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Chapter 4

Question 2

$$f(r) = -\int_0^{+\infty} P[r]\, \mathrm{Log}_2(P[r])\, dr$$

$$\longrightarrow \mathrm{Max}\left\{ \underbrace{f(r) + \lambda g(r) + \mu h(r)}_{A(r)} \right\}$$

Constraint : $\begin{cases} g(r) = \int_0^{+\infty} P[r]\, dr = 1 \\ h(r) = \int_0^{+\infty} r\, P[r]\, dr = C \end{cases}$

$$\Rightarrow \frac{\partial A(r)}{\partial r} = -P[r]\, \mathrm{Log}_2(P[r]) + \lambda P[r] + \mu r P[r] = 0$$

$$\longrightarrow \mathrm{Log}_2(P[r]) = \lambda + \mu r \longrightarrow \boxed{P[r] = 2^{\lambda}\, 2^{\mu r}}$$

$$g(r) = \int_0^{+\infty} 2^{\lambda}\, 2^{\mu r}\, dr = 1$$

$$\longrightarrow \int_0^{+\infty} 2^{\mu r}\, dr = 2^{-\lambda}$$

$2 = e^{\mathrm{Ln}\, 2}$

$$\longrightarrow \int_0^{+\infty} e^{(\mathrm{Ln}2)\mu r}\, dr \quad : \quad (\mu \mathrm{Ln} 2) r = u \longrightarrow \begin{cases} (\mu \mathrm{Ln}2)\, dr = du \\ \mu < 0 \end{cases}$$

$$\longrightarrow \frac{-1}{(\mu \mathrm{Ln} 2)} \underbrace{\int_{-\infty}^{0} e^{u}\, du}_{e^{u}\big]_{-\infty}^{0} = 1 - 0 = 1} \longrightarrow \boxed{\frac{-1}{\mu \mathrm{Ln} 2} = 2^{-\lambda}} \quad \text{I}$$

$$h(r) = \int_0^{+\infty} r\, 2^{\lambda}\, 2^{\mu r}\, dr = C$$

$$\rightarrow \int_0^{+\infty} r\, 2^{\mu r}\, dr = C \times 2^{-\lambda}$$

$$\rightarrow \int_0^{+\infty} r\, e^{(\mu Ln2) r}\, dr \quad : \quad (\mu Ln2)\, r = x \rightarrow \begin{cases} (\mu Ln2)\, dr = du \\ r = \dfrac{x}{\mu Ln2} \\ \mu < 0 \end{cases}$$

$$\rightarrow \frac{-1}{(\mu Ln2)^2} \int_{-\infty}^{0} \underbrace{x}_{u}\, \underbrace{e^{x}\, dx}_{dv} \quad : \quad \begin{cases} x = u \rightarrow dx = du \\ e^{x} dx = dv \rightarrow e^{x} = v \end{cases}$$

$$\rightarrow \frac{-1}{(\mu Ln2)^2} \left[\underbrace{x e^{x} \Big]_{-\infty}^{0}}_{(0-0)=0} - \underbrace{\int_{-\infty}^{0} e^{x}\, dx}_{e^{x} \big]_{-\infty}^{0} = (1-0) = 1} \right]$$

$$\Rightarrow \boxed{\frac{+1}{(\mu Ln2)^2} = C \times 2^{-\lambda}} \quad \mathrm{II}$$

$$\rightarrow \frac{\mathrm{II}}{\mathrm{I}} = \frac{\dfrac{1}{(\mu Ln2)^2}}{\dfrac{-1}{(\mu Ln2)}} = C \rightarrow \frac{-1}{\mu Ln2} = C \rightarrow \mu Ln2 = -\frac{1}{C}$$

$$\boxed{\mu = \frac{-1}{C Ln2}}$$

$$\mathrm{I} \rightarrow \frac{-1}{\dfrac{-1}{C Ln2} \times Ln2} = 2^{-\lambda} \rightarrow C = 2^{-\lambda} \rightarrow \boxed{2^{\lambda} = \frac{1}{C}}$$

$$\Rightarrow P[r] = 2^{\lambda} \times 2^{\mu r} = \boxed{\frac{1}{C} \times 2^{\left(\frac{-r}{C Ln2}\right)}}$$