Garbage Collection

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A program's objects (V) and references (E) are represented by a directed graph $P = (V, E)$. Each program has at least $\varphi \in V$.

Math	English
$\dfrac{a \in V}{P \vdash a}$	If a is a node in V, then we can say it's "in" P.
$\dfrac{}{P \vdash \varphi}$	The node φ always exists in P.
$\dfrac{(a, b) \in E}{P \vdash a \to b}$	If there is a directed edge from a to b, we write $a \to b$.
$\dfrac{P \vdash (a \to b) \land (b \to c)}{P \vdash a \rightsquigarrow c}$	Intr' of a new notation $\rightsquigarrow$.
$\dfrac{P \vdash a}{P \vdash a \rightsquigarrow a}$	Reflexivity of $\rightsquigarrow$ shown.
$\dfrac{P \vdash (a \rightsquigarrow b) \land (b \rightsquigarrow c)}{a \rightsquigarrow c}$	Transitivity of $\rightsquigarrow$ shown.
$\therefore P \vdash a \rightsquigarrow b \text{ iff } \exists x_1 ... x_n \in V \ni:$ $(a = x_1) \land (b = x_n) \land (x_1 \to ... \to x_n)$	Therefore, the meaning of $a \rightsquigarrow b$ is "there exists some path from a to b".
$\dfrac{P \vdash \varphi \rightsquigarrow a}{P \vdash R(a)}$	Intr' of a new predicate $R(x \in V)$. If $R(a)$ holds, we say that a is referenced by the program.
$\dfrac{P \vdash b}{P \vdash b^n \text{ where } n = \lvert\{a \mid a \to b \land R(a)\}\rvert}$	Intr' of a new notation a^n. It can be thought of as the number of "meaningful references to a."
$\dfrac{P \vdash R(a)}{P \vdash a^n \text{ where } n > 0}$	If $R(a)$ holds, what does that say about a^n?
$\dfrac{P \vdash a^0}{P \vdash \cancel{R(a)}}$	If a has no "meaningful references", does $R(a)$ hold?
$\dfrac{P \vdash ([a \to b] \twoheadrightarrow [a \not\to b]), R(a)}{P \vdash b^n \twoheadrightarrow b^{n-1}}$	Intr' of new notation $\twoheadrightarrow$. When $a \twoheadrightarrow b$, we say "a steps to b".