Ahmad Fallahpour

CSCI567 2017 Homework Assignment 1 09/24/2017

1 Linear Regression

1.1 $X^T X$ is not invertible

$X^T X$ is not invertible if $rank(X) = rank(X^T X) \leq N < D + 1$

1.2 Bias solution

$$RSS(\boldsymbol{\omega}) = \sum_n \left[y_n - f(x_n)\right]^2 = \sum_n \left[y_n - \left(\omega_0 + \sum_d \omega_d x_{nd}\right)\right]^2 , \tag{1}$$

$$\frac{\partial RSS(\boldsymbol{\omega})}{\partial \omega_0} = \sum_n -2\left[y_n - \left(\omega_0 + \sum_d \omega_d x_{nd}\right)\right] = 0 , \tag{2}$$

$$\sum_n y_n = \omega_0 \sum_n 1 + \sum_n \sum_d \omega_d x_{nd} , \tag{3}$$

$$\sum_n y_n = \omega_0 \times N + \sum_n \sum_d \omega_d x_{nd} , \tag{4}$$

$$\frac{1}{N} \sum_n y_n = \omega_0 + \frac{1}{N} \sum_n \sum_d \omega_d x_{nd} , \tag{5}$$

$$\frac{1}{N} \sum_n y_n = \omega_0 + \sum_d \omega_d \frac{1}{N} \sum_n x_{nd} , \tag{6}$$

As it is mentioned in the question itself, if we have following condition:

$$\frac{1}{N} \sum_n x_{nd} = 0, \forall d = 1, 2, ..., D \tag{7}$$

In this case, we can have bias as the mean of the samples as follow:

$$\omega_0 = \frac{1}{N} \sum_n y_n \tag{8}$$

2 Logistic Regression

2.1 Bias solution

$$\varepsilon(\omega, b) = \sum_n \big\{y_n \log \sigma(b) + (1 - y_n)\log[1 - \sigma(b)]\big\}, \tag{9}$$

$$\frac{\partial \varepsilon(\omega, b)}{\partial b} = \sum_n \{y_n \frac{1}{\sigma(b)} \frac{\partial \sigma(b)}{\partial b} + (1 - y_n) \frac{1}{[1 - \sigma(b)]} \frac{\partial [1 - \sigma(b)]}{\partial b}\}, \tag{10}$$

$$= \sum_n \{y_n \frac{\sigma(b)[1 - \sigma(b)]}{\sigma(b)} + (1 - y_n) \frac{-[1 - \sigma(b)]\sigma(b)}{[1 - \sigma(b)]}\}, \tag{11}$$

$$= \sum_n \big\{y_n[1 - \sigma(b)] - (1 - y_n)\sigma(b)\big\}, \tag{12}$$

$$= \sum_n \big\{y_n - y_n\sigma(b) - \sigma(b) + y_n\sigma(b)\big\}, \tag{13}$$

$$= \sum_n \big\{y_n - \sigma(b)\big\} = 0, \tag{14}$$

$$=> \sum_n y_n = \sigma(b) \sum_n 1, \tag{15}$$

$$\sum_n y_n = \sigma(b) \times N, \tag{16}$$

$$\sigma(b) = \frac{1}{N} \sum_n y_n, \tag{17}$$

From Eqn (17), we can see that the probability that a test sample is labeled as 1 is the mean of the samples similar to Q1.2

$$\sigma(b) = \frac{1}{1 + e^{-b}} = \frac{1}{N} \sum_n y_n, \tag{18}$$

$$b = \ln\left(\frac{\sum_n y_n}{N - \sum_n y_n}\right) \tag{19}$$