Saturated Semantics for Coalgebraic Logic Programming

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École Normale Supérieure de Lyon, France

CALCO 2013

Coalgebraic Logic Programming (the ground case)

[E. Komendantskaya, G. McCusker & J. Power 2010]

$$
\begin{array}{rcl}
\mathtt{r(b,c)} & \leftarrow & \mathtt{q(a),q(b),q(c)} \\
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\mathtt{r(b,b)} & \leftarrow & \mathtt{q(c)} \\
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\end{array}
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```
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```

Coalgebra in Set

$$p \quad : \quad At \to \mathcal{P}_f\mathcal{P}_f(At)$$

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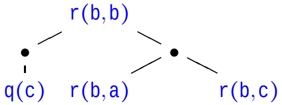

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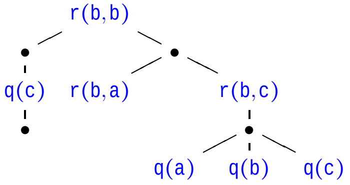

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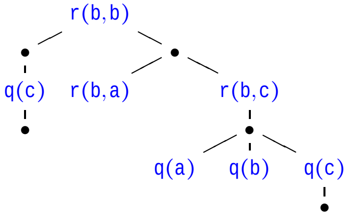

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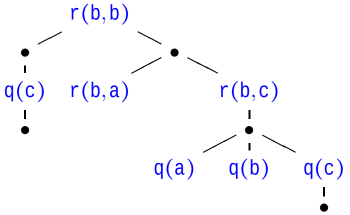
$$[\![\mathtt{r(b,b)}]\!]_p =$$



Coalgebra in **Set**

$$\begin{array}{rcl} p & : & At \to \mathcal{P}_f\mathcal{P}_f(At) \\ [\![-]\!]_p & : & At \to \mathcal{C}(\mathcal{P}_f\mathcal{P}_f)(At) \end{array}$$

Coalgebraic Logic Programming (the general case)

[E. KOMENDANTSKAYA & J. POWER, CALCO 2011]

$$
\begin{aligned}
\mathtt{List}(\mathtt{c}(\mathtt{x}_1,\mathtt{x}_2)) &\leftarrow \mathtt{Nat}(\mathtt{x}_1), \mathtt{List}(\mathtt{x}_2) \\
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$\boxed{\mathbf{L}_\Sigma}$ free Lawvere Theory on Σ

$\boxed{\textbf{objects}}$ natural numbers $(n \approx < x_1, \ldots, x_n >)$.

$\boxed{\textbf{arrow } \theta : n \to m}$ a substitution $[x_1 \mapsto t_1, \ldots, x_n \mapsto t_n]$, where $t_1, \ldots, t_n$ are Σ-terms on variables $x_1, \ldots, x_m$.

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$\boxed{At : \mathbf{L}_\Sigma \to \mathbf{Set}}$ space of $\mathbf{L}_\Sigma$-typed atoms

$\boxed{At(n)}$ the set of atoms on variables $x_1, \ldots, x_n$.

$\boxed{At(\theta : n \to m)}$ a function mapping A into $\theta(A)$.

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Coalgebra in $\mathbf{Set}^{\mathbf{L}_\Sigma}$

$$\begin{array}{rcl} p & : & At \to \widetilde{\mathcal{P}_f}\widetilde{\mathcal{P}_f}(At) \\ [\![-]\!]_p & : & At \to \mathcal{C}(\widetilde{\mathcal{P}_f}\widetilde{\mathcal{P}_f})(At) \end{array}$$

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Definition of p

[E. KOMENDANTSKAYA & J. POWER, CALCO 2011]

Term-Matching

$$\text{Atom } A \rightsquigarrow A = \tau(H) \overset{\exists\tau}{\leftsquigarrow} \text{Clause } \boxed{H \leftarrow B_1, \ldots, B_k}$$

$$\Downarrow$$

$$\{\tau(B_1), \ldots, \tau(B_k)\} \in p_n(A)$$

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\text{Atom } A \arrow[r, squiggly] & A = \tau(H) \arrow[d, Rightarrow] & \text{Clause } \boxed{H \leftarrow B_1, \ldots, B_k} \arrow[l, squiggly, "\exists\tau"'] \\
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$$

The problem

$p : At \to \widetilde{\mathcal{P}_f}\widetilde{\mathcal{P}_f}(At)$ is **not** a natural transformation.

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\begin{tikzcd}
\texttt{List}(\texttt{x}_1) \arrow[r, mapsto, "p_1"] \arrow[d, mapsto, "At([x_1 \mapsto nil])"'] & \varnothing \arrow[d, mapsto, "\widetilde{\mathcal{P}_f}\widetilde{\mathcal{P}_f}(At)([x_1 \mapsto nil])"] \\
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The final semantics is **not compositional**:

$$[[\theta(A)]]_p \neq \overline{\theta}([[A]]_p).$$

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Cfr.

$$\begin{tikzcd}
t = \overline{a} < x > | b(y) \arrow[r, mapsto] \arrow[d, mapsto, "{[b \mapsto a]}"'] \arrow[dr, phantom, "\neq"] & \alpha(t) \arrow[d, mapsto, "{\alpha([b \mapsto a])}"] \\
t' = \overline{a} < x > | a(y) \arrow[r, mapsto] & \alpha(t')
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Saturated Semantics

$$p \quad \mapsto \quad p^\sharp$$

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$$\begin{tikzcd} At \in \mathbf{Set}^{\mathbf{L}_\Sigma} \arrow[rr, bend left, "\mathcal{U}"] & & \mathbf{Set}^{|\mathbf{L}_\Sigma|} \end{tikzcd}$$

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$$\begin{tikzcd} & \mathcal{U}(At) \arrow[d, "p"] \\ \boxed{\mathbf{Set}^{|\mathbf{L}_\Sigma|}} & \hat{\mathcal{P}}_f\hat{\mathcal{P}}_f\mathcal{U}(At) \end{tikzcd}$$

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$$\begin{tikzcd} At \arrow[r, "\eta_{At}"] \arrow[dr, "p^\sharp"'] & \mathcal{K}\mathcal{U}(At) \arrow[d, "K(p)"] \\ & \mathcal{K}\hat{\mathcal{P}}_f\hat{\mathcal{P}}_f\mathcal{U}(At) \end{tikzcd}$$

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Coalgebra in $\mathbf{Set}^{\mathbf{L}_\Sigma}$

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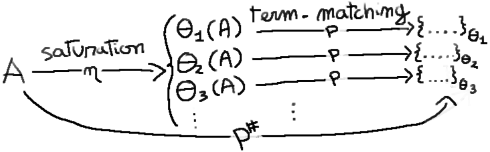
Saturated Semantics



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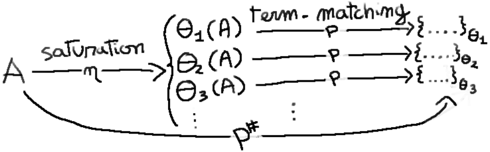

p ~ term-matching
$p^\sharp$ ~ unification

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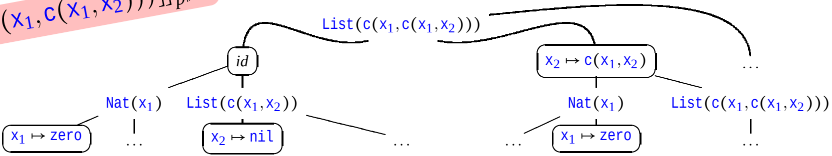
Saturated Derivation Trees

$[\![\mathtt{List}(\mathtt{c}(\mathtt{x}_1, \mathtt{c}(\mathtt{x}_1, \mathtt{x}_2)))]\!]_{p^\sharp} =$



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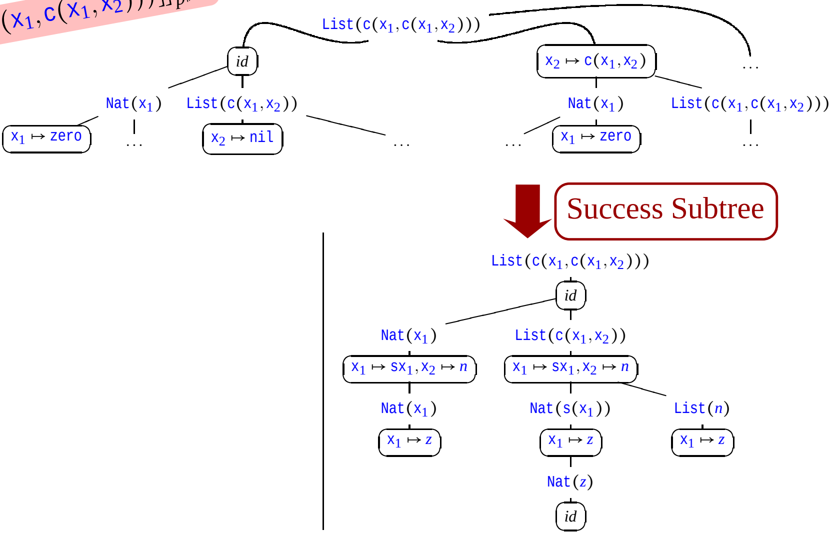

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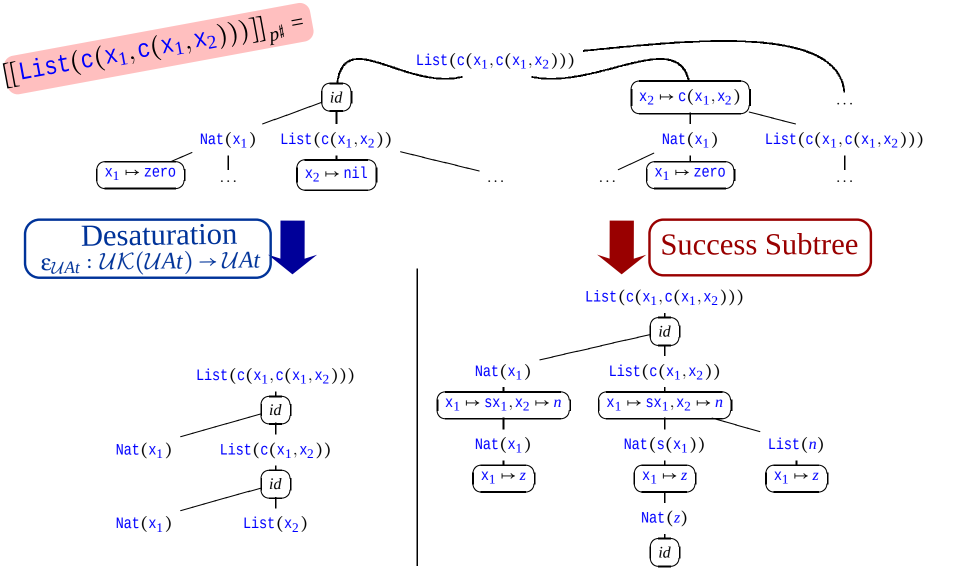

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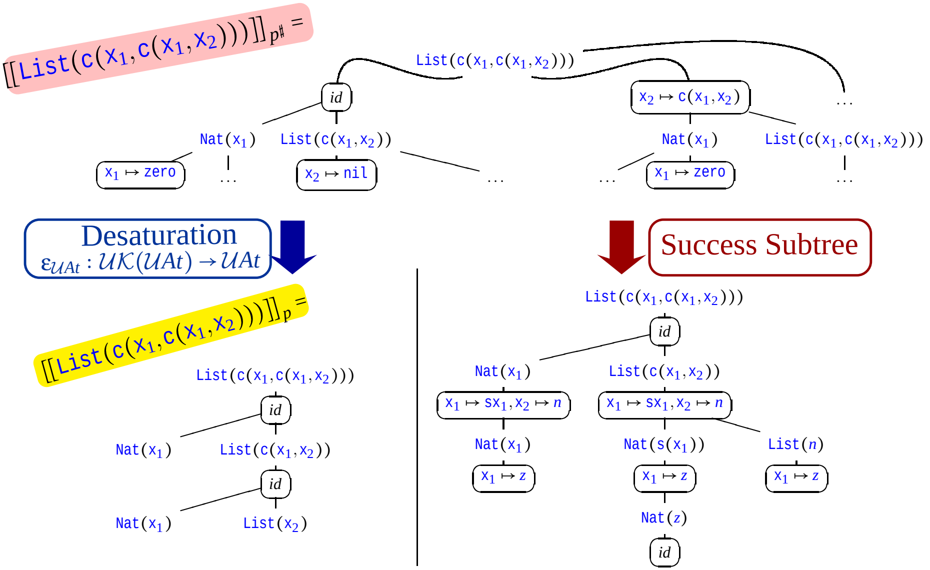
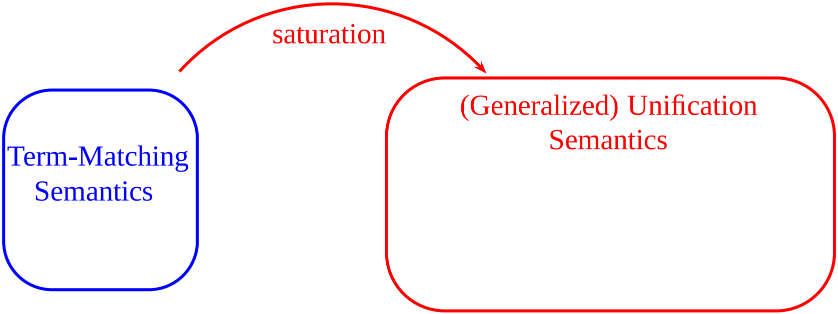


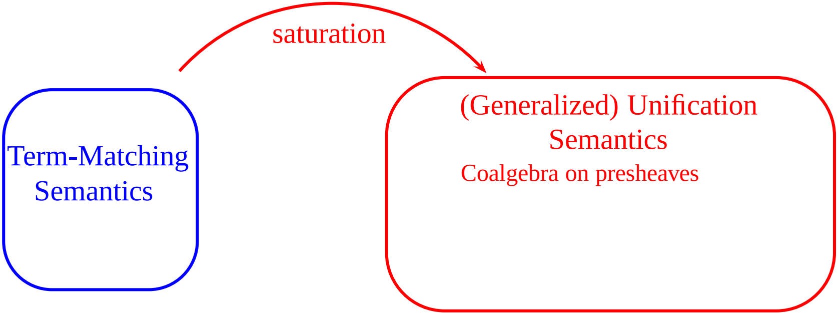
Summary

Term-Matching
Semantics

Summary


saturation
Term-Matching Semantics
(Generalized) Unification Semantics

Summary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Coalgebra on presheaves

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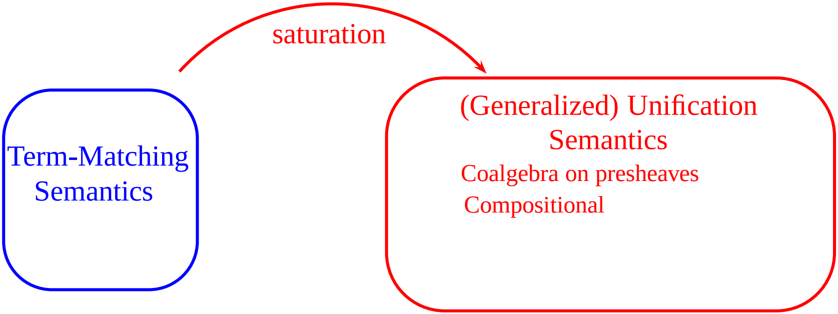
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Coalgebra on presheaves
Compositional

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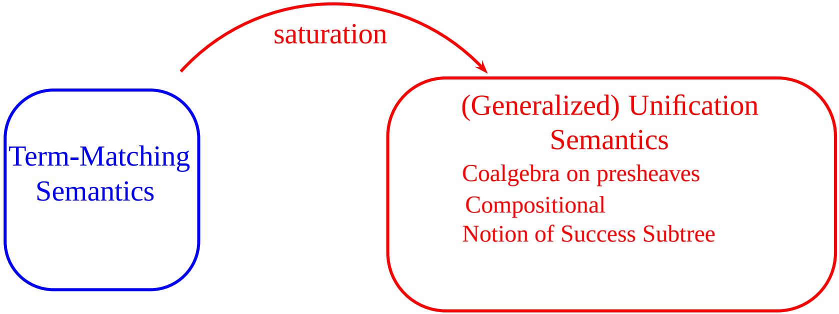
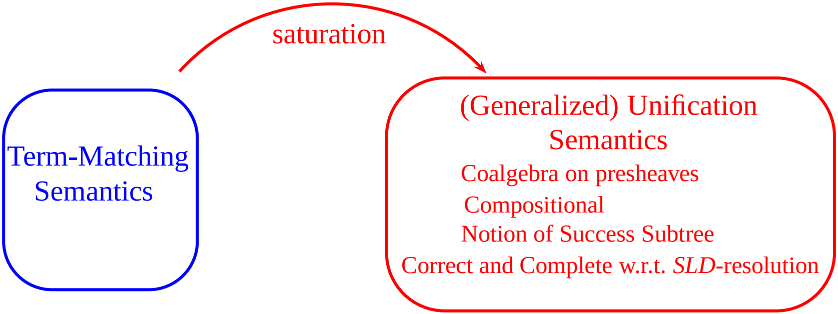

saturation
Term-Matching Semantics
(Generalized) Unification Semantics
Coalgebra on presheaves
Compositional
Notion of Success Subtree

Summary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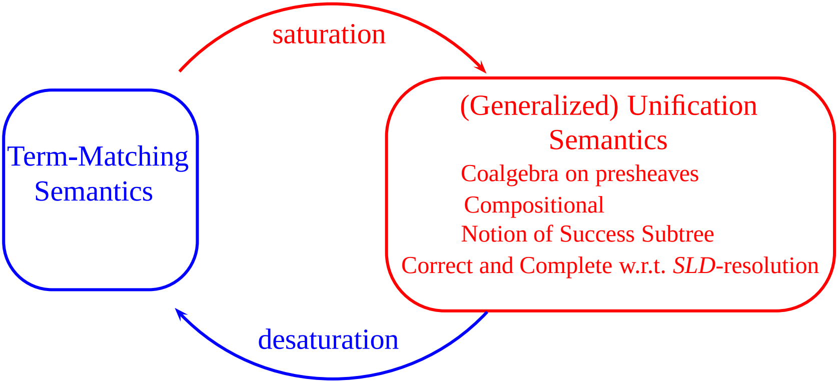
Summary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desaturation