CSCI547 Machine Learning
Homework 1

Zachary Falkner
Department of Computer Science
University of Montana

March 6, 2018

1 Linear Regression

1A

$(X^T X + \gamma I_N)\vec{W} = X^T Y$
$\vec{w} = [w_0, w_1, w_2, w_3]$
$X =$

$$\begin{bmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ ... & ... & ... & ... \\ 1 & x_m & x_m^2 & x_m^3 \end{bmatrix}$$

$\mathcal{J} = \frac{1}{2}(\sum_{i=1}^{m}(y_i - w_0 - w_1 x_i - w_2 x_i^2 - w_3 x_i^3))^2$

$\frac{\partial \mathcal{J}}{\partial w_0} = -\sum_{i=1}^{m}(y_i - w_0 - w_1 x_i - w_2 x_i^2 - w_3 x_i^3)$
$\frac{\partial \mathcal{J}}{\partial w_1} = -\sum_{i=1}^{m}(x_i(y_i - w_0 - w_1 x_i - w_2 x_i^2 - w_3 x_i^3)) + 2\gamma w_1$
$\frac{\partial \mathcal{J}}{\partial w_2} = -\sum_{i=1}^{m}(x_i^2(y_i - w_0 - w_1 x_i - w_2 x_i^2 - w_3 x_i^3)) + 2\gamma w_2$
$\frac{\partial \mathcal{J}}{\partial w_3} = -\sum_{i=1}^{m}(x_i^3(y_i - w_0 - w_1 x_i - w_2 x_i^2 - w_3 x_i^3)) + 2\gamma w_3$

$\frac{\partial \mathcal{J}}{\partial w_0} = \sum_{i=1}^{m}(-y_i + w_0 + w_1 x_i + w_2 x_i^2 + w_3 x_i^3)$
$\frac{\partial \mathcal{J}}{\partial w_1} = \sum_{i=1}^{m}(-y_i x_i + w_0 x_i - w_1 x_i^2 - w_2 x_i^3 - w_3 x_i^4) + 2\gamma w_1$
$\frac{\partial \mathcal{J}}{\partial w_2} = \sum_{i=1}^{m}(-y_i x_i^2 + w_0 x_i^2 + w_1 x_i^3 + w_2 x_i^4 + w_3 x_i^5) + 2\gamma w_2$

$\frac{\partial \mathcal{J}}{\partial w_3} = \sum_{i=1}^{m}(-y_i x_i^3 + w_0 x_i^3 + w_1 x_i^4 + w_2 x_i^5 + w_3 x_i^6) + 2\gamma w_3$

$\frac{\partial \mathcal{J}}{\partial w_0} = 0 = \sum_{i=1}^{m}(-y_i + w_0 + w_1 x_i + w_2 x_i^2 + w_3 x_i^3)$
$\frac{\partial \mathcal{J}}{\partial w_1} = 0 = \sum_{i=1}^{m}(-y_i x_i + w_0 x_i - w_1 x_i^2 - w_2 x_i^3 - w_3 x_i^4) + 2\gamma w_1$
$\frac{\partial \mathcal{J}}{\partial w_2} = 0 = \sum_{i=1}^{m}(-y_i x_i^2 + w_0 x_i^2 + w_1 x_i^3 + w_2 x_i^4 + w_3 x_i^5) + 2\gamma w_2$
$\frac{\partial \mathcal{J}}{\partial w_3} = 0 = \sum_{i=1}^{m}(-y_i x_i^3 + w_0 x_i^3 + w_1 x_i^4 + w_2 x_i^5 + w_3 x_i^6) + 2\gamma w_3$

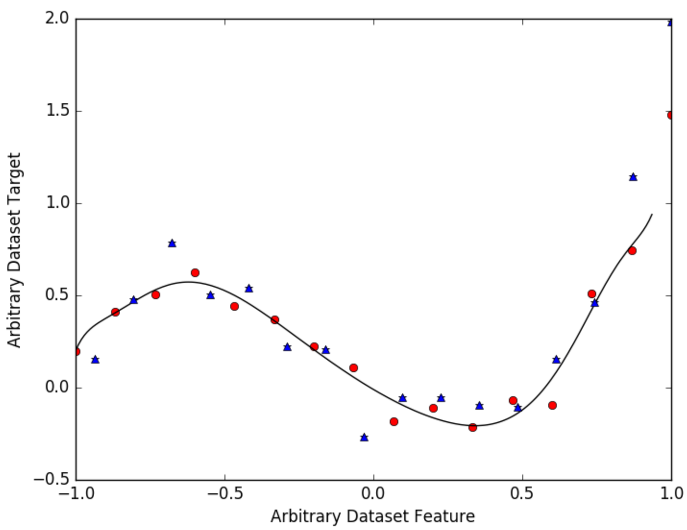
$\sum_{i=1}^{m} y_i = \sum_{i=1}^{m}(+w_0 + w_1 x_i + w_2 x_i^2 + w_3 x_i^3)$
$\sum_{i=1}^{m} y_i = \frac{\sum_{i=1}^{m}(w_0 x_i - w_1 x_i^2 - w_2 x_i^3 - w_3 x_i^4) + 2\gamma w_1}{x_i}$
$\sum_{i=1}^{m} y_i = \frac{\sum_{i=1}^{m}(w_0 x_i^2 + w_1 x_i^3 + w_2 x_i^4 + w_3 x_i^5) + 2\gamma w_2}{x_i^2}$
$\sum_{i=1}^{m} y_i = \frac{\sum_{i=1}^{m}(w_0 x_i^3 + w_1 x_i^4 + w_2 x_i^5 + w_3 x_i^6) + 2\gamma w_3}{x_i^3}$

1B

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

#       degree - an arbitrary degree of polynomial
#       gamma - regularaztion strength (constant)
#       dataset - a pandas dataframe
#
def fit_with_l2(degree, gamma, dataset, test_set):
        #data and target from padas frame
        x = dataset[0].as_matrix().astype(float)
        y = dataset[1].as_matrix().astype(float)

        x_test = test_set[0].as_matrix().astype(float)
        y_test = test_set[1].as_matrix().astype(float)

        xhat = np.linspace(x.min(),x.max(), 200)
        Xhat = np.vander(xhat, N=degree + 1, increasing=True)
        x = 2*(dataset[0] - x.min())/(x.max()-x.min()) - 1  #this is x_tilde
        x_test = 2*(test_set[0] - x.min())/(x.max()-x.min()) - 1

        # Vandermond matrix (design matrix)
        X = np.vander(x,degree+1,increasing=True)

        # Identitity matrix for regularization
        Eye = np.eye(X.shape[1])
        #Eye hat, to not regularize bias
        Eye[0, 0] = 0

        # Solve for weight's set (params)  (training)
        w = np.linalg.solve(np.dot(X.T,X) + gamma*Eye,np.dot(X.T,y))
```

```
        yhat = np.dot(Xhat,w) # 250x2, 16x1

        X_test = np.vander(x_test, N=degree+1, increasing=True)

        avg_rmse = np.sqrt(np.sum((np.dot(X,w) - y)**2)/len(y))
        avg_rmse_test = np.sqrt(np.sum((np.dot(X_test, w) - y_test)**2)/len(
            y_test))

        plt.plot(x,y,'ro')
        plt.plot(xhat,yhat,'k-')
        plt.plot(x_test,test_set[1],'b*')
        plt.plot(x_test,y_test,'b^')
        plt.xlabel('Arbitrary Dataset Feature')
        plt.ylabel('Arbitrary Dataset Target')
        plt.show()

if __name__ == "__main__":
        dataset_file = "P1C_training.csv"
        testset_file = "P1C_test.csv"
        dataset = pd.read_csv(dataset_file, header=None)
        testset = pd.read_csv(testset_file, header=None)
        fit_with_l2(15, 1e-4, dataset, testset)
```

1C

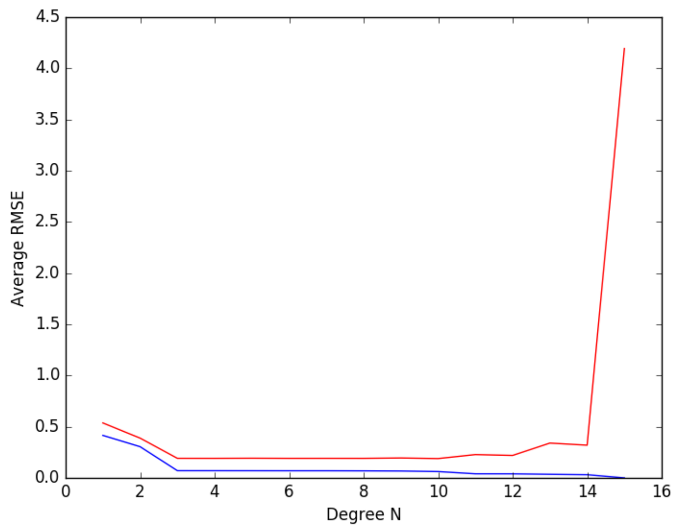


1D

```
import pandas as pd
import numpy as np
```

```python
import matplotlib as mpl
import matplotlib.pyplot as plt

def fit_with_l2(degree, gamma, dataset, test_set):
        #data and target from padas frame
        x = dataset[0].as_matrix().astype(float)
        y = dataset[1].as_matrix().astype(float)

        x_test = test_set[0].as_matrix().astype(float)
        y_test = test_set[1].as_matrix().astype(float)

        xhat = np.linspace(x.min(),x.max(), 200)
        Xhat = np.vander(xhat, N=degree + 1, increasing=True)
        x = 2*(dataset[0] - x.min())/(x.max()-x.min()) - 1  #this is x_tilde
        x_test = 2*(test_set[0] - x.min())/(x.max()-x.min()) - 1

        # Vandermond matrix (design matrix)
        X = np.vander(x,degree+1,increasing=True)

        # Identitity matrix for regularization
        Eye = np.eye(X.shape[1])
        #Eye hat, to not regularize bias
        Eye[0, 0] = 0

        # Solve for weight's set (params)  (training)
        w = np.linalg.solve(np.dot(X.T,X) + gamma*Eye,np.dot(X.T,y))

        yhat = np.dot(Xhat,w) # 250x2, 16x1

        X_test = np.vander(x_test, N=degree+1, increasing=True)

        avg_rmse = np.sqrt(np.sum((np.dot(X,w) - y)**2)/len(y))
        avg_rmse_test = np.sqrt(np.sum((np.dot(X_test, w) - y_test)**2)/len(
            y_test))
        return avg_rmse, avg_rmse_test

if __name__ == "__main__":
        dataset_file = "P1C_training.csv"
        testset_file = "P1C_test.csv"
        dataset = pd.read_csv(dataset_file, header=None)
        testset = pd.read_csv(testset_file, header=None)

        degs = []
        rmses = []
        rmses_test = []

        for i in range(1, 16):
                avg_rmse, avg_rmse_test = fit_with_l2(i, 0, dataset, testset
                    )
                degs.append(i)
                rmses.append(avg_rmse)
                rmses_test.append(avg_rmse_test)

        plt.plot(degs,rmses,'b-')
        plt.plot(degs, rmses_test, 'r-')
        plt.xlabel('Degree N')
        plt.ylabel('Average RMSE')
```

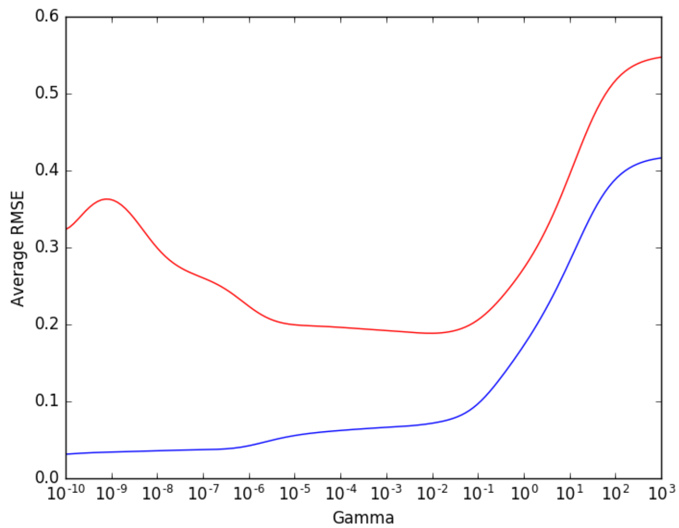
```
        plt.show()
```



1E

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

def fit_with_l2(degree, gamma, dataset, test_set):
        #data and target from padas frame
        x = dataset[0].as_matrix().astype(float)
        y = dataset[1].as_matrix().astype(float)

        x_test = test_set[0].as_matrix().astype(float)
        y_test = test_set[1].as_matrix().astype(float)

        xhat = np.linspace(x.min(),x.max(), 200)
        Xhat = np.vander(xhat, N=degree + 1, increasing=True)
        x = 2*(dataset[0] - x.min())/(x.max()-x.min()) - 1  #this is x_tilde
        x_test = 2*(test_set[0] - x.min())/(x.max()-x.min()) - 1

        # Vandermond matrix (design matrix)
        X = np.vander(x,degree+1,increasing=True)

        # Identitity matrix for regularization
        Eye = np.eye(X.shape[1])
        #Eye hat, to not regularize bias
        Eye[0, 0] = 0

        # Solve for weight's set (params)  (training)
```

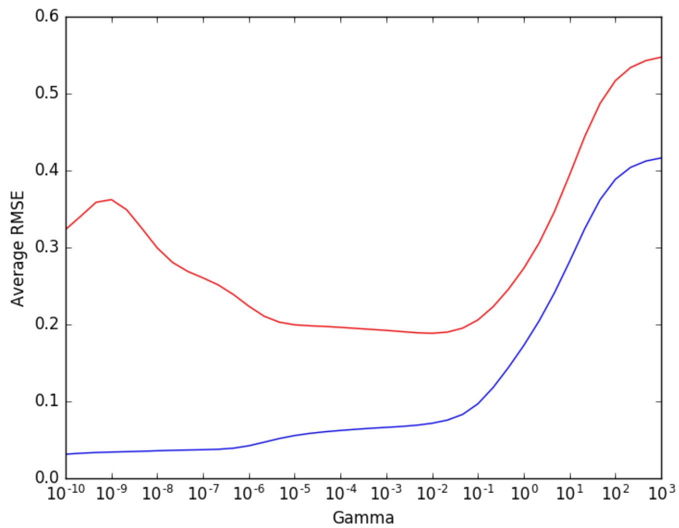
```
        w = np.linalg.solve(np.dot(X.T,X) + gamma*Eye,np.dot(X.T,y))

        yhat = np.dot(Xhat,w) # 250x2, 16x1

        X_test = np.vander(x_test, N=degree+1, increasing=True)

        avg_rmse = np.sqrt(np.sum((np.dot(X,w) - y)**2)/len(y))
        avg_rmse_test = np.sqrt(np.sum((np.dot(X_test, w) - y_test)**2)/len(
            y_test))
        return avg_rmse, avg_rmse_test

if __name__ == "__main__":
        dataset_file = "P1C_training.csv"
        testset_file = "P1C_test.csv"
        dataset = pd.read_csv(dataset_file, header=None)
        testset = pd.read_csv(testset_file, header=None)

        rmses = []
        rmses_test = []

        gammas = np.logspace(-10, 3, 150)

        for i in gammas:
                avg_rmse, avg_rmse_test = fit_with_l2(15, i, dataset,
                    testset)
                rmses.append(avg_rmse)
                rmses_test.append(avg_rmse_test)

        plt.semilogx(gammas,rmses,'b-')
        plt.semilogx(gammas, rmses_test, 'r-')
        plt.xlabel('Gamma')
        plt.ylabel('Average RMSE')
        plt.show()
```



1F

```python
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from sklearn import linear_model

def fit_with_l1(degree, gamma, dataset, test_set):
        #data and target from padas frame
        x = dataset[0].as_matrix().astype(float)
        y = dataset[1].as_matrix().astype(float)

        x_test = test_set[0].as_matrix().astype(float)
        y_test = test_set[1].as_matrix().astype(float)

        xhat = np.linspace(x.min(),x.max(), 200)
        Xhat = np.vander(xhat, N=degree + 1, increasing=True)
        x = 2*(dataset[0] - x.min())/(x.max()-x.min()) - 1  #this is x_tilde
        x_test = 2*(test_set[0] - x.min())/(x.max()-x.min()) - 1

        # Vandermond matrix (design matrix)
        X = np.vander(x,degree+1,increasing=True)

        # Identitity matrix for regularization
        Eye = np.eye(X.shape[1])
        #Eye hat, to not regularize bias
        Eye[0, 0] = 0

        # Solve for weight's set (params)  (training)
        w = np.linalg.solve(np.dot(X.T,X) + gamma*Eye,np.dot(X.T,y))

        yhat = np.dot(Xhat,w) # 250x2, 16x1

        X_test = np.vander(x_test, N=degree+1, increasing=True)

        lasso = linear_model.Lasso(alpha=gamma, max_iter=10000000)
        lasso.fit(X, y)

        avg_rmse = np.sqrt(np.sum((np.dot(X,w) - y)**2)/len(y))
        avg_rmse_test = np.sqrt(np.sum((np.dot(X_test, w) - y_test)**2)/len(
            y_test))
        return avg_rmse, avg_rmse_test


if __name__ == "__main__":
        dataset_file = "P1C_training.csv"
        testset_file = "P1C_test.csv"
        dataset = pd.read_csv(dataset_file, header=None)
        testset = pd.read_csv(testset_file, header=None)

        rmses = []
        rmses_test = []

        gammas = np.logspace(-10, 3, 40)

        for i in gammas:
```

```
            print("running lasso {}".format(i))
            avg_rmse, avg_rmse_test = fit_with_l1(15, i, dataset,
                testset)
            rmses.append(avg_rmse)
            rmses_test.append(avg_rmse_test)

    plt.semilogx(gammas,rmses,'b-')
    plt.semilogx(gammas, rmses_test, 'r-')
    plt.xlabel('Gamma')
    plt.ylabel('Average RMSE')
    plt.show()
```

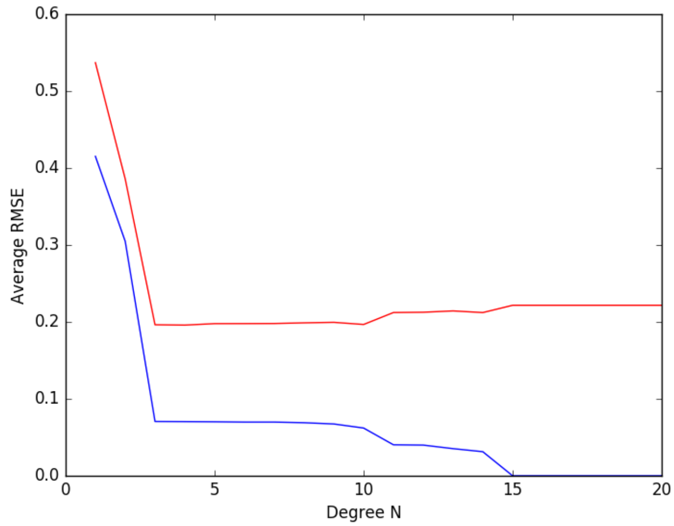


The plots look nearly identical to me with the exception that 1E is smoother because i used less points in 1F (Lasso takes a long time to run with small alphas)

1G*

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

def fit_with_l2(degree, gamma, dataset, test_set):
        #data and target from padas frame
        x = dataset[0].as_matrix().astype(float)
        y = dataset[1].as_matrix().astype(float)

        x_test = test_set[0].as_matrix().astype(float)
```

```
        y_test = test_set[1].as_matrix().astype(float)

        xhat = np.linspace(x.min(),x.max(), 200)
        Xhat = np.vander(xhat, N=degree + 1, increasing=True)
        x = 2*(dataset[0] - x.min())/(x.max()-x.min()) - 1  #this is x_tilde
        x_test = 2*(test_set[0] - test_set[0].min())/(test_set[0].max()-
            test_set[0].min()) - 1

        # Vandermond matrix (design matrix)
        X = np.polynomial.legendre.legvander(x,degree)

        # Identitity matrix for regularization
        Eye = np.eye(X.shape[1])
        #Eye hat, to not regularize bias
        Eye[0, 0] = 0

        # Solve for weight's set (params)  (training)
        w = np.linalg.solve(np.dot(X.T,X) + gamma*Eye,np.dot(X.T,y))

        yhat = np.dot(Xhat,w) # 250x2, 16x1

        X_test = np.polynomial.legendre.legvander(x_test, degree)

        avg_rmse = np.sqrt(np.sum((np.dot(X,w) - y)**2)/len(y))
        avg_rmse_test = np.sqrt(np.sum((np.dot(X_test, w) - y_test)**2)/len(
            y_test))
        return avg_rmse, avg_rmse_test

if __name__ == "__main__":
        dataset_file = "P1C_training.csv"
        testset_file = "P1C_test.csv"
        dataset = pd.read_csv(dataset_file, header=None)
        testset = pd.read_csv(testset_file, header=None)

        degs = []
        rmses = []
        rmses_test = []

        for i in range(1, 21):
                avg_rmse, avg_rmse_test = fit_with_l2(i, 0, dataset, testset
                    )
                degs.append(i)
                rmses.append(avg_rmse)
                rmses_test.append(avg_rmse_test)

        plt.plot(degs,rmses,'b-')
        plt.plot(degs, rmses_test, 'r-')
        plt.xlabel('Degree N')
        plt.ylabel('Average RMSE')
        plt.show()
```



Probability and Bayes Theorem

2A Bayesian Inference

$P(D) = 10^{-4}$
$P(\neg D) = 1 - 10^{-4}$
$P(T|D) = 0.99$
$P(T) = P(D)P(T|D) + P(\neg D)P(\neg T|D)$
$P(D|T) = \frac{P(T|D)P(D)}{P(T)}$
$P(D|T) = \frac{P(T|D)P(D)}{P(T)}$
$P(D|T) = \frac{P(T|D)P(D)}{P(D)P(T|D)+P(\neg D)P(\neg T|D)}$
$P(D|T) = \frac{0.99*10^{-4}}{10^{-4}*0.99+(1-10^{-4})*0.01}$
$P(D|T) = 0.0098039$
$P(T_N|D) = 0.99999$
$P(D|T_N) = \frac{0.99999*10^{-4}}{10^{-4}*0.99999+(1-10^{-4})*0.00001}$
$P(D|T) = 0.9090983$

2B Bayesian Linear Regression

2C* Updating Bayesian Linear Regression

Naive Bayes

3A

We do not need to compute the denominator in the classification rule for Naive Bayes because we are assuming the data is independent and identically distributed. That is, it is a normal distribution with no dependence between features.

3B

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

from sklearn.datasets import load_digits
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix


if __name__ == "__main__":
        digits = load_digits()
        data = np.round(digits['data']/16.0)

        X = data    # n x m matrix of features
        y = digits.target  # n vector of classes
        X,X_test,y,y_test = train_test_split(X,y,test_size=0.33,random_state
            =42) # Split into 33% test and 67% training sets

        classes = np.linspace(0,9,10)

        m = X.shape[0]  # Number of data instances
        m_test = X_test.shape[0] # Number of test data instances
        N = 10           # Number of classes
        n = X.shape[1]  # Number of features

        mu_array = np.zeros((n,N))
        sigma2_array = np.zeros((n,N))
        prior_array = np.zeros((N))

        #Learning phase
        for k in range(N):    #Loop over each class label
                C_k = classes[k]
```

```
        prior = sum(y==C_k)/float(y.shape[0])
                                    # Count the number of data
            where the label is C_k
        mu = np.sum(X[y==C_k],axis=0)/len(X[y==C_k])
                               # Take the mean of those features
            where the corresponding label is C_k
        mu_array[:,k] = mu
                                                   # Store in
            the arrays we created above
        prior_array[k] = prior

class_probabilities = np.zeros((m,N))  # The probabilities for

for i,x in enumerate(X):  # Loop over the training data instances
        for k in range(N):    # Loop over the classes
                prior = prior_array[k]
                mu = mu_array[:,k]
                # likelihood = np.prod(np.exp(-(x-mu)**2/(2*sigma2))
                    ) #change me
                likelihood = np.prod(np.power(mu, x) * np.power((1-
                    mu), 1-x) )
                posterior_k = prior*likelihood
                class_probabilities[i,k] = posterior_k

class_probabilities /= np.sum(class_probabilities,axis=1,keepdims=
    True)

y_pred_train = np.argmax(class_probabilities,axis=1)

cm_train = confusion_matrix(y,y_pred_train)
print(cm_train)
print("training accuracy:", 1 - sum(abs(y!=y_pred_train))/float(m))


# Test set predictions
class_probabilities = np.zeros((m_test,N))

for i,x in enumerate(X_test):
        for k in range(N):
                prior = prior_array[k]
                mu = mu_array[:,k]
                sigma2 = sigma2_array[:,k]
                likelihood = np.prod(np.power(mu, x) * np.power((1-
                    mu), 1-x) )
                posterior_k = prior*likelihood
                class_probabilities[i,k] = posterior_k

class_probabilities /= class_probabilities.sum(axis=1,keepdims=True)
y_pred_test = np.argmax(class_probabilities,axis=1)

cm_test = confusion_matrix(y_test,y_pred_test)
print(cm_test)
print("test accuracy:", 1 - sum(abs(y_test-y_pred_test))/float(
    m_test))
```

training confusion matrix:

$$\begin{bmatrix}
119 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \\
0 & 108 & 4 & 0 & 0 & 1 & 1 & 0 & 7 & 6 \\
1 & 2 & 116 & 1 & 0 & 0 & 0 & 0 & 2 & 3 \\
0 & 2 & 1 & 111 & 0 & 2 & 0 & 6 & 2 & 3 \\
0 & 1 & 0 & 0 & 108 & 0 & 1 & 4 & 3 & 0 \\
0 & 0 & 1 & 2 & 1 & 99 & 0 & 0 & 0 & 6 \\
0 & 3 & 0 & 0 & 1 & 0 & 119 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 115 & 1 & 0 \\
0 & 11 & 3 & 0 & 0 & 3 & 0 & 2 & 101 & 2 \\
0 & 0 & 0 & 2 & 0 & 2 & 0 & 3 & 2 & 103
\end{bmatrix}$$

training accuracy: 0.913549459684

testing confusion matrix:

$$\begin{bmatrix}
52 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\
0 & 37 & 5 & 0 & 1 & 0 & 0 & 0 & 7 & 5 \\
0 & 4 & 46 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 2 & 45 & 0 & 0 & 0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0 & 63 & 0 & 0 & 1 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 60 & 0 & 0 & 0 & 10 \\
0 & 0 & 0 & 0 & 0 & 1 & 55 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 58 & 0 & 1 \\
0 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 45 & 1 \\
0 & 0 & 0 & 4 & 0 & 1 & 0 & 4 & 2 & 57
\end{bmatrix}$$

test accuracy: 0.872053872054