21EES101T

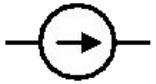
ELECTRICAL AND ELECTRONICS ENGINEERING

Unit-1

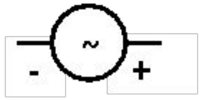
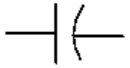
ELECTRIC CIRCUITS

Electric circuits are broadly classified as Direct Current (D.C.) circuits and Alternating Current (A.C.) circuits. The following are the various elements that form electric circuits.

D.C. Circuits

<u>**Elements**</u>	<u>**Representation**</u>
Voltage source	
Current source	
Resistor	

A.C. Circuits

<u>**Elements**</u>	<u>**Representation**</u>
Voltage source	
Current source	
Resistor	
Inductor	
Capacitor	

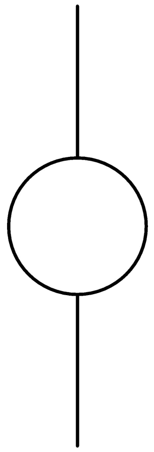
An active element is capable of generating energy while a passive element is not. Typical active elements include generators, batteries and operational amplifiers. Examples of passive elements are resistors, capacitors and inductors. Usually the voltage across a passive element is represented as $v(t)$ while that across a active element is represented either as $v(t)$ or $e(t)$.

We also will classified sources as **Independent** and **Dependent** sources

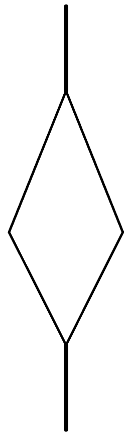
Independent source establishes a voltage or a current in a circuit without relying on a voltage or current elsewhere in the circuit

Dependent sources establishes a voltage or a current in a circuit whose value depends on the value of a voltage or a current elsewhere in the circuit

We will use circle to represent **Independent source** and diamond shape to represent **Dependent sources**



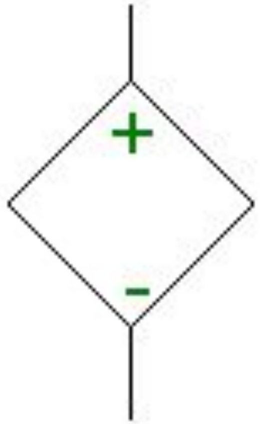
Independent source



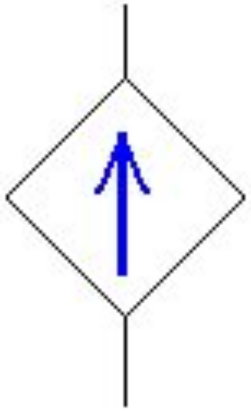
Dependent sources

Dependent Power Sources

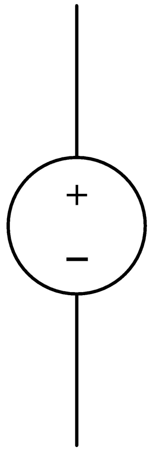
- Voltage controlled voltage source
  - (VCVS)
- Current controlled voltage source
  - (CCVS)



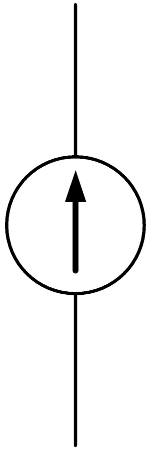
- Voltage controlled current source
  - (VCCS)
- Current controlled current source
  - (CCCS)



Independent and dependent voltage and current sources can be represented as



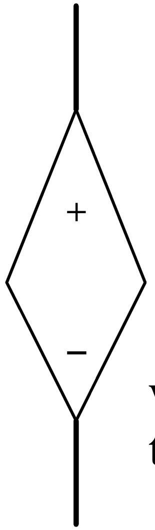
V 5



A 3

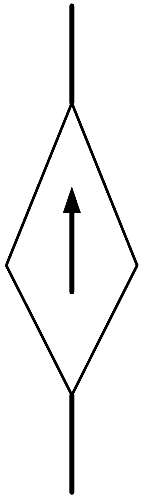
Independent voltage source

Independent current source



$\boldsymbol{i_x}$ ***V*** **4**

were i_x is some current through an element



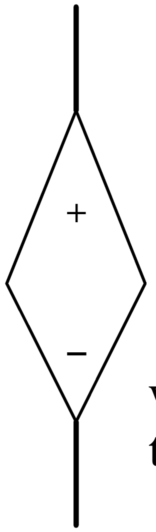
$\boldsymbol{v_x}$ ***A*** **4**

were v_x is some voltage across an element

Dedependent voltage source
Voltage depend on current

Dedependent current source
Current depend on voltage

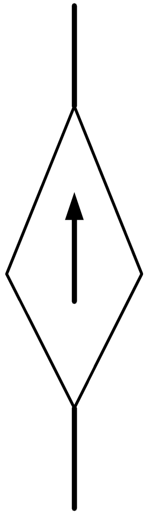
The dependent sources can be also as



v_x V 4

were v_x is some current through an element

Dedependent voltage source
Voltage depend on voltage



i_x A 7

were i_x is some voltage across an element

Dedependent current source
Current depend on current

DC CIRCUITS

The voltage across an element is denoted as E or V. The current through the element is I.

Conductor is used to carry current. When a voltage is applied across a conductor, current flows through the conductor. If the applied voltage is increased, the current also increases. The voltage current relationship is shown in Fig. 1.

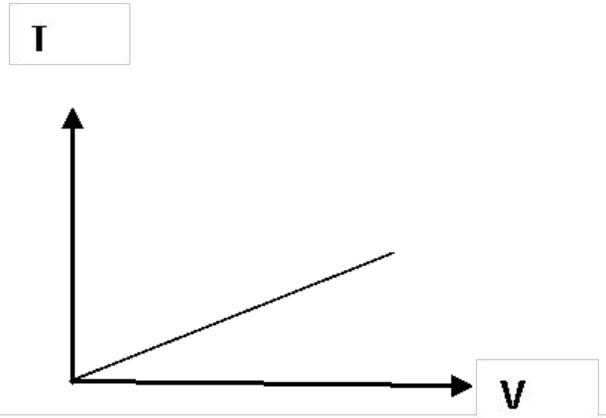


Fig. 1 Voltage – current relationship

It is seen that $I \infty V$. Thus we can write

$$I = G\,V \qquad (1)$$

where G is called the conductance of the conductor.

$$I = G\,V \qquad (1)$$

Very often we are more interested on RESISTANCE, R of the conductor, than the conductance of the conductor. Resistance is the opposing property of the conductor and it is the reciprocal of the conductance. Thus

$$R = \frac{1}{G} \quad \text{or} \quad G = \frac{1}{R} \qquad (2)$$

Therefore $$I = \frac{V}{R} \qquad (3)$$

The above relationship is known as OHM's law. Thus Ohm law can be stated as the current flows through a conductor is the ratio of the voltage across the conductor and its resistance. Ohm's law can also be written as

$$V = R\,I \qquad (4)$$

$$R = \frac{V}{I} \qquad (5)$$

The resistance of a conductor is directly proportional to its length, inversely proportional to its area of cross section. It also depends on the material of the conductor. Thus

$$R = \rho \; \frac{\ell}{A} \tag{6}$$

where ρ is called the specific resistance of the material by which the conductor is made of. The unit of the resistance is Ohm and is represented as Ω. Resistance of a conductor depends on the temperature also. The power consumed by the resistor is given by

$$P = V\,I \tag{7}$$

When the voltage is in volt and the current is in ampere, power will be in watt. Alternate expression for power consumed by the resistors are given below.

$$P = R\,I \times I = I^2 R \tag{8}$$

$$P = V \times \frac{V}{R} = \frac{V^2}{R} \tag{9}$$

KIRCHHOFF’s LAWS

There are two Kirchhoff’s laws. The first one is called Kirchhoff’s current law, KCL and the second one is Kirchhoff’s voltage law, KVL.

Kirchhoff’s current law deals with element currents meeting at a junction, which is a meeting point of two are more elements.

Kirchhoff’s voltage law deals with element voltages in a closed loop also called as closed circuit.

Kirchhoff's current law

Kirchhoff's current law states that the algebraic sum of element currents meeting at a junction is zero.

Consider a junction P wherein four elements, carrying currents I_1, I_2, I_3 and I_4, are meeting as shown in Fig. 2.

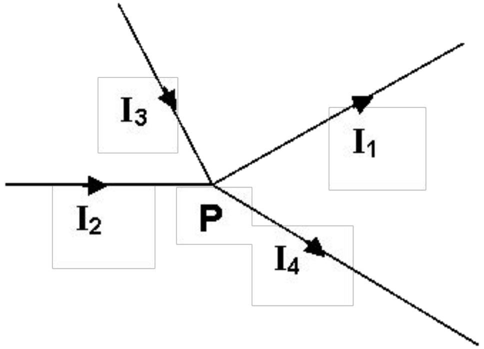


Fig. 2 Currents meeting at a junction

Note that currents I_1 and I_4 are flowing out from the junction while the currents I_2 and I_3 are flowing into the junction. According to KCL,

$$I_1 - I_2 - I_3 + I_4 = 0 \tag{10}$$

The above equation can be rearranged as

$$I_1 + I_4 = I_2 + I_3 \tag{11}$$

From equation (11), KCL can also stated as at a junction, the sum of element currents that flows out is equal to the sum of element currents that flows in.

Kirchhoff's voltage law

Kirchhoff's voltage law states that the algebraic sum of element voltages around a closed loop is zero.

Consider a closed loop in a circuit wherein four elements with voltages V_1, V_2, V_3 and V_4, are present as shown in Fig. 3.

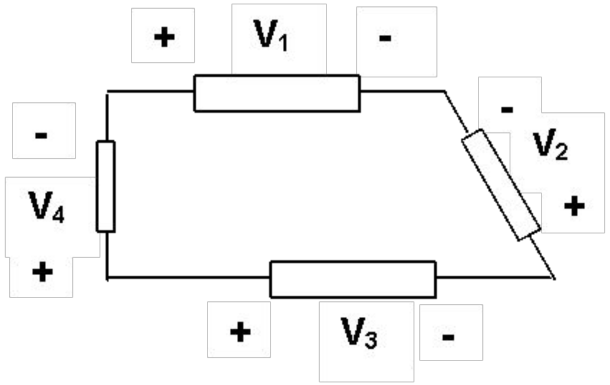


Fig. 3 Voltages in a closed loop

Assigning positive sign for voltage drop and negative sign for voltage rise, when the loop is traced in clockwise direction, according to KVL

$$V_1 - V_2 - V_3 + V_4 = 0 \qquad (12)$$

The above equation can be rearranged as

$$V_1 + V_4 = V_2 + V_3 \qquad (13)$$

From equation (13), KVL can also stated as, in a closed loop, the sum of voltage drops is equal to the sum of voltage rises in that loop.

Resistors connected in series

Two resistors are said to be connected in series when there is only one common point between them and no other element is connected in that common point. Resistors connected in series carry same current. Consider three resisters R_1, R_2 and R_3 connected in series as shown in Fig. 4. With the supply voltage of E, voltages across the three resistors are V_1, V_2 and V_3.

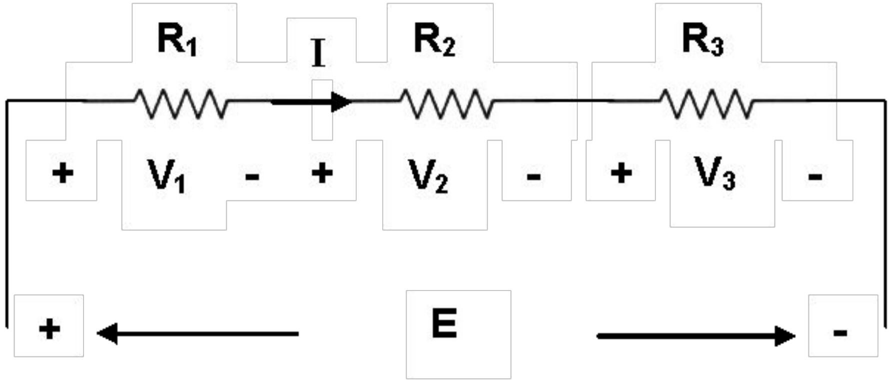


Fig. 4 Resistors connected in series

As per Ohm's law

$$\left.\begin{aligned} V_1 &= R_1 I \\ V_2 &= R_2 I \\ V_3 &= R_3 I \end{aligned}\right\} \quad (14)$$

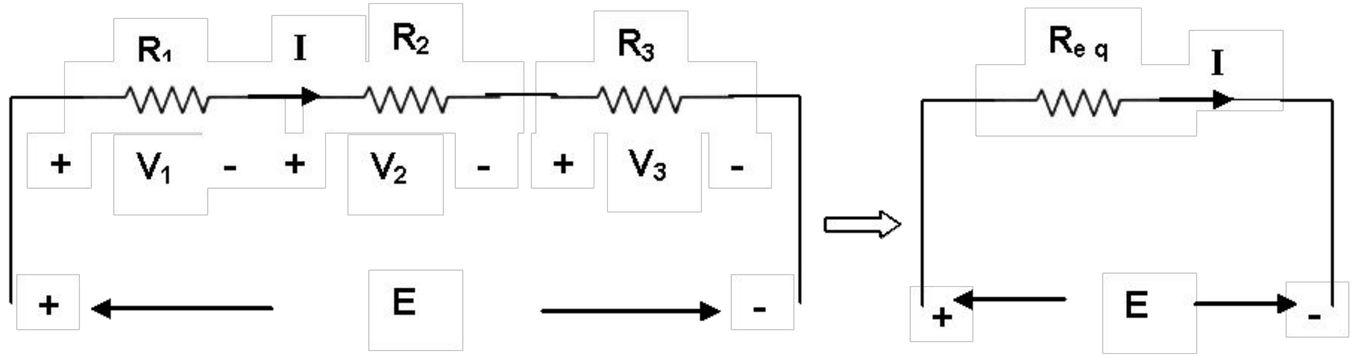


Fig. 4 Resistors connected in series

Applying KVL,

$$E = V_1 + V_2 + V_3 \tag{15}$$

$$= (R_1 + R_2 + R_3)\, I = R_{eq}\, I \tag{16}$$

Thus for the circuit shown in Fig. 4,

$$E = R_{eq}\, I \tag{17}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

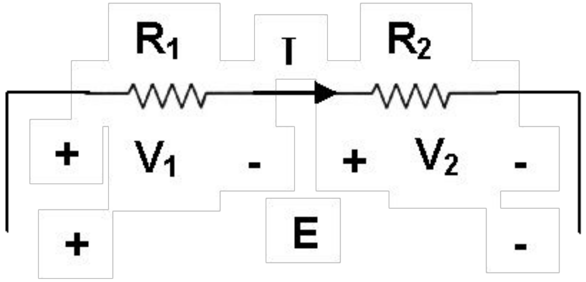
$$R_{eq} = R_1 + R_2 + R_3 \tag{18}$$

This is true when two are more resistors are connected in series. When n numbers of resistors are connected in series, the equivalent resistor is given by

$$R_{eq} = R_1 + R_2 + \dots\dots\dots\dots + R_n \tag{19}$$

Voltage division rule

Consider two resistors connected in series. Then



$V_1 = R_1 I$

$V_2 = R_2 I$

$E = (R_1 + R_2) I$ and hence $I = E / (R_1 + R_2)$

Total voltage of E is dropped in two resistors. Voltage across the resistors are given by

$$V_1 = \frac{R_1}{R_1 + R_2} E \quad \text{and} \tag{20}$$

$$V_2 = \frac{R_2}{R_1 + R_2} E \tag{21}$$

Resistors connected in parallel

Two resistors are said to be connected in parallel when both are connected across same pair of nodes. Voltages across resistors connected in parallel will be equal.

Consider two resistors R_1 and R_2 connected in parallel as shown in Fig. 5.

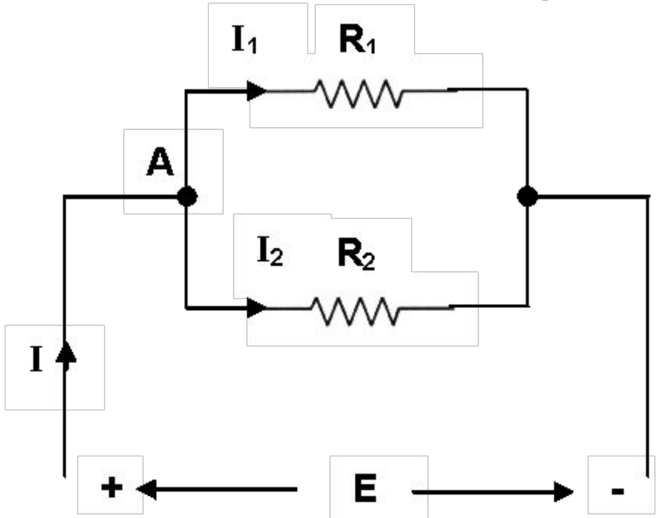
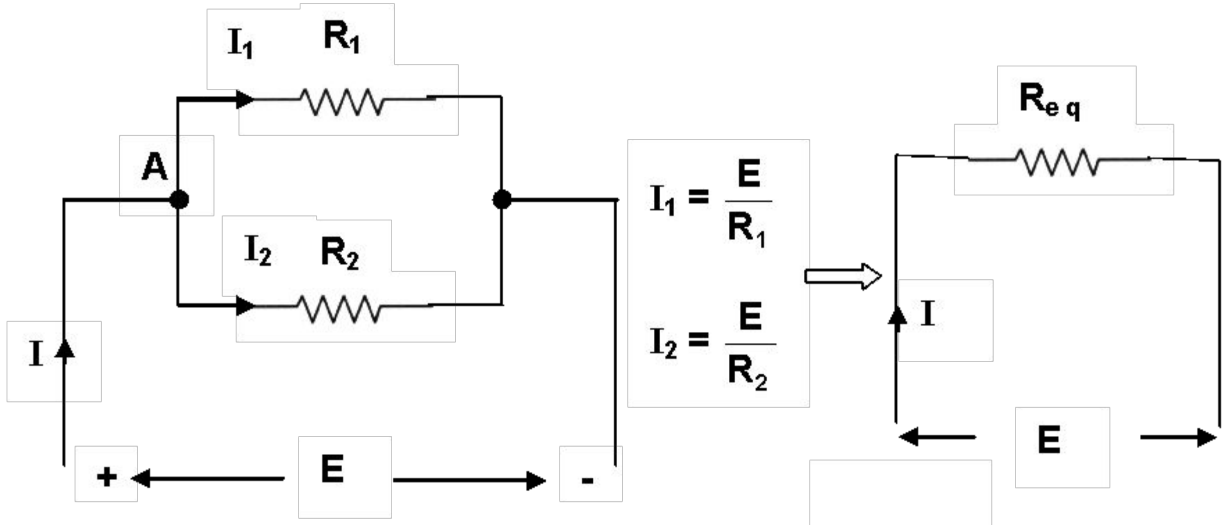


Fig. 5 Resistors connected in parallel

As per Ohm's law,

$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \qquad (22)$$



Applying KCL at node A

$$I = I_1 + I_2 = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_{eq}} \tag{23}$$

Thus for the circuit shown in Fig. 5

$$I = \frac{E}{R_{eq}} \tag{24}$$

where E is the circuit voltage, I is the circuit current and R_{eq} is the equivalent resistance. Here

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{25}$$

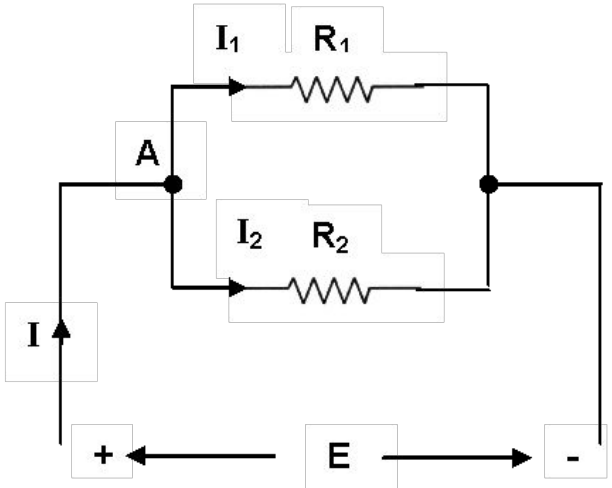
From the above $\dfrac{1}{R_{eq}} = \dfrac{R_1 + R_2}{R_1 R_2}$

Thus $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \tag{26}$$

When n numbers of resistors are connected in parallel, generalizing eq. (25), R_{eq} can be obtained from

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{R_n} \tag{27}$$

Current division rule



$$\left.\begin{aligned} I_1 &= \frac{E}{R_1} \\ I_2 &= \frac{E}{R_2} \end{aligned}\right\} \quad (22)$$

Fig. 5 Resistors connected in parallel

Referring to Fig. 5, it is noticed the total current gets divided as I_1 and I_2. The branch currents are obtained as follows.

From eq. (23)

$$E = \frac{R_1 R_2}{R_1 + R_2} I \quad (29)$$

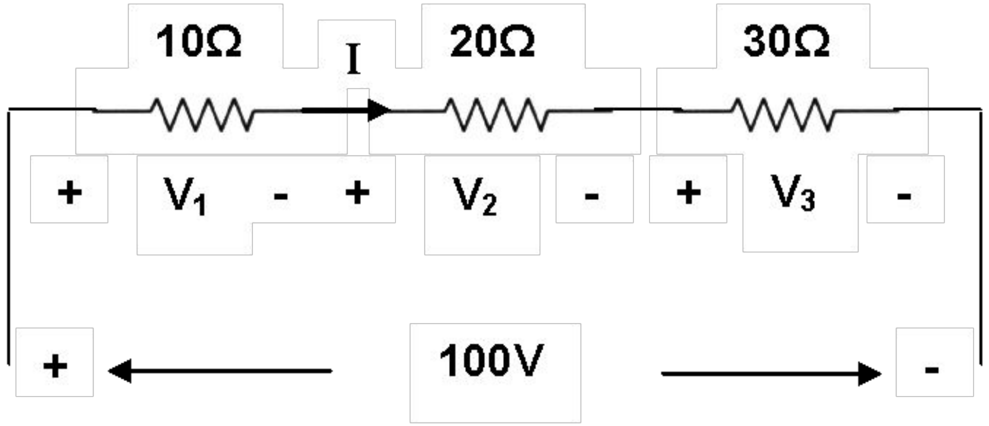
Substituting the above in eq. (22)

$$\begin{aligned} I_1 &= \frac{R_2}{R_1 + R_2} I \\ I_2 &= \frac{R_1}{R_1 + R_2} I \end{aligned} \quad (30)$$

Example 1

Three resistors 10Ω, 20Ω and 30Ω are connected in series across 100 V supply. Find the voltage across each resistor.

Solution



Current I = 100 / (10 + 20 + 30) = 1.6667 A

Voltage across 10Ω = 10 x 1.6667 = 16.67 V

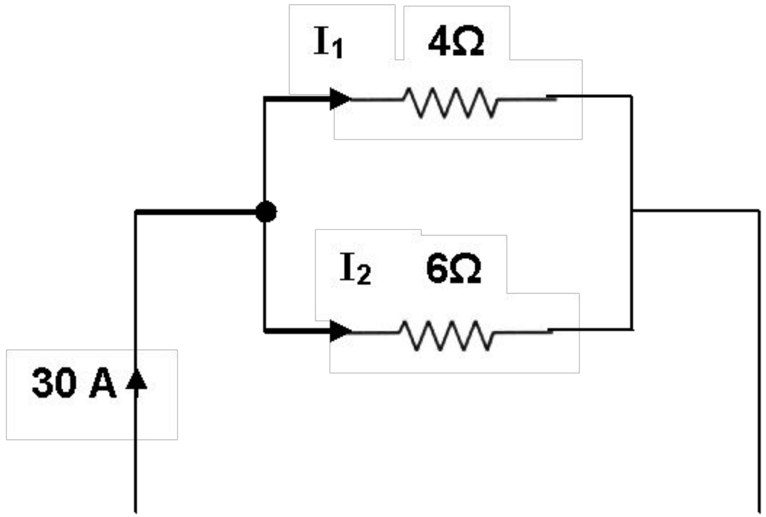
Voltage across 20Ω = 20 x 1.6667 = 33.33 V

Voltage across 30Ω = 30 x 1.6667 = 50 V

Example 2

Two resistors of 4Ω and 6Ω are connected in parallel. If the supply current is 30 A, find the current in each resistor.

Solution



Using the current division rule

Current through 4Ω = $\dfrac{6}{4+6} \times 30 = 18\,A$

Current through 6Ω = $\dfrac{4}{4+6} \times 30 = 12\,A$

Example 3

Four resistors of 2 ohms, 3 ohms, 4 ohms and 5 ohms respectively are connected in parallel. What voltage must be applied to the group in order that the total power of 100 W is absorbed?

Solution

Let R_T be the total equivalent resistor. Then

$$\frac{1}{R_T} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$$

Resistance $R_T = \frac{120}{154} = 0.7792\,\Omega$

Let E be the supply voltage. Then total current taken = E / 0.7792 A

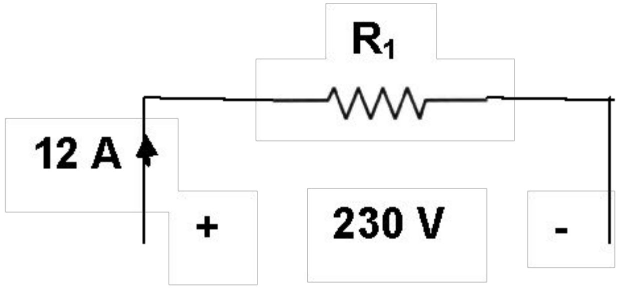
Thus $\left(\frac{E}{0.7792}\right)^2 \times 0.7792 = 100$ and hence $E^2 = 100 \times 0.7792 = 77.92$

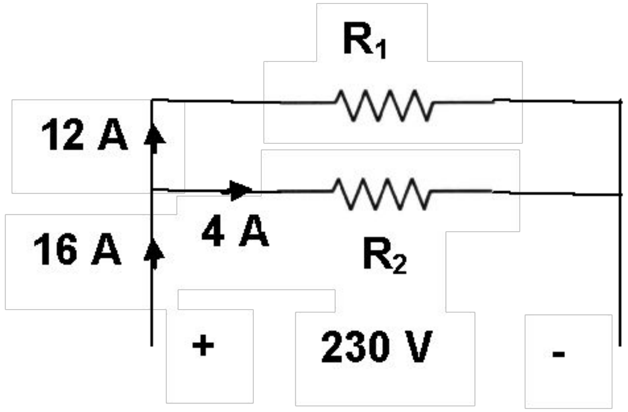
Required voltage = $\sqrt{77.92} = 8.8272$ V

Example 4

When a resistor is placed across a 230 V supply, the current is 12 A. What is the value of the resistor that must be placed in parallel, to increase the load to 16 A?

Solution





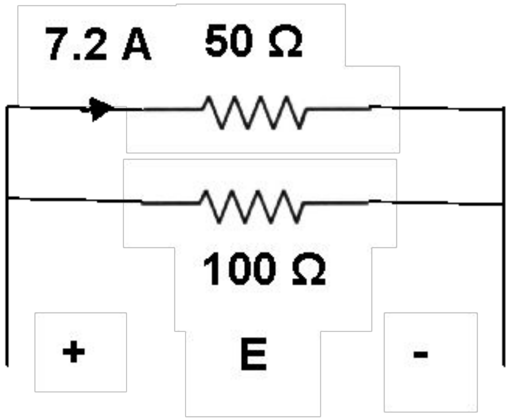
To make the load current 16 A, current through the second resistor = 16 –12 = 4 A

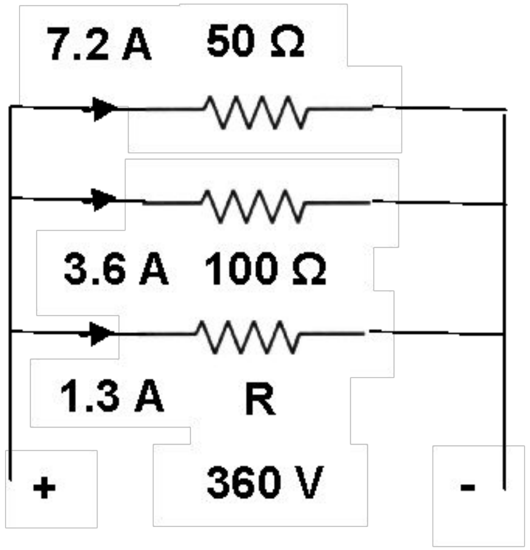
Value of second resistor R_2 = 230/4 = 57.5 Ω

Example 5

A 50 Ω resistor is in parallel with a 100 Ω resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistor to be added in parallel to make the line current as 12.1A?

Solution





Supply voltage E = 50 x 7.2 = 360 V

Current through 100 Ω = 360/100 = 3.6 A

When the line current is 12.1 A, current through third resistor = 12.1 – (7.2 + 3.6)

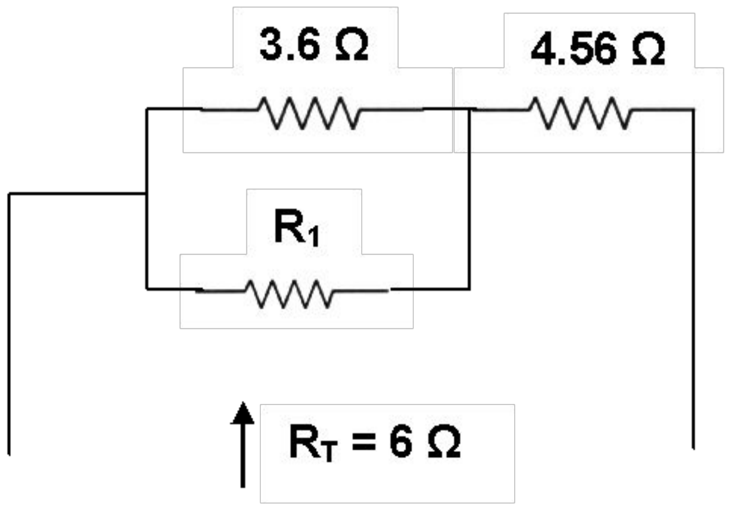
= 1.3 A

Value of third resistor = 360/1.3 = 276.9230 Ω

Example 6

A resistor of 3.6 ohms is connected in series with another of 4.56 ohms. What resistance must be placed across 3.6 ohms, so that the total resistance of the circuit shall be 6 ohms?

Solution



$3.6 \parallel R_1 = 6 - 4.56 = 1.44\ \Omega$

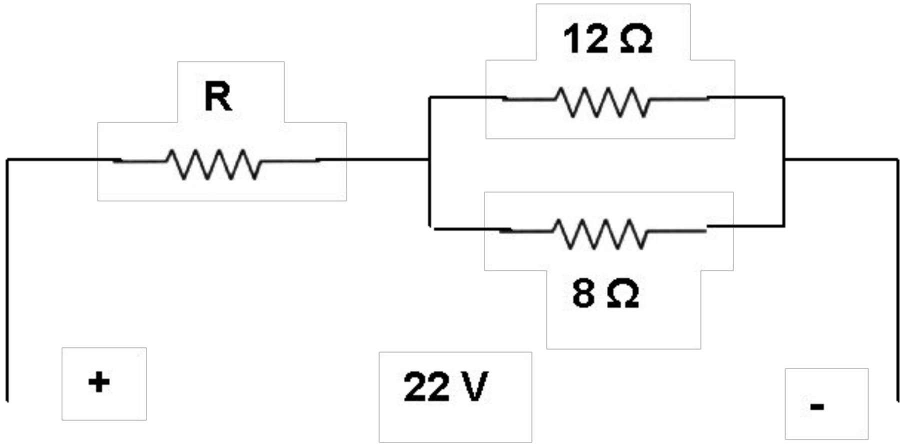
Thus $\frac{3.6 \times R_1}{3.6 + R_1} = 1.44$; Therefore $\frac{3.6 + R_1}{R_1} = \frac{3.6}{1.44} = 2.5$; $\frac{3.6}{R_1} = 1.5$

Required resistance $R_1 = 3.6/1.5 = 2.4\ \Omega$

Example 7

A resistance R is connected in series with a parallel circuit comprising two resistors 12 Ω and 8 Ω respectively. Total power dissipated in the circuit is 70 W when the applied voltage is 22 V. Calculate the value of the resistor R.

Solution



Total current taken = 70 / 22 = 3.1818 A

Equivalent of 12 Ω ‖ 8 Ω = 96/20 = 4.8 Ω

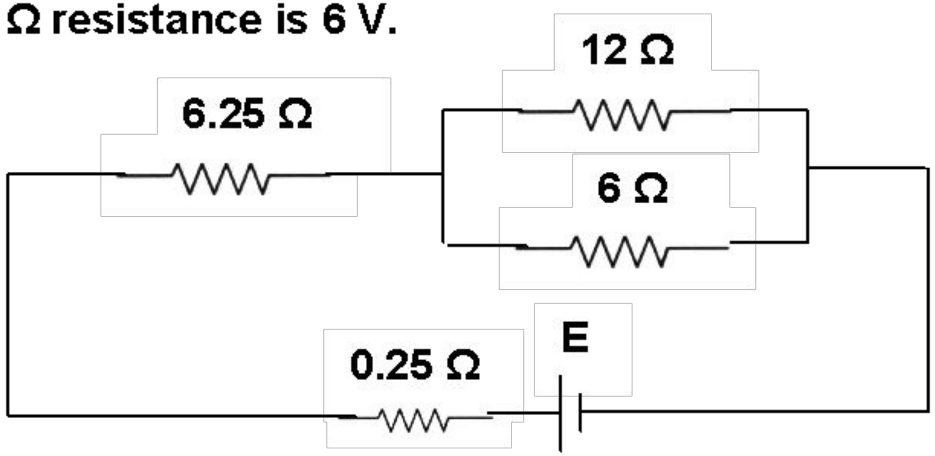
Voltage across parallel combination = 4.8 x 3.1818 = 15.2726 V

Voltage across resistor R = 22 – 15.2726 = 6.7274 V

Value of resistor R = 6.7274/3.1818 = 2.1143 Ω

Example 8

The resistors 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance and a battery which has an internal resistance of 0.25 Ω. Determine the emf of the battery if the potential difference across 6 Ω resistance is 6 V.



Voltage across 6 Ω = 6 V

Solution

Current in 6 Ω = 6/6 = 1 A

Current in 12 Ω = 6/12 = 0.5 A

Therefore current in 0.25 Ω = 1.0 + 0.5 = 1.5 A

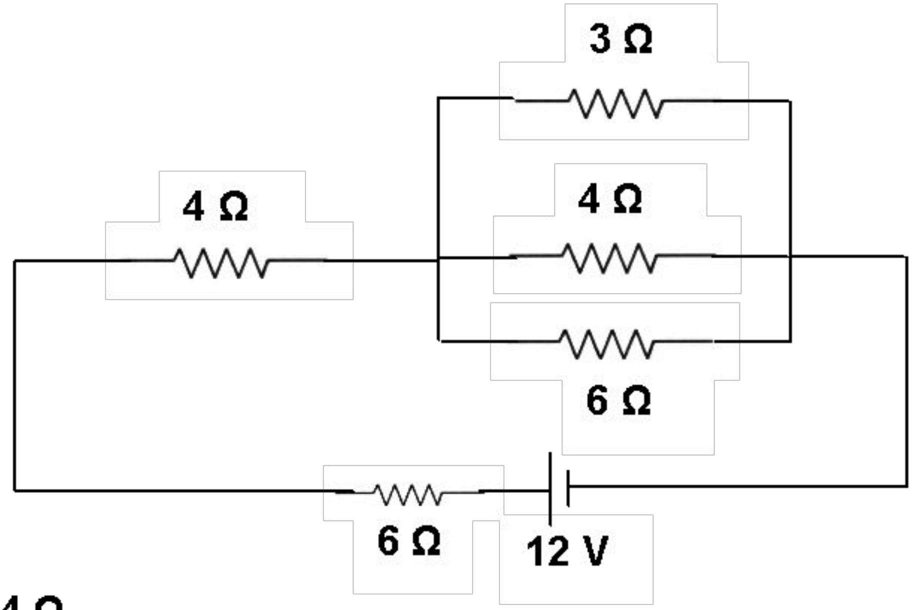
Using KVL   E = (0.25 x 1.5) + (6.25 x 1.5) + 6 = 15.75 V

Therefore battery emf E = 15.75 V

Example 9

A circuit consist of three resistors 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of 12 V and an internal resistance of 6 Ω is connected across the circuit. Find the total current in the circuit and the terminal voltage across the battery.

Solution



4 Ω ‖ 6 Ω = 24/10 = 2.4 Ω

2.4 Ω ‖ 3 Ω = 7.2/5.4 = 1.3333 Ω

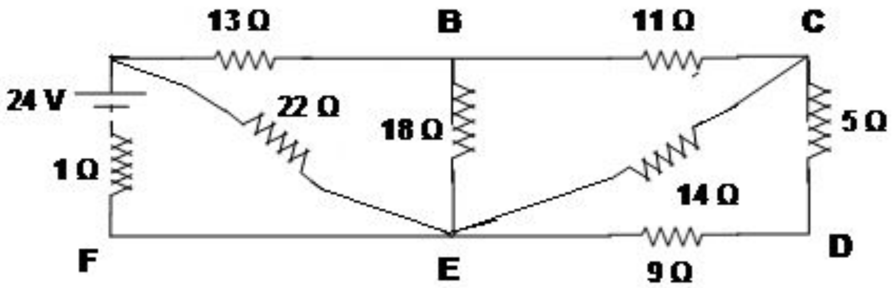
Total circuit resistance = 4 + 6 + 1.3333 = 11.3333 Ω

Circuit current = 12/11.3333 = 1.0588 A

Terminal voltage across the battery = 12 – (6 x 1.0588) = 5.6472 V

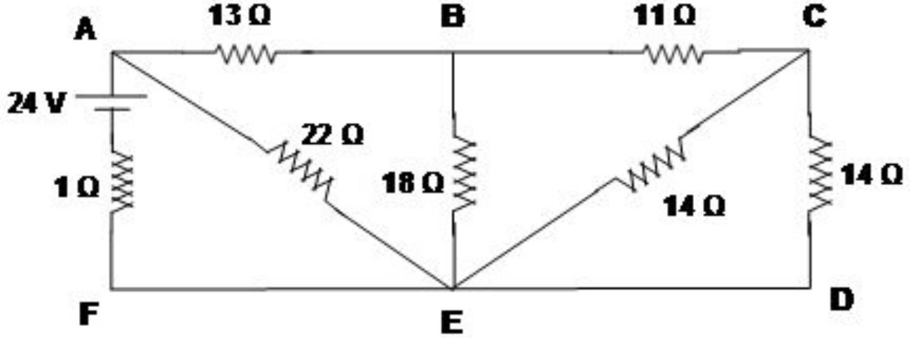
Example 10

An electrical network is arranged as shown. Find (i) the current in branch AF (ii) the power absorbed in branch BE

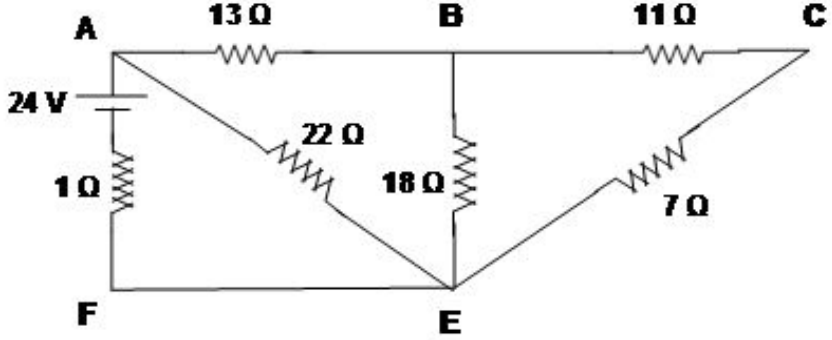


Solution

Various stages of reduction are shown.



1



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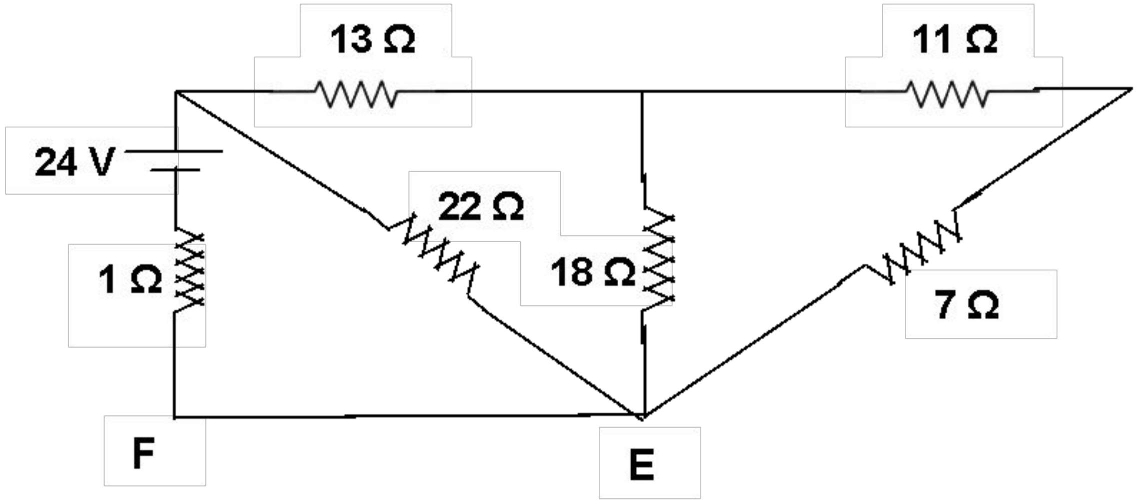
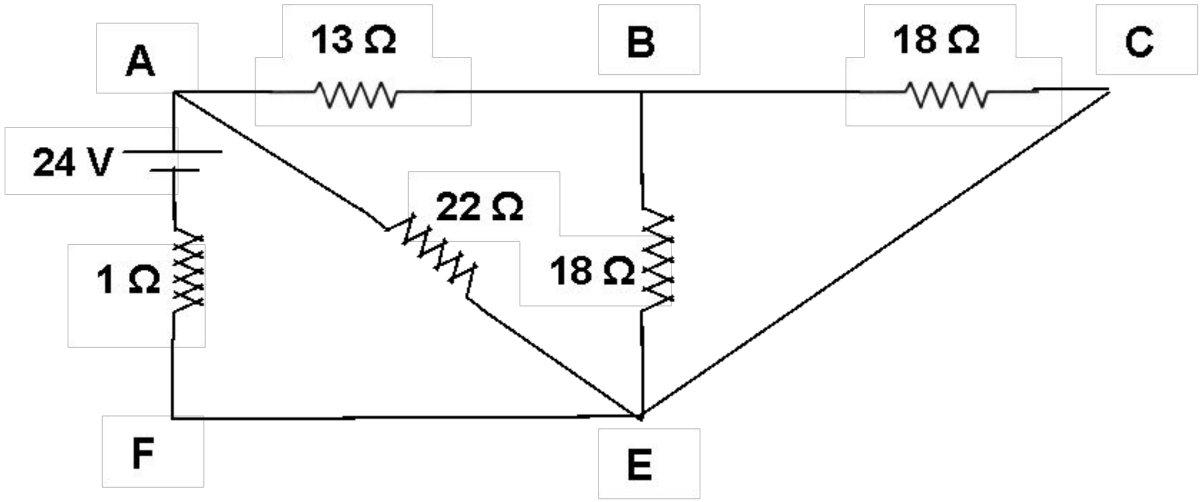
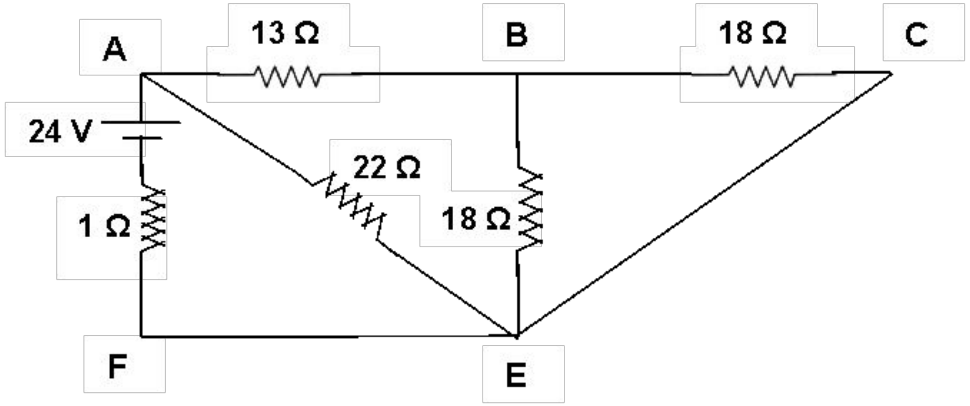


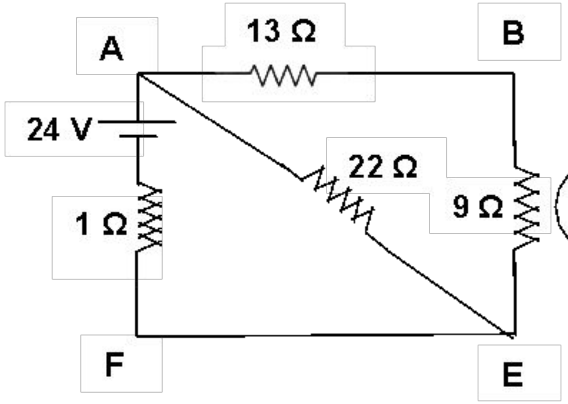
2



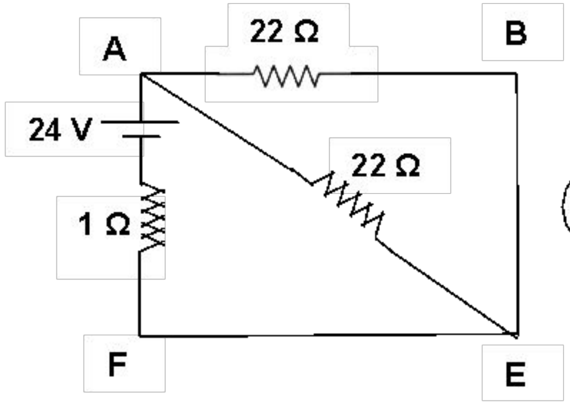
3


A
13 Ω
B
18 Ω
C
24 V
22 Ω
18 Ω
1 Ω
F
E

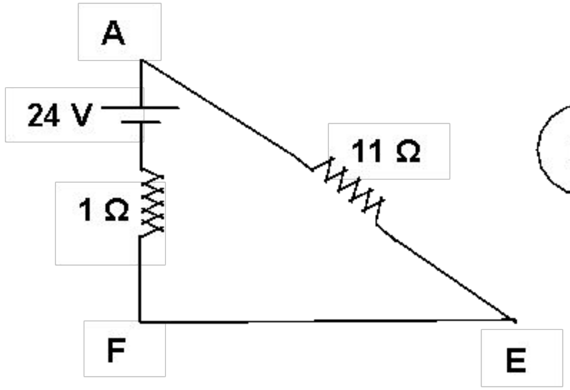
3


A
13 Ω
B
24 V
22 Ω
9 Ω
1 Ω
F
E

4


A
22 Ω
B
24 V
22 Ω
1 Ω
F
E

5


A
24 V
11 Ω
1 Ω
F
E

6

Current in branch AF = 24/12 = 2 A from F to A

Using current division rule current in 13 Ω in Fig. 4= 1 A

Referring Fig. 3, current in branch BE = 0.5 A

Power absorbed in branch BE = 0.5^2 x 18 = 4.5 W

3.2 MESH CURRENT METHOD (ALSO KNOWN AS LOOP ANALYSIS)

Element currents are there in reality while mesh currents are more useful and are imaginary. In any circuit, number of elements and hence the number of element currents, will be larger as compared to the number of independent meshes. All the elements currents can be calculated once we know the mesh currents. Here after independent meshes will be simply referred as meshes and independent mesh currents will be referred as mesh currents leaving the adjective independent.

Consider the circuit shown in Fig. 3.2. This circuit has two meshes and one set of mesh currents are marked.

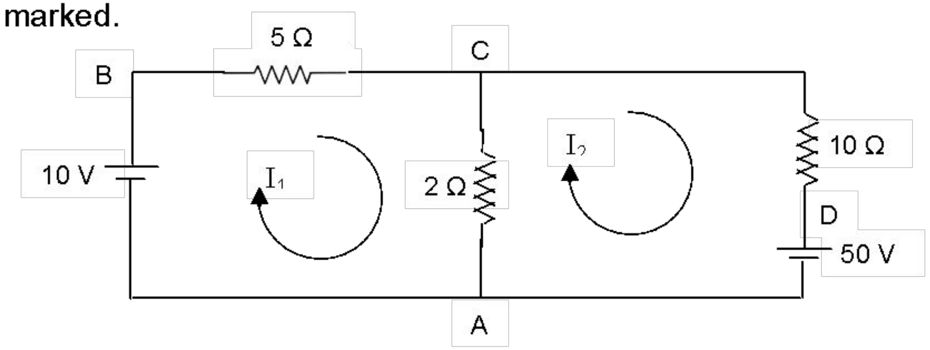
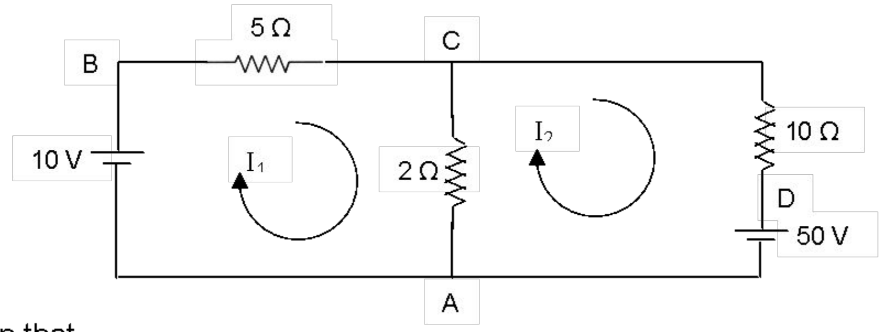


Fig. 3.2 Circuit with two meshes.



It can be seen that

Current in 10 V battery is I_1 from A to B;

Current in 5 Ω resistor is I_1 from B to C;

Current in 2 Ω resistor is $I_1 - I_2$ from C to A or $I_2 - I_1$ from A to C;

Current in 10 Ω is I_2 from C to D; and

Current in 50 V battery is I_2 from D to A or $- I_2$ from A to D.

Thus all the five element currents can be calculated from the two mesh currents. This is true for any general circuit.

Now let us apply KVL and write the mesh equations for the two meshes. Voltage rises are taken as positive and voltage drops are taken as negative.

$$- 10 + 5 I_1 + 2 (I_1 - I_2) = 0 \tag{3.1}$$

$$10 I_2 + 50 + 2 (I_2 - I_1) = 0 \tag{3.2}$$

$$-10 + 5 I_1 + 2 (I_1 - I_2) = 0 \quad (3.1)$$

$$10 I_2 + 50 + 2 (I_2 - I_1) = 0 \quad (3.2)$$

The above equations can be rearranged as

$$7 I_1 - 2 I_2 = 10 \quad (3.3)$$

$$-2 I_1 + 12 I_2 = -50 \quad (3.4)$$

Solving these equations, we get $I_1 = 0.25$ A and $I_2 = -4.125$ A

The currents in different elements are shown in Fig. 3.3.

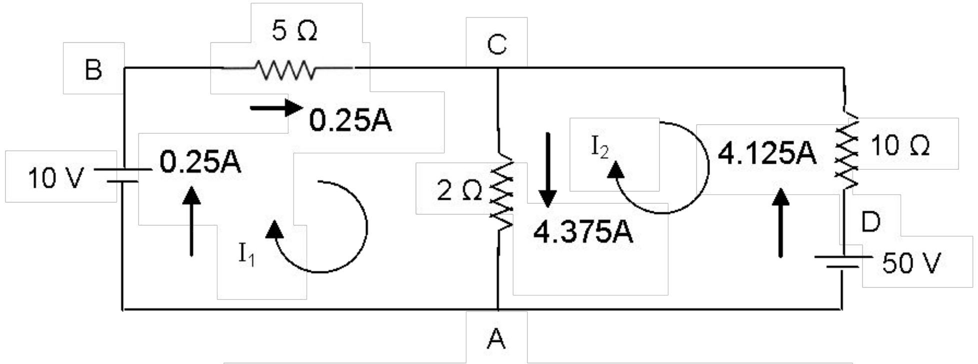
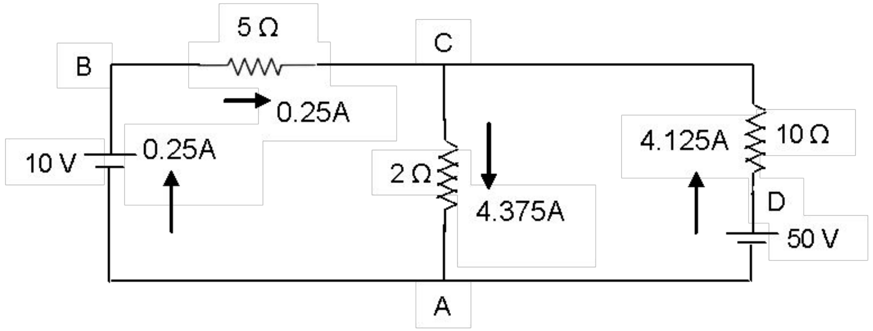


Fig. 3.3 Currents in different elements.



Powers associated with different elements are obtained as:

Power consumed by 5 Ω resistor $= 0.25^2 \times 5 = 0.3125$ W

Power consumed by 2 Ω resistor $= 4.375^2 \times 2 = 38.28125$ W

Power consumed by 10 Ω resistor $= 4.125^2 \times 10 = 170.15625$ W

Total power consumed by the resistors $= 208.75$ W

Power supplied by 10 V battery $= 10 \times 0.25 = 2.5$ W

Power supplied by 50 V battery $= 50 \times 4.125 = 206.25$ W

Total power supplied by the batteries $= 208.75$ W

The following procedure can be followed to solve circuits using mesh current method.

1. Find the number of independent meshes and identify one set of independent meshes.

2. Assume either clockwise or anti-clockwise direction for the mesh currents in the independent meshes. Once the mesh currents are assumed, element currents in the required direction can be obtained in terms of the mesh currents.

3. For each independent mesh, travel through the elements in the assumed mesh current direction and write the equation applying KVL.

4. Solve these equations for the mesh currents.

Other quantities of interest can now be computed.

Example 3.1

For the circuit shown in Fig. 3.4 (i) compute the mesh currents (ii) determine the total power loss in the resistors and (iii) find the power associated with the voltage sources and the total power supplied by them.

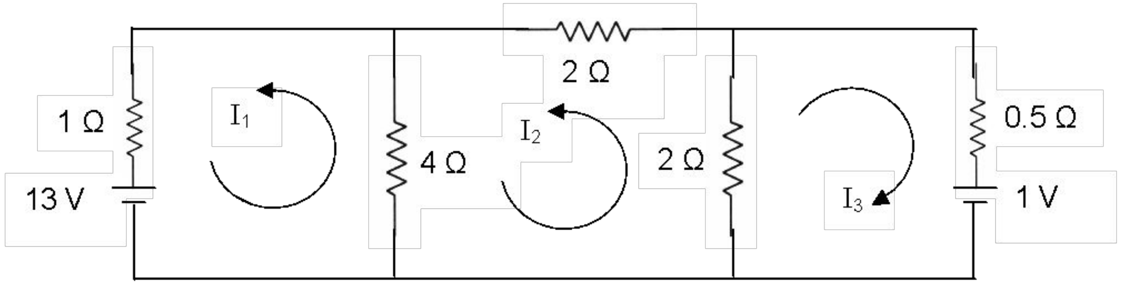


Fig. 3.4 Circuit for Example 3.1.

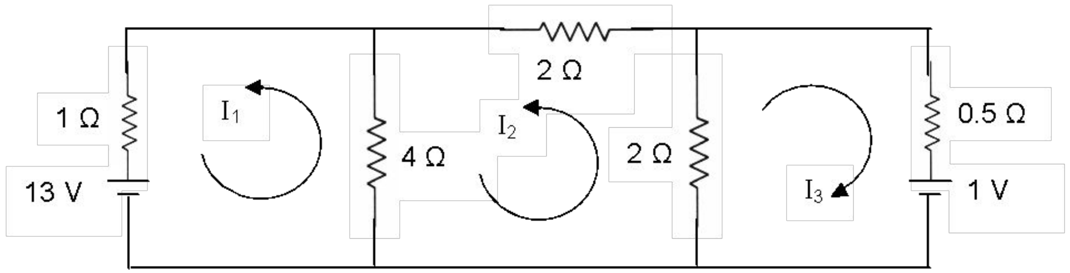
Solution:

Three mesh equations can be obtained as:

$I_1 + 13 + 4(I_1 - I_2) = 0$ i.e. $5\, I_1 - 4\, I_2 = -13$

$2\, I_2 + 4(I_2 - I_1) + 2(I_2 + I_3) = 0$ i.e. $-4\, I_1 + 8\, I_2 + 2\, I_3 = 0$

$0.5\, I_3 + 1 + 2(I_3 + I_2) = 0$ i.e. $2\, I_2 + 2.5\, I_3 = -1$



Matrix form of the mesh equations is

$$\begin{bmatrix} 5 & -4 & 0 \\ -4 & 8 & 2 \\ 0 & 2 & 2.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \\ -1 \end{bmatrix}$$

Mesh currents are: $I_1 = -5$ A ; $I_2 = -3$ A ; $I_3 = 2$ A

When the circuit contains only independent voltage sources, mesh equations in matrix form can be written directly by examining the circuit.

Total power loss = 25 + 16 + 18 + 2 + 2 = 63 W

Power supplied by 13 V battery = 65 W ;

It is to be noted that in 1 V battery, positive current leaves the negative terminal.

Therefore, power received by 1 V battery = 2 W

Thus 13 V battery is in discharge mode whereas 1 V battery is in charge mode.