

# Down the rabbit hole with hierarchical autoregressive neural networks

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today

# Neural network: what you think it is

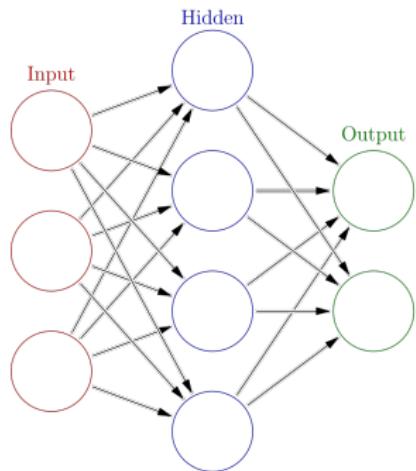


Figure: First Google Image search result for string "neural network"

# Neural network: what you think it is

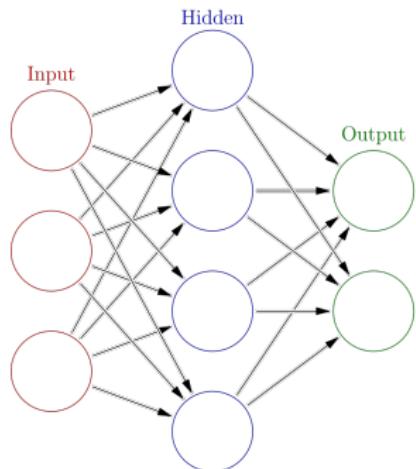


Figure: First Google Image search result for string "neural network"



Figure: Your reaction to that information

# Neural network: what it really is

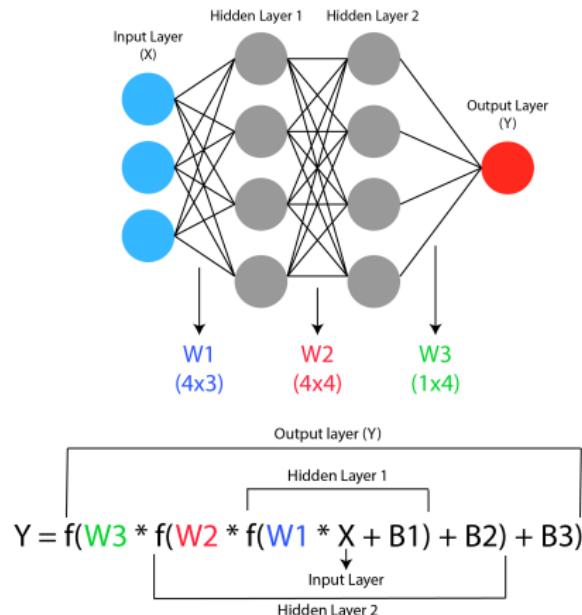


Figure: In physics, everything is a matrix

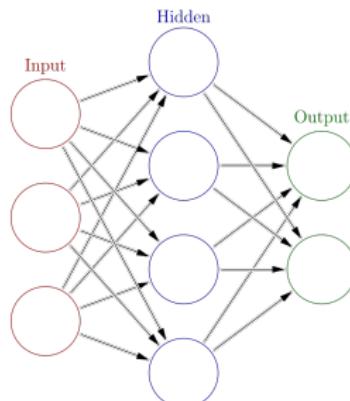
# Neural network: what it does



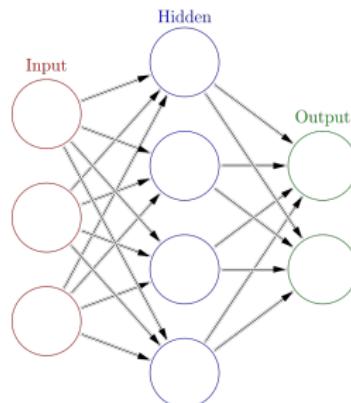
# Neural network: what it does



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Firlej	0.95
cushion	0.82
mug	0.53
hair	0.42
hand	0.34

# Autoregression

$$X_t = \sum_{i=1}^p \varphi_i X_{t-i} + \varepsilon_t$$

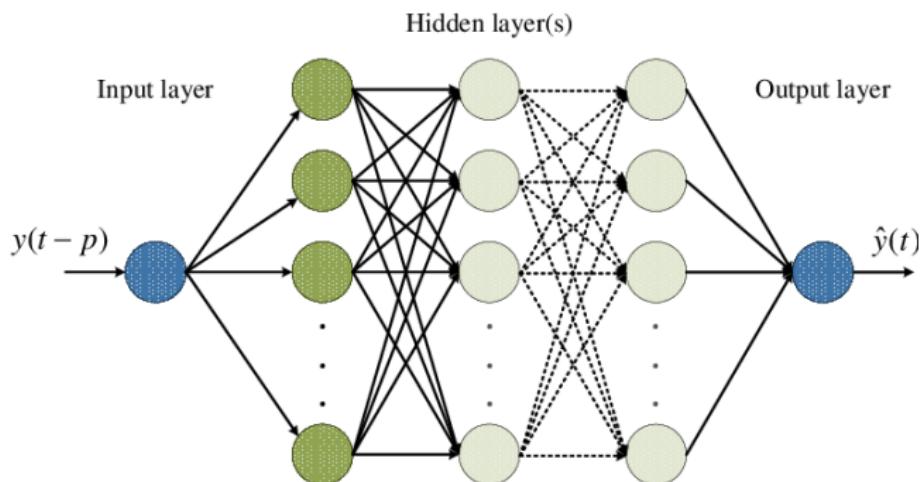


Figure: An example of autoregressive neural network

# Hierarchy

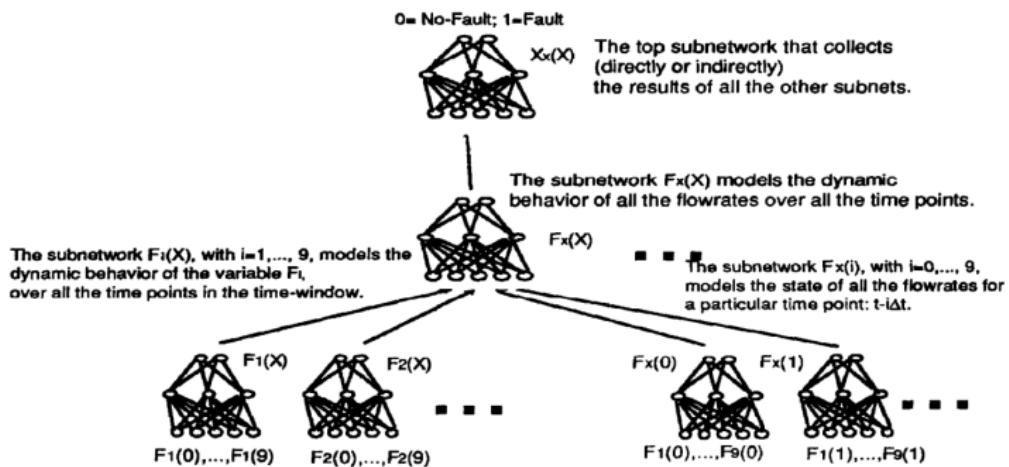
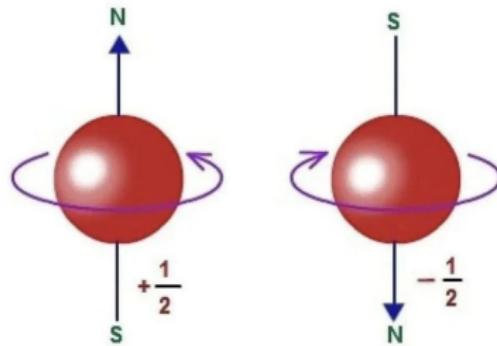


Figure: An example of hierarchical neural network

# What is spin

Electron spin explained: imagine a ball that's rotating, except it's not a ball and it's not rotating



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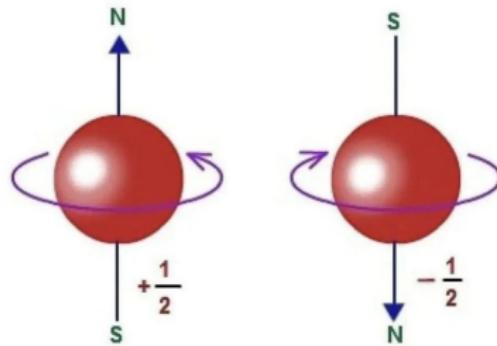


Figure: This is not what spin means

# What is spin, again

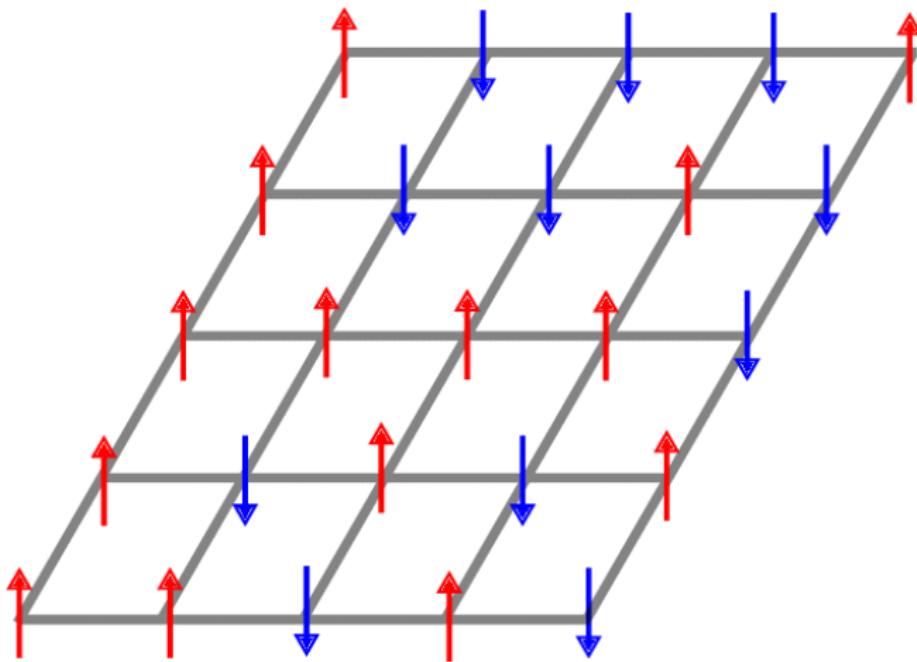


Figure: Illustration of 2D Ising model

# Some physics now

$$M(\sigma) = \sum_{i \in \text{spins}} s_i$$

$$E(\sigma) = -\frac{1}{2} \sum_{i,j \in \text{neighbours}} s_i s_j$$

$$p(\sigma) = \frac{e^{-\beta E(\sigma)}}{Z(\sigma)}$$

# How to apply hierarchy

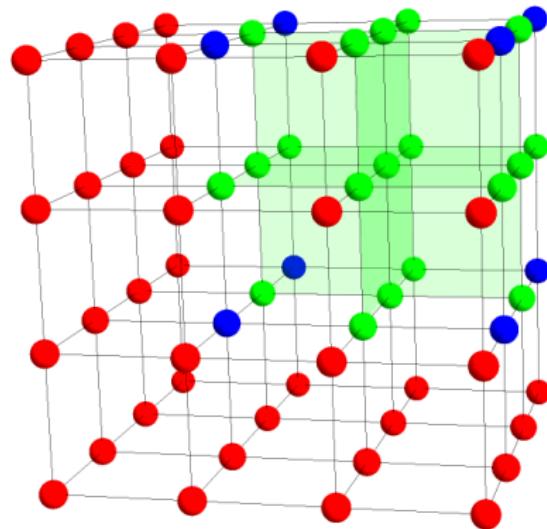


Figure: Hierarchy in 4x4x4 cube

# How to apply autoregression

$$F = -\frac{1}{\beta} \log Z$$

$$F = \frac{1}{\beta} \sum_{i \in \text{configurations}} q_\theta(\sigma_i) [\beta E(\sigma_i) + \log q_\theta(\sigma_i)]$$

# Magnetization: Monte Carlo vs neural network

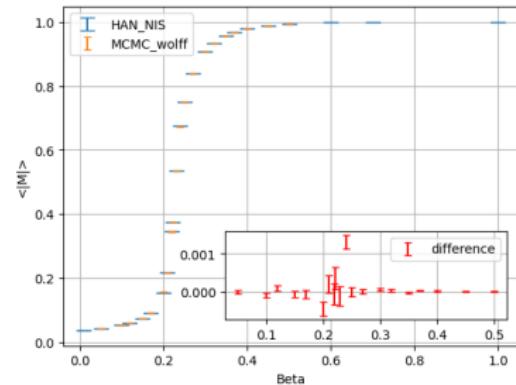
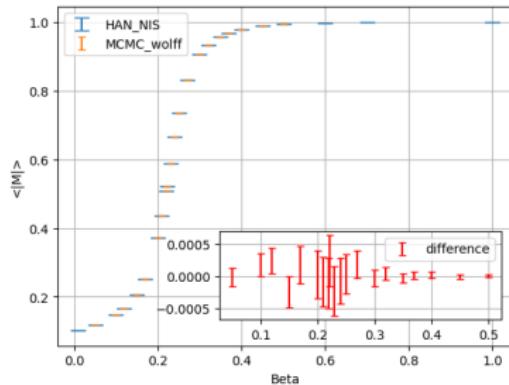
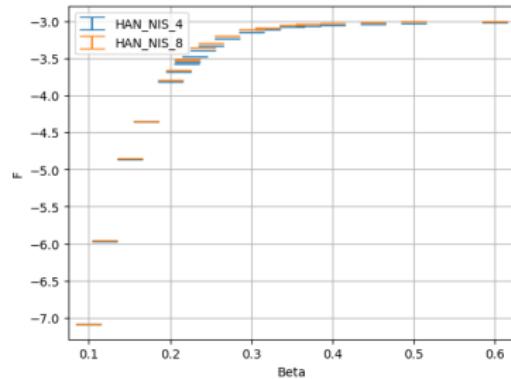
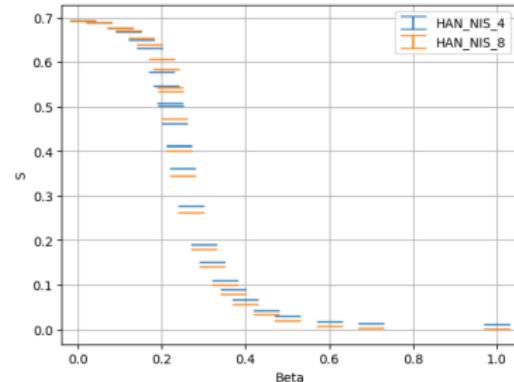


Figure: Magnetization vs temperature function  $\beta$

## Extensive properties: neural network



(a) free energy



(b) entropy

Figure: Extensive parameters vs temperature parameter  $\beta$  for  $4 \times 4 \times 4$  and  $8 \times 8 \times 8$  nets

# Bibliography

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-  Solving Statistical Mechanics Using Variational Autoregressive Networks, arXiv:1809.10606
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Thanks for attention



<https://indico.cern.ch/event/1361918/contributions/5785806/attachments/2840617/4965149/title.pdf>