by: fallahtp@gmail.com

chapter 4

Question 1

a)

$$f(r) = -\int_0^{r_{max}} P[r] \log_2(P[r])\, dr \longrightarrow Max\{\underbrace{f(r) + \lambda g(r)}_{u(r)}\}$$

constraint: $g(r) = \int_0^{r_{max}} P[r]\, dr = 1$

$$\Rightarrow \frac{\partial u(r)}{\partial r} = -P[r]\log_2(P[r]) + \lambda P[r] = 0 \longrightarrow \lambda = \log_2(P[r])$$

$$\longrightarrow \boxed{P[r] = 2^{\lambda}}$$

$$g(r) = \int_0^{r_{max}} 2^{\lambda}\, dr = 1$$

$$\longrightarrow 2^{\lambda}(r_{max} - 0) = 1 \longrightarrow 2^{\lambda} = \boxed{\frac{1}{r_{max}} = P[r]}$$

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b) equation 4.17 $\longrightarrow$ $H + \log_2 \Delta r = -\int_0^{r_{max}} P[r] \log_2(P[r])\, dr$

$$H + \log_2 \Delta r = -\int_0^{r_{max}} \frac{1}{r_{max}} \log_2\left(\frac{1}{r_{max}}\right) dr$$

$$H + \log_2 \Delta r = -\frac{\log_2\left(\frac{1}{r_{max}}\right)}{r_{max}} \times (r_{max} - 0)$$

$$H + \log_2 \Delta r = +\log_2(r_{max})$$

$$\Rightarrow H = \log_2(r_{max}) - \log_2 \Delta r = \boxed{\log_2\left(\frac{r_{max}}{\Delta r}\right)}$$