chapter 3

Question 5

Theorical NeuroSience, Dayan - Abbott (2001) .
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a) $$E = \int L(s, s_b)\, P[s|r]\, ds$$

$$L(s, s_b) = (s - s_b)^2$$

$$E = \int (s - s_b)^2 P[s|r]\, ds \longrightarrow \frac{dE}{ds_b} = \frac{d}{ds_b} \int (s - s_b)\, P[s|r]\, ds$$

$$= \int \frac{d}{ds_b} \left[(s - s_b)^2 P[s|r] \right] ds$$

$$\boxed{\frac{dE}{ds_b} = \int -2(s - s_b)\, P[s|r]\, ds}$$

$$\xrightarrow{\text{Minimize}} \frac{dE}{ds_b} = 0 \longrightarrow \int -2(s - s_b)\, P[s|r]\, ds = 0$$

$$\xrightarrow{\text{seperate integrals}} \int s\, P[s|r]\, ds - \int s_b\, P[s|r]\, ds = 0$$

$$\longrightarrow \int s\, P[s|r]\, ds = s_b \underbrace{\int P[s|r]\, ds}_{=1}$$

$$\longrightarrow \boxed{s_b = \int s\, P[s|r]\, ds}$$

$\boxtimes$

b) $E = \int L(s, s_b)\, P[s|r]\, ds$

$$L(s, s_b) = |s - s_b|$$

s_{min} and s_{max} are limits of S

$s_{max} > s_b$

$s_{min} < s_b$

$$E = \int |s - s_b|\, P[s|r]\, ds = \int_{s_{min}}^{s_b} -(s - s_b)\, P[s|r]\, ds + \int_{s_b}^{s_{max}} (s - s_b)\, P[s|r]\, ds$$

Libniz integral rule

minimization

$$\rightarrow \frac{dE}{ds_b} = \int_{s_{min}}^{s_b} P[s|r]\, ds - \int_{s_b}^{s_{max}} P[s|r]\, ds = 0$$

$$\rightarrow \boxed{\int_{s_{min}}^{s_b} P[s|r]\, ds = \int_{s_b}^{s_{max}} P[s|r]\, ds}$$

s_b is the Median