end if**
12: $a_{ij} := a_i + b_{ij}\frac{\partial \xi_i^{(k)}}{\partial \dot{m}_{ij}'^{(k)}}$.
13: Send a_{ij} to each PHN neighbour $j \in \mathcal{N}_i^h$.
14: **(Decentralized calculation of the step size)**
15: Wait for d_{ij} and $s_{ij}^{\max}$ from each $j \in \mathcal{N}_i^h$ except u_i.
16: $s_i'^{\max} := \max\{s|\underline{\dot{m}}_i \le \dot{m}_i'^{(k)} - s c_i \le \overline{\dot{m}}_i\}$
17: $s_i^{\max} := \min\{s_i'^{\max}, s_{ij}^{\max}\}$ for all $j \in \mathcal{N}_i^h$
18: **if** i is NOT the slack node l_0 **then**
19: $d_{u_i i} := b_{u_i i}(t_i c_i + \sum_{j\in\mathcal{N}_i^h, j\neq u_i} b_{ij} d_{ij})$
20: $s_{u_i i}'^{\max} := \max\{s|\underline{\dot{m}}_{u_i i} \le \dot{m}_{u_i i}'^{(k)} + s d_{u_i i} \le \overline{\dot{m}}_{u_i i}\}$.
21: $s_{u_i i}^{\max} := \min\{s_{u_i i}'^{\max}, s_i^{\max}\}$.
22: Send $d_{u_i i}$ and $s_{u_i i}^{\max}$ to node u_i
23: **else**
24: $s := \min\{s_i'^{\max}, s^{\text{ref}}\}$, broadcast s.
25: **end if**
26: **(Gradient descent)** $\dot{m}_i^{(k+1)} := \dot{m}_i'^{(k)} - s c_i$
27: **(Average)** $\dot{m}_i'^{(k+1)} := 1/l \sum_{v=k-l+1}^{k} \dot{m}_i^{(v)}$
28: **(Update $\mathbf{Y}'^{(k+1)}$)** Wait for $\dot{m}_j'^{(k+1)}$ from $j \in \mathcal{N}_i^h$ except u_i, update $\dot{m}_{ij}'^{(k+1)}$ according to mass conservation, and send $\dot{m}_i'^{(k+1)}$ to u_i
29: **(Residuals)**: $R_{\mathbf{Y}_i}^{(k+1)} := \|\mathbf{Y}_i'^{(k+1)} - \mathbf{Y}_i^{(k+1)}\|_\infty$
30: **end for**

of DESs connected to the PHN. For brevity, we define the terms in the brackets as $\frac{\partial \xi_j^{(k)}}{\partial \mathbf{Y}_j^{(k)}}$. In **Algorithm 2**, they are first locally calculated at step 5. Then auxiliary variables a_i, a_{ij}, t_i and b_{ij} are introduced to calculate c_i in a fully decentralized manner.

In the step size calculation part, each agent i estimates the maximum step size from step 14 to 21 and sends it to u_i. To ensure the primal feasibility of $\mathbf{Y}_i$, the minimum of candidate step sizes is calculated at the slack node as s and broadcast to the entire system in step 24.

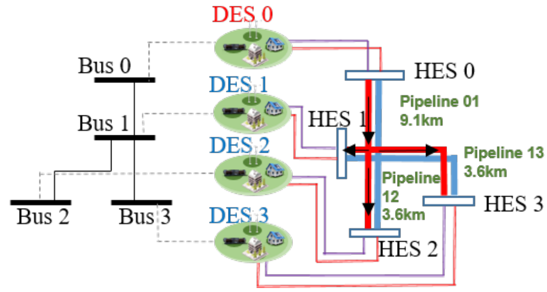

Fig. 3. The sketch of 4-DES system.

In step 26-29, every DES node performs the gradient descent for $\dot{m}_i$ and update $\dot{m}_{ij}$ according to the constraints.

A detailed illustration of how these calculation processes are designed is presented in the supplementary materials [17]. It should be noted that, in **Algorithm 2**, some calculations, e.g., step 10 and 16, must be performed after a certain message is received. These communications and calculations are asynchronized and not completely parallelable. However, they only involve waiting for several real numbers and simple addition/subtraction/comparison operations. Therefore, the asynchronization would not lead to expensive communication cost under the decentralized framework.

The proposed method can also be interpreted as solving the following saddle point problem in a decentralized manner:

$$\min_{\mathbf{Y}} \max_{\boldsymbol{\lambda}} \psi(\mathbf{Y}, \boldsymbol{\lambda}) \tag{53}$$

where the value of $\psi(\mathbf{Y}, \boldsymbol{\lambda})$ and its gradients is approximated through ADMM. It is worth noting that $\psi(\mathbf{Y}, \boldsymbol{\lambda})$ is concave in $\boldsymbol{\lambda}$ and $\frac{\partial \psi}{\partial \mathbf{Y}}$ is Lipschitz continuous since $\mathbf{Y}$ is bounded. Therefore, a convergence analysis is possible. However, due to the complexity, the problem is left open and [25] provides some information on recent mathematical developments for solving such nonconvex saddle point problems.

IV. Case Study

A. System Configuration

Fig. 3 shows the sketch of a test system composed of 4 DESs, each possibly equipped with a micro gas-fired CHP, a HP or renewable sources. The EPN and PHN are strongly coupled by the DESs, which are coordinated in a decentralized manner. DES 0 is a HS node and provides heat to the other 3 DESs through the PHN. The other DESs serving as heat customers can also flexibly adjust their heat extraction from the PHN by running local CHPs and HPs as a supplementary. The time step of day-ahead dispatch is 1 hour.

Table I lists the system parameters, including generation limits of CHPs and HPs, the heat-to-power ratios and COPs, historical fluid flow rates along with their adjustment ranges, and some parameters used in the decentralized numerical calculation. More detailed data is provided in [17]. Profiles of total power and heat loads and renewable energy forecasts are shown in Fig. 4. The subproblem of each DES is solved with ECOS [26] on a 3.4GHz PC computer with 8GB of memory.

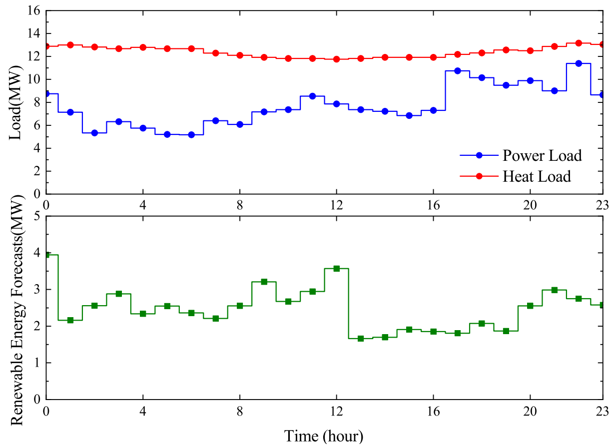

Fig. 4. Power and heat loads and renewable energy forecasts.

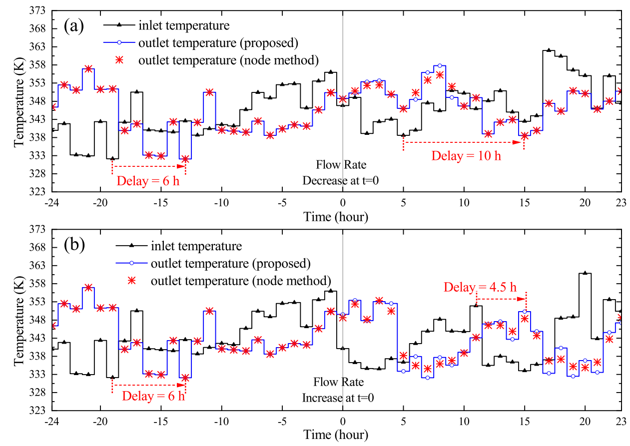

Fig. 5. Calculated water temperature from the proposed model and the node method for pipeline 01. The fluid flow rates and the inlet temperatures are manually given. (a) fluid flow rates decrease at t=0. (b) fluid flow rates increase at t=0.

TABLE I
PARAMATERS USED IN THE CALCULATION

Item	Value	Item	Value
$[\underline{P}_0^{\text{CHP}}, \overline{P}_0^{\text{CHP}}]$	$[2, 8]$MW	k_0^{CHP}	2
$[\underline{P}_1^{\text{CHP}}, \overline{P}_1^{\text{CHP}}]$	$[0.4, 1.6]$MW	k_1^{CHP}	1
$[\underline{P}_2^{\text{CHP}}, \overline{P}_2^{\text{CHP}}]$	$[0.8, 3.2]$MW	k_2^{CHP}	1
$[\underline{P}_3^{\text{CHP}}, \overline{P}_3^{\text{CHP}}]$	$[0.4, 1.6]$MW	k_3^{CHP}	1
$[\underline{P}_1^{\text{HP}}, \overline{P}_1^{\text{HP}}]$	$[0, 0.43]$MW	COP_1^{HP}	1.4
$[\underline{P}_2^{\text{HP}}, \overline{P}_2^{\text{HP}}]$	$[0, 0.43]$MW	COP_2^{HP}	1.4
$[\underline{P}_3^{\text{HP}}, \overline{P}_3^{\text{HP}}]$	$[0, 0.43]$MW	COP_3^{HP}	1.4
$f_0^{\text{CHP}}(\cdot)$	$2x^2 + 440x$	$f_1^{\text{CHP}}(\cdot)$	$25x^2 + 370x$
$f_2^{\text{CHP}}(\cdot)$	$25x^2 + 370x$	$f_3^{\text{CHP}}(\cdot)$	$25x^2 + 370x$
$\dot{m}_1^{\text{his}}$	50.89kg/s	$[\underline{\dot{m}}_1, \overline{\dot{m}}_1]$	$[25.45, 76.33]$kg/s
$\dot{m}_2^{\text{his}}$	50.89kg/s	$[\underline{\dot{m}}_2, \overline{\dot{m}}_2]$	$[25.45, 76.33]$kg/s
$\dot{m}_3^{\text{his}}$	50.89kg/s	$[\underline{\dot{m}}_3, \overline{\dot{m}}_3]$	$[25.45, 76.33]$kg/s
$[\underline{\dot{m}}_{01}, \overline{\dot{m}}_{01}]$	$[76.34, 229.02]$kg/s	ρ	50
$[\underline{\dot{m}}_{12}, \overline{\dot{m}}_{12}]$	$[25.45, 76.34]$kg/s	ϵ_1^{tol}	0.2MW/0.2K
$[\underline{\dot{m}}_{13}, \overline{\dot{m}}_{13}]$	$[25.45, 76.34]$kg/s	ϵ_2^{tol}	0.005MW/0.01K
s^{ref}	0.025	$\overline{T}_i$	363K(90°C)

B. Comparisions of the proposed model and the node method

Fig. 5 compares the simulation results of water temperatures for supply pipeline 01 given by the proposed model and the node method. The PHN fluid flow rates and the inlet temperatures are manually given. It can be observed that the results of the two methods are quite close to each other. When the pipeline delay time is an integer multiple of the dispatch time step, e.g., when t<0 or t=10-23 in Fig. 5(a), the outlet flow temperatures given by different methods are identical. When the delay time is not an integer multiple of Δt, or when the fluid flow rates encounter a sudden change, the decision of flow temperatures may have a max difference of 2-3K. Nevertheless, the trends of the curves are highly consistent.

C. Performance of the Decentralized Optimization Method

Fig. 6 displays the convergence trajectories when the proposed decentralized dispatch method is employed. According to Fig. 6(a), the objective cost function and lagrangian function converges in 1500 iterations. The residuals also drop to the thresholds in about 1200 iterations as illustrated by Fig. 6(c) and (d), and each iteration takes about 0.04s to solve the subproblems and perform gradient descents. The decisions of fluid flow rates are shown in Fig. 6(b). In the first 400 iterations, the fluid flow rates present a stairlike variation because the high residuals prevent frequent executions of algorithm 2. As the iteration continues, the residuals reach a low level and the complicated decision variables are then updated in company with the multipliers and simple decision variables in each iteration.

A direct application of parallel ADMM [24] to solve the nonconvex dispatch problem is also realized. Different from the proposed method, the reference ADMM further includes the consensus terms of $\mathbf{Y}_i$ into the local objective compared to (50) and simultaneously optimizes all of the primal variables, i.e., $\mathbf{X}_i$ and $\mathbf{Y}_i$ in each subproblem. To solve the nonconvex subproblem, Generalized Benders Decomposition(GBD) [27] is employed to locally decompose and optimize $\mathbf{X}_i$ and $\mathbf{Y}_i$. To enhance the astringency of the reference method, different relaxation factors have been introduced to the information exchange step (c.f. step 4 in algorithm 1) and proximal terms [28] have been involved. However, none of the trials is observed to converge even to an approximately workable solution. A graphical representation of the convergence trajectories is given in the supplementary materials [17]. The oscillations of the decision variables are very strong and are inferred to mainly result from the bilinear terms in constraint (21), where the fluid flow rates, the flow temperatures and the heating power related to their products are all decision variables.

D. Impact of Hybrid Quality and Quantity Regulation

To illustrate the effectiveness of the proposed method, we investigate the reference case where fluid flow rates remain historical values. The operation status of the DESs is compared in Fig. 7. Some important indices are compared in Table II. It can be observed that with a comprehensive regulation of heat transport through hybrid quality and quantity dispatch, the

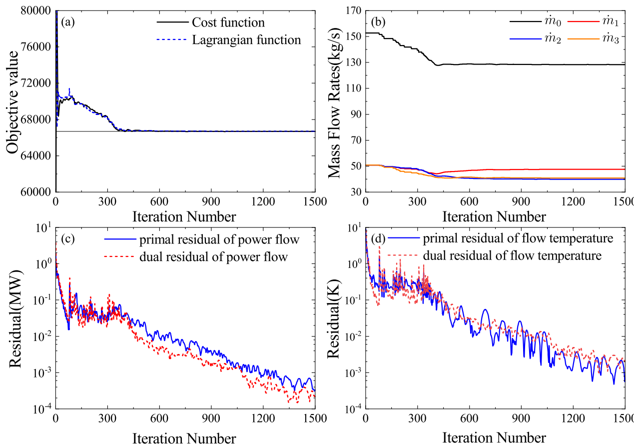

Fig. 6. Convergence trajectories of: (a) Objective function value. (b) fluid flow rates. (c) Residuals of transmission line power flow. (d) Residuals of flow temperatures in PHN

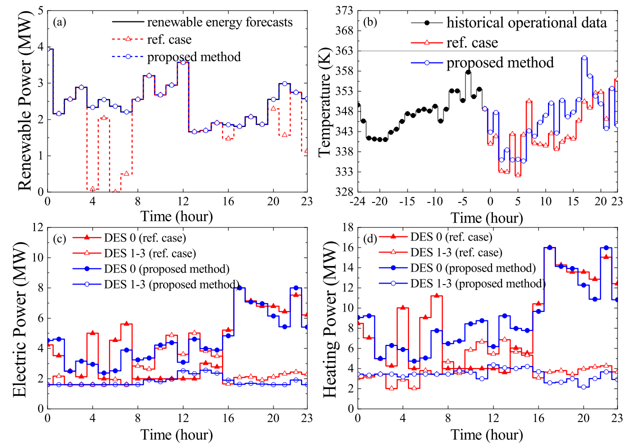

Fig. 7. Operation status of the DESs: (a) Power output of renewable energy sources. (b) Historical and dispatched supply water temperature at HES 0. (c) Electric power output of the CHPs. (d) Heat power output of the CHPs and HPs.

renewable energy is completely accommodated, and the total cost decreases by 5.3%. This is accomplished by decreasing the PHN fluid flow rates by about 5%-20% and appropriately increasing the supply water temperature. A higher temperature does not lead to a larger heat loss in the pipelines. Instead, the lower flow rates weaken the heat transfer from high-temperature water flow to the environment.

The improvement in heat losses is subtle and cannot explain the great economic impact of renewable accommodation. Actually, we can note that the DESs serving as HC nodes have lower heat generation and extract more heat from the PHN compared to the reference case, which means the DESs throughout the entire system can work in a more complementary manner. Remembering that in Table I CHP 0 seems expensive in terms of electricity generation but has a higher heat-to-power ratio, increasing its output and decreasing those of CHP 1, 2 and 3 can better make a way for the renewables. The failure of such attempts without fluid flow rates regulation may be explained by the time delay. In the reference case, the pipeline delay time between the DES 0 and the other HC DESs is about 7 hours. If the HC nodes are going to increase heat exchange with the PHN at time section 21 and 23 to provide generation flexibility to the renewables, a certain amount of heat must be stored to the PHN by DES 0 in advance. According to the delay time, appropriate candidate time sections would include time sections 14-16 and 0-5. However, these time sections are load valleys and increasing output of CHP 0 to store the heat may lead to curtailment of the renewables. Another option is to exploit the heat stored in the PHN before the dispatched day. However, as seen from the historical operation data in Fig. 7 (b), the supply water temperature beyond time sections -5-0 is relatively low and cannot be easily extracted from the PHN. The proposed method increases the average delay time to approximately 9 hours, which enables DES 0 to store heat to the PHN at load peaks and exploit the heat stored before the dispatched day. Therefore, the economy of the connected DESs is significantly increased through a better synergy.

In addition to the delay time, fluid flow rates also relate to the heat capacity flow rates and the thermal resistances of the HESs. Analogic to conventional heat storage devices, the fluid flow rates would affect the heat storage capacities and maximum charging or discharging rates, which are also important but not discussed in detail in the test case. It is obvious that higher fluid flow rates would definitely lead to higher storage capacities and higher maximum charging or discharging rates, and thus benefit the entire system. Yet according to the proposed method, the fluid flow rates should better be decreased, which indicates a dominant effect of delay time in this particular case.

TABLE II
COMPARISONS OF THE PROPOSED METHOD AND THE REFERENCE CASE

	Reference Case	Proposed Method
Cost	70408.50	66684.06(-5.3%)
Renewable curtailment	17.3%	0.0%
Heat losses(MWh,%)	3.07 (1.03%)	2.06 (0.7%)
$\dot{m}_1$(kg/s)	50.89	47.61
$\dot{m}_2$(kg/s)	50.89	39.90
$\dot{m}_3$(kg/s)	50.89	41.87
Delay time of pipeline 01 (hour)	6	7.13
Delay time of pipeline 12 (hour)	2	2.49
Delay time of pipeline 13 (hour)	2	2.55
Heat injected to the PHN by DES 0 (MWh)	202.47	218.28
Local Heat generation of DES 1, 2, and 3(MWh)	97.88	81.28

V. Conclusion

By applying trigonometric expansion to nodal temperatures, a new model is established to model pipeline dynamics in DHN considering regulation of fluid flow rates. Besides, the heat transfer processes throughout the entire heating system are analyzed and the heat current method is employed to comprehensively model them. On this basis, an optimization model of multiple DESs connected through the EPN and the DHN is established.

To deal with the non-convexity of the model, a primal-decomposition-based decentralized gradient descent method in company with ADMM is proposed to optimize the power and heat transport among multi-agent DESs considering hybrid quality and quantity regulation. The method can be considered as equivalent to solving a saddle-point problem in a fully decentralized manner.

Case study on an integrated system validates the effectiveness of the proposed model and method. A direct application of ADMM with various relaxation methods does not converge, while the proposed method manages to adjust the fluid flow rates and reduces the curtailment rate of renewable energy from 17.3% to 0 compared to the case without quantity regulation.

Our future work will further investigate the effect of uncertainties in the highly integrated user-side energy systems and attempt to propose both a robust day-ahead optimization method and fast on-line adjusting strategies.

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