

# Chapter 3

## Question 7

Theoretical neuroscience, Dayan-Abbott (2001)  
fallah.tp@gmail.com

$$\begin{aligned}
 a) \quad \langle \Delta Z \rangle &= \int \frac{\partial \ln(P[r|s])}{\partial s} \left( \underbrace{P[r|s+\Delta s] - P[r|s]}_{\approx P[r|s] + \Delta s \frac{\partial P[r|s]}{\partial s}} \right) dr \\
 &= \int \frac{\partial \ln(P[r|s])}{\partial s} \left( \underbrace{\Delta s \frac{\partial P[r|s]}{\partial s}}_{\substack{\text{previous question} \\ \frac{\partial \ln(P[r|s])}{\partial s} \times P[r|s]}} \right) dr \\
 &= \Delta s \int \underbrace{\left( \frac{\partial \ln(P[r|s])}{\partial s} \right)^2 P[r|s]}_{I_F(s)} dr \\
 &\rightarrow \boxed{\langle \Delta Z \rangle = \Delta s \times I_F(s)}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \langle Z \rangle &= \int \underbrace{\frac{\partial \ln(P[r|s])}{\partial s}}_{\frac{\partial P[r|s]}{\partial s} \times \frac{1}{P[r|s]}} P[r|s] dr \\
 \langle Z \rangle &= \int \frac{\partial P[r|s]}{\partial s} dr = \frac{\partial}{\partial s} \underbrace{\int P[r|s] dr}_1 = \frac{\partial(1)}{\partial s} = 0
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \text{Var}(Z(r)) &= \overset{\text{expected value } \langle \cdot \rangle}{E} \left[ \underbrace{(Z(r) - \langle Z \rangle)^2}_{\downarrow 0} \right] = \langle Z(r)^2 \rangle \xrightarrow{\text{eq. (3.19)}} \\
 \text{Var}(Z) &= \left\langle \left( \frac{\partial \ln(P[r|s])}{\partial s} \right)^2 \right\rangle \overset{\text{eq. (3.43)}}{=} I_F(s)
 \end{aligned}$$