Chapter 3

Question 9

(theorical neuroscience, Dayan - Abbott (2001)
fallahtp@gmail.com)

$$E = \frac{1}{T}\int_0^T \left\langle \left(\int_{-\infty}^{+\infty} (\rho(t-\tau) - \langle r \rangle) K(\tau)\, d\tau - s(t-t_0) \right)^2 \right\rangle dt$$

suppose $\rightarrow$ $\begin{cases} \rho_{i-k} = \rho((i-k)\Delta t) \\ K_k = K(k\Delta t) \\ S_i = s(i\Delta t - \tau_0) \end{cases}$

$$\Rightarrow E = \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \left(\Delta t \sum_{k=-\infty}^{+\infty} (\rho_{i-k} - \langle r \rangle) K_k - S_i \right)^2$$

$\longrightarrow$ optimiz $\Rightarrow \frac{\partial E}{\partial k_j} = 0 \longrightarrow$

$$\rightarrow \frac{\partial E}{\partial k_j} = \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \left[\cancel{2} \times \left(\Delta t \sum_{k=-\infty}^{+\infty} (\rho_{i-k} - \langle r \rangle) K_k - S_i \right) \rho_{i-j} \cancel{\Delta t} \right] = 0$$

$$\Rightarrow \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \left[\Delta t \sum_{k=-\infty}^{+\infty} (\rho_{i-k} - \langle r \rangle) K_k \rho_{i-j} - \rho_{i-j} S_i \right] = 0$$

$$\rightarrow \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \Delta t \sum_{k=-\infty}^{+\infty} (\rho_{i-k} - \langle r \rangle) K_k \rho_{i-j} = \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \rho_{i-j} S_i$$

change order $\longrightarrow$ $\Delta t \sum_{k=-\infty}^{+\infty} K_k \, \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} (\rho_{i-k} - \langle r \rangle)\rho_{i-j} \;=\; \frac{\Delta t}{T} \sum_{i=0}^{T/\Delta t} \rho_{i-j} S_i$

$$\begin{cases} i\Delta t = t \\ j\Delta t = \tau \\ k\Delta t = \tau' \end{cases}$$

$$\rightarrow \int_{-\infty}^{+\infty} K(\tau') \times \frac{1}{T} \underbrace{\int_0^T (\rho(t-\tau') - \langle r \rangle)(\rho(t-\tau))\, dt}_{Q_{\rho\rho}(\tau-\tau')} \, d\tau' = \underbrace{\frac{1}{T}\int_0^T \rho(t-\tau)\, S(t-\tau_0)\, dt}_{Q_{rs}(\tau-\tau_0)}$$

$$\rightarrow \boxed{\int_{-\infty}^{+\infty} K(\tau')\, Q_{\rho\rho}(\tau-\tau')\, d\tau' = Q_{rs}(\tau-\tau_0)}$$